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# A Multi-Class Dynamic User Equilibrium Model for Queuing Networks with Advanced Traveler Information Systems

HAI-JUN HUANG[1,★] and WILLIAM H. K. LAM[2]
[1]*School of Management, Beijing University of Aeronautics and Astronautics, Beijing 100083, China, and Graduate School of The Chinese Academy of Sciences, Beijing 100039, China.*
*e-mail: hjhuang@mail.nsfc.gov.cn*
[2]*Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, China*

**Abstract.** This paper presents a formulation and solution algorithm for a composite dynamic user-equilibrium assignment problem with multi-user classes, in order to assess the impacts of Advanced Traveler Information Systems (ATIS) in general networks with queues. Suppose that users equipped with ATIS will receive complete information and hence be able to choose the best departure times and routes in a deterministic manner, while users not equipped with ATIS will have incomplete information and hence may make decisions on departure times and routes in a stochastic manner. This paper proposes a discrete-time, finite-dimensional variational inequality formulation that involves two criteria regarding the route and departure time choice behaviors, i.e., the deterministic dynamic user equilibrium and the nested logit-based stochastic dynamic user equilibrium. The formulation is then converted to an equivalent "zero-extreme value" minimization problem. A heuristic algorithm based on route/time-swapping process is proposed, which iteratively adjusts the route and departure time choices to reach closely to an extreme point of the minimization problem. A numerical example is used to demonstrate the effectiveness of the proposed approach for assessing the ATIS impacts such as changes in individual travel costs, departure times, route inflows, queuing peaks and total network travel cost.

**Mathematics Subject Classifications (2000):** 58E35, 90B10, 90B20, 91B50.

**Key words:** advanced traveler information systems, composite dynamic user-equilibrium assignment, route and departure time choices, queuing network.

## 1. Introduction

Advanced Traveler Information Systems (ATIS) that provide travelers with up-to-date traffic information, are generally believed to be efficient in many aspects such as improving individual trip planning, alleviating road congestion and enhancing traffic network performance. There have been substantial developments for modeling and assessing qualitatively and quantitatively the effects of ATIS on travelers and the transportation system, see the review papers or reports by Boyce [13], Gardes and May [16], Abdel-Aty *et al.* [1], Bonsall [12], Emmerink *et al.* [18]

★ Author for correspondence.

and Watling [38]. These studies include field experiments, route choice surveys, computer microscopic (or macroscopic) simulations and analytical models. Various multi-class traffic models have been developed to differentiate travelers who receive information versus travelers who don't have information. It is believed that travelers equipped and unequipped with ATIS will behave differently in departure time and route choices. Dafermos [15] was the first to realize the importance of incorporating multiple user classes in a modeling framework.

However, most of the previous multi-class traffic models for assessing the impacts of ATIS on networks with recurrent congestion are static (see, for example, [20, 35, 36, 11, 24, 7, 40]). These models generally do not allow the study of other ATIS impacts such as changes in departure times, traffic queuing locations and duration. And, the benefits of ATIS demonstrated by these static models may be limited or even nonexistent in dynamic road networks. Arnott *et al.* [4] adopted a simple model to show that traffic information may cause travelers to change their departure times in such a way as to exacerbate road congestion during the rush hour. Therefore, there is a need to derive a formulation of the dynamic traffic assignment that takes into account the day-to-day adjustment process for the case with ATIS.

Some preliminary studies on the evaluation of ATIS benefits under dynamic traffic conditions have been conducted. Ben-Akiva *et al.* [9] proposed a dynamic formulation to model the day-to-day adjustment process concerning the informed travelers' departure time and route choices in a network with parallel links for one Origin–Destination (OD) pair. This dynamic formulation was extended by Vythoulkas [37] to the case for general network and modified by Ben-Akiva *et al.* [10] for studying the travelers' information acquisition process. However, their modeling framework did not treat multiple user classes explicitly in the mathematical formulation, where the proportion of informed travelers who review their choices each day was given and the uncertainty of the route and departure time choices for all travelers was expressed by the two parameters adopted in the nested logit model.

Emmerink and his collaborators have contributed substantially in studying the economic impacts of driver information system by using a dynamic, stochastic route choice modeling framework (see, for example, [18, 19, 17]). Not only the benefits but also some adverse impacts of ATIS were assessed and evaluated. The Emmerink's analytical models that explicitly distinguish the informed and uninformed travelers, are suitable for simple networks with single route or parallel routes or for some multi-OD pair networks with special settings. For the case of general network, the simulation approach was however used [19], following Mahmassani and his co-authors' works (see a summary by Mahmassani and Herman [25]). Similar studies were carried out by Al-Deek *et al.* [2] on a simplified two-route queuing corridor with non-recurrent congestion due to incident.

Ran *et al.* have made a significant progress in developing analytical approaches for modeling the multi-class dynamic traffic assignment problems in general networks [28]. They divided the users into three classes, namely fixed route travelers, stochastic dynamic user-optimal and dynamic user-optimal, and integrated their traffic behaviors into one formulation using a variational inequality (VI) approach. A combination of various solution techniques, such as relaxation, Frank-Wolfe and method of successive averages, was suggested for solving the equivalent VI problem. However, Ran *et al.* [28] did not consider the departure time choice problem and the dynamic queuing phenomenon in networks with ATIS.

Huang and Lam [21] have recently proposed a discrete-time modeling approach for dealing with the Simultaneous Route and Departure time choice (SRD) dynamic user-equilibrium problem in network with queues. The queues were assumed to develop vertically when link inflow rates exceed link capacities and the link travel times were calculated properly so as to ensure the first-in-first-out discipline for average traffic flows at intersections. An equivalent "zero-extreme value" minimization problem was formulated and solved by a novel algorithm based on route/time-swapping process.

This paper aims to extend the work of Huang and Lam [21] for the case of multiple user classes in a queuing network with ATIS. We classify travelers into two classes, those who follow (i) dynamic user-equilibrium (DUE) principle and (ii) stochastic dynamic user-equilibrium (SDUE) principle. Class (i) represents travelers who are equipped with ATIS and therefore able to receive real-time traffic information so as to assess the route and departure time attributes identically and without error. Realizing deterministic departure time choice may not be absolutely true in the real world. However, the departure times can be assumed more or less deterministically particularly for commuter trips under recurrent congestion conditions. In this paper, the departure times are assumed deterministically for travelers equipped with ATIS so as to ensure the tractability of the model formulation. Class (ii) represents travelers who are not equipped with ATIS and hence have partial traffic information (perhaps from past experiences). It is assumed that class (ii) travelers would determine their routes and departure times based on *perceived* rather than actual travel times (or costs). In this paper, the SDUE simultaneous route and departure time choice problem is modeled by a nested logit formulation and further described by an equivalent VI formulation. This VI formulation is different from that developed by Ran *et al.* [28] because the proposed VI here is directly established on the basis of the so-called *augmented travel costs*. We show that the nested logit formulation for governing route and departure time choices can be derived from equalizing the augmented travel costs, firstly proposed for the SDUE simultaneous route and departure time choice problems. The two VI formulations corresponding to the DUE and SDUE route/time choice criteria, respectively, are finally combined into one framework on a discrete-time basis.

The remainder of this paper is organized as follows. Section 2 shows how these two classes of dynamic equilibria can be formulated in an integrated framework. The functions and methods to calculate the link travel times, path travel times and path travel costs under dynamic environment are also presented in this section. In

Section 3, we firstly introduce an equivalent "zero-extreme value" minimization condition and then propose a heuristic algorithm that may solve the captioned minimization problem via a route/time-swapping process. Section 4 shows some computational results of applying the proposed approach to a capacity-constrained grid network, with the objective of investigating the ATIS impacts on travelers and system performance. Finally, concluding remarks are given in Section 5 together with suggestions for further research. It should be noted that the model proposed in this paper is mainly used for planning and evaluation purposes particularly for assessing the impacts of ATIS in general networks with queues. Therefore, some of the assumptions adopted in the model may be appropriate under certain circumstances such as under recurrent congestion conditions without spillback effects at intersections. This may be true for commuter trips during normal peak hour periods.

## 2. The Variational Inequality Formulation

We consider a network $G(N, L)$ composed of a finite set of nodes, $N$, and a finite set of directed links, $L$. Let $a$ denote a link, and let $p$ denote a path (or route). Both path and route are used without difference in this paper. A path is simply an acyclic ordered set of links, $\{a_1, a_2, \ldots, a_m\}$, that connects an origin $r$ ($r \in R \subset N$) and a destination $s$ ($s \in S \subset N$). $R$ and $S$ are the sets of origins and destinations, respectively. Let $P_{rs}$ be the set of all feasible paths for travel between an OD pair $(r, s)$. The time period $T$ of interest is discretized to a finite set of equal time intervals, $K = \{k : k = 1, \ldots, \underline{K}\}$, and $\delta$ is the interval length with $\delta\underline{K} = T$. Note that we do not deal with continuous time variation in this paper since the model will finally be implemented on the basis of time slices. We assume that the value of $T$ is large enough to ensure that all travelers are entering into and exiting from the network during the study period $[0, T]$. On the other hand, it is also assumed that the value of $\delta$ is small enough so that the proposed discrete-time modeling framework is close to its continuous-time counterpart. We further assume that a flow rate, either specified by path or by link, is constant during a given time interval. Let

$f_p^{rs}(k)$ = the flow rate of *equipped travelers* on path $p$ that enters the network from origin $r$ to destination $s$ during interval $k$;

$\hat{f}_p^{rs}(k)$ = the flow rate of *unequipped travelers* on path $p$ that enters the network from origin $r$ to destination $s$ during interval $k$;

$c_p^{rs}(k, \mathbf{f}, \hat{\mathbf{f}})$ = the unit travel cost (on an average basis) incurred by travelers departing from $r$ and selecting path $p$ for travel to $s$ during interval $k$;

$c_{\min}^{rs}(\mathbf{f}, \hat{\mathbf{f}})$ = the minimum unit travel cost between OD pair $(r, s)$, where $c_{\min}^{rs}(\mathbf{f}, \hat{\mathbf{f}}) = \mathrm{Min}\{c_p^{rs}(k, \mathbf{f}, \hat{\mathbf{f}}) : p \in P_{rs}, k \in K\}$;

$F^{rs}$ = the given demand of *equipped travelers* between OD pair $(r, s)$ during period $[0, T]$;

$\hat{F}^{rs}$ = the given demand of *unequipped travelers* between OD pair $(r, s)$ during period $[0, T]$.

For simplicity, the vectors $\mathbf{f}$ and $\hat{\mathbf{f}}$ represent the set of path inflow rates associated with equipped and unequipped travelers respectively. These two classes of travelers are assigned onto the same road network. Their interactions are reflected by the path travel cost functions that are dependent on both $\mathbf{f}$ and $\hat{\mathbf{f}}$. In order to make it short, we hereinafter use "$\cdot$" to represent "$\mathbf{f}, \hat{\mathbf{f}}$".

### 2.1. THE DUE ROUTE AND DEPARTURE TIME CHOICE CONDITIONS FOR EQUIPPED TRAVELERS

The equipped travelers have complete real-time information about the road traffic condition. This class of travelers is assumed to make route and departure time choices in a deterministic dynamic user-equilibrium (DUE) manner. The DUE conditions can be written as below:

$$c_p^{rs}(k, \cdot) \begin{cases} = c_{\min}^{rs}(\cdot), & \text{if } f_p^{rs}(k) > 0, \\ \geqslant c_{\min}^{rs}(\cdot), & \text{if } f_p^{rs}(k) = 0, \end{cases} \quad \forall p \in P_{rs},\ r \in R,\ s \in S,\ k \in K, \tag{1}$$

$$\sum_{p \in P_{rs}} \sum_{k \in K} \delta f_p^{rs}(k) = F^{rs}, \quad \forall r \in R,\ s \in S, \tag{2}$$

$$f_p^{rs}(k) \geqslant 0, \quad \forall p \in P_{rs},\ r \in R,\ s \in S,\ k \in K. \tag{3}$$

Equation (2) represents the OD flow conservation constraints and Equation (3) is the non-negativity constraint.

Equation (1) defines a deterministic dynamic user equilibrium for equipped travelers. Only those paths and departure times (referred to as time-dependent paths later) for travel between an OD pair that have minimum travel costs are used, and those time-dependent paths that are not used should have costs that are higher than or equal to these minimum travel costs.

Following Wie *et al.* [39], the above discrete-time SRD equilibrium conditions can be expressed by a finite-dimensional variational inequality formulation as follows:

Find a vector $\mathbf{f}^* \in \Omega$ such that for all $\mathbf{f} \in \Omega$

$$\sum_{k \in K} \sum_{r \in R} \sum_{s \in S} \sum_{p \in P_{rs}} c_p^{rs}(k, \mathbf{f}^*, \hat{\mathbf{f}}) \lfloor f_p^{rs}(k) - f_p^{rs^*}(k) \rfloor \geqslant 0, \tag{4}$$

where $\Omega$ is a closed convex set $\Omega \subset \Re^n$ $(n = \underline{K} \times \sum_{rs} |P_{rs}|)$

$$\Omega = \left\{ \mathbf{f} \geqslant 0 : \sum_{p \in P_{rs}} \sum_{k \in K} \delta f_p^{rs}(k) = F^{rs},\ \forall r \in R,\ s \in S \right\};$$

or in the following vector form for simplicity:

Find a vector $\mathbf{f}^* \in \Omega$ such that

$$\mathbf{c}(\mathbf{f}^*, \hat{\mathbf{f}}) \circ (\mathbf{f} - \mathbf{f}^*) \geqslant 0, \quad \forall \mathbf{f} \in \Omega, \tag{5}$$

where "$\circ$" denotes the inner product in $\Re^n$.

It should be reminded that, for equipped travelers, their travel costs are assumed to be the measured or actual travel costs.

### 2.2. THE SDUE ROUTE AND DEPARTURE TIME CHOICE CONDITIONS FOR UNEQUIPPED TRAVELERS

For unequipped travelers, their route and departure time choices are random to certain extent since they only have partial information for decision-making. Suppose that an individual first decides what time to depart and then which route to follow. This sequential decision-making process, under the utility maximization principle, leads to a nested logit formulation as below. The probability that an unequipped traveler, travelling from $r$ to $s$, departs at time interval $k$ and selects route $p$ [8] can be obtained

$$\Pr_p^{rs}(k) = \frac{\exp(\mu_2 U_p^{rs}(k))}{\sum_{p \in P_{rs}} \exp(\mu_2 U_p^{rs}(k))} \times \frac{\exp(\mu_1 U_*^{rs}(k))}{\sum_{k \in K} \exp(\mu_1 U_*^{rs}(k))}, \tag{6}$$

where $U_p^{rs}(k)$ is the measured utility experienced by a traveler from $r$ to $s$, departing at interval $k$ for choosing alternative route $p$; $\mu_1$, $\mu_2$ are the scale parameters associated with the departure time and route choice decisions, respectively; $U_*^{rs}(k)$ is a composite variable that expresses the expected maximum utility experienced by a traveler from $r$ to $s$, who departs at interval $k$ and select alternative paths, and is defined as:

$$U_*^{rs}(k) = \frac{1}{\mu_2} \ln \sum_{p \in P_{rs}} \exp(\mu_2 U_p^{rs}(k)). \tag{7}$$

A necessary condition, which must be satisfied in order to ensure the validity of the above formulation, is $\mu_1/\mu_2 \leqslant 1$. The positive values of $\mu_1$ and $\mu_2$ are related to the variances of the unobserved or random utility components attributable to the departure time and route choices, respectively. For the case $\mu_1 = \mu_2 = \mu$, we obtain a joint logit model as follows:

$$\Pr_p^{rs}(k) = \frac{\exp(\mu U_p^{rs}(k))}{\sum_{k \in K} \sum_{p \in P_{rs}} \exp(\mu U_p^{rs}(k))}. \tag{8}$$

Without loss of generality, the measured utility can be simplified to the following form:

$$U_p^{rs}(k) = d^{rs} - c_p^{rs}(k, \cdot), \tag{9}$$

where $d^{rs}$ is a constant specified to OD pair $(r, s)$, it could be the individual's welfare for trip-making between origin $r$ and destination $s$. Substituting Equation (9) into Equation (6), we have

$$\mathrm{Pr}_p^{rs}(k) = \frac{\exp(-\mu_2 c_p^{rs}(k, \cdot))}{\sum_{p\in P_{rs}} \exp(-\mu_2 c_p^{rs}(k, \cdot))} \times \frac{\exp(-\mu_1 c_*^{rs}(k, \cdot))}{\sum_{k\in K} \exp(-\mu_1 c_*^{rs}(k, \cdot))}, \tag{10}$$

where

$$c_*^{rs}(k, \cdot) = \frac{1}{-\mu_2} \ln \sum_{p\in P_{rs}} \exp(-\mu_2 c_p^{rs}(k, \cdot)). \tag{11}$$

We now view the nested logit-based stochastic user-equilibrium problem from a different angle. For each measured time-dependent path travel cost, the perceived travel cost is defined as:

$$C_p^{rs}(k, \cdot) = c_p^{rs}(k, \cdot) - \frac{1}{\mu_1}\xi_T^{rs} - \frac{1}{\mu_2}\xi_{TR}^{rs}, \tag{12}$$

where the perception error terms $\xi_{TR}^{rs}$ attributable to the uncertain route choices are independent and identically Gumbel distributed with a scalar parameter $\mu_2$, and the terms $\xi_T^{rs}$ are distributed so that $\min_{p\in P_{rs}}\{C_p^{rs}(k, \cdot)\}$ are Gumbel distributed with a scalar parameter $\mu_1$ [8]. As shown in Sheffi [29], a satisfaction function between OD pair $(r, s)$ can be defined as below:

$$S^{rs}(\cdot) = E\Big[\min_{k\in K, p\in P_{rs}} \{C_p^{rs}(k, \cdot) | \mathbf{c}^{rs}(\cdot)\}\Big]. \tag{13}$$

This satisfaction function has the following property:

$$\frac{\partial S^{rs}(\cdot)}{\partial c_p^{rs}(k, \cdot)} = \mathrm{Pr}_p^{rs}(k). \tag{14}$$

Integrating Equation (14) with Equation (10) leads to

$$S^{rs}(\cdot) = -\frac{1}{\mu_1} \ln \sum_{k\in K} \exp(-\mu_1 c_*^{rs}(k, \cdot)). \tag{15}$$

Re-arranging Equations (10), (11) and (15), and defining $\mathrm{Pr}_p^{rs}(k) = \delta \hat{f}_p^{rs}(k)/\hat{F}^{rs}$, we have

$$\begin{aligned} S^{rs}(\cdot) \;=\; & \frac{\mu_2}{\mu_1} c_p^{rs}(k, \cdot) + \frac{\mu_2 - \mu_1}{\mu_1\mu_2} \ln \sum_{p\in P_{rs}} \exp(-\mu_2 c_p^{rs}(k, \cdot)) + \\ & + \frac{1}{\mu_1} \ln \frac{\delta \hat{f}_p^{rs}(k)}{\hat{F}^{rs}}. \end{aligned} \tag{16}$$

Equation (16) is an important formulation derived in this paper. For each measured time-dependent path travel cost $c_p^{rs}(k,\cdot)$, if we define a so-called *augmented travel cost* that is equal to the right-hand side of (16), i.e.,

$$\hat{c}_p^{rs}(k,\cdot) = \frac{\mu_2}{\mu_1} c_p^{rs}(k,\cdot) + \frac{\mu_2-\mu_1}{\mu_1\mu_2} \ln \sum_{p\in P_{rs}} \exp(-\mu_2 c_p^{rs}(k,\cdot)) + \\ + \frac{1}{\mu_1} \ln \frac{\delta \hat{f}_p^{rs}(k)}{\hat{F}^{rs}}, \tag{17}$$

then we can find the equalization of $\hat{c}_p^{rs}(k,\cdot)$ for all actually used paths $p$ and selected departure time intervals $k$ implies a nested logit-based stochastic user-equilibrium solution as the one given by (10). This can be verified by solving the equalized Equations (17) for all actually used paths and selected departure times, together with the OD flow conservation constraints $\sum_{p\in P_{rs}} \sum_{k\in K} \delta \hat{f}_p^{rs}(k) = \hat{F}^{rs}$.

Therefore, the SDUE route and departure time choice conditions for unequipped travelers can be expressed as:

$$\hat{c}_p^{rs}(k,\cdot) \begin{cases} = \hat{c}_{\min}^{rs}(\cdot), & \text{if } \hat{f}_p^{rs}(k) > 0, \\ \geqslant \hat{c}_{\min}^{rs}(\cdot), & \text{if } \hat{f}_p^{rs}(k) = 0, \end{cases} \quad \forall p \in P_{rs},\ r\in R,\ s\in S,\ k\in K, \tag{18}$$

where $\hat{c}_{\min}^{rs}(\cdot) = \mathrm{Min}\{\hat{c}_p^{rs}(k,\cdot) : p \in P_{rs},\ k\in K\}$. Under the SDUE conditions, only those paths and departure time intervals for travel between an OD pair that have minimum augmented travel costs are used, and those time-dependent paths that are not used should have costs that are higher than or equal to these minimum augmented travel costs.

Similar to that presented in Subsection 2.1 for equipped travelers, the above discrete-time SRD stochastic dynamic user-equilibrium conditions can further be expressed as follows by a finite-dimensional variational inequality formulation:

Find a vector $\hat{\mathbf{f}}^* \in \hat{\Omega}$ such that for all $\hat{\mathbf{f}} \in \hat{\Omega}$

$$\sum_{k\in K}\sum_{r\in R}\sum_{s\in S}\sum_{p\in P_{rs}} \hat{c}_p^{rs}(k,\mathbf{f},\hat{\mathbf{f}}^*)\big[\hat{f}_p^{rs}(k) - \hat{f}_p^{rs^*}(k)\big] \geqslant 0, \tag{19}$$

where $\hat{\Omega}$ is a closed convex set $\hat{\Omega} \subset \Re^n$ $(n = \underline{K} \times \sum_{rs} |P_{rs}|)$

$$\hat{\Omega} = \left\{ \hat{\mathbf{f}} \geqslant 0 : \sum_{p\in P_{rs}} \sum_{k\in K} \delta \hat{f}_p^{rs}(k) = \hat{F}^{rs}, \forall r\in R, s\in S \right\};$$

or in vector form as below:

Find a vector $\hat{\mathbf{f}}^* \in \hat{\Omega}$ such that

$$\hat{c}(\mathbf{f},\hat{\mathbf{f}}^*) \circ (\hat{\mathbf{f}} - \hat{\mathbf{f}}^*) \geqslant 0, \quad \forall \hat{\mathbf{f}} \in \hat{\Omega}. \tag{20}$$

It is important to note that, for unequipped travelers, the *augmented travel costs* defined in (17) must be adopted when using the VI formulation (20) or the dynamic generalization of Wardrop's user-equilibrium principle (18) to describe the SDUE route and departure time choice behaviors. The augmented travel cost is neither the measured travel cost nor the perceived travel cost. The purpose of introducing it is to present the SDUE condition in a deterministic manner and then obtain an integrated VI framework in formulating the composite dynamic equilibrium assignment problem studied in this paper.

### 2.3. THE INTEGRATED VI FRAMEWORK

The integrated variational inequality formulation, which is equivalent to the SRD-DUE/SDUE conditions (1) and (6) or (1) and (10), can be established by intergrating Equations (4) and (19) as follows:

*The dynamic traffic flow pattern* ($\mathbf{f}^* \in \Omega, \hat{\mathbf{f}}^* \in \hat{\Omega}$) *is a discrete-time SRD-DUE/SDUE state if and only if it satisfies the variational inequality problem:*

$$\sum_{k\in K}\sum_{r\in R}\sum_{s\in S}\sum_{p\in P_{rs}} c_p^{rs}(k,\mathbf{f}^*,\hat{\mathbf{f}}^*)\big[f_p^{rs}(k)-f_p^{rs^*}(k)\big]+ \\ +\sum_{k\in K}\sum_{r\in R}\sum_{s\in S}\sum_{p\in P_{rs}} \hat{c}_p^{rs}(k,\mathbf{f}^*,\hat{\mathbf{f}}^*)\big[\hat{f}_p^{rs}(k)-\hat{f}_p^{rs^*}(k)\big]\geqslant 0, \tag{21}$$

*for all* $\mathbf{f}\in\Omega$ *and all* $\hat{\mathbf{f}}\in\hat{\Omega}$, *here* $\Omega$ *and* $\hat{\Omega}$ *are the sets of all feasible time-dependent path inflow rates associated with equipped and unequipped travelers as defined by Equations* (4) *and* (19), *respectively. Or in vector form as below for simplicity:*

$$\mathbf{c}(\mathbf{f}^*,\hat{\mathbf{f}}^*)\circ(\mathbf{f}-\mathbf{f}^*)+\hat{c}(\mathbf{f}^*,\hat{\mathbf{f}}^*)\circ(\hat{\mathbf{f}}-\hat{\mathbf{f}}^*)\geqslant 0, \tag{22}$$

*for all* $\mathbf{f}\in\Omega$ *and all* $\hat{\mathbf{f}}\in\hat{\Omega}$.

As shown in (17), $\hat{c}(\mathbf{f},\hat{\mathbf{f}})$ are continuous functions of $\mathbf{c}(\mathbf{f},\hat{\mathbf{f}})$. Therefore, the existence of the DUE/SDUE solutions is dependent on the properties of the measured path travel cost functions $\mathbf{c}(\mathbf{f},\hat{\mathbf{f}})$. Huang and Lam [21] proved that for the SRD-DUE problem with single user class, the specific path travel cost functions are continuous in path inflow rates and there then exists at least one DUE solution according to the Brouwer's fixed point theory. This paper adopts the same approach with that in Huang and Lam [21]. Consequently, the continuity of path travel costs is guaranteed since it is not dependent on the number of user classes. Hence, the DUE/SDUE solutions for the problem studied in this paper do exist. It should be noted that the continuity of path travel costs is associated with the assumptions made for travel behavior and the method used for model formulation. In reality, the continuity of path travel costs may not always be guaranteed because of the complex dynamics and temporal/spatial effects of traffic flows at intersections. However, spillback effects at intersections are generally not considered for planning and/or evaluation purposes in which the capacity of intersection is always assumed to be greater than the passing traffic demand. In this paper, the proposed model is mainly used to assess the impacts of ATIS under recurrent congestion conditions during normal peak hour periods for commuter trips. In the next subsection, we establish the path travel cost functions in details and show how the given path inflows spread through the network dynamically. It starts from formulation of the dynamic link travel times.

### 2.4. LINK TRAVEL TIME, PATH TRAVEL TIME AND PATH TRAVEL COST

In this paper, the point queue (or vertical queue) assumption is employed [22, 23] as the proposed model is mainly used for planning and evaluation purposes. In other words, the spillback effect of queue length is not considered explicitly, which may weaken the ability of the proposed model in capturing traffic realism under overflow conditions. However, modeling spillback effect analytically is still an open question in transportation science. On the other hand, we also assume that at the end of each link there is a bottleneck with the maximum exit rate $s_a$, $\forall a \in L$. Let

$u_a(k), \hat{u}_a(k)$ = the inflow rate of *equipped* and *unequipped* travelers on link $a$ during interval $k$, respectively;

$U_a(k), \hat{U}_a(k)$ = the cumulative arrivals of *equipped* and *unequipped* travelers on link $a$ until interval $k$, respectively;

$v_a(k), \hat{v}_a(k)$ = the departure rate of *equipped* and *unequipped* travelers from link $a$ during interval $k$, respectively;

$V_a(k), \hat{V}_a(k)$ = the cumulative departures of *equipped* and *unequipped* travelers from link $a$ until interval $k$, respectively;

$q_a(k)$ = the queue length experienced by travelers entering link $a$ at interval $k$;

$t_a(k)$ = the travel time on link $a$ for travelers entering this link at interval $k$;

$s_a$ = the maximum exit rate of the bottleneck on link $a$.

With the assumption of constant flow rates during each time interval, we have

$$U_a(k) = U_a(k-1) + \delta u_a(k), \quad \forall a \in L,\ k \in K, \tag{23a}$$

$$\hat{U}_a(k) = \hat{U}_a(k-1) + \delta \hat{u}_a(k), \quad \forall a \in L,\ k \in K, \tag{23b}$$

where $U_a(k)$ and $\hat{U}_a(k)$ are zero unless $k \in K$. The class-specified flows entering link $a$ at interval $k-1$ would leave the link before the end of interval $k-1+t_a(k-1)$ at the departure rates $v_a(k-1+t_a(k-1))$ and $\hat{v}_a(k-1+t_a(k-1))$ for equipped travelers and unequipped travelers, respectively. On the other hand, the flows entering at the next interval, $k$, would leave the link during $[k-1+t_a(k-1), k+t_a(k)]$ at the departure rates $v_a(k+t_a(k))$ and $\hat{v}_a(k+t_a(k))$ for the two classes of travelers respectively. It is assumed that the departure rates during $[k-1+t_a(k-1), k+t_a(k)]$ are fixed. We then have

$$\begin{aligned} V_a(k+t_a(k)) &= V_a(k-1+t_a(k-1)) + \\ &\quad + \delta[k+t_a(k)-(k-1+t_a(k-1))]v_a(k+t_a(k)), \\ &\quad \forall a \in L,\ k \in K, \end{aligned} \tag{24a}$$

$$\begin{aligned} \hat{V}_a(k+t_a(k)) &= \hat{V}_a(k-1+t_a(k-1)) + \\ &\quad + \delta[k+t_a(k)-(k-1+t_a(k-1))]\hat{v}_a(k+t_a(k)), \\ &\quad \forall a \in L, k \in K. \end{aligned} \tag{24b}$$

Under the first-in-first-out discipline, on average a group of vehicles must leave link $a$ in the same order as its order of arrival at the link $a$. So, $U_a(k) = V_a(k+t_a(k))$ and $\hat{U}_a(k) = \hat{V}_a(k+t_a(k))$ must hold for all $k$. Together with (23) and (24), this leads to

$$u_a(k) = v_a(k+t_a(k))[1+t_a(k)-t_a(k-1)], \tag{25a}$$

$$\hat{u}_a(k) = \hat{v}_a(k+t_a(k))[1+t_a(k)-t_a(k-1)]. \tag{25b}$$

In Equation (25), it is important to note that, for each class, the equipped travelers for example, the departure rate $v_a(k+t_a(k))$ is controlled not only by its inflow rate $u_a(k)$ but also by the difference between its travel time $t_a(k)$ and the travel time $t_a(k-1)$ for travelers entering at the last interval. Suppose that $t_a(k) = t_a(k-1)$ for simplicity or other reasons, then the first-in-first-out discipline would be violated such as the one in Chen and Hsueh [14].

Let $t_a^0$ denote the free-flow travel time required to traverse the line haul section of link $a$, then the total travel time of traversing through the whole link $a$ for travelers entering at interval $k$ can be computed by

$$t_a(k) = t_a^0 + \frac{q_a(k)}{\delta s_a}, \quad \forall a \in L, k \in K. \tag{26}$$

By applying (26) into (25), we have

$$u_a(k) = v_a(k+t_a(k))\left[1 + \frac{q_a(k)-q_a(k-1)}{\delta s_a}\right], \quad \forall a \in L,\ k \in K, \tag{27a}$$

$$\hat{u}_a(k) = \hat{v}_a(k+t_a(k))\left[1 + \frac{q_a(k)-q_a(k-1)}{\delta s_a}\right], \quad \forall a \in L,\ k \in K. \tag{27b}$$

According to the deterministic queuing theory, the departure rate from link $a$ is evaluated independently of the downstream traffic flow as follows:

$$\begin{aligned} &v_a(k+t_a(k)) + \hat{v}_a(k+t_a(k)) \\ &\quad = \begin{cases} s_a, & \text{if } t_a(k) > t_a^0 \text{ or } u_a(k)+\hat{u}_a(k) > s_a, \\ u_a(k)+\hat{u}_a(k), & \text{otherwise.} \end{cases} \end{aligned} \tag{28}$$

Combining Equations (27) and (28), together with the non-negativity condition $q_a(k) \geqslant 0$ for all $a$ and $k$, we obtain the key result as below:

$$q_a(k) = \max\{[q_a(k-1) + \delta(u_a(k) + \hat{u}_a(k)) - s_a)], 0\}. \tag{29}$$

Equation (29) shows that if $q_a(0)$ ($= 0$ as assumed in this paper) and $(u_a(k) + \hat{u}_a(k))$ for all $k$ are given, then the queues for all $k$ can be obtained recursively. And then, the link travel times for all $k$ can be determined by (26).

From Equation (29), it can also be seen that there is a highly asymmetric nature in the proposed dynamic assignment model: the queues $q_a(k)$ are affected by $(u_a(l) + \hat{u}_a(l))$, for $l \leqslant k$, but not by $(u_a(l) + \hat{u}_a(l))$, for $l \geqslant k$. This will cause the Jacobian matrix of link travel times to be asymmetric. As a result, an equivalent minimization problem based on the Beckmann *et al.* [5] standard objective function cannot be defined for the captioned problem.

Now, we establish the path travel time and then the path travel cost functions. Each path flow pattern $(\mathbf{f}, \hat{\mathbf{f}})$, gives rise to a generalized path cost $c_p^{rs}(k, \cdot)$, for each path $p \in P_{rs}$, $\forall r \in R$, $s \in S$, and departure interval $k \in K$. This path cost is a function of the total travel time on the path and the arrival time at the destination. In this paper, the travel time required to traverse path $p = \{a_1, a_2, \dots, a_m\}$ for vehicles entering the network at interval $k$, is calculated using the nested function as below:

$$t_p^{rs}(k) = t_{a_1}(k) + t_{a_2}(k + t_{a_1}(k)) + \cdots + t_{a_m}(k + t_{a_1} + t_{a_2} + \cdots + t_{a_{m-1}}), \tag{30}$$

where $t_{a_1} = t_{a_1}(k)$, $t_{a_2} = t_{a_2}(k + t_{a_1}(k)), \dots$, for short. Equation (30) can be rewritten as

$$t_p^{rs}(k) = \sum_{a \text{ on path } p} \sum_{l(\geqslant k) \in K} t_a(l) \delta_{apk}^{rs}(l), \tag{31}$$

where $\delta_{apk}^{rs}(l)$ is equal to 1, if the flow on path $p$ for OD pair $(r, s)$ entering the network at interval $k$ arrives link $a$ at interval $l$; otherwise, 0. This is written as

$$\delta_{a_i pk}^{rs}(l) = \begin{cases} 1, & \text{if } k + t_{a_1} + t_{a_2} + \cdots + t_{a_{i-1}} = l, \\ 0, & \text{otherwise,} \end{cases} \tag{32}$$

and for any link $a$ on path $p$,

$$\sum_{l(\geqslant k) \in K} \delta_{apk}^{rs}(l) = 1, \quad \forall p \in P_{rs},\ r \in R,\ s \in S,\ k \in K. \tag{33}$$

Thus, the path travel time is simply the sum of all the link travel times on the path concerned, and the link travel times are computed based on the link traffic conditions when the vehicles enter that link. As indicated by Bell *et al.* [6], the difficulty of dynamic equilibrium assignment in comparison to the steady state equilibrium assignment arises because $\delta_{apk}^{rs}(l)$ depends on link travel times which in turn depend on link inflows. Consequently, the path travel times computed from (31) are essentially non-linear and non-convex.

Equations (26) and (29) are used to compute the link travel times, where the link inflow rates are given by the following relationships:

$$u_a(k) = \sum_{r\in R}\sum_{s\in S}\sum_{p\in P_{rs}} u_a^{rsp}(k), \quad \forall a \in L,\ k \in K, \tag{34a}$$

$$\hat{u}_a(k) = \sum_{r\in R}\sum_{s\in S}\sum_{p\in P_{rs}} \hat{u}_a^{rsp}(k), \quad \forall a \in L,\ k \in K, \tag{34b}$$

$$u_a^{rsp}(k) = f_p^{rs}(k)\zeta_a^{rsp} + v_b^{rsp}(k)\zeta_{ba}^{rsp}, \quad \forall p \in P_{rs},\ a\,\&\,b \in L,\ k \in K, \tag{35a}$$

$$\hat{u}_a^{rsp}(k) = \hat{f}_p^{rs}(k)\zeta_a^{rsp} + \hat{v}_b^{rsp}(k)\zeta_{ba}^{rsp}, \quad \forall p \in P_{rs},\ a\,\&\,b \in L,\ k \in K, \tag{35b}$$

$$v_b^{rsp}(k) = \begin{cases} u_b^{rsp}(k - t_b^0), & \text{if the queue is null at interval } k, \\ s_b \dfrac{u_b^{rsp}(i)}{u_b(i) + \hat{u}_b(i)}, & \text{otherwise, where } i + t_b(i) = k, \end{cases} \tag{36a}$$

$$\hat{v}_b^{rsp}(k) = \begin{cases} \hat{u}_b^{rsp}(k - t_b^0), & \text{if the queue is null at interval } k, \\ s_b \dfrac{\hat{u}_b^{rsp}(i)}{u_b(i) + \hat{u}_b(i)}, & \text{otherwise, where } i + t_b(i) = k, \end{cases} \tag{36b}$$

where $u_a^{rsp}(k)$ and $\hat{u}_a^{rsp}(k)$ are the flow rates of equipped and unequipped travelers, respectively, that enter link $a$ of path $p$ during interval $k$; $\zeta_a^{rsp} = 1$ if link $a$ is the first arc of path $p$ and $\zeta_a^{rsp} = 0$ otherwise; $v_b^{rsp}(k)$ and $\hat{v}_b^{rsp}(k)$ are the flow rates of equipped and unequipped travelers, respectively, that exit from link $b$ of path $p$ during interval $k$; $\zeta_{ba}^{rsp} = 1$ if link $b$ is the predecessor of link $a$ on path $p$ and $\zeta_{ba}^{rsp} = 0$ otherwise. Equations (35a) and (35b) describe the flow conservation of the two classes of travelers respectively, at the tail node of link $a$ (i.e., the head node of link $b$). Equation (36a) indicates that the path-specified link exit rate associated with the equipped travelers is either the entry rate at interval $k - t_b^0$ (if there are no vehicles in the queue) or a portion of the link capacity (if there are vehicles in the queue). For the latter case, the portion given in (36a) results from applying the first-in-first-out condition (25a) for all equipped vehicles travelling on the same path $p \in P_{rs}$, i.e.,

$$u_b^{rsp}(i) = v_b^{rsp}(k)[1 + t_b(i) - t_b(i-1)]. \tag{37}$$

Dividing the above Equation (37) by $u_b(i) + \hat{u}_b(i) = s_b[1 + t_b(i) - t_b(i-1)]$, we have

$$v_b^{rsp}(k) = s_b \frac{u_b^{rsp}(i)}{u_b(i) + \hat{u}_b(i)}. \tag{38}$$

We here assume that the vehicles belonging to different paths and different user classes are randomly mixed in the queue provided that they arrive at the link at the same time. Similarly, the same explanation can be made for Equation (36b) that is associated with the unequipped travelers.

Considering the schedule delay costs of arriving times at destinations, we may now define the generalized path travel cost functions (measured) as follows:

$$c_p^{rs}(k,\cdot) = \alpha t_p^{rs}(k) + \begin{cases} \beta[k_{rs}^* - \Delta_{rs} - k - t_p^{rs}(k)], \\ \quad \text{if } k + t_p^{rs}(k) < k_{rs}^* - \Delta_{rs}, \\ \gamma[k + t_p^{rs}(k) - k_{rs}^* - \Delta_{rs}], \\ \quad \text{if } k + t_p^{rs}(k) > k_{rs}^* + \Delta_{rs}, \\ 0, \quad \text{otherwise}, \end{cases} \tag{39}$$

where $\alpha$ is the unit cost of travel time, $\beta$ is the unit cost of schedule delay time-early, $\gamma$ is the unit cost of schedule delay time-late, and $[k_{rs}^* - \Delta_{rs}, k_{rs}^* + \Delta_{rs}]$ is the window of arrival times at destination $s$ without penalty for travel from origin $r$. $k_{rs}^*$ is the middle point of the time window and may represent the official work start time for commuter trips between OD pair $(r, s)$. $\Delta_{rs}$ is the half length of the window and is the non-penalty tolerance of arrival times for commuters. In accordance with the previous empirical results [30] and the theoretical conditions for stability [31], we assume that $\gamma > \alpha > \beta$.

Up to now, we have completely formulated the proposed modeling framework in a precise discrete-time form. Once the link travel times, $t_a(k)$ for all $a$ and $k$, were estimated, the indicator variables, $\delta_{apk}^{rs}(l)$ for all $r$, $s$, $a$, $p$, $k$ and $l$, would be determined accordingly. Then, the *measured* and *augmented* path travel costs would be computed. As a result, the time-dependent path inflow rates of both the user classes can be updated *according to a certain rule*, and the link inflow rates can be obtained by (34)–(36), and the link queues by (29), and the link travel times by (26). This process constitutes an iterative mechanism and may converge to an equilibrium solution if a certain rule for updating the path inflow rates is employed.

The monotonicity of the path travel cost functions can guarantee that some reasonable iterative adjustment process of $(\mathbf{f}, \hat{\mathbf{f}})$, starting from arbitrary point, will converge to an equilibrium. In a queuing network with single link, Smith and Ghali [32, 33] proved that the link travel time is a monotonic function of link inflow rates. However, in complicated dynamic networks (e.g., there are more than two OD pairs and/or more than two links with active bottlenecks on a path), we are unable to deduce monotonicity of the *path* travel times (costs) from monotonicity of the *link* travel times. Therefore, we could not ensure that an algorithm would certainly converge to the set of multi-class dynamic equilibria. In the next section, we first formulate an equivalent "zero-extreme value" minimization problem to the proposed discrete-time DUE/SDUE route and departure time choice conditions (1) and (10). Then, an iterative algorithm for solving the minimization problem is presented.

## 3. The Solution Algorithm

Define a function $W(\mathbf{f}, \hat{\mathbf{f}})$ as follows:

$$W(\cdot) = \sum_{k\in K}\sum_{r\in R}\sum_{s\in S}\sum_{p\in P_{rs}} \big\{ f_p^{rs}(k)[c_p^{rs}(k,\cdot) - c_{\min}^{rs}(\cdot)] + \\ + \hat{f}_p^{rs}(k)[\hat{c}_p^{rs}(k,\cdot) - \hat{c}_{\min}^{rs}(\cdot)]\big\}. \quad (40)$$

It has been shown in Subsection 2.2 that the equilibrium condition (18) is equivalent to the nested logit-based SDUE formulation (10). We further observe that conditions (1) and (18) are manifestly true if and only if $W(\mathbf{f}, \hat{\mathbf{f}}) = 0$. This is because all terms in Equation (40)'s summation are non-negative. Then, finding $(\mathbf{f}^*, \hat{\mathbf{f}}^*)$ that satisfies (1) and (18), or (5) and (20), is equivalent to solve

$$W(\mathbf{f}, \hat{\mathbf{f}}) = 0, \quad \text{subject to} \quad \mathbf{f} \in \Omega \text{ and } \hat{\mathbf{f}} \in \hat{\Omega}. \quad (41)$$

Since $W(\mathbf{f}, \hat{\mathbf{f}}) \geqslant 0$ for all $\mathbf{f}$ and $\hat{\mathbf{f}}$, it would be possible to seek a DUE/SDUE solution by moving $\mathbf{f}$ and $\hat{\mathbf{f}}$ along the descent direction of $W$ so as to force $W$ to zero. The conventional descent direction methods can be used to solve this "zero-extreme value" minimization problem if the path cost functions are differentiable. Such methods require the first-order partial derivatives of $W$ with respect to $(\mathbf{f}, \hat{\mathbf{f}})$, i.e., $\partial W/\partial \mathbf{f}$ and $\partial W/\partial \hat{\mathbf{f}}$. In other words, in each iteration the following eight Jacobian matrices must be determined:

$$[\partial \mathbf{c}(\cdot)/\partial \mathbf{f}]_{n\times n}, \quad [\partial \mathbf{c}(\cdot)/\partial \hat{\mathbf{f}}]_{n\times n}, \quad [\partial \mathbf{c}_{\min}(\cdot)/\partial \mathbf{f}]_{(|R|\times|S|)\times n},$$
$$[\partial \mathbf{c}_{\min}(\cdot)/\partial \hat{\mathbf{f}}]_{(|R|\times|S|)\times n}, \quad [\partial \hat{c}(\cdot)/\partial \mathbf{f}]_{n\times n}, \quad [\partial \hat{c}(\cdot)/\partial \hat{\mathbf{f}}]_{n\times n},$$
$$[\partial \hat{c}_{\min}(\cdot)/\partial \mathbf{f}]_{(|R|\times|S|)\times n}, \quad \text{and} \quad [\partial \hat{c}_{\min}(\cdot)/\partial \hat{\mathbf{f}}]_{(|R|\times|S|)\times n}.$$

The computational demands to these matrices are gigantic since $n$ may be a large number even in small-size networks. Hence, this paper does not consider any conventional descent direction methods, but proposes a natural adjustment mechanism that iteratively forces $W$ to zero.

This iterative algorithm is analogous to the "day-to-day" route-swapping process suggested by Smith and Wisten [34], or the route choice adjustment process based on projected dynamical systems introduced by Nagurney and Zhang [26, 27]. These authors, however, did not consider the equilibrium when departure time choices and multiple user classes are allowed. The basic idea of the proposed algorithm is:

For equipped travelers, inflows on the non-cheapest time-dependent paths are moved to the cheapest time-dependent paths; the volumes moved are proportional to $f_p^{rs}(k) \times [c_p^{rs}(k,\cdot) - c_{\min}^{rs}(\cdot)]$, i.e., travelers farther from the equilibrium and on paths with larger flow rates are more strongly inclined to change route and departure time choices than those on paths with smaller flow rates and with travel costs closer to the minimal cost. Note that the costs mentioned here are the *measured* or *actual* travel costs and the equilibrium is defined on these costs.

For unequipped travelers, inflows on the non-cheapest time-dependent paths are moved to the cheapest time-dependent paths; the volumes moved are proportional to $\hat{f}_p^{rs}(k) \times [\hat{c}_p^{rs}(k,\cdot) - \hat{c}_{\min}^{rs}(\cdot)]$, i.e., travelers farther from the equilibrium and on paths with larger flow rates are more strongly inclined to change route and departure time choices than those on paths with smaller flow rates and with travel costs closer to the minimal cost. Note that the costs mentioned here are the *augmented* travel costs and the equilibrium is defined on these costs.

The above flow rate adjustment rule (i.e., route/time-swap rule) is modeled in a computer program that constitutes an iterative algorithm. It is in fact a route/time-swapping process for each user class. The step-by-step procedure of the proposed algorithm is given below.

*Step 1.* Set the iteration index $\tau = 1$.
Choose the initial path inflow rates $f_p^{rs}(k)_\tau$ in $\Omega$ and $\hat{f}_p^{rs}(k)_\tau$ in $\hat{\Omega}$.

*Step 2.* Determine $\delta_{apk}^{rs}(l)_\tau$(under free-flow condition when $\tau = 1$).

*Step 3.* Calculate the link inflow rates $u_a(l)_\tau$ and $\hat{u}_a(l)_\tau$ by Equations (34)–(36).

*Step 4.* Compute the link queues $q_a(l)_\tau$ by Equation (29) and the link travel times $t_a(l)_\tau$ by Equation (26).

*Step 5.* Compute the path times $t_p^{rs}(k)_\tau$ by Equation (31), the actual path travel costs $c_p^{rs}(k,\cdot)_\tau$ by Equation (39) and the augmented path travel costs $\hat{c}_p^{rs}(k,\cdot)_\tau$ by Equation (17).
Find $c_{\min}^{rs}(\cdot)_\tau = \text{Min}\{c_p^{rs}(k,\cdot)_\tau : p \in P_{rs}, k \in K\}$ and let $P_\tau^* = \{(p,k) : c_p^{rs}(k,\cdot)_\tau = c_{\min}^{rs}(\cdot)_\tau, p \in P_{rs}, k \in K\}$.
Find $\hat{c}_{\min}^{rs}(\cdot)_\tau = \text{Min}\{\hat{c}_p^{rs}(k,\cdot)_\tau : p \in P_{rs}, k \in K\}$ and let $\hat{P}_\tau^* = \{(p,k) : \hat{c}_p^{rs}(k,\cdot)_\tau = \hat{c}_{\min}^{rs}(\cdot)_\tau, p \in P_{rs}, k \in K\}$.

*Step 6.* Update the path inflow rates as below:

$$f_p^{rs}(k)_{\tau+1} = f_p^{rs}(k)_\tau - \rho_\tau f_p^{rs}(k)_\tau [c_p^{rs}(k,\cdot)_\tau - c_{\min}^{rs}(\cdot)_\tau], \quad p \in P_{rs}, k \in K, \tag{42}$$

$$f_p^{rs}(k)_{\tau+1} = f_p^{rs}(k)_\tau + \frac{\psi_\tau}{|P_\tau^*|}, \quad \text{for } (p,k) \in P_\tau^*, \tag{43}$$

where

$$\psi_\tau = \sum_{p \in P_{rs}, k \in K} \rho_\tau f_p^{rs}(k)_\tau [c_p^{rs}(k,\cdot)_\tau - c_{\min}^{rs}(\cdot)_\tau];$$

$$\hat{f}_p^{rs}(k)_{\tau+1} = \hat{f}_p^{rs}(k)_\tau - \rho_\tau \hat{f}_p^{rs}(k)_\tau [\hat{c}_p^{rs}(k,\cdot)_\tau - \hat{c}_{\min}^{rs}(\cdot)_\tau], \quad p \in P_{rs}, k \in K, \tag{44}$$

$$\hat{f}_p^{rs}(k)_{\tau+1} = \hat{f}_p^{rs}(k)_\tau + \frac{\hat{\psi}_\tau}{|\hat{P}_\tau^*|}, \quad \text{for } (p,k) \in \hat{P}_\tau^*, \tag{45}$$

where

$$\hat{\psi}_\tau = \sum_{p \in P_{rs}, k \in K} \rho_\tau \hat{f}_p^{rs}(k)_\tau [\hat{c}_p^{rs}(k, \cdot)_\tau - \hat{c}_{\min}^{rs}(\cdot)_\tau].$$

*Step 7.* The iteration is terminated if some convergence criterion (see Equation (46) below) is satisfied; otherwise, set $\tau = \tau + 1$ and return to Step 2.

In Step 1, we may choose $f_p^{rs}(k)_1 = F^{rs}/[\delta(\underline{K} - J)|P_{rs}|]$, $\hat{f}_p^{rs}(k)_1 = \hat{F}^{rs}/[\delta(\underline{K} - J)|P_{rs}|]$ for $k = 1, 2, \ldots, \underline{K} - J$, and $f_p^{rs}(k)_1 = 0$, $\hat{f}_p^{rs}(k)_1 = 0$ for $k = \underline{K} - J + 1, \ldots, \underline{K}$, where $J$ intervals are used so that travelers entering before $\underline{K} - J$ would leave the network not later than $\underline{K}$. Clearly, this initial solution is feasible since the OD demand for each user class is simply averaged on all possible alternatives by time and by path. This simple method for setting initial solution is adopted in our numerical experiments presented in the next section.

Operations in Steps 2–5 directly follow the contents presented in Section 2. In Step 5, the subsets $P_\tau^*$ and $\hat{P}_\tau^*$ contains the time-specified paths with the minimum actual travel cost associated with equipped travelers and the minimum augmented travel cost associated with unequipped travelers, respectively. The number of the time-specified paths in these two subsets is expected to increase as the iteration continues and to reach its maximum at equilibrium points (if possible).

In Step 6, either for equipped travelers or for unequipped travelers, the algorithm first deducts flows from the time-specified paths with higher costs than the minimum cost paths, then distributes (induces) equally these flows onto the minimum cost paths (i.e., the ones contained in $P_\tau^*$ or $\hat{P}_\tau^*$). The sum of deduction (i.e., $\psi_\tau$ for equipped travelers or $\hat{\psi}_\tau$ for unequipped travelers) should however be equal to the sum of induction so as to ensure the feasibility of the solution from the adjustment process. Note that Equations (42) and (44) are subject to all time-specified paths associated with equipped and unequipped travelers, respectively, but does not change the inflow rates on the corresponding minimum cost paths. In addition, the parameter $\rho_\tau$ should be chosen to be sufficiently small in order to prevent undue oscillation and negative inflow rates.

It is known from the current theories developed so far that there exists a decreasing sequence $\{\rho_\tau,\ \tau = 1, 2, 3, \ldots\}$ to ensure the asymptotic stability of the iterative process, under the assumption that the path travel cost functions are strictly monotonic. In the proposed SRD-DUE/SDUE model, the path travel cost functions are however not monotonic in complicated networks. So, global stability is not guaranteed. Huang and Lam [21] showed a local stability analysis for the algorithm proposed for solving the single user-class SRD-DUE problem based on the results given by Zhang and Nagurney [41] and by Nagureney and Zhang [26, 27]. The algorithm presented in this paper is analogous to the one proposed by Huang and Lam [21] but extended to multiple user classes. Therefore, our algorithm would converge to a DUE/SDUE solution if the path cost functions are strictly monotonic; otherwise, the algorithm may not lead to converged solution. Smith and Wisten [34] proved that the path travel cost function is monotonic when no path contains more than one active bottleneck (i.e., the bottleneck behind which queue builds up). Of course, the monotonic assumption for path cost function is only true under certain circumstances but this assumption has been generally been adopted in models for planning and evaluation purposes.

## 4. Numerical Example

In this section, a numerical example is used to illustrate the application of the proposed model and algorithm to solve the multi-class dynamic user-equilibrium route and departure time choice problem in a grid network. The algorithm was coded in FORTRAN together with a convergence indicator defined as below:

$$W(\mathbf{f}_\tau, \hat{\mathbf{f}}_\tau) / \sum_{k\in K}\sum_{r\in R}\sum_{s\in S}\sum_{p\in P_{rs}} \left[ f_p^{rs}(k)_\tau c_{\min}^{rs}(\cdot)_\tau + \hat{f}_p^{rs}(k)_\tau \hat{c}_{\min}^{rs}(\cdot)_\tau \right], \quad (46)$$

which measures how closely a solution of the $\tau$th iteration satisfies the discrete time SRD-DUE/SDUE condition $W(\mathbf{f}, \hat{\mathbf{f}}) = 0$. Set $\{\rho_\tau\} = 0.0001\{1^{(1\to1000)}, 1/2^{(1001\to2000)}, 1/3^{(2001\to3000)}, \ldots\}$, i.e., $\rho_\tau = 0.0001$ when $\tau$ is from 1 to 1000 and $\rho_\tau = 0.00005$ when $\tau$ is from 1001 to 2000, etc.

The example grid network, shown in Figure 1, consists of nine nodes, twelve links and one OD pairs (i.e., from A to B). All free-flow link travel times and bottleneck exit capacities are also given in this figure. The study period is composed of four hours, from 6:00 am to 10:00 am, that is discretized into 400 time intervals with 0.6 minute each. Other input data are: $\mu_1 = 0.5$, $\mu_2 = 0.6$, $\alpha = 12$ (\$/h), $\beta = 8$ (\$/h), $\gamma = 20$ (\$/h), $\Delta = 0.2$ h, $k^* = $ 9:00 am, and $F + \hat{F} = 30{,}000$ veh. Let $J = 100$ for setting the initial feasible solution.

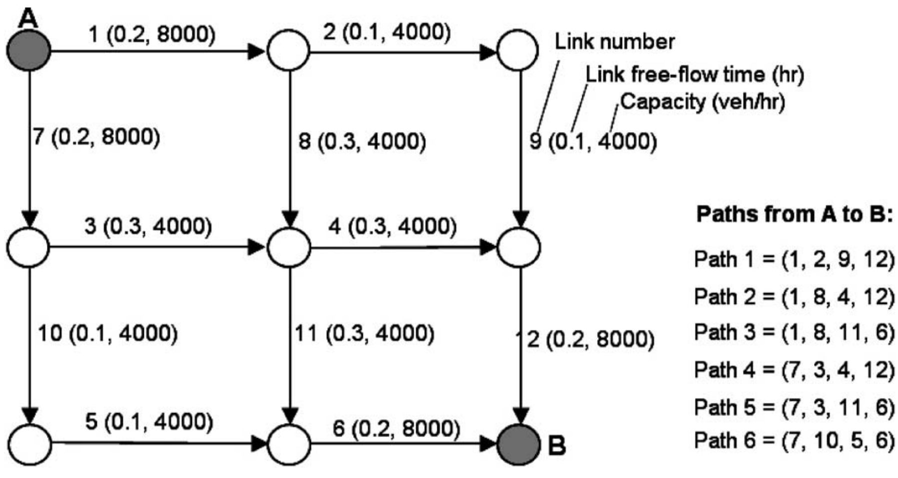


*Figure 1.* Test network.

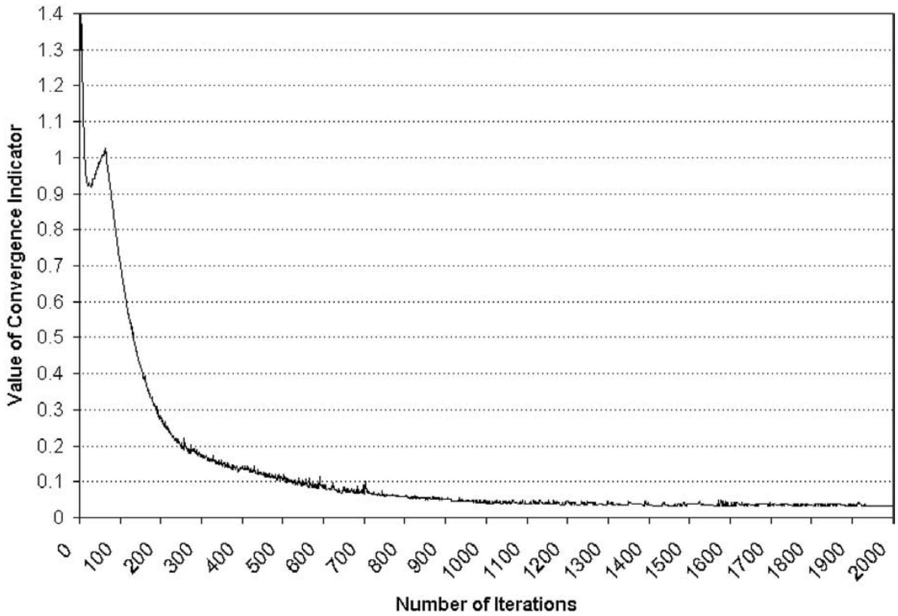


*Figure 2.* Convergence of the algorithm.

The symmetric structure of the example network implies that some paths will have the same inflow rates and travel costs, and some links will have the same traffic conditions such as inflow rates, travel times and queue lengths. This symmetry is particularly designed for presentation of the essential ideas with the numerical results. The identical outputs on paths 1 and 6, and on paths 2, 3, 4 and 5, were indeed obtained from the computation. Hence, for simplicity, we will discuss as follows the numerical results on paths 1 and 2, as well as the relevant links.

Firstly, we set 50% travelers with ATIS and then examine the effectiveness of the proposed approach for solving the dynamic user-equilibrium problem under such a market penetration of ATIS. Figure 2 shows that the value of the convergence indicator is decreasing generally as the iteration proceeds, although oscillations occur particularly for iterations 100–200. It was found from experiments that the convergence value decreased more smoothly but more slowly than the curve shown in Figure 2 if a smaller step size or sequence $\{\rho_\tau\}$ was used. The following analyses, including the ones under various ATIS market penetrations, are based on the outputs after 100,000 iterations (the value of the convergence indicator approaches to 0.01).

Figure 3 shows the inflow rates and actual travel costs on paths 1 and 2 for equipped travelers. It demonstrates that an approximate dynamic equilibrium solution has been obtained for the path inflow pattern of equipped travelers between origin A and destination B. Similarly, Figure 4 shows the results for unequipped travelers. It can be seen that the *augmented travel costs* for actually used paths and selected departure times is nearly equal to each other. In other words, the SDUE

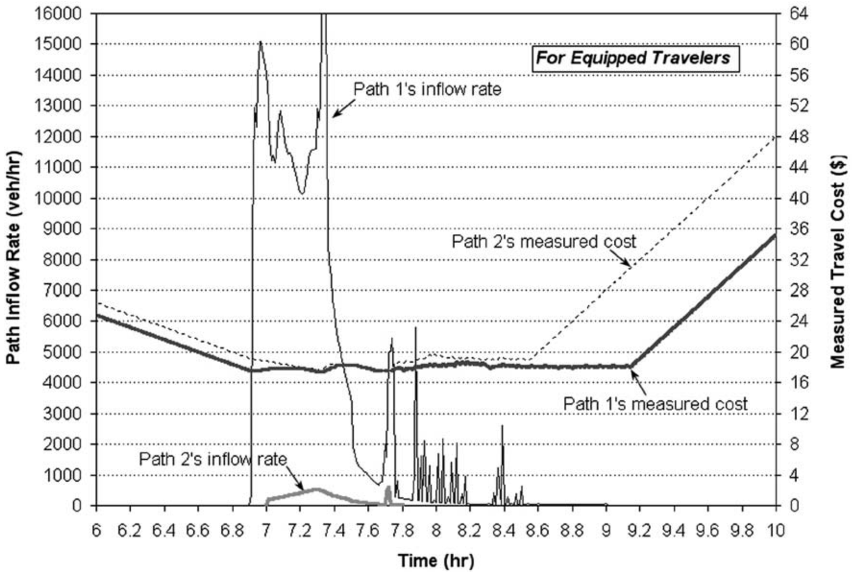


*Figure 3.* Equipped travelers' path inflow rates and the actual travel costs on paths 1 and 2.

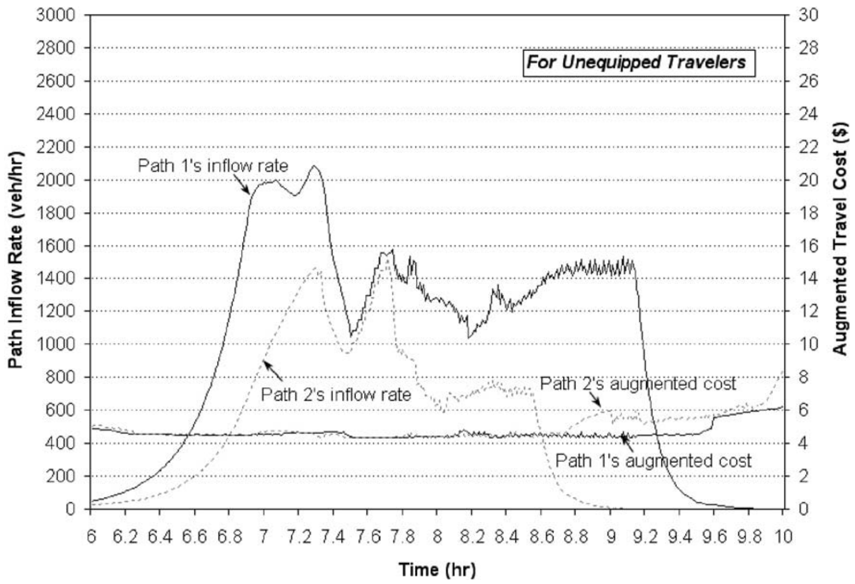


*Figure 4.* Unequipped travelers' path inflow rates and the augmented travel costs on paths 1 and 2.

path inflow pattern of unequipped travelers has been obtained. By comparing Figure 3 with Figure 4, it can be found that the unequipped travelers have much more scattered departure time and diverse route choices than the equipped travelers. Only 1.36% of equipped travelers select path 2 that has the highest free-flow travel time among all paths, but so do 12.09% of unequipped travelers.

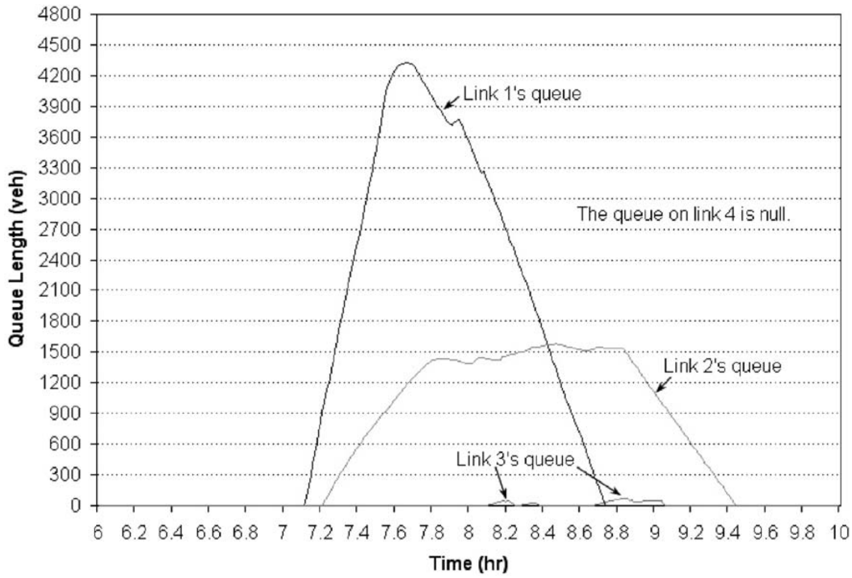


*Figure 5.* Queue lengths on links 1, 2 and 3.

Figure 5 depicts the formation and dissipation of the queues on some links. With the symmetry of the example network, the queuing peaks heavily occur on four links, namely links 1, 2, 7 and 10. Link 3 (also link 8) has few queues while link 4 (also link 11) has none. There are no queues on other links since the exiting rates of their upstream links are less than or equal to their capacities.

The above numerical results show that the model and algorithm proposed in this paper perform well in solving the multi-class dynamic user-equilibrium route and departure time choice problem in queuing networks. We now investigate the ATIS impacts on travelers and system performance by changing the ATIS market penetration, i.e., the value of $F/(F + \hat{F})$.

Figure 6 depicts the individual average travel costs of both equipped and un-equipped travelers against the ATIS market penetration. It is shown that the curve for non-equipped travelers always lies above that for equipped travelers, implying that using ATIS always benefits individual travelers if the cost for purchasing the information system itself is negligible. However, the average travel cost saving obtained by equipped travelers decreases monotonically with the ATIS market penetration level. This implies that a marginal equipped traveler adversely affects the already equipped travelers. It can be seen from Figure 6 that when the market penetration of ATIS is less than 43% or greater than 97.5%, the average travel cost of equipped travelers is always less than that of all travelers without ATIS (i.e. the do-nothing scenario). In other words, under these ranges of market penetration, travelers equipped with ATIS will benefit from savings in their average travel costs as compared to the do-nothing scenario. On the other hand, Figure 6 shows that both curves are ascending before the level of ATIS market penetration exceeds 80

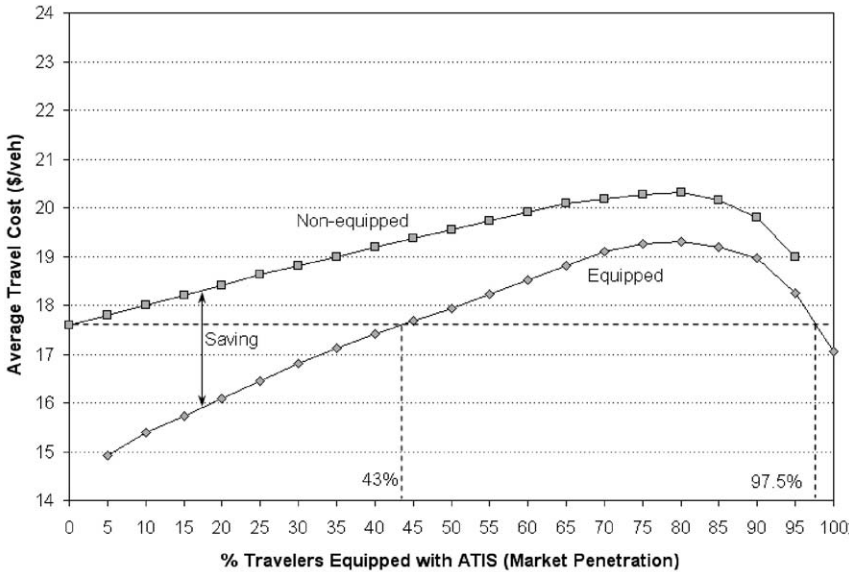


*Figure 6.* Average travel costs for equipped and unequipped travelers.

per cent, implying that the ATIS effects to both equipped and unequipped travelers are negative. This could be explained as follows.

With the increase of the market penetration of ATIS, more equipped travelers would choose the departure times and paths that lead to the minimal and/or shorter travel cost for their travel. This may affect the unequipped travelers to change their original choices on departure times and paths due to the limited link exiting capacities. Figures 7 and 8 verify this speculation. Figure 7 shows that the unequipped travelers have a wider range of departure times, while Figure 8 indicates that lower % of unequipped travelers have been assigned onto the attractive paths 1 and 6 than that of the equipped travelers. Travelers show favor towards these two paths as they have the lowest free-flow travel time. However, travelers have to choose other departure times and paths if the capacities of these two paths during the preferred time periods for departure are occupied. Consequently, the average travel costs for both equipped and unequipped travelers are increased.

When there are more than 80% travelers equipped with ATIS, both the average travel costs of equipped and unequipped travelers start to decline as shown in Figure 6. It is because most of the travelers are now equipped with ATIS and they can make a reasonable decision on departure time and route. On the other hand, the disturbance caused by the unequipped travelers' random travel behavior becomes very small, as the number of unequipped travelers is very few and most of the preferred departure times and paths have been taken up by equipped travelers. An obvious adjustment on the proportion of equipped travelers selecting paths 1 and 6, can be found in Figure 8 when the level of ATIS market penetration changes from 50% to 80%. After that, the proportions of equipped and unequipped travelers on

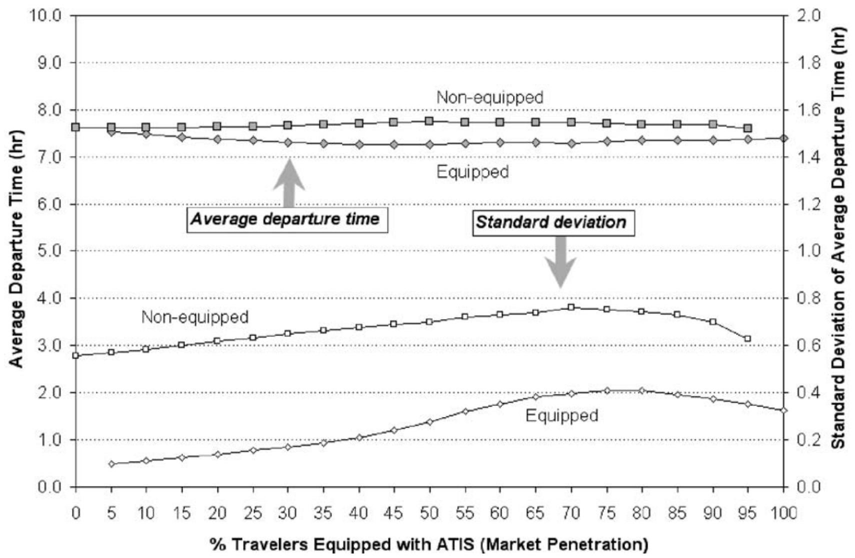


*Figure 7.* Average departure times and their standard deviations for equipped and unequipped travelers.

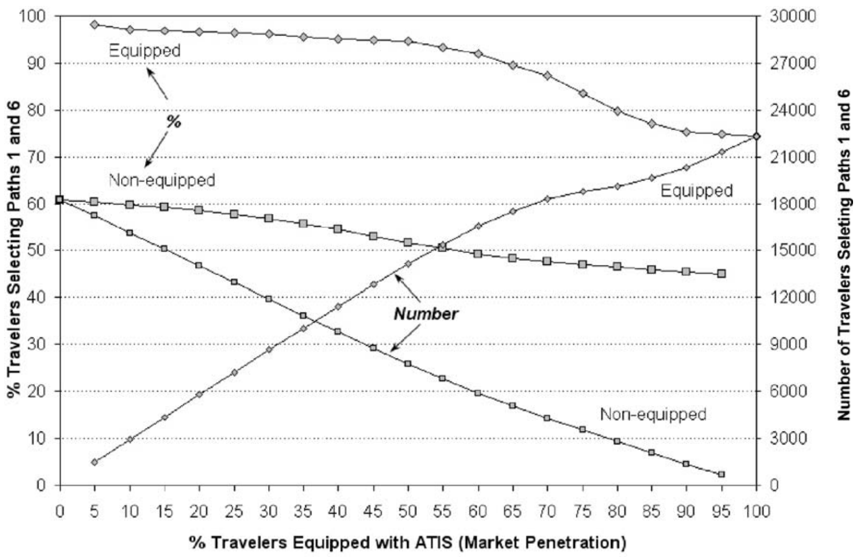


*Figure 8.* Proportions of equipped and unequipped travelers selecting paths 1 and 6.

these two paths tend to be stable (Figure 8) and the standard deviation of departure times falls (Figure 7), implying that the freedom on departure time and path choices is reducing.

Figures 9, 10 and 11 show the maximum queue length, the average queue length and the times when the queue begins and ends on links 1, 2 and 3, respectively. We do not show the results on the other links because of the symmetry of the network

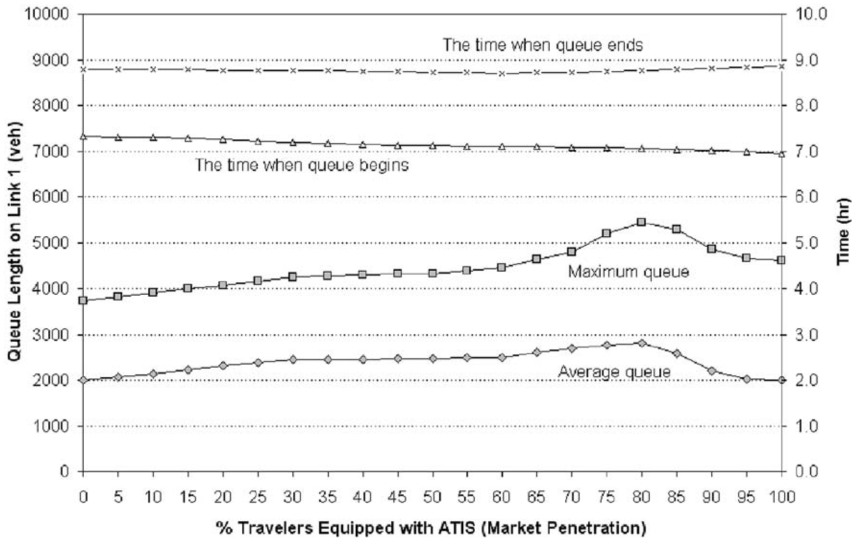


*Figure 9.* Average and maximum queue lengths on link 1 and the times when queue begins and ends.

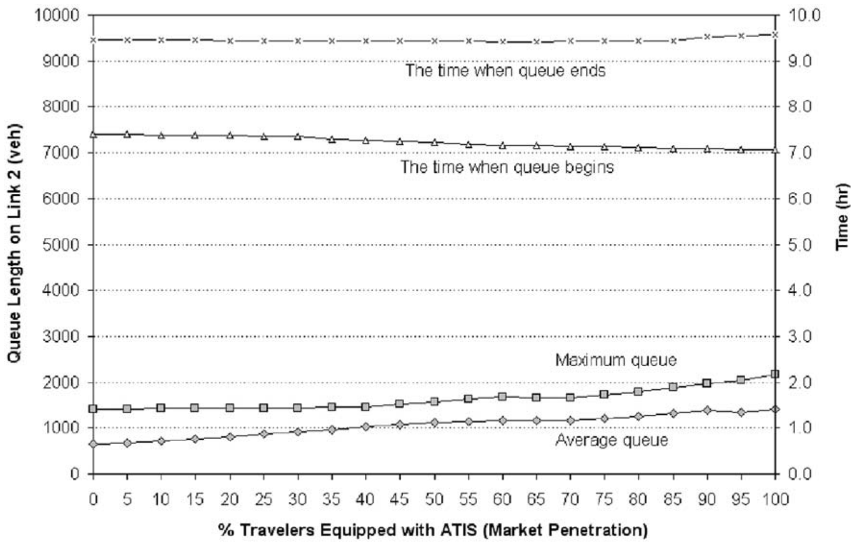


*Figure 10.* Average and maximum queue lengths on link 2 and the times when queue begins and ends.

and of no queues on some of these links. On link 1, the queuing duration becomes longer when more travelers are equipped with ATIS. Moreover, the queue length increases as the ATIS market penetration is increasing to 80%. Similar results were found on link 2, but the queue length continues to increase when the ATIS market penetration exceeds 80%. The reduction of queue length on link 1 is due to the fact

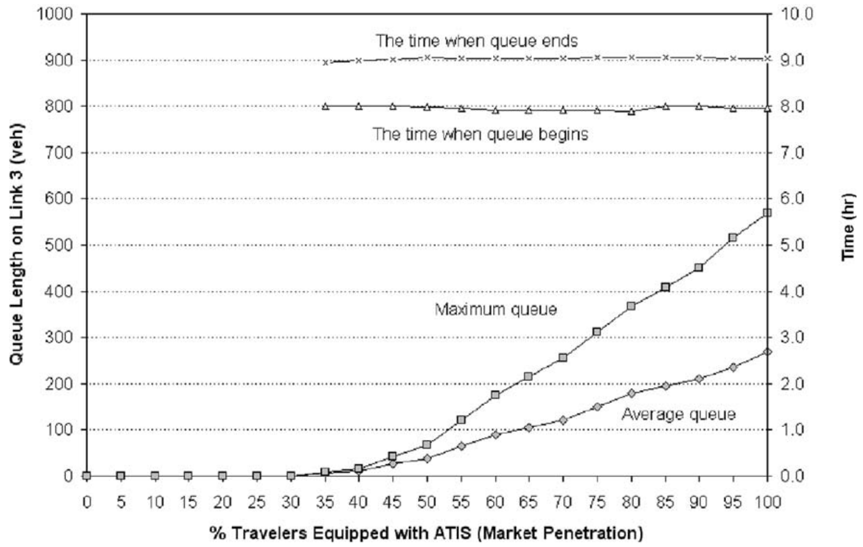


*Figure 11.* Average and maximum queue lengths on link 3 and the times when queue begins and ends.

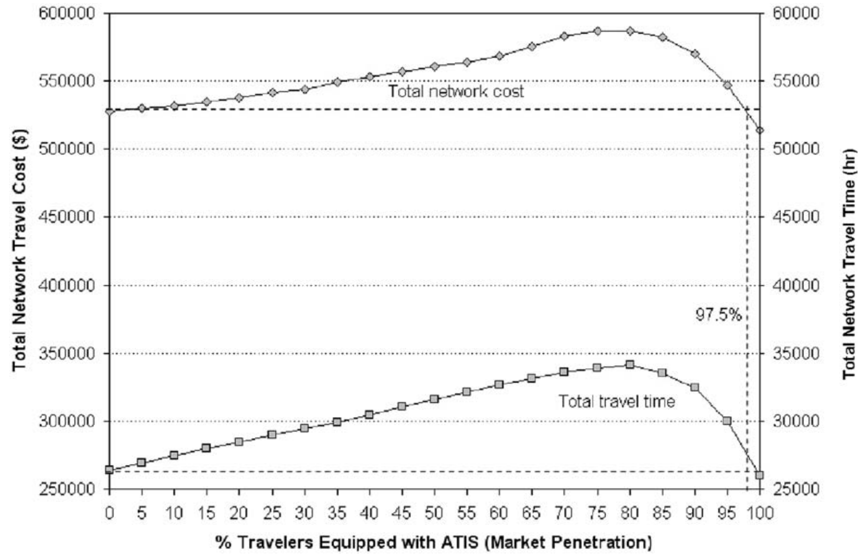


*Figure 12.* Total network travel cost and total network travel time by variation of ATIS market penetration.

that additional travelers move to the alternative paths 2, 3, 4 and 5, and/or select other departure times for their travel. The growing queue on link 3 verifies this.

Finally, we examine the effects of ATIS market penetration on the total network travel cost and the total network travel time. It can be seen in Figure 12 that 80% of travelers equipped with ATIS is a dividing point for the two curves, which coincides with the numerical results presented before. Figure 12 shows that, at most levels of market penetration, the ATIS is most likely to generate negative effect on the transport network from the systematic viewpoint, either on the total network travel cost or the total network travel time. It is noted that when the market penetration of ATIS is close to 100%, both the total network travel costs and times are only marginally better than that without ATIS. This may be true for commuter trips under recurrent congestion conditions.

In view of the above findings, one might argue that this conclusion is counter-intuitive for a transport network with ATIS to be worse off augmented to the situation without ATIS. We should make some explanations for this argument. Firstly, our conclusion is based on a specially designed network with particular input data; if we change the network structure and input data, we may obtain different results. For example, if we let links on paths 1 and 6 have larger capacities, then more opportunities (or spare capacities) are given to unequipped travelers for choosing these two paths during peak periods. As a result, the total network cost will be reduced. Secondly, this counter-intuitive conclusion is related to some assumptions made in the model proposed in this paper. For instance, the notion of deterministic departure times for equipped travelers may be too optimistic, which leads to a narrow time window for departure associated with these travelers (although this may be true for commuters). Consequently, the system optimization is not implemented even when a equilibrium of generalized path travel costs is approximately reached. Thirdly, our study reveals a possibility that travel time information, if provided without any control on market penetration of ATIS, may exacerbate road congestion in a dynamic traffic network when two travel choices are considered simultaneously, i.e. departure time and route choices.

It should be noted that our model is developed for planning and evaluation purposes under recurrent congestion conditions. Hence some of the assumptions adopted in our model may not be absolutely true under certain circumstances such as under non-recurrent congestion conditions with spillback effects. However, driver information systems are claimed to be able to solve traffic jams quickly in situations with non-recurrent congestion. In view of these circumstances and other issues discussed before, we believe that developing a modeling framework that incorporates dynamic system optimization principle under further relaxed assumptions is necessary.

## 5. Concluding Remarks

In this paper, a multi-class dynamic user-equilibrium assignment problem with simultaneous route and departure time choices in queuing networks is formulated as a discrete-time, finite-dimensional variational inequality model. The model involves two criteria regarding the travelers' behaviors, i.e., the deterministic dynamic user equilibrium for ATIS equipped travelers and the nested logit-based stochastic dynamic user equilibrium for unequipped travelers. The model is then converted to an equivalent "zero-extreme value" minimization problem that may be solved by a route/time-swapping process. It iteratively adjusts the route and departure time choices to reach closely to an extreme point of the minimization problem. However, it should be noted that the model proposed in this paper is mainly used to assess the impacts of advanced traveler information systems in general networks with queues for planning and evaluation purposes. Therefore, some of the assumptions adopted in the model may be appropriate under certain circumstances such as under recurrent congestion conditions without spillback effects at intersections. The numerical results on a test network are presented, including the ATIS impacts on individual travel costs, departure times, route inflows, queuing peaks and total network travel cost. Some insights about the effects of ATIS are obtained on the basis of this special example, e.g., ATIS is unlikely to improve network travel cost (or time) significantly. This may be true for commuter trips during normal peak hour periods.

Further studies on this topic are suggested as follows: (i) to investigate the sensitivities of the model parameters, including the scaling factors of the nested logit-based assignment, the value of time and the schedule delay penalty; (ii) to extend the model to capture the characteristics of non-recurrent congestion and the spillback effects on upstream links; (iii) to consider non-deterministic departure time choices for both equipped and unequipped commuters; (iv) to develop dynamic system optimization model and compare it with dynamic user-equilibrium model; and (v) to examine alternative efficient solution algorithms.

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