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Probability of Heavy Traffic Period in Third Generation CDMA Mobile Communication

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Abstract. The paper considers a problem of deriving the multidimensional distribution of a segment of a long-range dependent traffic in the third generation mobile communication network. An exact expression for the probability is found when a self-similar process from [8] models the traffic. The probability of heavy-traffic period, the outage probability, and the level-crossing probability are found. It is shown that the level crossing probability depends on the average call length only. Further, this probability for traffic with dependent samples is lower than for traffic with independent samples. Also, it is shown that there is a linear dependence between the average heavy traffic interval and the average call length.

Keywords: wireless communication, self-similar traffic, outage probability

1. Introduction

Consider a mobile communication stationary traffic as a sequence $\ldots, Y(-1), Y(0), Y(1), \ldots$ where $Y(t)$ is the number of actually transmitting mobile stations (MS) at time t ($t \in \mathbf{Z} \mathrel{\hat{=}} \{\ldots, -1, 0, 1, \ldots\}$) (see figure 1). Let $Y(t)$, $t \in \mathbf{Z}$, be a long-range dependent traffic as it is expected in the third generation of CDMA mobile communication networks capable to transmit not only voice but also data and file messages.

A problem is to find the probability that at times t and $t + N$ ($N \geqslant 2$), the traffic is not heavy (this means that $Y(t) \leqslant M$ and $Y(t+N) \leqslant M$ where M is a given threshold) but in the whole interval between t and $t + N$, the traffic is heavy (i.e., $Y(i) > M, t < i < t + N$) (see figure 2). This probability is called the *probability of heavy-traffic period of length* $N - 1$ and denoted as $P_{\text{hvy}}(N - 1)$.

Another problem is to find the *outage probability* $P_{\text{out}}(N)$ [6,9–11], that is, the probability that at time t, the traffic is not heavy but in an interval, which begins at $t + 1$ and lasts a given time N at least, it is heavy, i.e., $P_{\text{out}}(N) \mathrel{\hat{=}} \Pr\{Y(t) \leqslant M, Y(t + 1) > M, \ldots, Y(t + N) > M\}$(see figure 3).

In a special case of $N = 1$, it is interesting to give an expression for the outage probability called the *level crossing probability* and denoted as P_{cross}. P_{cross} is the probability that at time t the traffic is not heavy but at time $t + 1$, it is heavy, $P_{\text{cross}} \mathrel{\hat{=}} P_{\text{out}}(1) = \Pr\{Y(t) \leqslant M, Y(t + 1) > M\}$ (see figure 4). The level crossing probability is related, in its turn, to another important traffic characteristic. Consider an interval $t + 1, \ldots, t + T$ (where T is a random variable) such that $Y(t) \leqslant M, Y(t + 1) > M, \ldots, Y(t + T) > M, Y(t + T + 1) \leqslant M$. This interval is called the heavy traffic interval and the random variable T is the length of the interval. An interesting value is the average of T, i.e., $\mathrm{E}T$. The following equation relates $\mathrm{E}T$ and the level crossing probability:

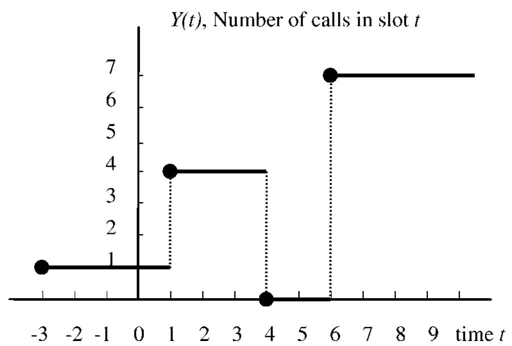

Figure 1. Mobile communication traffic.

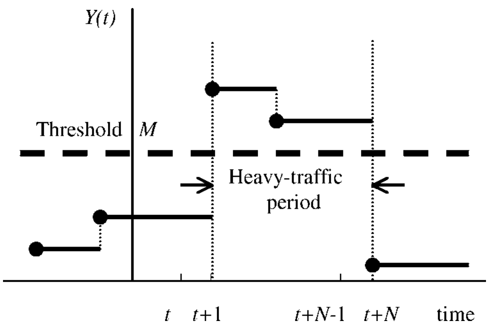

Figure 2. Heavy-traffic period.

$$\mathrm{E}T = \frac{\Pr\{Y(t) > M\}}{P_{\text{cross}}}. \tag{1}$$

A more general problem is to find the probability distribution of traffic segment $Y(t), \ldots, Y(t + N)$. Having this distribution, one can find, in principle, not only the probability of heavy-traffic period but also all other traffic characteristics depending on the traffic behavior in interval from t to $t + N$. However, the calculation of these characteristics is not a simple problem. There are a number of papers

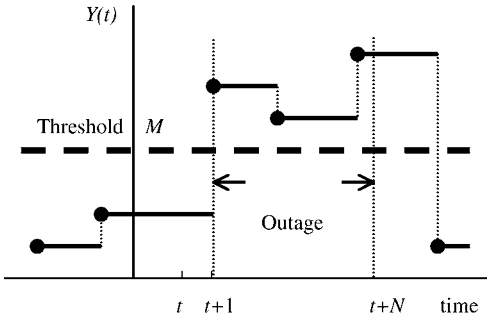

Figure 3. Outage event.

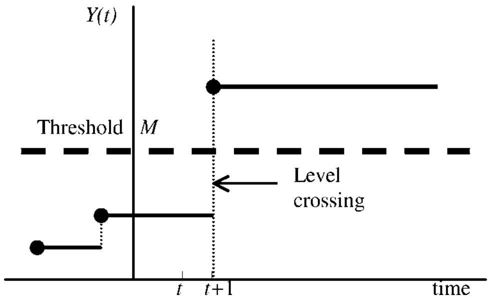

Figure 4. Level crossing.

and books on the subject (for example, [1,4]). We note that even for the most considered Gauss, Rayleigh, and Markov processes, the problem of calculation of P_{hvy} and P_{out} is not generally solved.

If $Y(t), t \in \mathbf{Z}$, are independent, the problem is a trivial one since for finding

$$P(y_t, \ldots, y_{t+N}) = \Pr\{Y(t) = y_t, \ldots, Y(t+N) = y_{t+N}\}, \tag{2}$$

it is necessary to know only the one-dimensional distribution $\Pr\{Y(t) = y\}$. On contrary, if $Y(t), t \in \mathbf{Z}$, is a long-range dependent process, the problem is not trivial and if $P(y_t, \ldots, y_{t+N})$ can be found, it would not be expected to have a simple analytical expression.

In the paper, we use a model for long-range dependent traffic from [8] (section 2). The model is motivated by the real-time traffic measurements in corporate LANs, Variable-bit-rate video sources, WWW-network, and other communication systems [2,3,5]. In fact, the considered traffic model represents a self-similar stochastic process (section 3). In section 4, we give a representation of traffic values $Y(t), \ldots, Y(t+N)$ in terms of some random variables which, in section 5, are used for deriving a formula for the multidimensional distribution $P(y_t, \ldots, y_{t+N})$. The distribution will be derived by using a splitting argument and showing with its application that the segment $Y(t), \ldots, Y(t+N)$ of this long-range dependent traffic can be represented by a certain number of independent but not identically distributed Poisson random variables. Using $P(y_t, \ldots, y_{t+N})$, we get $\mathrm{E}T$ and some numerical results. It is shown that for the considered traffic, the level crossing probability depends only on the average call length but not on other behavior of the distribution of call length and that the probability P_{cross} for traffic with dependent $Y(t)$ is lower than for traffic with the same $\mathrm{E}Y(t)$ but independent $Y(t)$. Also, it is shown that there is a linear dependence between the average heavy traffic interval $\mathrm{E}T$ and the average call length when $\mathrm{E}Y(t)$ is kept as given. Section 6 gives a Gaussian approximation to the probability of heavy-traffic period. An extension of the obtained results to a more general traffic model is given in section 7. Section 8 is a conclusion.

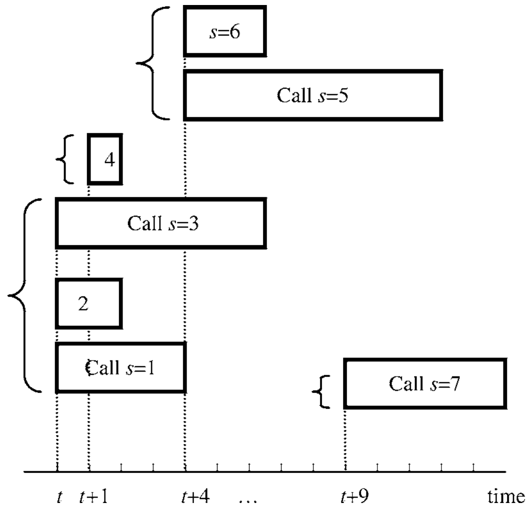

Figure 5. The traffic as an aggregation of calls. There are $\xi = 3$ call arrivals at t, $\xi_{t+1} = 1$ call arrivals at $t+1$, $\xi_{t+4} = 2$ call arrivals at $t+4$, $\xi_{t+9} = 1$ call arrivals at $t+9$, and no other call arrivals between t and $t+9$. The call $s = 1$ has 4 packets, the call $s = 2$ has 2 packets and so on.

2. Input traffic

The considered traffic $Y(t), t \in \mathbf{Z}$, is assumed to be a stream of packets. The packets have equal lengths accepted as length 1. The packets are assigned to mobile station calls, so the traffic is an aggregation of packets generated by mobile stations during their calls (see figure 5). The calls are enumerated by $s \in \mathbf{Z}$. (A number s is not a telephone number of MS. The enumeration by s is introduced here only to order calls in the traffic process. Thus, the same MS can appear in different segments of traffic being numerated by different s; each its s represents a new call of the MS.) We say that a call s "starts to generate its packets" at time denoted by ω_s ($\omega_s \leqslant \omega_{s+1}$). The moment ω_s is called the time of call s arrival. The call s generates one packet at each time $\omega_s + i - 1$ in time interval $\omega_s, \ldots, \omega_s + \tau_s - 1$, $i \in \{1, \ldots, \tau_s\}$.

The time interval $\omega_s, \ldots, \omega_s + \tau_s - 1$ is called the *call period* s and $\tau_s \in \mathbf{N}$ is called the length of call s. It is clear that before time ω_s and after time $\omega_s + \tau_s - 1$, the call s does not generate any packets. At any time moment $t \in \mathbf{Z}$, more than one call can be started (in other words, we say that more than one call arrival can occur). By ξ_t, we denote the number of calls arrived at t, that is, $\xi_t \in \mathbf{Z}_+$ is the number of MS started their calls at t.

Thus,

$$Y(t)=\sum_{s\in\mathbf{Z}}\theta_s(t-\omega_s+1),\quad t\in\mathbf{Z},\tag{3}$$

where $\theta_s(i)=1$ for $i\in\{1,\dots,\tau_s\}$ and $\theta_s(i)=0$ for $i\leqslant 0$ and $i\geqslant\tau_s+1$. This means that $Y(t)$ is a total number of packets generated by all active MS at time t.

It is assumed $\tau_s,\ s\in\mathbf{Z}$, are i.i.d.; the numbers of call arrivals, $\xi_t\in\mathbf{Z}_+\hat{=}\{0,1,\dots\}$, $t\in\mathbf{Z}$, are i.i.d. with $0<\lambda\hat{=}\mathrm{E}\xi_t<\infty$ and $\Pr\{\xi_t=0\}<1$; the random variables τ_s are mutually independent of ξ_t and ω_s. Let τ (let ξ) be a generic symbol for τ_s (for ξ_t).

The call traffic $Y(t)$ is specified by two distributions, the distribution of number of new call arrivals at time t ($\Pr\{\xi=k\}$) and the distribution of call length ($\Pr\{\tau=l\}$).

In what follows, it is assumed that ξ is a Poisson random variable, i.e.,

$$\Pr\{\xi=k\}=\frac{\mathrm{e}^{-\lambda}\lambda^k}{k!},\quad 0<\lambda<\infty,\ k\in\mathbf{Z}_+,\tag{4}$$

where $\lambda=\mathrm{E}\xi$ is a parameter of the Poisson distribution. With this assumption, the call traffic $Y(t)$ is specified by one distribution ($\Pr\{\tau=l\}$) and one parameter (λ).

Remark. We note, although it will not be used later, that when it is known that actual call traffic is described by (3) with Poisson ξ, it is possible to find its $\Pr\{\tau=l\}$ and λ by measuring the autocovariance $w(k)\hat{=}\mathrm{cov}\{Y(t),Y(t+k)\}$ of the traffic $Y(t)$. Namely,

$$\Pr\{\tau\geqslant k+1\}=\frac{w(k)-w(k+1)}{w(0)-w(1)},\tag{5}$$

$$\lambda=w(0)-w(1).\tag{6}$$

In turn, the autocovariance $w(k)$ can be expressed in terms of λ and $\Pr\{\tau\geqslant l\}$ by the following equation:

$$w(k)=\lambda\sum_{l=k+1}^{\infty}\Pr\{\tau\geqslant l\}.\tag{7}$$

As to distribution of call length, the most important case is such that τ has a Pareto-type distribution

$$\Pr\{\tau=l\}=c_0l^{-\alpha-1},\quad c_0\hat{=}\left(\sum_{l=1}^{\infty}l^{-\alpha-1}\right)^{-1},\quad 1<\alpha<2,\quad l\in\mathbf{N},\tag{8}$$

where α is an only parameter of the distribution. Table 1 gives the values of the normalization constant c_0 and the average call length $\mathrm{E}\tau$ for several values of α.

The traffic $Y(t)$, which has Pareto-type distributed τ and Poisson ξ, is a stationary (in narrow sense) and ergodic process. It is specified by only two parameters, λ and α. These parameters can be measured with application of (5) and (6) when actual call traffic is described by (3).

Our results below hold true for general distribution $\Pr\{\tau=l\}$ with finite mean. However, when we want to consider a long-range dependent traffic, we shall assume that (8) is satisfied, that is, τ is Pareto-type distributed. A sufficiency of the condition (8) for traffic self-similarity will be given by a theorem in section 3.

Table 1

α	c_0	$\mathrm{E}\tau$
1.1	0.6409	6.784
1.2	0.6709	3.751
1.3	0.6981	2.745
1.4	0.7229	2.245
1.5	0.7454	1.947
1.6	0.7660	1.751
1.7	0.7848	1.612
1.8	0.8019	1.509
1.9	0.8176	1.431
2.0	0.8319	1.368

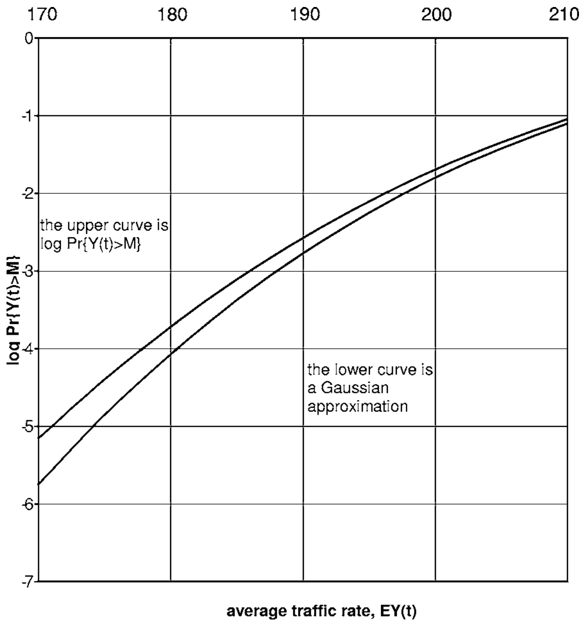

Figure 6. The curve of $\log\Pr\{Y(t)>M=230\}$ as a function of $\mathrm{E}Y(t)$.

Note. When $Y(t)$ is Poissonian, the one-dimensional probability

$$\Pr\{Y(t)>M\}=\mathrm{e}^{-\mathrm{E}Y(t)}\sum_{m=M+1}^{\infty}\frac{(\mathrm{E}Y(t))^m}{m!}$$

plays an important role in finding the Erlang capacity of reverse links for CDMA mobile networks (see [9], equation (6.47) and figure 6.4 there). As shown in [9], for a single cell and perfect power control network (as well as for some other networks), it is necessary to find $\Pr\{Y(t)>230\}$ numerically as a function of $\mathrm{E}Y(t)$. To calculate $\Pr\{Y(t)>230\}$, it is possible to use the following expression for the large deviation of Poissonian $Y(t)$:

$$\Pr\bigl\{Y(t)>230\bigr\}=\sum_{m=230}^{\infty}\frac{\mathrm{e}^{m[1+\ln(\mathrm{E}Y(t)/m)]-\mathrm{E}Y(t)}}{\sqrt{2\pi m}\bigl(1+\frac{1}{12m}+\frac{1}{288m^2}+\dots\bigr)},$$

where the series

$$\left(1 + \frac{1}{12m} + \frac{1}{288m^2} + \cdots\right)$$

is from the Stirling formula.

Figure 6 shows the probability $\Pr\{Y(t) > 230\}$ as a function of $\mathrm{E}Y(t)$. Also, figure 6 shows how close to $\Pr\{Y(t) > 230\}$ is the Gaussian approximation

$$\int_{(230-\mathrm{E}Y(t))/\sqrt{\mathrm{E}Y(t)}}^{\infty} \frac{\exp\{-x^2/2\}}{\sqrt{2\pi}}$$

given in [9].

3. Traffic self-similarity

Under some conditions, the traffic $Y(t),\ t \in \mathbf{Z}$, defined by (3) is actually long-range dependent and, actually, asymptotically self-similar. To present such a condition, which is taken from [8], we first recall a definition of second-order asymptotic self-similarity.

Let $X = (\ldots, X_{-1}, X_0, X_1, \ldots)$ be a second-order-stationary real-number random process of discrete time $t \in \mathbf{Z}$. Let $X^{(m)} \mathrel{\hat{=}} (\ldots, X_{-1}^{(m)}, X_0^{(m)}, X_1^{(m)}, \ldots)$, where

$$X_t^{(m)} \mathrel{\hat{=}} \frac{X_{tm-m+1} + \cdots + X_{tm}}{m}, \quad m \in \mathbf{N} \mathrel{\hat{=}} \{1, 2, \ldots\},\ t \in \mathbf{Z}, \tag{9}$$

be the X process averaged over blocks of length m. The process X is called second-order asymptotically self-similar with the Hurst parameter $H = 1 - (\beta/2)$, $0 < \beta < 1$, if

$$\lim_{m\to\infty} r_m(k) = \frac{1}{2}\delta^2\big(k^{2-\beta}\big), \quad k \in \mathbf{N}, \tag{10}$$

where $r_m(k)$ is the correlation coefficient of $X^{(m)}$ and $\delta^2(f(x))$ is the central second difference operator applied to the function $f(x)$, so

$$\delta^2(k^{2-\beta}) = (k+1)^{2-\beta} - 2k^{2-\beta} + (k-1)^{2-\beta}. \tag{11}$$

Theorem 1 [8]. The process $Y(t)$ is second-order asymptotically self-similar (as-s) with parameter $H = 1 - (\beta/2)$, $0 < \beta < 1$ if ξ is the Poisson random variable and

$$\Pr\{\tau = l\} \sim L(l) l^{-(\beta+2)}, \quad l \to \infty, \tag{12}$$

where $L(l)$ is a slowly varying function at infinity. (A positive measurable function $L(x)$ is called slowly varying at infinity if $L(ux)/L(x) \to 1$, $x \to \infty$ for each $u > 0$.)

According to this theorem, the traffic $Y(t), t \in \mathbf{Z}$, with Pareto-type distributed τ, (8), and Poisson ξ, (4), is an as-s process with the Hurst parameter $H = (3-\alpha)/2$ [8].

4. Random variables representing $Y(t), \ldots, Y(t+N)$

Here, we express $Y(t), \ldots, Y(t+N)$ in terms of the following random variables:

- $\xi_t(l)$: the number of calls with given length $\tau = l$, started at t ($t \in \mathbf{Z}, l \in \mathbf{N}$);
- $\xi_t^{(l)} \mathrel{\hat{=}} \xi_t(l) + \xi_t(l+1) + \xi_t(l+2) + \cdots$: the number of calls with lengths $\tau \geqslant l$, started at t;
- $\rho_t^{(l)} \mathrel{\hat{=}} \xi_t(l) + \xi_{t-1}(l+1) + \xi_{t-2}(l+2) + \cdots$: the number of calls started at t or earlier and still continued in the interval $[t, t+1)$;
- $\eta_t^{(l)} \mathrel{\hat{=}} \xi_t^{(l)} + \xi_{t-1}^{(l+1)} + \xi_{t-2}^{(l+2)} + \cdots$: the random variable which can be interpreted via the above defined variables.

The random variables $\{\xi_t(l),\ l \in \mathbf{N},\ t \in \mathbf{Z}\}$ are independent and Poissonian with parameters

$$\lambda_1(l) \mathrel{\hat{=}} \mathrm{E}\xi_t(l) = \lambda \Pr\{\tau = l\} \tag{13}$$

which are independent of t and dependent on l. Using a splitting argument applied to a Poisson random variable ξ_t one can easily check this fact. Namely, for each t, one should split ξ_t into $\xi_t(l)$, $l \in \mathbf{N}$ by sending all calls with length l into $\xi_t(l)$ and get $\xi_t = \sum_{l=1}^{\infty} \xi_t(l)$.

The other random variables introduced above, namely, $\xi_t^{(l)}, \rho_t^{(l)}, \eta_t^{(l)}$, are also Poissonian since they are defined as the sums of Poisson random variables. Their averages are independent of t and dependent on l,

$$\lambda_2(l) \mathrel{\hat{=}} \mathrm{E}\xi_t^{(l)} = \mathrm{E}\rho_t^{(l)} = \lambda \sum_{j=l}^{\infty} \Pr\{\tau = j\}, \tag{14}$$

$$\lambda_3(l) \mathrel{\hat{=}} \mathrm{E}\eta_t^{(l)} = \lambda \sum_{i=0}^{\infty} \sum_{j=l+i}^{\infty} \Pr\{\tau = j\}. \tag{15}$$

We note also that $\xi_t^{(1)} \equiv \xi_t$.

Now, we express the successive traffic random variables $Y(t), \ldots, Y(t+N)$ in terms of $\xi_t(l), \xi_t^{(l)}, \rho_t^{(l)}, \eta_t^{(l)}$. We have

$$\begin{aligned} Y(t+N) &= \left(\sum_{l=1}^{N} \xi_{t+N-l+1}^{(l)}\right) + \eta_t^{(N+1)}, \\ Y(t+N-i) &= \sum_{l=1}^{N-i} \sum_{j=0}^{i-1} \xi_{t+N-l-i+1}(l+j) \\ &\quad + \sum_{l=1}^{N-i} \xi_{t+N-l-i+1}^{(l+i)} \\ &\quad + \sum_{l=N-i+1}^{N} \rho_t^{(l)} + \eta_t^{(N+1)}, \quad 0 < i < N, \\ Y(t) &= \left(\sum_{l=1}^{N} \rho_t^{(l)}\right) + \eta_t^{(N+1)}. \end{aligned} \tag{16}$$

It is important that in each equation in (16), all summands are *independent Poisson* random variables. Dependence between $Y(t), \ldots, Y(t+N)$ is due to the fact that the different equations among (16) contain the intersecting summands. For example, the random variable $\eta_t^{(N+1)}$ is an intersecting summand; it is contained in each of those equations.

Let us write now (16) in special cases of $N = 1$ and $N = 2$ for a simple illustration.

For $N = 1$, we have

$$Y(t) = \rho_t^{(1)} + \eta_t^{(2)}, \tag{17}$$

$$Y(t+1) = \xi_{t+1}^{(1)} + \eta_t^{(2)}. \tag{18}$$

The equations (17) and (18) give a representation of $Y(t)$, $Y(t+1)$ in terms of three independent Poisson random variables, $\xi_{t+1}^{(1)}$, $\rho_t^{(1)}$, $\eta_t^{(2)}$, with parameters given by (13)–(14). Dependence between $Y(t)$ and $Y(t+1)$ exists because of $\eta_t^{(2)}$ contributes in both $Y(t)$ and $Y(t+1)$.

For $N = 2$, we have

$$Y(t) = \rho_t^{(1)} + \rho_t^{(2)} + \eta_t^{(3)}, \tag{19}$$

$$Y(t+1) = \xi_{t+1}(1) + \xi_{t+1}^{(2)} + \rho_t^{(2)} + \eta_t^{(3)}, \tag{20}$$

$$Y(t+2) = \xi_{t+2}^{(1)} + \xi_{t+1}^{(2)} + \eta_t^{(3)}. \tag{21}$$

The equations (19)–(21) give a representation of $Y(t)$, $Y(t+1)$, $Y(t+2)$ in terms of six independent Poisson random variables $\xi_{t+1}(1)$, $\xi_{t+1}^{(2)}$, $\xi_{t+2}^{(1)}$, $\rho_t^{(1)}$, $\rho_t^{(2)}$, $\eta_t^{(3)}$ with parameters given by (13)–(15). Dependence between $Y(t)$, $Y(t+1)$, $Y(t+2)$ exists because of $\eta_t^{(3)}$ is in each of these equations; $\rho_t^{(2)}$ is in (19) and (20), and $\xi_{t+1}^{(2)}$ is in (20) and (21).

5. Distribution $P(y_t, \ldots, y_{t+N})$

Now, we are prepared to present the multidimensional distribution $P(y_t, \ldots, y_{t+N})$. To make a less complicated presentation, first we give $P(y_t, y_{t+1})$ and $P(y_t, y_{t+1}, y_{t+2})$.

Let $\xi_{t+1}^{(1)}$, $\rho_t^{(1)}$, $\eta_t^{(2)}$ take on the values denoted as $k_{t+1}^{(1)}$, $r^{(1)}$, $n^{(2)}$, respectively. All $k_{t+1}^{(1)}$, $r^{(1)}$, $n^{(2)}$ are from $\mathbf{Z}_+$.

The distribution $P(y_t, y_{t+1})$ is given by

$$P(y_t, y_{t+1}) = \sum_{\substack{k_{t+1}^{(1)},\ r^{(1)},\ n^{(2)} \in \mathbf{N}: \\ r^{(1)}+n^{(2)}=y_t, \\ k_{t+1}^{(1)}+n^{(2)}=y_{t+1},}} \mathrm{e}^{-2\lambda_2(1)-\lambda_3(2)} \frac{[\lambda_2(1)]^{k_{t+1}^{(1)}+r^{(1)}}[\lambda_3(2)]^{n^{(2)}}}{(k_{t+1}^{(1)}!)(r^{(1)}!)(n^{(2)}!)}, \tag{22}$$

where the sum is taken over $k_{t+1}^{(1)}$, $r^{(1)}$, $n^{(2)}$ in the shown region which is denoted now as Δ_1. (We could present (22) in simpler notation omitting most of indexes but we have to keep the used notation for our later presentation of general case.)

We can give another expression for $P(y_t, y_{t+1})$if we use (22) and note that $\lambda_2(1) = \lambda$, $\lambda_3(2) = \lambda[(\mathrm{E}\tau) - 1]$, and

$$\Delta_1 = \{k_{t+1}^{(1)},\ r^{(1)},\ n^{(2)}\colon 0 \leqslant n^{(2)} \leqslant y_t,\ r^{(1)} = y_t - n^{(2)},\ k_{t+1}^{(1)} = y_{t+1} - n^{(2)}\}.$$

We have

$$P(y_t, y_{t+1}) = \mathrm{e}^{-\lambda(1+\mathrm{E}\tau)}\lambda^{y_t+y_{t+1}} \times \sum_{n^{(2)}=0}^{y_t} \frac{[((\mathrm{E}\tau)-1)/\lambda]^{n^{(2)}}}{[(y_t - n^{(2)})!][(y_{t+1} - n^{(2)})!][n^{(2)}!]}.$$

Let $\xi_{t+1}(1)$, $\xi_{t+1}^{(2)}$, $\xi_{t+2}^{(1)}$, $\rho_t^{(1)}$, $\rho_t^{(2)}$, $\eta_t^{(3)}$ take on the values denoted as $k_{t+1}(1)$, $k_{t+1}^{(2)}$, $k_{t+2}^{(1)}$, $r^{(1)}$, $r^{(2)}$, $n^{(3)}$, respectively. All $k_{t+1}(1)$, $k_{t+1}^{(2)}$, $k_{t+2}^{(1)}$, $r^{(1)}$, $r^{(2)}$, $n^{(3)}$ are from $\mathbf{Z}_+$.

The distribution $P(y_t,\ y_{t+1},\ y_{t+2})$ is given by

$$\begin{aligned} P(y_t, y_{t+1}, y_{t+2}) &= \sum \Pr\{\xi_{t+1}(1) = k_{t+1}(1)\} \Pr\{\xi_{t+1}^{(2)} = k_{t+1}^{(2)}\} \\ &\quad \times \Pr\{\xi_{t+2}^{(1)} = k_{t+2}^{(1)}\} \Pr\{\rho_t^{(1)} = r^{(1)}\} \\ &\quad \times \Pr\{\rho_t^{(2)} = r^{(2)}\} \Pr\{\eta_t^{(3)} = n^{(3)}\} \\ &= \sum \mathrm{e}^{-\lambda-2\lambda_2(1)-2\lambda_2(2)-\lambda_3(3)} \\ &\quad \times \frac{[\lambda_1(1)]^{k_{t+1}(1)}[\lambda_2(1)]^{k_{t+2}^{(1)}+r^{(1)}}}{(k_{t+1}(1)!)(k_{t+1}^{(2)}!)(k_{t+2}^{(1)}!)(r^{(1)}!)} \\ &\quad \times \frac{[\lambda_2(2)]^{k_{t+1}^{(2)}+r^{(2)}}[\lambda_3(3)]^{n^{(3)}}}{(r^{(2)}!)(n^{(3)}!)}, \end{aligned} \tag{23}$$

where the sums are taken over $k_{t+1}(1)$, $k_{t+1}^{(2)}$, $k_{t+2}^{(1)}$, $r^{(1)}$, $r^{(2)}$, $n^{(3)}$ in the region

$$\begin{aligned} \Delta_2 = \{&k_{t+1}(1),\ k_{t+1}^{(2)},\ k_{t+2}^{(1)},\ r^{(1)},\ r^{(2)},\ n^{(3)} \in \mathbf{Z}_+, \\ &r^{(1)} + r^{(2)} + n^{(3)} = y_t, \\ &k_{t+1}(1) + k_{t+1}^{(2)} + r^{(2)} + n^{(3)} = y_{t+1}, \\ &k_{t+2}^{(1)} + k_{t+1}^{(2)} + n^{(3)} = y_{t+2}\}. \end{aligned}$$

We conclude with a theorem giving the multidimensional distribution $P(y_t, \ldots, y_{t+N})$.

Theorem 2. Distribution $P(y_t, \ldots, y_{t+N})$ is the sum of the products of Poisson distributions of the following independent random variables:

$$\begin{aligned} &\{\{\xi_{t+n-l-i+1}(l+j),\ 1 \leqslant l \leqslant N-i,\ 1 \leqslant i \leqslant N-1,\ 0 \leqslant j \leqslant i-1\}, \\ &\{\xi_{t+N-l-i+1}^{(l+i)},\ 1 \leqslant l \leqslant N,\ 0 \leqslant i \leqslant N-1\}, \\ &\{\rho_t^{(l)},\ 1 \leqslant l \leqslant N\}, \eta_t^{(N+1)}\}. \end{aligned}$$

The sum is taken over the region $\Delta_N = \{Y(t) = y_t, \ldots, Y(t+N) = y_{t+N}\}$, where $Y(t), \ldots, Y(t+N)$ have to be replaced with the right-hand sides of the equations (16).

Using the theorem, it is easy to put down a formula for the probability of heavy-traffic period of length $N-1$ (denoted as $P_{\text{hvy}}(N-1)$). Namely,$P_{\text{hvy}}(N-1)$ is the same sum as

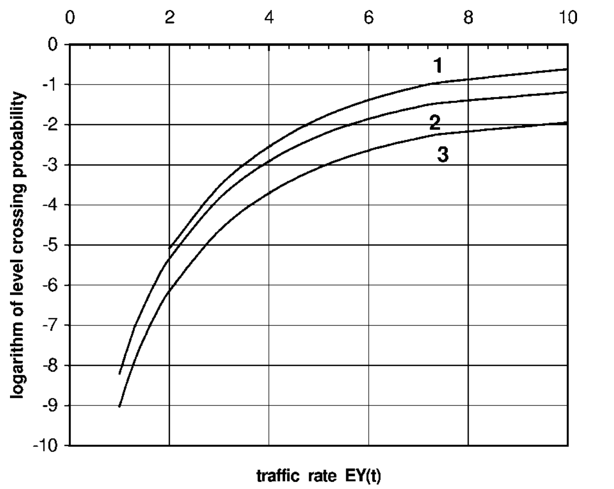

Figure 7. Logarithm of the level crossing probability as a function of traffic rate. The curve 1 is for traffic with independent $Y(t)$. The curves 2 and 3 for traffic with $\mathrm{E}\tau = 10$ and $\mathrm{E}\tau = 100$, respectively.

given in the theorem with only difference that now the region is

$$\Delta_N^* = \{Y(t) \leqslant M, Y(t+1) > M, \ldots, Y(t+N-1) > M, \\ Y(t+n) \leqslant M\}.$$

Similarly, the outage probability $P_{\text{out}}(N)$ is the same sum as given in the theorem with only difference that the region is

$$\Delta_N^{**} = \{Y(t) \leqslant M, Y(t+1) > M, \ldots, Y(t+N) > M\}.$$

For example, if $N = 1$,

$$\begin{aligned} P_{\text{out}}(1) \equiv P_{\text{cross}} &= \sum_{y_t=0}^{M} \sum_{y_{t+1}=M+1}^{\infty} P(y_t, y_{t+1}) \\ &= \mathrm{e}^{-\lambda(1+\mathrm{E}\tau)} \sum_{y_t=0}^{M} \lambda^{y_t} \sum_{y_{t+1}=M+1}^{\infty} \lambda^{y_{t+1}} \\ &\quad \times \sum_{n^{(2)}=0}^{y_t} \frac{[((\mathrm{E}\tau) - 1)/\lambda]^{n^{(2)}}}{[(y_t - n^{(2)})!][(y_{t+1} - n^{(2)})!][n^{(2)}!]} \end{aligned} \tag{24}$$

We note that the level crossing probability P_{cross} depends on $\mathrm{E}\tau$ only but not on the entire behavior of $\Pr\{\tau = l\}$. (P_{cross} as function of $\mathrm{E}Y(t)$ is shown in figure 7.) For $Y(t)$, we have $\mathrm{E}Y(t) = \lambda\, \mathrm{E}t$. Hence, $\lambda = \mathrm{E}Y(t)/\mathrm{E}t$ in (24).

The same is true for $\mathrm{E}T$, i.e., $\mathrm{E}T$ depends on $\mathrm{E}\tau$ only. Figure 8 shows $\mathrm{E}T$ as function of $\mathrm{E}\tau$ for different values of $\mathrm{E}Y(t)$ and $M = 10$. It can be observed with the figure that $\mathrm{E}T$ is a linear function of $\mathrm{E}\tau$. The 3-dimensional plot which shown in figure 9, gives $\mathrm{E}T$ as function of $\mathrm{E}\tau$ and $\mathrm{E}Y(t)$.

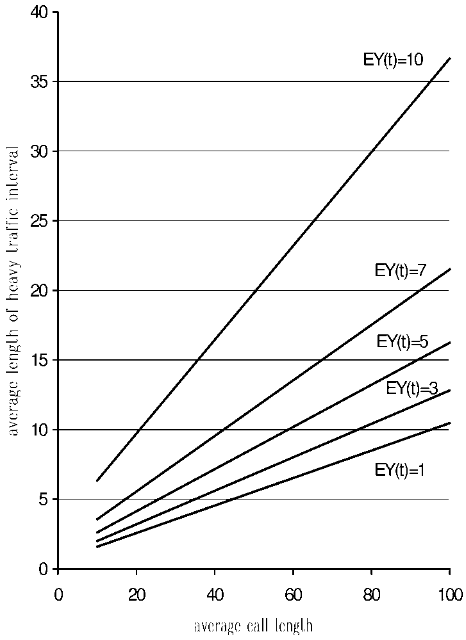

Figure 8. Average length of the heavy-traffic interval as a function of average call length.

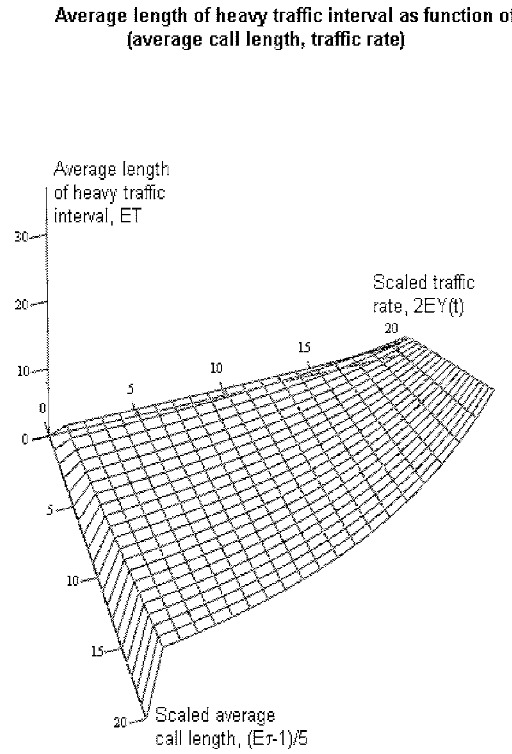

Figure 9. Average length of the heavy-traffic interval as a function of the pair (average call length, traffic rate).

6. Gaussian approximation

Here we give a way of an approximate calculation of $q \mathrel{\hat{=}} \Pr\{Y(t) \leqslant M, Y(t+1) > M, \ldots, Y(t+N-1) > M, Y(t+N) \leqslant M\}$, the probability of heavy-traffic period of length $N-1$. We do not know how the approximation (denoted as q_0) is closed to the exact value of the probability q and cannot say whether q_0 is an upper or lower bound to q or it is not any bound at all.

To get q_0, we use the following fact: the function

$$g(k) \mathrel{\hat{=}} (\sigma^2/2)\delta^2(k^{2-\beta}), \quad k \in \mathbf{N}, \qquad g(0) = \sigma^2$$

is the covariance of exactly second-order self-similar process with the variance σ^2 and the Hurst parameter $H = 1-(\beta/2)$, $0 < \beta < 1$.

To obtain q_0, we take the Gaussian multidimensional distribution having the density

$$f_N(x_t, \dots, x_{t+N}) = \frac{|A|^{1/2}}{(2\pi)^{n/2}} \exp\left\{-\frac{1}{2}\sum_{i,j=0}^{N} a_{ij}(x_{t+i}-m)(x_{t+j}-m)\right\},$$

where a_{ij} are the elements of the matrix A which is the inverse of the covariance matrix

$$R = \sigma^2 \begin{Vmatrix} g(0) & g(1) & \dots & g(N) \\ g(1) & g(0) & \dots & g(N-1) \\ \vdots & \ddots & \ddots & \vdots \\ g(N) & g(N-1) & \dots & g(0) \end{Vmatrix}$$

and $\sigma^2 = \operatorname{var} Y(t) = \lambda \mathrm{E}\tau,\ m = \mathrm{E}Y(t) = \lambda \mathrm{E}\tau$.

Now, we put

$$q_0 = \int_{-\infty}^{M} \mathrm{d}x_t \int_{M}^{\infty} \mathrm{d}x_{t+1} \dots \int_{M}^{\infty} \mathrm{d}x_{t+N-1} \int_{-\infty}^{M} \mathrm{d}x_{t+N} \times f_N(x_t, \dots, x_{t+N}).$$

This is the suggested approximation to q.

7. Extension to a more general model of traffic

In this section, we introduce a more general model of long-range dependent traffic [7], than is presented in section 2, and extend the above results to this new model. The only difference of the new model from that presented in section 2 is that in the new model, a call generates one packet at each time $\omega_s + i - 1$(see section 2) with probability p and does not generate any packet with probability $1-p$ independently of other generations in this call and other calls. The model considered in section 2 is a special case of the new model when $p = 1$. The new model matches a model considered in [9, section 6.6.2].

It is easy to notice that all results presented above hold for the new model if λ is substituted for $p\lambda$.

8. Conclusion

The problem of deriving of the multidimensional distribution of a segment of a long-range dependent traffic was considered for 3-G mobile communication network. An exact formula was obtained for this probability when a self-similar process from [8] models the traffic. It was shown how to obtain the probability of heavy-traffic period with every given length and the outage probability. As it was expected, in general case, these probabilities have cumbersome expressions caused by the long-range dependence in the traffic.

It is shown that for the considered traffic, the level crossing probability depends only on the average call length but not on other behavior of the distribution of call length and that the probability P_{cross} for traffic with dependent $Y(t)$ is lower than for traffic with the same $\mathrm{E}Y(t)$ but independent $Y(t)$. Also, it is shown that there is a linear dependence of average heavy traffic interval $\mathrm{E}T$ on the average call length when $\mathrm{E}Y(t)$ is kept as given.

Also, we presented an analytical Gaussian approximation to the probability of heavy-traffic period.

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