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FRACTAL DIMENSION OF FRACTURE SURFACES

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INTRODUCTION

Latest research has proved the self-similarity of fracture surfaces and the applicability of the powerful mathematical apparatus of fractal geometry to the description of self-similar structures of dimple fractures [1].

It is assumed that the fracture process (propagation of a crack) is accompanied by a cascade transfer of the freed elastic energy from larger scales to smaller ones and, finally, to the microscale where the energy dissipates being spent for the formation of a new fracture surface [2].

It is interesting to study not only the relationship between the fractal dimension of the fracture surface and the mechanical properties of the material, but also the dynamics of the fracture process, because the type of the relaxation of stresses can change within one fracture. This should be allowed for in the determination of the fractal dimension of fractures. The phenomenon of degeneration of the dimension of a microcrack upon the development of ductile fracture is described in [3]. The degeneration of a microcrack into a two-dimensional cluster determined by the Fourier spectra meets the scaling laws.

When choosing the objects for our study we proceeded from the necessity for testing the new approach on simple fracture surfaces.

The most important publications in the world literature on titanium research are devoted to the formation of its structure. Searching for the way to increase the coefficient of stress intensity in spacecraft alloys researchers try to provide such a structure in the metal that would yield the "roughest" fracture surface [4].

We have shown that the critical crack size for technically pure titanium corresponds to the size of a dimple and increases with the temperature. There are data that the degree of deformation and the distance between slip bands depend linearly. It has been shown that the strain transferred by a single slip band is constant, after which the band is locked [5]. A neck starts to form at the places where slip bands come out to the boundaries separating cells and grains. Then the strain develops in the neck due to the nucleation and the outgrowth of new slip bands to the formed pores. We can infer from this fact that a fracture surface is self-similar. This means that relation

$$M_\delta = M_0 \delta^{-(D_f - D)} \tag{1}$$

holds in the range of scales limited by the minimum and maximum sizes of dimples [1]. Here M_δ is a measure, M_0 is a constant, δ is the measured element, D_f is the fractal dimension, and D is a constant quantity (the Euclidean dimension for plane $D = 2$).

We know several algorithms for determining the fractal dimension of actual fracture surfaces. The dimension can be evaluated from the data of an analysis of the profile of the line obtained by studying the section of the surface by a plane normal to the base (the method of cross sections) [6], and by analyzing the sections formed by planes parallel to the base (the method of cutoff islands) [7]. The method of cross sections is also used in two varieties, i.e., the method of variations [8] and the method of determining the dimension from the correlation function of the Fourier spectrum [9]. Taking into account that researchers commonly possess distorted surface profiles provided by a scanning electron microscope as the initial data, we resorted in the present work to the method of cross sections.

METHOD OF STUDY

We obtained fractal characteristics of surfaces by the method of scanning electron microscopy studying fracture surfaces in molybdenum single crystal (groove fracture), technical-purity titanium (dimple fracture), and sintered powdered iron (intermediate fracture).

We obtained titanium of technical purity by magnetically controlled electroslag remelting (MER) and titanium of elevated purity by electron beam remelting of titanium iodide (IEBR). In order to determine the fractal dimension and the size of the dimple, we studied the fracture surface under a scanning electron microscope orienting the fracture normally to the axis of the primary beam; the maximum size of the dimples responsible for the fracture was determined from the most protruding ridges (observed upon inclining the specimens in the column of the microscope).

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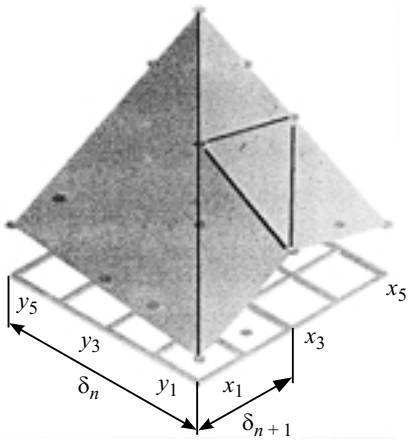

Fig. 1. Diagram of the variation of fractal dimension.

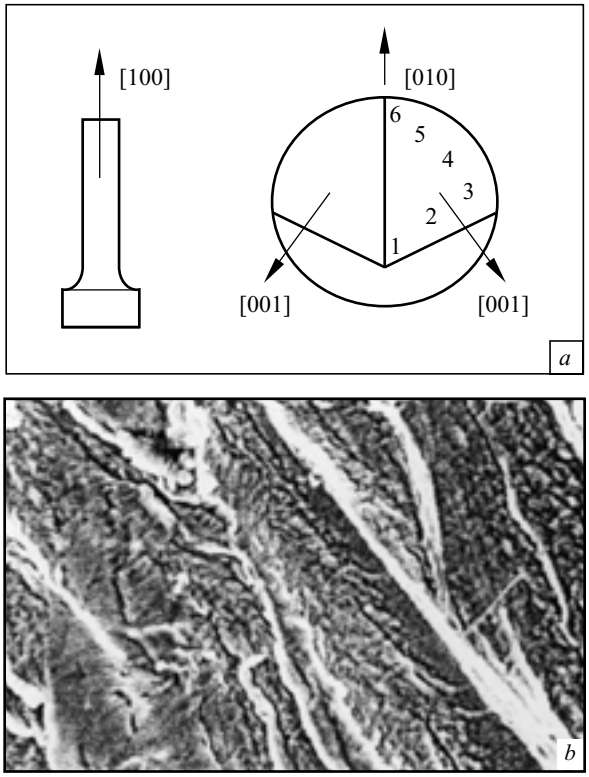

Fig. 2. Orientation of a molybdenum specimen: diagram of the regions of measurement of the dimension of its fracture (*a*) and fracture surface (× 2000) in region 5 (*b*).

The fractograms were examined with the help of a specially created set of programs FRSJA [10] based on the computation of surface areas in a wide (three orders of magnitude) range of scales. The measure M_δ was determined by summing the areas of elementary triangles *ABC*, i.e., $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_1, y_2, z_3)$ (Fig. 1) at $\delta \to 0$ and $M_\delta \to \infty$. The fractal dimension was determined by approximating the dependence of M_δ on δ by the method of least squares.

RESULTS AND THEIR DISCUSSION

A fracture in single crystal molybdenum formed as a result of the propagation of a cleavage crack is presented schematically in Fig. 2*a*.

The striated relief formed upon a comparatively slow propagation of a cleavage crack with periodic relaxation is presented in Fig. 2*b*. In the process of the tensile test the crack periodically crosses its own zone of plastic relaxation, and the load grows periodically until the crack attains the critical length.

The axis of the specimen coincides with direction 110. A stress concentrator is positioned in the region of point *6*.

The fracture front slowly moves in the direction *6* → *2* to the boundary *3 – 1*, where the stresses attain a maximum value. Then the fracture evolves with the formation of a mirror zone.

The central zone (the ridge) lies at the intersection of two crystallographic planes and therefore does not possess primary striae.

The lowering of the fractal dimension with the motion from point *6* to point *3* agrees with the existing notion of the processes that occur in propagation of a cleavage crack with periodic relaxation. Every new jump occurs with a lower expenditure of energy.

We used the FRSJA software to study the fracture surface the photograph of which is presented in [11]. The fracture in a specimen of porous iron was initiated under the conditions of static loading at 20°C and finished by final dynamic breaking at – 196°C. At first the fracture developed by the ductile mechanism with the formation of dimples. As the crack propagated, the fracture surface gradually acquired a cleavage nature with the formation of a groove relief (see Fig. 3). The fractal dimension was determined in five equidistant zones. The results of the computation of the fractal dimension are presented below.

Zone*	D_f
1	2.3377
2	2.2980
3	2.2713
4	2.2990
5	2.3067
6	2.3281

* See Fig. 2*a*.

It turned out that the fractal dimension decreased with the transition from a ductile dimple fracture mechanism ($D_f = 2.266$) to brittle fracture ($D_f = 2.179$). However, the computation of the dimension by this method does not allow us to draw parallels between the numerical value of D_f and the kind of fracture. The values of D_f measured on fractograms obtained under different conditions are hard to compare because equation (1) involves the constant M_0 that depends on the brightness, the contrast of the image, and other conditions of photography.

Fig. 3. Fracture surface in powder iron [11] obtained under the conditions of static loading at 20°C and dynamic final breakage at – 196°C. × 100.

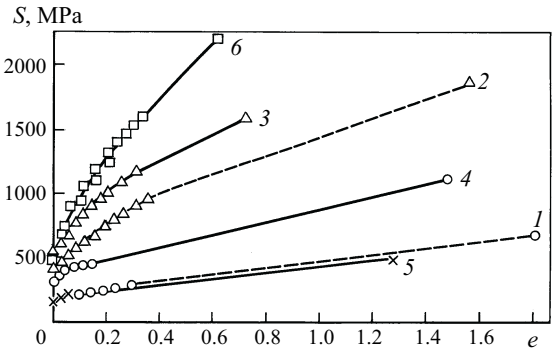

Fig. 4. Dependence of the true stress S on the true strain ε in titanium: solid lines) technical purity titanium (MER); dashed lines) titanium of elevated purity (IEBR); *1*, *4*) tested at $t_{test} = +20°C$; *2*, *3*) tested at $t_{test} = -196°C$; *5*) $t_{test} = +200°C$; 6) $t_{test} = -253°C$.

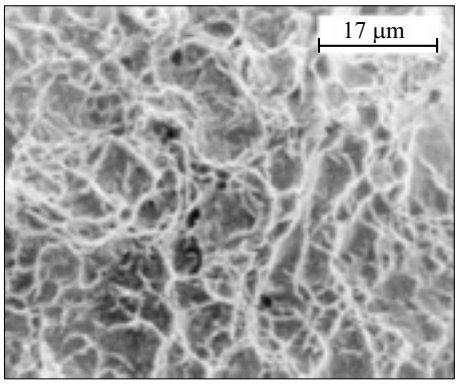

Fig. 5. Fracture surface in titanium of elevated purity tested at 20°C ($D_f = 2.29$, $S_k = 606$ MPa).

The study of the special features of deformation, fracture, and fractal characteristics of fractures was performed for technically pure titanium with various contents of admixtures. The empirical equation for the equivalent parameter of the content of interstitial elements c_0 makes it possible to evaluate the total effect of oxygen, nitrogen, and carbon on the true stress S [12], i.e.,

$$c_0 = 1\ (\%\ \mathrm{O}) + 2\ (\%\ \mathrm{N}) + \frac{3}{4}\ (\%\ \mathrm{C}). \qquad (2)$$

The stress levels are lower for materials with a lower content of interstitial elements at the same test temperature (curves *2* and *3* in Fig. 4). For titanium with enhanced purity ($c_0 = 202 \times 10^{-4}$) at + 20°C the true stresses in the zone of uniform strain virtually coincide with the value of S in technically pure titanium ($c_0 = 420 \times 10^{-4}$) at + 200°C (curves *1* and *5* in Fig. 4, respectively). Such a behavior of titanium with different contents of interstitial elements is explainable by the special features of the occurrence of slip in it [5].

Photographs of fracture surfaces of the central zone of titanium specimens were obtained in one scale (× 1000). The size distribution of the dimples was uniform. All the fractograms were similar to that presented in Fig. 5. We determined the fractal dimension for all the images of the fracture. The primary curves of the $\ln M_\delta\,(\ln \delta)$ dependences for titanium with different purities are presented in Fig. 6.[2] The fractal dimension was computed from the slopes of these curves.

Analyzing the results of the study we established that the fractal dimension D_f increased with the decrease in size of the dimples (i.e., the measured area increased).

Since the strength of metallic materials increases with increase in the size of the dimples, we can infer that the fractal dimension correlates quantitatively with the variation of the strength. In order to check the computation algorithm we studied the plane and the fractal surface. The straight lines in

[2] The authors express their gratitude to A. V. Perepelkin and Yu. V. Podrezov for the supplied materials.

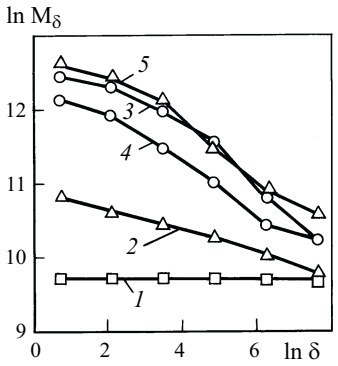

Fig. 6. Curves $\ln M_\delta(\ln \delta)$ for actual fractograms and model surfaces: *1*) synthesized surface with specified $D_f = 2.13$; *2*) plane with $D_f = 2$ (nonfractal); *3*, *5*) technical purity titanium, $t_{test} = -196$ and $+20$°C; *4*) titanium of elevated purity, $t_{test} = +20$°C.

Fig. 6 (*2* and *1*) correspond to only these objects. For actual surfaces the slope of the dependence $\ln M_\delta(\ln \delta)$ changes, i.e., the fractal dimension is not constant and the fracture surfaces are multifractal. The creation of a fractal fracture surface would require an infinitely high energy expenditure because, mathematically, the area of the fractal surface tends to infinity. The variation of D_f indicates the absence of scale invariance of the self-similarity of fracture surfaces and is connected with the presence of some characteristic dimple sizes.

This is explainable by the fact that the elastic energy in the fracture process scatters primarily at the places of the intersections of grain boundaries and slip bands [13], which causes the formation of dimples. The appearance of a primary pore at the place of the interaction between a slip band and an interface causes a series of accommodation shifts that are finally responsible for the formation of a neck and the dimple nature of the fracture.

It is shown in [14] that the content of the admixture primarily influences the dynamics of the evolution of the dislocation structure. Titanium iodide contains less interstitial elements than technically pure titanium (the equivalent contents of C, N, and O are, respectively, 202×10^{-4} and 420×10^{-4} %). The fractal dimension of titanium iodide decreases more intensely with increase in the temperature. Such a material is characterized by a higher activity of the relaxation processes, which seems to affect the behavior of the factor of growth of the total (accumulated) surface area in the transition from one scale of the fractal dimension to another.

CONCLUSIONS

1. The developed programs based on the approaches of fractal geometry allowed us to study the occurrence of fracture processes from images of fractures in specimens of molybdenum and porous iron.

2. The study of fracture surfaces in cast titanium tested under the conditions of uniaxial stretching has shown that the fractal dimension increases with decrease in the size of the dimple and the test temperature.

3. The fractal dimension of a fracture is sensitive to the content of interstitial admixtures in titanium, because this factor determines the dynamics of the evolution of the dislocation structure in the deformation process.

4. The assumption about the multifractal (statistically self-similar) nature of the studied fractures has been confirmed.

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