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EFFECT OF GEOMETRIC DRIVE PARAMETERS ON DYNAMICS OF PLANETARY MILLS WITH INTERNAL ROLLING

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Planetary mills represent a new generation of pulverizing equipment, a characteristic feature of the design of which consists in the fact that they normally possess two-four drums that rotate about the central axis and their own axes simultaneously. The drums are therefore involved in complex motion, which is usually divided into transient (rotation of the carrier) and relative (rotation of the drum) motions. Significant inertial forces, which are a basic source of action on the material being fractured and which contribute to improvement of comminution efficiency, develop as a result of a similar motion [1]. Structurally, planetary mills can be classed as horizontal or vertical in terms of axis positioning, and as mills with external and internal rolling in terms of method of rolling over a stationary annular surface.

Complex research on planetary mills was begun in the mid twentieth century [4]. Industrial and laboratory investigations of this type of mills is being conducted in such countries as the United States, France, Germany, Japan, and Russia. The first theoretical works involving the motion dynamics of grinding bodies in a planetary mill were founded on the postulate concerning equivalence between the forms of motion of the charge in drum and planetary mills [2], while the charge was represented as a single whole, disregarding the dynamics of the individual grinding bodies. This approach is subject to critique, and is recognized as unsubstantiated [3]. The fact that the effect of gravitational forces on the pattern of motion is disregarded is, in our opinion, highly incorrect. At the moment when the grinding bodies separate from the walls of the drum, and when transient and relative inertial forces may be in mutual equilibrium, gravitational forces will exert a decisive influence on their motion.

Considering these deficiencies, Vaitekhovich et al. [4, 5] examined the motion of a single grinding body subject to inertial and gravitational forces. Here, the constraint reaction at the point of contact between the grinding body and walls of the drum was adopted as a determining parameter. It is possible to determine not only the pattern of motion of the grinding body (with or without separation), but also to evaluate the pressure of the grinding body against the wall of the drum, and, consequently, on the material being ground on the basis of the magnitude of this reaction.

The angular velocity of the drums about the central (the angular velocity of the carrier) axis and its own axis, the constraint reaction N, and the frictional force F_f characterizing the crushing and pulverizing capacities of grinding bodies are basic parameters on which the operating efficiency of planetary mills depends. The ratio of the radii of the drum and nonstationary annular surface along which the rolling occurs will exert, in turn, an influence on the enumerated parameters. The geometric criterion $k = r_d/R$, where r_d is the radius of the drum, and R is the radius of the stationary annular surface, is introduced for evaluation of this influence.

The formulas for basic kinematic and dynamic characteristics of planetary mills with internal rolling, which were obtained in [4, 5] with consideration of the fact that the drums of the mill are in direct contact with a nonstationary annular surface, are as follows:

$$N_{rel} = \Omega^2 R(1-k)\left[\frac{1-k}{k} - \cos\frac{\varphi}{k}\right] - g\sin\left(\frac{1-k}{k}\varphi\right); \tag{1}$$

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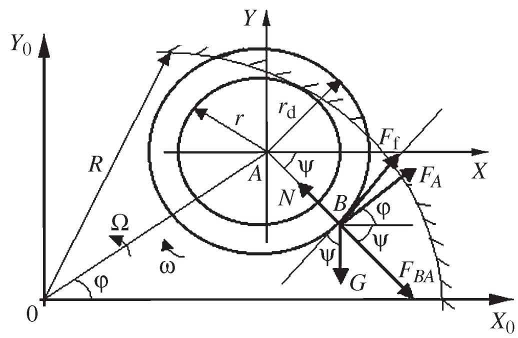

Fig. 1. Computational diagram for planetary mill.

$$F_f^{rel} = g\cos\left(\frac{1-k}{k}\varphi\right) - \Omega^2 R(1-k)\sin\frac{\varphi}{k}; \tag{2}$$

$$\omega = \sqrt{\frac{g(1-k)}{kR(1-2k)}}, \tag{3}$$

where N_{rel} is the relative constraint reaction; Ω is the angular velocity of the carrier; φ is the angle of rotation of the carrier; g is the acceleration of free fall; F_f^{rel} is the relative frictional force; and ω is the limiting angular velocity of the drum about its own axis, which characterizes the transition from a cascade to a centrifugal regime.

The drums may be rolled, however, by friction, gear, or other transmissions mounted on milling drums or their shafts [6, 7]. In this connection, formulas (1)–(3) are needed for adjustment, since in addition to the radii of the drum and stationary annular surface, it is now necessary to consider the radius r of the drive wheel (see Fig. 1), which will roll over the stationary annular surface. Here, the radius r may be larger or smaller than the radius of the drum.

The purpose of this study is to determine basic kinematic and dynamic characteristics of planetary mills with internal rolling with allowance for the radius of the drive wheel.

The geometric criterion $b = r_d/r$ was introduced additionally for realization of the stated problem.

Considering what has been stated above, let us examine the position of a grinding body (ball) at arbitrary point B (Fig. 1).

Let us assume that the carrier begins to move counterclockwise from axis X, and is turned at an angle φ. At this time, the drum is turned in the opposite direction by the angle ψ.

The motion of the grinding body is represented in moving coordinate system XAY, which is displaced forward around the circumference relative to stationary coordinate system $X_0 0 Y_0$. In that case, gravitational force G, frictional force F_f, constraint reaction N, and transient F_A and relative F_{BA} inertial forces act on the ball [8].

For the case in question, the inertial forces can be determined from the formulas

$$F_A = m\Omega^2 R(1-k); \tag{4}$$

$$F_{BA} = m\Omega^2 \frac{(1-k)^2}{k} bR, \tag{5}$$

where m is the mass of the grinding body.

For planetary mills with internal rolling, the relation between the angular velocities of the carriage and drum is established from the formulas

$$\omega = \Omega\frac{1-k}{k}; \qquad \psi = \varphi\frac{1-k}{k}.$$

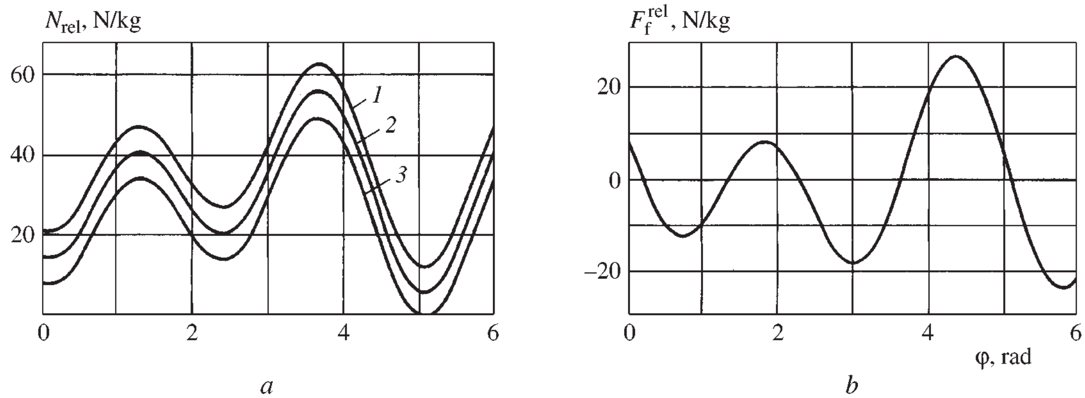

Fig. 2. Dependence of relative constraint reaction N_{rel} (*a*) and frictional force F_f^{rel} (*b*) on turn angle φ of carriage when $k = 0.4$: *1*) $b = 1.50$; *2*) $b = 1.25$; *3*) $b = 1.00$.

Using the d'Alambert principle [8], and setting up the equation for equilibrium on the radial and tangential axes, which pass through point *B*, we analyzed the motion of a single grinding body in the drum of a planetary mill. After a number of transformations, we obtained formulas for determination of the relative constraint reaction N_{rel} and the relative frictional force F_f^{rel} with allowance for geometric criterion *b*. The formula for the relative reaction takes on the form

$$N_{rel} = \Omega^2 R(1-k)\left[\frac{b(1-k)}{k} - \cos\frac{\varphi}{k}\right] - g\sin\left(\frac{1-k}{k}\varphi\right).$$

The formula for determination of the relative frictional force is derived identical to formula (2), since the frictional force is independent of geometric criterion *b*. The explanation for this is readily found in analyzing computational relationships (4) and (5) for the inertial forces. Since the relative inertial force F_{BA} always remains perpendicular to frictional force F_f, only the transient inertial force F_A, which is independent of geometric criterion *b*, will affect its variation.

Analysis of the operation of a planetary mill [9] indicates that the worst conditions under which the grinding body is retained against the surface of the drum are created when the latter is in the lower position, and the body is at the upper point of the grinding chamber.

Setting up the equation of equilibrium of all forces at point *B*, it is possible to determine the angular velocity, which is required to develop the inertial force and which is sufficient to maintain the body at the upper point:

$$\omega = \sqrt{\frac{g(1-k)}{kR[b(1-k)-k]}}. \qquad (6)$$

To determine the range of application of criterion *b*, it is necessary to solve the inequality $b(1 - k) - k \geq 0$; hence,

$$b \geq \frac{k}{1-k}. \qquad (7)$$

It is apparent from inequality (7) that criterion *b* may vary within rather broad limits. It must be considered, however, that for planetary mills with internal rolling, geometric criterion *k* may vary from 0 to 0.5.

The influence exerted by criterion *b* on basic characteristics of a mill was established by computational means using the Mathcad software package. Calculations were performed for the following fixed parameters: $k = 0.4$, $\Omega = 12$ rad/sec, and $R = 0.2$ m. In the calculations, the turn angle φ of the carriage was varied within the limits from zero to 2π, and the criterion *b* from 1.0 to 1.5, and the relative constraint reaction N_{rel} and frictional force F_f^{rel} were determined (Fig. 2).

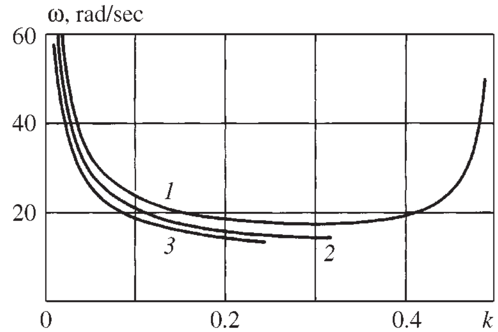

Fig. 3. Dependence of limiting angular velocity ω during transition from cascade to centrifugal operating regime on criterion *k*: *1*) *b* = 1.00; *2*) *b* = 1.25; *3*) *b* = 1.50.

It is apparent from the computational relationships that variation in the constraint reaction and frictional force is cyclical in nature. The relative constraint reaction for a centrifugal regime always remains positive. The point where it comes in contact with the horizontal axis corresponds to separation of the grinding body from the surface of the drum. The frictional force changes its direction several times during one cycle of the carriage. Variation in criterion *b* exerts a major influence on relative constraint reaction N_{rel}, which increases with increasing criterion *b*.

It is possible from results of analysis of the relative frictional force and constraint reaction to conclude that variation in criterion *b* has no effect on the abrasive component, and affects the pulverizing component of the breaking forces to a significant degree.

Figure 3 shows the dependence of the limiting angular velocity on transition from a cascade to a centrifugal regime, which was calculated from formula (6), on geometric criteria *k* and *b*. Relationship *1*, which was obtained in [10], indicates that there is a stable range of *k* values from 0.1 to 0.4 for which this criterion has no appreciable effect on the transition from one regime to the other. Analysis of computational relationships *2* and *3* indicates that the limiting angular velocity decreases with increasing criterion *b*, but this effect is not as vigorous in this case as for the normal constraint reaction.

In analyzing the kinematic and dynamic characteristics of planetary mills, it is therefore necessary to consider the geometric relationships of their drive elements, which exert a considerable influence on the pulverizing component of the breaking forces.

REFERENCES

1. V. Duda, *Cement* [Russian translation], Stroiizdat, Moscow (1981).
2. L. P. Bushuev, “On the analogy between the motion of a mass of particles in ball and planetary mills,” *Izv. Akad. Nauk SSSR, OTN: Mekh. Mashinostr.*, No. 4, 153–155 (1959).
3. Yu. V. Dmitrak, “Theory of the motion of a grinding mass and improving equipment efficiency for fine grinding of rocks,” *Author’s Abstract of Dissertation for Doctor of Technical Sciences*, Moscow (2000).
4. P. E. Vaitekhovich, D. V. Gapanyuk, and D. V. Semenenko, “Analysis of kinematic and dynamic characteristics of planetary mills,” in: *Materials Presented at the International Scientific-Technical Conference “Interstroimex-2002”* [in Russian], Mogilev (2002), pp. 322–323.
5. P. E. Vaitekhovich, D. V. Gapanyuk, and D. V. Semenenko, “Conditions for separation of a ball from the walls of a drum in a planetary mill with internal rolling,” *Tr. Belarus. Gosudar. Tekh. Univ.: Ser. Khim. Tekhnol. Neorgan. Veshch.*, No. 10, 223–225 (2002).
6. Russian Federation Patent No. 2036009, MKI B 02 C 17/08, “Planetary mill,” *Byull. Izobr.*, No. 15 (1995).

7. Russian Federation Patent No. 2056169, MKI B 02 C 17/08, “Planetary mill,” *Byull. Izobr.*, No. 8 (1996).
8. S. M. Targ, *Short Course in Theoretical Mechanics* [in Russian], Nauka, Moscow (1967).
9. P. E. Vaitekhovich, A. V. Vavilov, and G. M. Khves’ko, “Determination of critical rotational speeds for a planetary mill,” *Vestn. Belarus. Nauchno-Tekh. Univ*., No. 2, 34–39 (2003).
10. P. E. Vaitekhovich, “Kinematics of planetary mills with internal rolling,” *Tr. Belarus. Gosudar. Tekh. Univ.: Ser. Khim. Tekhnol. Neorgan. Veshch.*, No. 10, 217–223 (2002).