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APPROXIMATE CALCULATION OF MIXING PARAMETERS FOR HIGHLY VISCOUS COMPOSITIONS IN PLANETARY-MOTION MIXERS

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One of the basic factors preventing quality mixing of rapidly charged systems is the existence or formation in the systems of solid-phase agglomerates, which are extremely stable during mixing, during the initial stages of the process.

During the initial stage of mixing in planetary-motion mixers, vigorous fracturing of solid-phase agglomerates takes place most actively in accordance with the mechanism of displacement of their individual parts in the gap between the edge of the mixer blade and the wall or bottom of the mixer. Here, failing shear deformations develop [1], if the tangential stresses τ_w over a certain cross-sectional area of an agglomerate exceeds the limiting shear stress $[\tau_{sh}]$ which gives rise to its bulk failure, i.e., when $\tau_w > [\tau_{sh}]$. In the remaining volume of the planetary-motion mixer, agglomerates are failed preferentially in accordance with the mechanism of separation of solid-phase particles or fragments of an agglomerate from its surface. Here, the agglomerates fail [1], if the tangential stresses τ_w that develop on the contact surface between the liquid and solid phases exceed the limiting shear stress $[\tau_{wa}]$, which causes individual particles to separate from the agglomerate.

The possibility and rate of mixing of the components of rapidly charged systems are determined by providing for local shear deformations that are sufficient for bulk failure of the agglomerates ($\tau_w > [\tau_{sh}]$) on the one hand, and by ensuring sufficient shear deformations for the failure of fine agglomerates along their surface ($[\tau_{wa}] < \tau_w < [\tau_{sh}]$) and averaging of severed solid-phase particles in a liquid-like matrix on the other. Consequently, the condition $\tau_w > [\tau_{sh}]$ is a necessary and adequate condition for failure of the agglomerates, and an increase in the degree of uniformity of the components being mixed.

The purpose of the investigations that we conducted was to determine the velocity regimes for the mixing of rapidly charged systems, which ensure failure of the agglomerates that are formed in the stage when highly disperse solid components are introduced to a liquid phase. The investigations were conducted on a laboratory mixer fitted with a planetary drive with internal gearing formed from crown and satellite gears and two helical-strip agitators (Fig. 1*a*). The planetary drive of the industrial mixer may be built with external gearing consisting of a crown gear and a satellite gear (Fig. 1*b*).

In vessels containing agitators with a planetary drive, highly viscous systems are mixed preferentially in accordance with the mechanism of laminar mixing, i.e., under forces of internal friction. Here, circulation flows that are extremely complex in terms of structure and that vary in rate develop in the mass being mixed. Since zones of dispersive agitation (vigorous shearing and impression) are small in this type of mixers, it is obvious that basic power outlays for mixing are associated with the creation of developed circulation flows of the mass. In view of their complexity, the structure of these flow does not submit to analytical description; the rate and effectiveness of the mixing can therefore be estimated approximately from power outlays.

It is established experimentally that when highly viscous compositions are mixed in planetary-motion mixers, the change in power consumption over time is clearly oscillatory in nature.

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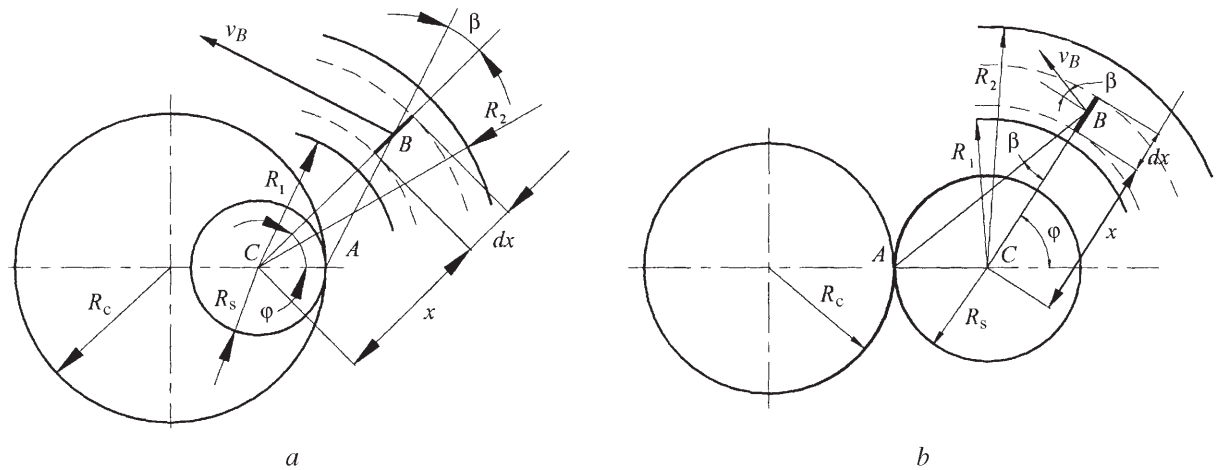

Fig. 1. Diagram showing planetary drive of mixer with internal (*a*) and external (*b*) gearing consisting of crown and satellite gears.

In first approximation, we can determine the power consumed by assuming that the force dT resisting the displacement of a blade element of a mixer can be defined, as in the problem of a liquid flow past a flat plate, from the formula

$$dT = \xi \rho_{sys} v_B^2 dF \sin\left(\frac{\pi}{2} - \beta\right) \sin\alpha,$$

where ξ is the coefficient of lateral resistance; ρ_{sys} is the density of the system being mixed; v_B is the local displacement rate of a surface element of the agitator; $dF = xd\varphi dx$ is the surface area of the blade element;

$$\beta = \arcsin\left[\frac{R_s}{AB}\sin(\pi - \varphi)\right] = \arcsin\left[\frac{R_s}{AB}\sin\varphi\right]$$

is the angle between the normal to the surface element of the blade and the vector of the linear velocity v_B of its displacement, R_s is the radius of the pitch circle of the satellite gear, AB is the radius vector of the center of instantaneous rotation, φ is an angle characterizing the position of the blade element, $\alpha = \arctan(S/2\pi x)$ is the current helix angle of the helical line of the blade, and S is the travel of the helical line of the blade.

The local displacement rate v_B of a surface element of the mixer for a known angular velocity ω of the carrier can be determined proceeding from geometric considerations. Since point A of the tangency of the pitch circles of the crown and satellite gears is the center of instantaneous rotation, the circumferential velocity of a surface element of a blade rotating about this center can be determined from the expression

$$v_B = \Omega_0 AB, \tag{1}$$

where Ω_0 is the angular velocity of instantaneous rotation of a surface element of the blade, which is positioned near point B.

Depending on the type of gearing composed of a crown gear and a satellite gear, the radius vector AB can be determined in accordance with the law of cosines:

$$AB = \sqrt{R_s^2 + x^2 - 2R_s x\cos(\pi - \varphi)} = \sqrt{R_s^2 + x^2 + 2R_s x\cos\varphi} \tag{2}$$

for external gearing, and

$$AB = \sqrt{R_s^2 + x^2 - 2R_s x\cos\varphi} \tag{3}$$

for internal gearing.

If it is assumed that the "±" signs in the formulas correspond to external and internal gearing consisting of a crown gear and satellite gear, expressions (2) and (3) can be rewritten as

$$AB = \sqrt{R_s^2 + x^2 \pm 2R_s x \cos\varphi}\,. \tag{4}$$

The linear displacement rate v_C of the shaft of the agitator (point C) is determined from the formula

$$v_C = \Omega_0 R_s. \tag{5}$$

Since point C pertains to one member (carrier), it rotates about the shaft of the carrier with the angular velocity ω of the carrier, and the relationship

$$v_C = \omega(R_c + R_s) \tag{6}$$

is then valid.

Equating the right sides of formulas (1) and (6), we obtain

$$\Omega_0 = \frac{R_c \pm R_s}{R_s}\omega, \tag{7}$$

where R_c is the radius of the pitch circle of the crown gear.

Substituting expressions (4) and (7) in relationship (1), we obtain

$$v_B = \frac{R_c \pm R_s}{R_s}\omega\sqrt{R_s^2 + x^2 \pm 2R_s x \cos\varphi}\,.$$

The maximum displacement of the leading edge of the blade can be found from the formula

$$v_B^{\max} = \omega\frac{(R_c \pm R_s)(R_m \pm R_s)}{R_s},$$

where R_m is the radius of the outer leading edge of the blade.

The angular velocity of the agitator can be determined from the expression

$$\omega_m = \omega\frac{R_c \pm R_s}{R_s}.$$

Since the elementary power consumption for the displacement of a blade element $dN = v_B dT$,

$$dN = \xi\rho_{sys}\omega^3\left(\frac{R_c}{R_s} \pm 1\right)^3 (R_s^2 + x^2 \pm 2R_s x\cos\varphi)^{3/2}\sin\left(\arctan\frac{S}{2\pi x}\right)\sin\left[\frac{\pi}{2} - \arcsin\left(\frac{R_s}{AB}\sin\varphi\right)\right] x\,dx\,d\varphi =$$

$$= \xi\rho_{sys}\omega^3\left(\frac{R_c}{R_s} \pm 1\right)^3 (R_s^2 + x^2 \pm 2R_s x\cos\varphi)^{3/2}\frac{S}{\pi x\sqrt{4 + \dfrac{S^2}{\pi^2 x^2}}}\sqrt{1 - \frac{R_s^2}{R_s^2 + x^2 \pm 2R_s x\cos\varphi}\sin^2\varphi}\; x\,dx\,d\varphi,$$

where $\xi = A\vartheta_{eff}^{m_1}$; $\vartheta_{eff} = \mu_{eff}/\rho_{sys}$; and μ_{eff} is the average effective viscosity of the mass being agitated.

As a result, we obtain

$$N = \xi\rho_{\text{sys}}\omega^3\left(\frac{R_c}{R_s} \pm 1\right)^3 \times$$

$$\times \int_{R_1}^{R_2}\int_{\varphi_1}^{\varphi_1+\varphi_2} (R_s^2 + x^2 \pm 2R_s x\cos\varphi)^{3/2} \frac{S}{\pi x\sqrt{4 + \dfrac{S^2}{\pi^2 x^2}}} \sqrt{1 - \frac{R_s^2}{R_s^2 + x^2 \pm 2R_s x\cos\varphi}\sin^2\varphi}\; x dx d\varphi \qquad (8)$$

for pug mills and helical-strip agitators, where R_1 and R_2 are the radii of the inner (for a pug mill – the radius of its shaft), and outer edges of the blade of a agitator, respectively, $\varphi_1 = 0–2\pi n_2$ is the initial angle characterizing the position of the agitator at an arbitrary time τ, n_2 is the rotational speed of the agitator during the time period examined, $\varphi_2 = 2\pi m$ is the angle corresponding to the number of turns of an agitator blade, and m is the number of blade turns.

It is obvious that for helical-strip agitators, the width of which, as a rule, is small, the expression in question can be simplified as

$$N = \xi\rho_{\text{sys}}\omega^3\left(\frac{R_c}{R_s} \pm 1\right)^3 \frac{R_2^2 - R_1^2}{2} \frac{S}{\pi x_{\text{av}}\sqrt{4 + \dfrac{S^2}{\pi^2 x_{\text{av}}^2}}} \times$$

$$\times \int_{\varphi_1}^{\varphi_1+\varphi_2} (R_s^2 + x_{\text{av}}^2 \pm 2R_s x_{\text{av}}\cos\varphi)^{3/2} \sqrt{1 - \frac{R_s^2\sin^2\varphi}{R_s^2 + x_{\text{av}}^2 \pm 2R_s x_{\text{av}}\cos\varphi}}\, d\varphi, \qquad (9)$$

since $dF = x_{\text{av}}(R_2 - R_1)d\varphi = (R_2^2 - R_1^2)d\varphi/2$, where $x_{\text{av}} = (R_2 + R_1)/2$.

The analytical expression obtained can be used for approximate calculation by numerical methods of the power outlays required to displace the a worm or spiral-strip blade as a function of its spatial orientation at a given time $\tau_1 = f(\varphi_1)$.

Simplification of expressions (8) and (9) will also make it possible to obtain formulas for calculation of the power consumed in displacing the shafts of the agitators N_{sh} and the cross ties N_{cr} between the blades and shaft of helical-strip agitators:

$$dN = \xi\rho_{\text{sys}}\omega^3 h\left(\frac{R_c}{R_s} \pm 1\right)^3 (R_s^2 + x^2 \pm 2R_s x\cos\varphi)^{3/2} \sin\left[\frac{\pi}{2} - \arcsin\left(\frac{R_s}{AB}\sin\varphi\right)\right] dx =$$

$$= \xi\rho_{\text{sys}}\omega^3 h\left(\frac{R_c}{R_s} \pm 1\right)^3 (R_s^2 + x^2 \pm 2R_s x\cos\varphi)^{3/2} \sqrt{1 - \frac{R_s^2}{R_s^2 + x^2 \pm 2R_s x\cos\varphi}\sin^2\varphi}\; dx;$$

$$N = \xi\rho_{\text{sys}}\omega^3 h\left(\frac{R_c}{R_s} \pm 1\right)^3 \times$$

$$\times \int_{R_1}^{R_2} (R_s^2 + x^2 \pm 2R_s x\cos\varphi)^{3/2} \sqrt{1 - \frac{R_s^2\sin^2\varphi}{R_s^2 + x^2 \pm 2R_s x\cos\varphi}}\, dx,$$

where h is the height of the cross tie or shaft.

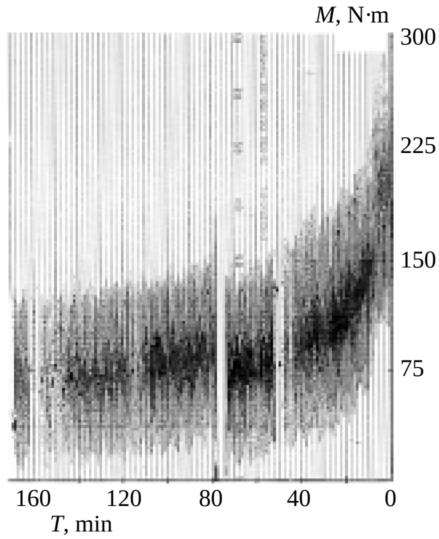

Fig. 2. Diagram showing variation in twisting moment M on carrier shaft of planetary-motion mixer as function of mixing time T.

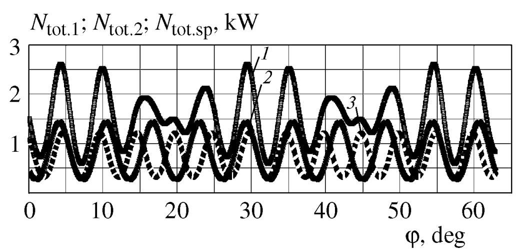

Fig. 3. Dependence of power consumed in mixing for first and second spiral agitators $N_{tot.1}$ and $N_{tot.2}$ and total power consumed $N_{tot.sp}$ on angle φ: *1)* $N_{tot.sp} = N_{tot.1} + N_{tot.2}$; *2)* $N_{tot.1}$; *3)* $N_{tot.2}$.

The total power outlays for agitating a mass in a planetary-motion mixer equipped, for example, with a combination of worm and spiral-strip agitators, can be calculated from the formula

$$N_{tot}(\tau_1) = N_{sh} + N_{cr}(\tau_1) + N_{wo}(\tau_1) + N_{sp}(\tau_1) + N_{ed} + N_{dis}, \tag{10}$$

where N_{wo} and N_{sp} are the power outlays required to displace the worm and spiral blades, respectively, N_{ed} are the power outlays required to displace and crush the mass in the narrowing gaps between the edges of the blades and the wall of the mixer (the outlays N_{ed} can be neglected in view of the locality of these regions), and N_{dis} are the additional power outlays required to displace the edge of the partition of the spiral-strip agitator, which is situated near its bottom.

Experimental relationships between the twisting moment on the shaft of the carrier on the mixing time for the laboratory mixer confirm the dependencies of the twisting moment and power required for agitation on the mutual orientation of the agitator blades when the latter are in planetary motion. Figure 2 shows an experimental diagram of the relationship between the twisting moment on the carrier shaft of the mixer and the mixing time, while Fig. 3 presents the computed curve of the power consumed for mixing (for two spiral agitators) versus the angle of rotation.

Engineering analysis of rational parameters of the mixing process in planetary-motion mixers is possible if the parameters A and m_1 in the equation for determination of the coefficient of lateral resistance of a blade element $\xi = A(\mu_{eff}/\rho_{sys})^{m_1}$ are known. The values of these parameters can be established experimentally for a specific composition. Results of experimental investigations of the mixing process of a highly viscous composition on a model planetary mixer, which were conducted in 1972 by the Moscow Institute of Chemical Machine Building, enabled us to establish that the value of the exponent m_1 can be assumed equal to 0.4 with an accuracy sufficient for calculations.

Experimental curves of the twisting moment on the carrier shaft, which were obtained on the laboratory mixer with the carrier operating at a fixed rotational speed, were analyzed to determine the numerical value of the parameter A. Numerical calculations performed using relationships (8)–(10) made it possible to establish that variation of the parameter A for the composition under investigation ranges from $0.3 \cdot 10^3$ to $1.0 \cdot 10^3$.

It should be pointed out that the calculations performed using Eqs. (8)–(10) are approximate in nature, since the mutual effect of circulation flows that develop during mixing as the agitator blades experience planetary displacement were disregarded in their derivation. The results obtained were used in developing a procedure for engineering analysis of rational parameters of the mixing process in industrial planetary-motion mixers.

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