



Why Isn't Convergence Instantaneous? Young Workers, Old Workers, and Gradual Adjustment

MICHAEL KREMER

Massachusetts Institute of Technology, Department of Economics, E52-251C, Cambridge, MA 02142-1347 USA

JAMES THOMSON

The University of Melbourne, Department of Economics, Parkville Victoria 3052 Australia

The human capital of young and old workers are imperfect substitutes both in production and in providing on-the-job training. This helps explain why capital does not flow from rich to poor countries, causing instantaneous convergence of per capita output. If each generation chooses its human capital optimally, given that of the preceding and succeeding generations, human capital follows a unique rational-expectations path. For moderate substitutability, human capital within each sector oscillates relative to that in other sectors, but aggregate human capital converges to the steady state monotonically.

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1. Introduction

In this article, we examine how the optimal level of human capital for each generation depends on the human capital of the previous and succeeding generations. For a moderate degree of substitutability between human capital of young and old workers, human capital within each sector of the economy oscillates relative to that in other sectors. Aggregate human capital, however, converges monotonically to the steady state, at rates consistent with those observed empirically by Barro and Sala-i-Martin (1992), Dowrick and Nguyen (1989), and Mankiw, Romer, and Weil (1992). Imperfect substitutability between the human capital of young and old workers can thus help explain why per capital output does not converge instantaneously, as the open-economy, neoclassical growth model predicts.

The human capital of young and old workers are likely to be imperfect substitutes in production because young and old workers have comparative advantages in different, complementary tasks. Thus young workers are computer programmers, production line workers, and athletes, whereas old workers are managers, production supervisors, and coaches. In general, managers are likely to be older due to a need for experience, the ability to command respect from younger workers, and gradual learning about manager quality. The human capital of young and old workers are also likely to be imperfect substitutes in training new workers. Formal education may prepare people to learn more quickly the specific

production skills taught by older workers in on-the-job training. Hence, people with more formal education also receive more on-the-job training. Lawyers and doctors, for example, generally obtain extensive formal education and on-the-job training, whereas gas station attendants usually obtain little of either.

Empirical evidence supports the view that the human capital of young and old workers are imperfect substitutes. At the macro level, Murphy and Welch (1992) find that an increase in the number of young college-educated workers in the United States has either a small negative or a positive effect on the wage of old workers with college degrees. At the micro level, Pierce (1990) finds that an increase in the number of young lawyers has a positive (although not statistically significant) effect on the wages of old lawyers. While these estimates must be treated with caution given the short length of available time-series and the possible presence of other confounding factors, the differing movements of returns to young and old human capital provide evidence that they cannot be perfect substitutes. From 1979 to 1987, for example, the wage differential between U.S. high school and college graduates increased by 31 percent among young workers but by only 5 percent among old workers (Katz and Murphy, 1992).¹ Similarly, Goldin and Margo (1992) find that the increase in supply of young educated workers in the 1940s reduced their relative wage considerably but had little impact on the wages of old educated workers.

We take this imperfect substitutability between young and old human capital as given and explore how it affects the dynamic path of human capital. Since each generation's optimal level of human capital depends on that of the preceding and succeeding generations, the rational-expectations path of human capital satisfies a second-order difference equation. Assuming there are decreasing returns to accumulable factors, this path will be unique and will converge to a steady state. We argue that, for a moderate degree of complementarity between young and old human capital, the path of aggregate human capital will converge monotonically to the steady state but that human capital in each sector of the economy will oscillate relative to that in other sectors.

This contrast between individual sectors and the aggregate economy arises because increases in old human capital within an individual sector will tend to reduce the price of that sector's output and thus reduce the return to young human capital within that sector. For example, as the number of old doctors increases, total output of medical services will increase, and this will drive down the price of medical services, causing the wage for competing young doctors to fall. Thus, within an individual sector, the optimal amount of human capital for any generation is likely to fall with the human capital of the preceding and succeeding generations. (Freeman, [1975a, 1975b, 1976], argues that the behavior of human capital in law, physics, and engineering is consistent with cycles, though he interprets this in terms of a cobweb model rather than in terms of a rational-expectations model.)

Aggregate output does not oscillate, however, since an increase in old human capital spread across sectors, rather than concentrated in a particular sector, will not reduce the relative price of any particular good or service. If young and old human capital are complements in production, higher aggregate human capital of the preceding and succeeding generations will increase the incentive for the current generation to accumulate human capital. The better the professors in the last generation, the greater the incentive to become a research assistant, and the better the next generation of research assistants, the greater the incentive

to become a professor. We show that a moderate degree of imperfect substitutability implies that aggregate human capital will converge to the steady state monotonically.

Barro and Sala-i-Martin (1992) find convergence rates of about 2 percent a year between countries with the same steady-state income. Under standard open-economy growth models, capital flows would create instantaneous equalization of output among countries with similar institutions. Under imperfect substitutability, however, convergence is not immediate because it is not optimal to invest too heavily in young human capital or physical capital given a fixed supply of old human capital. Imperfect substitutability can be interpreted as slowing convergence by creating a form of adjustment cost in human capital, since under imperfect substitutability total output depends positively on each generation's human capital but negatively on the change in human capital between generations. As Alwyn Young has suggested in comments on Barro and Sala-i-Martin (1992), adjustment costs in human capital may help explain slow convergence.

This approach to explaining slow convergence can be contrasted with that of Cohen and Sachs (1986) and of Barro, Mankiw, and Sala-i-Martin (1995), who explain the failure of per capita output to converge instantaneously through capital-market imperfections that make countries behave like closed economies. In their models, some share of investment has to be financed from domestic saving, so consumption smoothing causes gradual convergence to the steady state, just as in the closed-economy, neoclassical model. While capital market imperfections certainly exist, they do not seem sufficient to explain the huge income differences between rich and poor countries. Per capita output was not equalized between countries in the days when loan contracts were enforced by gunboats, and Puerto Rico's output per capita remains substantially below that of the rest of the United States, despite substantial fiscal transfers and U.S. laws. Under the neoclassical model, observed income differences would imply large differences in the marginal product of capital, and it is not clear why mechanisms would not develop to allow greater capital flows. In 1913, 48 percent of Argentina's capital stock was foreign-owned (Taylor, 1992), suggesting that capital-market institutions capable of handling large flows can arise given sufficient incentives.

Previous work by Benhabib and Rustichini (1991) discusses how nonexponential depreciation can lead to cycles in investment, as capital is periodically replaced. Pierce (1990) estimates that young and old lawyers are poor substitutes, with an elasticity of substitution of about 0.25, and develops a rational-expectations model of cycles in the acquisition of human capital in law. This article differs from Benhabib and Rustichini in allowing for complementarity between different vintages of human capital and from Pierce in using a general equilibrium analysis to examine the path of economywide human capital.

Chari and Hopenhayn (1991) show that diffusion of technology will be gradual if there is complementarity between experienced and inexperienced workers in each technology. They solve for the steady-state distribution of workers across various vintages of technology. This article, on the other hand, focuses on the dynamic path of investment in human capital outside the steady state and on the determinants of the speed of convergence to the steady state. In fact, transition periods of several generations seem more relevant to the process by which poor countries become rich than to the diffusion of a particular new technology within an advanced country.

The remainder of the article is organized as follows. In Section 2 we present a basic, aggregate model with imperfect substitutability between the human capital of young and old workers in production. In Section 3 we examine the movement of human capital at the sectoral level, and in Section 4 we integrate aggregate and sectoral dynamics. In these sections, we identify the parameter values for which there will be cycles in human capital at the sectoral level and for which there will be monotonic convergence to the steady state at the aggregate level. In Section 5 we argue that for reasonable parameter values, imperfect substitutability of young and old human capital can generate convergence at rates consistent with those observed empirically by Barro and Sala-i-Martin (1992), Dowrick and Nguyen (1989), Mankiw, Romer, and Weil (1992), and more recently Islam (1995) and Caselli, Esquivel, and Lefort (1996). Section 6 concludes with a discussion of implications for investment in human capital, focusing on the case of African countries that have rapidly expanded human capital from a low base.

2. Aggregate Dynamics

In this section, we investigate the path of aggregate human capital in a simple model in which young and old human capital are imperfect substitutes in production. We show that if there is complementarity between young and old human capital in the sense that an increase in the supply of old human capital will cause an increase in the wage of young human capital, holding constant the quantities (rather than prices) of other factors, then there will be monotonic convergence of aggregate human capital, whereas if there is substitutability, there will be gradually dampening oscillations. This section thus follows the work of Pierce (1990). In the next section, we show that if there is sufficient complementarity between goods in different sectors, there will be cycles in human capital in individual sectors, even though there is monotonic convergence of aggregate human capital.

2.1. *The Competitive-Equilibrium Path of Human Capital*

Consider a small, open economy with a large number of perfectly competitive firms producing a single, nonstorable, tradable good. There is a large number of identical workers, each of whom lives for two periods and supplies one unit of labor in each period. For now, we assume that legal or cultural barriers to international mobility of workers are absolute.

Assume that production in period t depends on three inputs—human capital of young workers (those born in period t), human capital of old workers (those born in period $t - 1$), and a third factor that is in fixed supply for the economy as a whole and that might represent land or “raw” labor.² We normalize the supply of the fixed factor to one. Technology is characterized by the constant returns-to-scale production function:

$$Y_t = f(H_t, H_{t-1}, L_t), \tag{1}$$

where Y_t is aggregate production, H_t is young human capital used in period t , H_{t-1} is old human capital used in period t (as well as young human capital in period $t - 1$), and $L_t = 1$ is the quantity of the fixed factor. We assume that f has continuous second derivatives and

that output is increasing and concave in human capital ($f_1 > 0$, $f_2 > 0$, $f_{11} < 0$, $f_{22} < 0$, and $f_{11}f_{22} - f_{12}^2 > 0$) and in the fixed factor. As we show below, the cross-partial derivative of the production function with respect to young and old human capital f_{12} is a key quantity, and its sign determines whether the path of aggregate human capital is monotonic or exhibits oscillations.

The supply of human capital is chosen by workers given the expected returns to investment in human capital. We abstract from imperfections in international capital markets in order to focus on the effect of imperfect substitutability in slowing convergence. Hence, we assume that workers can borrow to finance human capital investment at a fixed world interest rate, r , subject to a no-Ponzi-game constraint.³ Workers have perfect foresight; each worker chooses a level of human-capital investment when young, and this determines the human capital he supplies in both periods of this life.⁴ In equilibrium, the discounted return to human-capital investment must equal the cost, which we normalize to one unit of output. That is,

$$q_t^y + \delta q_{t+1}^o = 1, \quad (2)$$

where q_t^y and q_t^o are the wages paid to young and old human capital, and δ is the value of one dollar one period ahead, which in turn equals $1/(1+r)$, where r is the world interest rate. The return on human capital when young, plus the discounted return on human capital when old, must equal one, the marginal cost of human capital investment.

Since agents can borrow in perfect capital markets, it is unnecessary to consider agents' intertemporal preferences in analyzing production. They simply maximize the value of wages minus education costs, discounted at the world interest rate, and then allocate consumption over time based on their own intertemporal preferences.

Profit maximization and free entry imply that each factor will be paid its marginal revenue product. Thus the equilibrium condition for human capital investment can be rewritten as a second-order difference equation:

$$\frac{\partial f(H_t, H_{t-1}, 1)}{\partial H_t} + \delta \cdot \frac{\partial f(H_{t+1}, H_t, 1)}{\partial H_t} = 1. \quad (3)$$

That is, the marginal revenue product of education when young, plus the discounted marginal revenue product when old, must equal one, the cost of education. This second-order difference equation defines the path of aggregate human capital. The unique steady-state level of human capital is the value H that solves

$$f_1(\bar{H}, \bar{H}, 1) + \delta f_2(\bar{H}, \bar{H}, 1) = 1. \quad (4)$$

2.2. *Equivalence to the Social Planner's Problem*

The path of human capital in the decentralized economy will be the same as that chosen by a social planner. To see this, note that a social planner seeking at time t to maximize

the present discounted value of output net of the cost of education will choose values of $H_t, H_{t+1}, H_{t+2}, \dots$, to solve

$$\text{MAX} \sum_{s=0}^{\infty} \delta^s [f(H_{t+s}, H_{t+s-1}, 1) - H_{t+s}], \quad (5)$$

subject to a no-Ponzi-game condition on the economy's borrowing from the rest of the world and given initial human capital H_{t-1} .

Since the social planner's problem is concave, the first-order conditions are necessary and sufficient for a maximum, given the no-Ponzi-game condition, and the maximum will be unique. The first-order conditions are the same as those given by equation (3), and the no-Ponzi-game condition on the social planner is equivalent to the no-Ponzi-game condition on each agent's borrowing in the competitive equilibrium. Thus the competitive equilibrium outcome is identical to the social planner's solution; for any positive initial level of human capital of the old, there will be a unique path of aggregate human capital, and along this path aggregate human capital will converge to its steady-state level.⁵

2.3. *Monotonic Versus Oscillatory Convergence*

The path of aggregate human capital converges monotonically or exhibits damped oscillations depending on the sign of the cross-partial derivative f_{12} —that is, depending on the degree of complementarity between young and old human capital in production. Further, since output is strictly monotonically increasing in the levels of both old and young human capital, income will show the same pattern of convergence as human capital.⁶

Proposition 1: *If $f_{12} > 0$, then aggregate human capital converges monotonically to the steady state; if $f_{12} < 0$, then the path of aggregate human capital exhibits damped oscillations.*⁷

*Proof of Proposition 1*⁸. The social planner's problem can be rewritten as

$$v(H_{t-1}) = \text{MAX}_x [f(x, H_{t-1}, 1) - x + \delta v(x)], \quad (6)$$

where $v(H_{t-1})$ is the present discounted value of future output net of the cost of investment in human capital (see Stokey and Lucas, 1989, sec. 4.1). If we define $g(x, H_{t-1}) \equiv f(x, H_{t-1}, 1) - x$, when $g(\cdot, \cdot)$ has continuous second derivatives, is strictly concave, and is strictly increasing in H_{t-1} . Thus, $v(\cdot)$ is continuously differentiable, strictly increasing, and strictly concave (see Stokey and Lucas, 1989, theorems 4.11, 4.7, 4.8).

Now, first-order condition for the maximization problem is $f_1(H_t, H_{t-1}) + \delta v'(H_t) = 1$. This implicitly defines H_t as a function of H_{t-1} and, using the implicit function theorem,

$$\frac{dH_t}{dH_{t-1}} = - \frac{f_{12}(H_t, H_{t-1})}{f_{11}(H_t, H_{t-1}) + \delta v''(H_t)}. \quad (7)$$

Both f and v are concave, so the denominator is negative and the derivative has the sign of f_{12} .

Therefore, if $f_{12} > 0$, then $H_t(H_{t-1})$ is increasing in H_{t-1} for all t , so $H_{t-1} < H_t$ implies $H_t(H_{t-1}) < H_{t+1}(H_t)$ and by induction, $H_{t+1} < H_{t+2} < H_{t+3} < \dots$. Convergence then implies that $H_{t-1} < H_t < H_{t+1} < \dots < H$. Similarly, if $H_{t-1} > H_t$ then $H_{t-1} > H_t > H_{t+1} > \dots > H$.

If $f_{12} < 0$, then H_t is decreasing in H_{t-1} . Thus $H_{t-1} < H$ implies $H_t(H_{t-1}) > H_t(H) = H$, and $H_{t-1} > H$ implies $H_t(H_{t-1}) < H_t(H) = H$. Hence human capital will exhibit damped oscillations.⁹ ■

Note that the production function can be interpreted as incorporating a form of adjustment cost. It can be rewritten in terms of young human capital in the current period H_t and the change in human capital between cohorts $\Delta H_t \equiv H_t - H_{t-1}$ as $f(H_t, H_t - \Delta H_t, 1)$. In the absence of adjustment costs, young human capital in period t would jump to the steady-state level H , the level at which the marginal benefit from an extra unit of young human capital in period t equals the marginal cost of acquiring human capital. With adjustment costs, the marginal benefits and costs are no longer equal for this choice of young human capital. For example, if f_{12} is positive and H_{t-1} is less than H (so that ΔH_t is positive), then f_1 at H_{t-1} is lower than if H_{t-1} were equal to H , and the marginal benefit of young human capital is less than the cost. Equilibrium thus implies a value of H_t below the steady-state level of human capital in this case. Conversely, if f_{12} is negative, then f_1 will be higher than if H_{t-1} were equal to H , and equilibrium implies a value of H_t above the steady-state level of human capital.

An example of a production function with the properties we have assumed is one that is Cobb-Douglas in the fixed factor and in a CES aggregate of young and old human capital:

$$f(H_t, H_{t-1}, L_t) = A[\alpha H_t^{1-\rho} + \beta H_{t-1}^{1-\rho}]^{\frac{\theta}{1-\rho}} L_t^{1-\theta}, \quad \alpha + \beta = 1, \quad - < \theta < 1, \quad (8)$$

where ρ , the reciprocal of the elasticity of substitution, represents the degree of complementarity between young and old human capital. When $\rho = 0$, they are perfect substitutes. Taking the limit as $\rho \rightarrow 1$ gives the Cobb-Douglas case, and taking the limit as $\rho \rightarrow \infty$ gives a Leontief production function in young and old human capital (that is, they are perfect complements).

For this CES production function, the cross-partial derivative is given by

$$f_{12}(H_t, H_{t-1}, 1) = \alpha\beta\theta(\rho - \theta + 1)A[\alpha H_t^{1-\rho} + \beta H_{t-1}^{1-\rho}]^{\frac{\theta}{1-\rho}-2} \quad (9)$$

and has the same sign as $\rho - 1 + \theta$. Thus, if ρ is greater than the share of the fixed factor $1 - \theta$, convergence to the steady state will be monotonic, and if the initial level of human capital is less than the steady-state level, human capital will be monotonically increasing.¹⁰ If young and old human capital are perfect substitutes ($\rho = 0$), there will be cycles in human capital, and if the production function is Cobb-Douglas in young and old human capital ($\rho = 1$), there will be monotonic convergence. Young and old human capital are complements within the human capital aggregate, but they are substitutes in that they compete to work with the fixed factor. Thus, the more important the fixed factor (the larger $1 - \theta$), the more likely it is that young and old human capital are substitutes and the path of human capital exhibits oscillations.

Note that for simplicity we have examined the process of human capital accumulation in an overlapping generations model in which workers live for two periods and each worker has young human capital in the first period of his life, which automatically becomes old human capital in the second period. In practice worker's human capital will gradually acquire more of the characteristics of old human capital as he gains experience, maturity, seniority, and so on over his working life. In addition, the amount of old human capital a worker has might not be a function of age alone. For example, when old human capital is scarce, young workers are likely to be promoted sooner to senior positions and gain management experience more quickly. For these reasons, the human capital of the old and the young in our two-period framework will be closer substitutes—that is, the appropriate value of the complementarity parameter ρ will be lower—than would be suggested by an extreme example in which all the old were forty-five-year-old master craftsmen and all the young were seventeen-year-old apprentices. The considerations make it more likely that, as we would expect, there will be monotonic convergence rather than oscillations in the path of aggregate human. They also mean that there will be fewer examples of oscillations in sectoral human capital of the type we discuss in the next section.

3. Imperfect Substitutability Between Young and Old Human Capital in Production: Sectoral Dynamics

In this section we consider the behavior of human capital at the sectoral level. We show that human capital within each sector may cycle relative to that in other sectors even for parameter values that would generate monotonic convergence in aggregate human capital. In the next section, we integrate aggregate and sectoral dynamics and show that, for a moderate degree of substitutability, human capital in individual sectors will cycle about a monotonically converging path of aggregate human capital, at least close to the steady state.

Consider a small, open economy that produces a single consumption good from a large number of intermediate inputs. As in the previous section, we assume that the consumption good is tradable, so that we can examine how convergence would be slowed in an economy with perfect capital markets. We assume, however, that the intermediate goods are nontradable.¹¹ Each input is produced from human capital and a fixed factor in a sector that contains a large number of perfectly competitive firms that produce only that good. We assume that human capital is sector-specific, and we normalize both the aggregate supply of the fixed factor and the number of sectors to one. Technology is identical across firms in a sector and is characterized by the constant returns-to-scale production function:

$$y_{j,t} = f^j(h_{j,t}, h_{j,t-1}, l_{j,t}), \quad (10)$$

where $y_{j,t}$ is production of good j , $h_{j,t}$ is young human capital used by sector j in period t , $h_{j,t-1}$ is old human capital used by sector j in period t , and $l_{j,t}$ is the quantity of the fixed factor used by sector j . We assume that f^j has the same properties as the aggregate production function f in Section 2.

The aggregate production function in this sectoral model has the Cobb-Douglas form

$$\log(Y_t) = \int_J \log(y_{j,t}) dj, \quad (11)$$

where Y_t is production of the consumption good, and the sectors are indexed on the set J .¹² (We present below results for the more general case of a CES production function, which allows for an arbitrary degree of complementarity between inputs.)

As in the case of aggregate human capital in the previous section, the path of human capital in each sector in the decentralized economy will be the same as that chosen by a social planner.¹³ The social planner will choose values of $h_{j,t+s}$ and $l_{j,t+s}$ to maximize the present discounted value of output net of the cost of education—that is, to solve:

$$\text{MAX} \sum_{s=0}^{\infty} \delta^s \left\{ \exp \left[\int_J \log(f(h_{j,t+s}, h_{j,t+s-1}, l_{j,t+s})) dj \right] - \int_J h_{j,t+s} dj \right\}, \quad (12)$$

subject to a sequence of constraints on the fixed factor, $\int l_{j,t+s} dj \leq 1$ for $s = 0, 1, 2, \dots$, and a no-Ponzi-game condition on the economy's borrowing from the rest of the world.

The first-order conditions for the social planner's problem are

$$\frac{\partial y_{j,t}}{\partial h_{j,t}} \cdot \frac{Y_t}{y_{j,t}} + \delta \cdot \frac{\partial y_{j,t+1}}{\partial h_{j,t}} \cdot \frac{Y_{t+1}}{y_{j,t+1}} = 1 \quad (13)$$

and

$$\frac{\partial y_{j,t}}{\partial l_{j,t}} \cdot \frac{Y_t}{y_{j,t}} = \mu_t, \quad (14)$$

where μ_t is the Lagrange multiplier associated with the constraint on the fixed factor for period t . These implicitly defined the path of $h_{j,t}$, human capital in sector j , given output of the consumption good (Y_t) and the amount of the fixed factor in sector j .

Since the social planner's problem is concave, the first-order conditions are necessary and sufficient for a maximum, given the no-Ponzi-game condition, and the maximum is unique. For any positive initial level and distribution of human capital of the old, there will be unique paths of human capital in each sector, and along these paths, sectoral human capital will converge to the steady-state level.

Under the assumptions that the production function for the consumption good is Cobb-Douglas across inputs, and the production function of a given sector j is multiplicatively separable between human capital and the fixed factor—that is, that f^j has the form

$$f^j(h_{j,t}, h_{j,t-1}, l_{j,t}) = g(h_{j,t}, h_{j,t-1}) \cdot G(l_{j,t}), \quad (15)$$

whether human capital in a sector oscillates relative to another sector with an identical production function depends on the sign of $f_{12}^j - f_1^j f_2^j / f^j$.¹⁴ We can summarize this result in the following proposition.

Proposition 2: *Suppose that two sectors have the same production function f , and the sign of $f_{12} - f_1 f_2 / f$ is the same for all values of young and old human capital. Then the sector that uses more human capital of generation $t - 1$ will also use more human capital of generation t if $f_{12} - f_1 f_2 / f > 0$; and the sector that uses more human capital of generation $t - 1$ will use less human capital of generation t if $f_{12} - f_1 f_2 / f < 0$.*

Proof of Proposition 2. With a Cobb-Douglas production function for the consumption good and a separable production function for the output of the sectors being compared, the first-order condition given by equation (14) becomes

$$\frac{G'(l_t) \cdot Y_t}{G(l_t)} = \mu_t. \quad (16)$$

Since there are a large number of sectors, Y_t and μ_t are determined by the aggregate economy and are unaffected by the human capital of any one sector. Thus sectoral demand for the fixed factor is independent of the level of human capital in the sector, and we can follow the same approach as in the proof of Proposition 1. Focusing on sector k , the social planner's problem given the path of aggregate output can be written as

$$v_{k,t}(h_{k,t-1}) = \text{MAX}_x \left\{ \exp \left[\int_J \log(y_{j,t}) dj \right] - \int_J h_{j,t} dj + \delta v_{k,t+1}(x) \right\}, \quad (17)$$

where $y_{j,t} = g(h_{j,t}, h_{j,t-1})G(l_{j,t})$, $v_{k,t}(h_{k,t-1})$ is the discounted value in period t of consumption conditional on the paths of human capital and output in all sectors except sector k , and the maximization on the right-hand side is with respect to the value of $h_{k,t}$. The first-order condition for the maximization problem is

$$g_1(h_{k,t}, h_{k,t-1}) \cdot \frac{Y_t}{g(h_{k,t}, h_{k,t-1})} - 1 + \delta v'_{k,t+1}(h_{k,t}) = 0. \quad (18)$$

If the derivative of the term $g_1 Y_t / g$ with respect to $h_{k,t-1}$ is positive, then lowering $h_{k,t-1}$ while holding $h_{k,t}$ fixed would make the left-hand side of the equation negative. Since the left-hand side is decreasing in $h_{k,t}$, this implies that a sector with less human capital of generation $t - 1$ would also have less human capital of generation t . Conversely, if the derivative is negative, a sector with less human capital of generation $t - 1$ will have more human capital of generation t . Now, the derivative is $Y_t(g_{12}g - g_1g_2)/g^2$, so its sign is determined by the sign of $g_{12} - g_1g_2/g$ or equivalently, $f_{12} - f_1f_2/f$. ■

If the amount of young human capital demanded by each sector is increasing in the amount of old human capital used by the sector, then a sector that has higher human capital relative to another sector in one period will also have higher human capital in the next and all subsequent periods in this Cobb-Douglas case. As in the aggregate case, demand for young human capital is influenced by the amount of old human capital through the degree of complementarity between young and old human capital in production, which is reflected by f_{12} . However, at the sectoral level, demand for young human capital is also influenced by the amount of old human capital through its effect on the price of the sector's output, which is reflected by $-f_1f_2/f$.

Thus, if $f_{12} - f_1f_2/f$ is positive for sectors with identical production functions, then the rank order of human capital across those sectors will be preserved over time. If, on the other hand, $f_{12} - f_1f_2/f$ is negative, then the rank order of human capital across the sectors will be reversed each period.¹⁵ Thus like Pierce (1990), we find possible cycles at the sectoral level. If human capital is high in generation $t - 1$ and young and old human

Table 1. Sectoral and aggregate dynamics.

	$\eta < \rho\theta/[1 - \rho(1 - \theta)]$	$\eta > \rho\theta/[1 - \rho(1 - \theta)]$
$\rho < 1 - \theta$	Sector: monotonic Aggregate: oscillations	Sector: oscillations Aggregate: oscillations
$\rho > 1 - \theta$	Sector: monotonic Aggregate: monotonic	Sector: oscillations Aggregate: monotonic

capital are substitutes, then generation t has little incentive to accumulate human capital. But if generation t chooses low human capital, that will provide generation $t + 1$ with an incentive to choose high human capital, and a new cycle will begin. For example, a large number of people entered academics after World War II, so few people entered in subsequent decades. Now that a large number of professors are retiring, more people are again entering academics.¹⁶

Note that if the sectoral production function has the mixed CES and Cobb-Douglas form,

$$f(h_{j,t}, h_{j,t-1}, l_{j,t}) = A[\alpha h_{j,t}^{1-\rho} + \beta h_{j,t-1}^{1-\rho}]^{\frac{\theta}{1-\rho}} l_{j,t}^{1-\theta}, \quad \alpha + \beta = 1, \quad 0 < \theta < 1, \quad (19)$$

then $f_{12} - f_1 f_2 / f$ has the same sign as $\rho - 1$. Thus, if the degree is complementarity between young and old human capital is greater than one, then the rank order of human capital across sectors will be preserved, and if the degree of complementarity is less than one, the rank order will be reversed each period.

For a more general, CES aggregate production function for the consumption good of the form

$$Y_t = \left[\int_J y_{j,t}^{1-\eta} dj \right]^{\frac{1}{1-\eta}}, \quad (20)$$

where η measures the degree of complementarity between the intermediate inputs, the rank order of human capital across sectors will be reversed if $\rho < \eta/[\theta + \eta(1 - \theta)]$. Thus rank reversal is more likely if intermediate goods are close complements in the production of the consumption good (η is high).¹⁷ When good j is a close complement for other inputs, production of good j will move closely with the path of aggregate production so that a relatively low level of human capital in sector j in one generation will be followed by a high level of human capital relative to other sectors in the next generation.¹⁸

Our results should not be taken to imply that human capital either converges monotonically in every sector or oscillates in every sector. Most sectors' goods may be reasonably close substitutes (η low), and young and old human capital may be complementary for most sectors (ρ relatively high), so that human capital converges monotonically in these sectors. But human capital may exhibit relative oscillations in those sectors in which young and old human capital are less complementary or whose output is strongly complementary with other goods in the economy.¹⁹

Table 1 summarizes the behavior of sectoral and aggregate human capital for various values of ρ , the degree of complementarity between young and old human capital.

We do not know the values of the parameters, but the absence of cycles at the aggregate level, indicates that ρ is greater than $1 - \theta$. Evidence of cycles within at least some sectors from Freeman (1975a, 1975b, 1976) suggests that η is greater than $\rho\theta/[1 - \rho(1 - \theta)]$ in those sectors.

4. Integrated Sectoral and Aggregate Dynamics Near the Steady-State

For the case in which sectors have identical production functions, it is possible to analyze sectoral and aggregate dynamics simultaneously by linearizing around the steady state. We consider the case of the CES production function and begin by aggregating across sectors and examining the equilibrium path of human capital for the economy as a whole. Then we show that the paths of sectoral human capital can be interpreted as movements about the path of aggregate human capital.

The equilibrium condition for human capital in each sector, given output of the aggregate consumption good in periods t and $t + 1$, is given by equation (13). If we use the production function for the consumption good and take a first-order Taylor series approximation of this first-order condition, we get a linear second-order difference equation in $\hat{h}_{j,t}$ and \hat{H}_t , the deviations of sectoral and aggregate human capital from their steady-state levels:

$$\hat{h}_{j,t+1} - c_{\text{sec}}\hat{h}_{j,t} + \frac{1}{\delta}\hat{h}_{j,t-1} + \frac{\theta}{\rho - 1} \cdot \left[\hat{H}_{j,t+1} + c_{\text{agg}}\hat{H}_{j,t} + \frac{1}{\delta}\hat{H}_{j,t-1} \right] = 0, \quad (21)$$

where

$$c_{\text{sec}} \equiv \frac{\alpha\beta\rho(1 + \delta) + (\alpha^2 + \beta^2\delta)}{\alpha\beta\delta(\rho - 1)}, \quad (22)$$

and $c_{\text{agg}} \equiv (\alpha^2 + \delta\beta^2)/(\alpha\beta\delta)$. Summing over sectors eliminates the terms in \hat{h} , the sectoral deviations, and gives a difference equation that defines the path of deviations of aggregate human capital from the steady-state level:

$$\hat{H}_{t+1} - \left[\frac{\alpha\beta\rho(1 + \delta) + (\alpha^2 + \beta^2\delta)(1 - \theta)}{\alpha\beta\delta(\rho - 1 + \theta)} \right] \hat{H}_t + \frac{1}{\delta}\hat{H}_{t-1} = 0. \quad (23)$$

Since this is a second-order, linear-homogeneous difference equation, its solution has the form $B_1(\Lambda_1)^t + B_2(\Lambda_2)^t$, where Λ_1 and Λ_2 are the roots of the corresponding characteristic equation, and B_1 and B_2 are constants determined by the initial value of aggregate human capital and the condition that human capital must converge to its steady-state level. The product of the roots is the coefficient on \hat{H}_{t-1} —that is, $1/\delta$, which is greater than one. Thus the two roots have the same sign, and one root (Λ_2 , say) will be greater than one in absolute value. Convergence requires that Λ_1 , the other root, is less than one in absolute value and that the coefficient on the larger root (B_2) is zero. The sum of the roots is the negative of

the coefficient on \hat{H}_t . Thus, Λ_1 will be positive if the term in square brackets in equation (23) is positive. That is, as we found before, convergence of aggregate human capital will be monotonic if $\rho > 1 - \theta$. Conversely, human capital will exhibit damped oscillations about its steady-state level if $\rho < 1 - \theta$.²⁰

Substituting the solution for the path of aggregate human capital $B_1 \Lambda_1^t$ back into equation (21) gives a second-order difference equation in deviations of sectoral human-capital from the steady-state level:

$$\hat{h}_{j,t+1} - c_{\text{sec}} \hat{h}_{j,t} + \frac{1}{\delta} \hat{h}_{j,t-1} + \frac{\theta}{\rho - 1} \cdot \left[\Lambda_1 + c_{\text{agg}} + \frac{1}{\delta \Lambda_1} \right] B_1 \Lambda_1^t = 0. \quad (24)$$

This has solution $\hat{h}_{j,t} = b_j \lambda_j^t + B_1 \Lambda_1^t$, where λ_j is the smaller root in absolute value of the characteristic equation corresponding to the left-hand side of equation (24), b_j is determined by the initial level of human capital in sector j relative to aggregate human capital, and $\int_j b_j = 0$. Thus, close to the steady state, the path of human capital in any sector is equivalent to the sum of two terms: one in Λ_1 representing the path of the aggregate economy and one in λ_j representing the path of the individual sector relative to aggregate human capital.

Human capital in sector j will exhibit damped oscillations about the path of aggregate human capital if λ_j is negative. For a Cobb-Douglas production function for the aggregate consumption good, this occurs when $\rho < 1$. Thus, when $1 - \theta < \rho < 1$, close to the steady state, human capital in each sector will exhibit damped oscillations about a path of aggregate human capital that is converging monotonically to its steady-state level. This strengthens our earlier result that the rank order of human capital across sectors will be reversed each period if $\rho < 1$.

For the more general, CES production function, human capital in sector j exhibits damped oscillations if $\rho < \eta / [\theta + \eta(1 - \theta)]$ —that is, if $\eta > \rho\theta / [1 - \rho(1 - \theta)]$. Thus, if good j is sufficiently complementary with other goods in the economy and if young and old human capital are not too complementary in production, there will be a unique sequence of human-capital levels in sector j that converge to the steady-state level, but that exhibit damped oscillations about the path of aggregate human capital.²¹ This is illustrated in Figure 1.²²

It is difficult to work out analytically what happens further from the steady state, but we conjecture that the further from the steady state, the more aggregate production is reduced by the imbalance in human capital between sectors. Thus, far from the steady state, the aggregate growth rate will be boosted by the reduction in imbalances between sectors.

Note that for this model to explain the path of consumption, as well as production, capital markets cannot be completely perfect. If they were perfect, poor countries would want to borrow to smooth their consumption. In reality, it may be difficult to borrow for consumption purposes and to pass the debts on to one's children. If enforcement mechanisms rely on trade sanctions that reduce income, they may be ineffective in enforcing repayment of consumption loans. Habit formation in consumption might also help explain

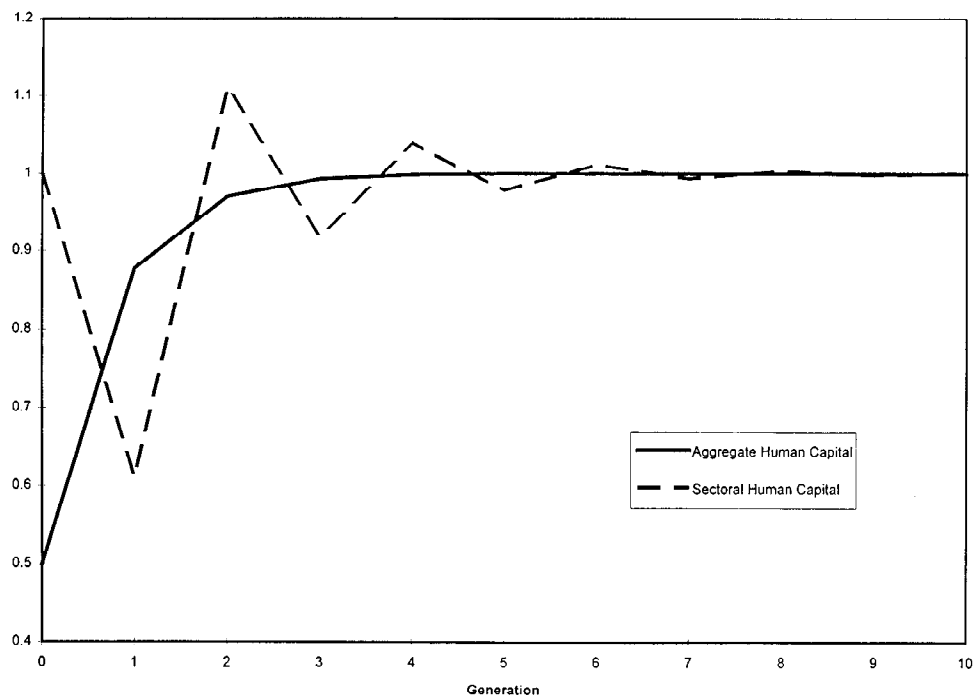


Figure 1. Path of human capital.

why countries spread consumption growth over time rather than borrow to raise consumption instantaneously to its steady-state level.

In addition to its implications for convergence, the model carries implications for relative wages and for migration. Since human capital is more scarce in poor regions, and capital earns the same return everywhere, unskilled labor will be relatively abundant in poor regions and will therefore earn a lower return. Furthermore, since human capital increases more rapidly in poor regions than in rich ones, the human capital of the young will be relatively abundant in poor countries and, hence, will earn a lower return. The model is thus consistent with migration of unskilled labor and young human capital from poor countries to rich countries. The model also predicts that old human capital will be scarce in poor countries and will earn a high return. Thus old educated people will have an incentive to migrate from rich to poor countries. To some extent, this phenomenon is observed empirically. Multinationals post experienced older managers from developed countries in developing

countries, and some people emigrate from developing countries when young and return when old. In the real world, however, the tendency for old educated people to migrate to poor countries is mitigated in two ways. First, older migrants would leave behind families and established networks of business and social contacts and would need to adjust to a different culture. Second, the reason that old human capital is highly valued is because old workers are needed to work with young workers, but cultural and language differences between old and young workers might make this difficult.

Nevertheless, the implication that old human capital will earn a higher absolute wage in poor countries than rich countries seems an unattractive feature of this model. However, models of capital-market imperfections, such as Barro, Mankiw, and Sala-i-Martin (1995), yield the similar implication that educated workers will earn a higher absolute amount in poor countries than in rich countries.²³

5. Convergence in a Generalized Model

In this section, we extend the model of Section 2 in order to determine speeds of convergence of young and old human capital. We extend the CES production function to allow for physical capital, technological progress, and population growth. We then examine how the speed of convergence is related to the parameters of the model. In particular, we show how the speed of convergence in general decreases as the degree of complementarity between young and old human capital (ρ) increases, and as the share of human capital in production (θ) increases relative to the share of the fixed factor.

In extending the model, we identify the fixed factor with raw labor and we assume that the number of workers born each generation grows by a constant factor n so that $L_t = L_0 n^t$. We suppose that there is labor-augmenting technological progress with intergenerational growth factor g , and we introduce physical capital K_t , which we assume is rented on the world capital market at the constant rate $r + d$, where r is the world interest rate and d is the rate of depreciation of physical capital.

With these extensions, the aggregate production function for goods in the economy is given by

$$Y_t = A[\alpha H_t^{1-\rho} + \beta H_{t-1}^{1-\rho}]^{\frac{\theta}{1-\rho}} K_t^\gamma (L_0 n^t g^t)^\phi, \quad \theta + \gamma + \phi = 1, \quad (25)$$

where H_t represents young human capital in period t , and H_{t-1} is old human capital in period t . As before, the competitive-equilibrium outcome is the same as the solution to the social planner's problem. The social planner chooses a path of human capital to solve

$$\text{MAX} \sum_{s=0}^{\infty} \delta^s [Y_{t+s} - H_{t+s}]. \quad (26)$$

This yields the first-order condition,

$$\begin{aligned} & \alpha \left[\alpha Z_t^{1-\rho} + \beta \left(\frac{Z_{t-1}}{ng} \right)^{1-\rho} \right]^{v-1} + \beta \delta (ng)^\rho \left[\alpha Z_{t+1}^{1-\rho} + \beta \left(\frac{Z_t}{ng} \right)^{1-\rho} \right]^{v-1} \\ & = \left[\frac{r+d}{\gamma} \right]^{\frac{\gamma}{1-\gamma}} \frac{Z^\rho}{\theta A^{\frac{1}{1-\gamma}}}, \end{aligned} \quad (27)$$

where $Z_t \equiv H_t / (L_0 g^t n^t)$ is aggregate human capital per efficiency unit of labor, and $v \equiv \theta / [(1-\rho)(1-\gamma)]$.

As in the case of the basic model of Section 2, for each initial level of human capital per efficiency unit there is a unique path of human capital that converges to the steady-state level, and this path will be chosen by the economy.²⁴ The path is monotonic if $\rho > \phi / (\theta + \phi)$, and exhibits damped oscillations if $\rho < \phi / (\theta + \phi)$. Note that $\phi / (\theta + \phi)$ is the share of the fixed factor relative to the fixed factor plus human capital. Thus convergence is “more likely” to be monotonic if the share of the fixed factor is small and will always be monotonic when ρ is one (or greater). The share of physical capital does not affect the speed of convergence because physical capital adjusts to the level of human capital (This is also true in Barro, Mankiw, and Sala-i-Martin, 1995).

We can calibrate the model and compare the results with empirically observed rates of convergence. In calibrating the model, our aim is to illustrate some of the effects and get a sense of their quantitative magnitude, rather than to test the model. We calibrate the model assuming no capital market imperfections and no adjustment costs other than those created by intergenerational complementarities. In reality, convergence will be further slowed by these other factors as well. The most plausible configurations of parameter values are therefore not ones that generate the speeds of convergence observed empirically but values that generate somewhat faster convergence.

To determine rates of convergence close to the steady state, we derive a linearized version of the model using the same procedure as in Section 4. Close to the steady state, deviations of human capital from their steady-state level ($Z_t \equiv Z_t - Z$) satisfy the second-order difference equation,

$$\hat{Z}_{t+1} - \frac{\frac{\phi(ng)^\rho}{\theta+\phi} [\alpha^2 (ng)^{1-2\rho} + \beta^2 \delta] + \alpha\beta\rho[1 + \delta ng]}{\alpha\beta\delta ng \left[\rho - \frac{\phi}{\theta+\phi} \right]} \hat{Z}_t + \frac{1}{\delta ng} \hat{Z}_{t-1} = 0. \quad (28)$$

The solution to this difference equation has the form $Z_t = B \Lambda^t$. If young and old human capital are close substitutes (ρ is small), then Λ is negative and human capital per efficiency unit will exhibit damped oscillations about the steady-state level; if young and old human capital are sufficiently complementary ($\rho > \phi / [\theta + \phi]$), then Λ is positive and human capital will converge monotonically to the steady-state level. We discuss the relationship between convergence and the parameters in our model in greater detail below.

We adopt the following baseline values for parameters: we assume a population growth rate of 1 percent a year, a productivity growth rate of 2 percent a year, and a discount rate of 4 percent a year. We assume people join the labor force at nineteen and leave at sixty-five, spending the first half of this period as young workers and the second half as old workers,

Table 2. Rate of convergence in percent per year for various values of ρ and θ .

	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.65$	$\theta = 0.69$
Rate of convergence near steady state for human capital and income:					
$\rho = 0.5$	12.93	7.90	4.68	2.99	1.06
$\rho = 1.0$	6.10	4.57	2.91	1.88	0.63
$\rho = 1.5$	4.58	3.51	2.27	1.45	0.47
Rate of convergence at 50 percent of steady state for human capital:					
$\rho = 0.5$	12.11	7.13	4.05	2.51	0.84
$\rho = 1.0$	5.24	3.86	2.40	1.52	0.48
$\rho = 1.5$	3.88	2.95	1.86	1.17	0.35
Rate of convergence at 50 percent of steady-state human capital for income:					
$\rho = 0.5$	^a	^a	3.19	2.01	0.70
$\rho = 1.0$	5.04	3.62	2.17	1.33	0.43
$\rho = 1.5$	4.32	3.15	1.81	1.08	0.32

^aInitial income is not defined because the rate of convergence of human capital is so high that it implies a negative value for old human capital.

so the length of a period is 23 years. The share of physical capital (γ) is assumed to be 0.3. The relative weights of young and old human capital (α and β) are both set to 0.5. This choice is somewhat arbitrary, but it is consistent with empirical observation of higher wages for older workers, since in the presence of population growth and technological change, old human capital is scarcer than young human capital.

The two most important free parameters are ρ , the degree of complementarity between young and old human capital, and θ , the share of human capital. Although Murphy and Welch (1992) have attempted to estimate some elasticities of substitution ($1/\rho$) between different types of labor, as noted in Section 1, their results are sensitive to their specific assumptions. The minimum wage has been used to calculate the share of human capital (Barro and Sala-i-Martin, 1995), but this provides only a lower bound since even workers with no formal education have human capital in the form of training from their parents. Barro and Sala-i-Martin argue that a total share for physical and human capital of about 0.8 enables a closed-economy, neoclassical growth model to generate observed speeds of convergence, but this gives no guidance as to the appropriate share of capital in this adjustment-cost model. The fact that we observe a wide range of income levels in the world points to a high share of accumulable factors, which would allow small differences in tax rates or corruption, for example, to produce large differences in steady-state income.

In the top panel of Table 2 below, we show the rates of convergence close to the steady state of human capital and income (both per efficiency unit of labor) for a range of plausible values of the share of human capital θ and the degree of complementarity ρ .²⁵ Recall that the share of physical capital is assumed to be 0.3, so that the share of the fixed factor equals $(0.7 - \theta)$. The rate of convergence falls as the share of human θ rises, and as young and old human capital become stronger complements (ρ increases).

Away from the steady state, the linearizations are less accurate, and the rates of convergence of income and human capital are no longer identical. Simulations using multiple shooting suggest that convergence of human capital is a little slower away from the steady

state. For example, the middle and bottom panels of Table 2 show rates of convergence of young human capital and income, respectively, for human capital per efficiency unit at 50 percent of the steady-state level.²⁶

It is possible to make some general observations about the relationship between the rate of convergence and the values of the parameters in our model.²⁷ First, the rate of convergence slows as the relative share of human capital compared to the fixed factor $\theta/(\theta + \phi)$ rises, and the rate of convergence does not depend on the share of physical capital, γ . The intuition for this can be seen by considering the human capital decision of the young. To the extent that the share of raw labor $\phi/(\theta + \phi)$ is high, the relative returns from choosing a level of human capital close to the steady-state level of human capital are also high, and convergence to the steady state will be fast.

In general, the rate of convergence declines monotonically in the degree of complementarity, but it need not do so. It is easiest to consider the effect of ρ on convergence by first considering the case without population growth or technological change. In this case, we get the intuitive result that the greater the degree of complementarity between young and old human capital, the more the young tend to choose a level of human capital close to that of the old, and the slower is convergence. For $ng > 1$, however, the rate of convergence increases with ρ at high levels of ρ . With $ng > 1$, the effective size of each cohort is larger than that of the previous cohort. To take an extreme example, if ρ were infinite, and the production function were Leontief in young and old human capital, then in the steady state each cohort's human capital would be in excess supply and earn nothing when it was young but would appropriate all returns to human capital when it was old. Hence, the optimal choice of human capital for the young would not be affected by the level of human capital for the old, and the economy would move to the steady state immediately. In general, for $ng > 1$, there will be some level of ρ that minimizes the rate of convergence. For our baseline parameter values, this level of ρ is between 2.9 and 3.1. With higher values of n and g , a smaller value of ρ would minimize the rate of convergence.

Convergence is faster when either α , the relative share of young human capital, or β , the relative share of old human capital, is close to one and is slower when the relative shares are more equal. If, for example, the share of young human capital were one, any cohort's return to human capital would be received entirely when young, and the return would be independent of the human capital of the previous cohort so the human capital investment would move straight to the steady-state level. A similar argument applies when $\beta = 1$. In the simulation reported above, the rate of convergence is minimized for α around 0.39 when ρ is 0.5, 0.47 when ρ is 1.0, and 0.64 when ρ is 2.0. However, the rates of convergence for these values of α are not markedly different from those in the simulations reported.

The effects of changes in the population and productivity growth factors n and g on convergence are identical, but neither they nor the effect of changes in the value of income one period ahead δ are easy to sign analytically. For the parameter values we have chosen, the rate of convergence increases as n , g and δ increase (and r , the interest rate, falls). For example, when $\rho = 1$, $\theta = 0.6$, and interest rate (r) is four percent (as in Table 2), the rate of convergence close to the steady state is 2.91 percent each year. If the interest rate is 2 percent, the rate of convergence would be 3.90 percent each year, and if the interest rate is 6 percent, the rate of convergence would be 2.25 percent.

Since the open-economy, neoclassical model without adjustment costs predicts immediate convergence, the results we report above can be taken as indicating that adjustment costs created by complementarity between old and young human capital significantly slow convergence.

It is instructive to compare these rates of convergence to those found in empirical research. Barro and Sala-i-Martin (1992) argue that U.S. states may be taken as having similar steady states and find that they converge at around 2 percent a year. OECD countries converge at around 1 percent a year unconditionally (Dowrick and Nguyen, 1989).²⁸ Barro and Sala-i-Martin interpret this lower speed of convergence as indicating some difference in steady states among these economies, but it could also be due to lesser mobility of capital (in the neoclassical model) or labor (in this model) between countries than within countries. Countries do not converge unconditionally, and under either our model or a closed-economy version of the neoclassical model, this would be interpreted as evidence that they do not have the same steady states. Barro and Sala-i-Martin find that countries converge at around 2 percent per year after controlling for human capital and interpret human capital as a proxy for steady-state income. However, if human capital were interpreted as the value of a state variable, conditional convergence would be difficult to interpret within the neoclassical model. Conditional convergence might be at least qualitatively consistent with the model of this article, since it predicts that countries that have rapidly growing human capital would have high ratios of human capital to income. This could cause a growth regression to generate a negative coefficient on income and a positive coefficient on human capital.

6. Conclusion and Policy Implications

We have argued that complementarity between the human capital of young and old workers can explain why output per capita does not converge instantaneously across countries. We have concentrated on complementarity that arises from young and old workers' playing different roles in the production process, but complementarity also arises because workers with extensive formal education receive more on-the-job training from old workers. Under either type of complementarity, each generation's optimal human capital depends on the human capital of previous and succeeding generations, and there is a unique path to the steady-state level of human capital along which each generation's human capital is optimal given that of all past and future generations. For a moderate degree of substitutability, the path of human capital within individual sectors, such as law or academics, will display gradually dampening cycles. Aggregate human capital, however, will converge to the steady state monotonically. A linearized version of the model can generate empirically relevant rates of convergence.

Although this model is similar to models of credit constraints, such as Barro, Mankiw, and Sala-i-Martin (1995), in predicting slow convergence, it is driven by a fundamentally different mechanism. Under credit constraints, growth is slowed by the desire to smooth consumption, whereas under imperfect substitutability, growth is slowed by a form of adjustment costs in production.²⁹

The policy implications of imperfect substitutability between young and old human capital differ sharply from those of imperfect credit markets. Models driven solely by imperfect

credit markets imply there are large potential welfare improvements from policies that relax credit constraints and allow a rapid expansion of education. However, the view that credit constraints are all that prevent instantaneous convergence of human capital and per capita output between rich and poor countries is difficult to reconcile with the poor performance of many countries with substantial oil and mineral wealth. Moreover, the experience of many African countries suggests caution about the benefits for growth of government policies aimed at a rapid expansion of education, even when this is not constrained by a lack of infrastructure or trained teachers. Starting from a low base, sub-Saharan Africa had a much higher growth rate of education than developing countries as a whole. From 1965 to 1990 primary enrollment as a percentage of the relevant age group increased 65 percent in sub-Saharan Africa, compared with a 33 percent average among developing countries. Secondary enrollment increased 8.5 times, compared with 2.75 times in developing countries as a whole. Tertiary enrollment grew 100 percent as opposed to 40 percent in developing countries as a whole (World Bank, *World Development Report*, 1987, 1993). Yet Africa's rate of economic growth has been dismal relative to developing countries in other regions.

Kenya provides an instructive example. Some 30,000 students were enrolled in secondary schools in 1963, on the eve of independence. Ten years later the number had increased more than fivefold, to 175,000, and by 1978, Kenya had fifteen times the number of secondary school students it had fifteen years earlier (Killick, 1981). Higher education grew even more dramatically: in 1963, 370 Kenyans were enrolled in East African universities; twelve years later there were 5,104, for an annual growth rate of 24.5 percent (Lockhart, 1981). By 1976, the education sector accounted for 12 percent of measured GDP and nearly 30 percent of Kenya's population were students (Todaro, 1981). According to government figures, education comprised 30 percent of the national budget (Lockart, 1981).³⁰

Yet with this tremendous increase in education came an increase in unemployment among the educated. At the beginning of the 1980s the education system was producing 200,000 graduates each year, including primary-school graduates, but employment in the modern sector of the economy was expanding by only about 40,000 jobs a year (Lockhart, 1981). At least according to some researchers, there was little evidence that education significantly improved productivity for those remaining in agriculture (Hopcraft, 1974). As education expanded, the estimated rate of return dropped. In a study of over 4,000 workers that controlled for ability as measured by test scores, Thias and Carnoy (1972) found that the social return to primary education dropped steadily from 14 percent in 1960 to 5 percent in 1966.³¹

The experience of Africa and of mineral-rich countries suggests that much of the new growth literature may have been overly optimistic about the short-run benefits of rapid expansion in human capital and may have exaggerated the importance of credit-market imperfections in slowing convergence. The empirical data are consistent with a view in which convergence is significantly slowed by intergenerational complementarity and other forms of adjustment costs in human capital.

Notes

1. An alternative explanation of the small movements in education differentials for older workers is provided by the “active labor market” hypothesis of Freeman (1975b), in which older workers are insulated from labor market changes by unions, seniority rules, and firm-specific human capital.
2. When we calibrate the model in Section 5, we allow for physical capital, technological progress, and population growth.
3. We have assumed that human capital is financed by borrowing on international capital markets subject to a no-Ponzi-game constraint. Assuming altruistic links between generations, the old could finance the education of the young in the steady state. Out of the steady state, when intercountry borrowing is required, a generation in a less developed country, when young, could borrow from the old of a more developed country and then, when they were old, repay the loans to the lenders’ children. We might, alternatively, suppose that each generation lives for three periods, borrowing to acquire education as children and repaying loans when young. This would add an additional interest rate term to equation (2) but otherwise leave our model unchanged.
Every country will be subject to this no-Ponzi-game constraint because countries that have converged to steady-state levels will be prepared to lend only finite amounts, and any country that had unbounded growth would need to be a net borrower. The presence of the fixed factor implies that if human capital grows faster than the interest rate, then human-capital investment will eventually exceed current output and borrowing must also eventually grow faster than the interest rate. Therefore, the discounted value of human capital must also be nonpositive in the limit as t goes to infinity. If utility were concave, and many countries were below the steady-state level of human capital, then we would expect those countries that had already converged or that were closest to the steady state to be net lenders, and those countries that were furthest from the steady state to be net borrowers. The interest rate would be above the level implied by the intertemporal discount rate of households and would fall as the countries of the world approached their steady states. Here we focus on the simple case of a small country facing a constant world interest rate.
4. We assume that workers can invest in education only when young. However, even without this assumption, a worker would ordinarily choose to do all his investment when young so that he could reap the benefits in both periods of his life rather than only in the second period.
5. We thank Kiminori Matsuyama for suggesting this approach to the proof of uniqueness.
6. It is easy to see that monotonic convergence of human capital implies monotonic convergence of output. If, for example, $H_{t-1} < H_t < H_{t+1}$, then $Y_t = f(H_t, H_{t-1}, 1) < f(H_{t+1}, H_t, 1) = Y_{t+1}$ since both young and old human capital increase between period t and period $t - 1$. Things are less straightforward when human capital exhibits oscillations. See note 19 below for a discussion of the behavior of output close to the steady state for a CES production function.
7. Whether human capital converges monotonically or exhibits damped oscillations depends on the sign of f_{12} for levels of human capital on the equilibrium path: it is a sufficient, but not a necessary, condition that f_{12} has the same sign for all possible levels of human capital. As we note below, for the CES production function, f_{12} does have the same sign for all levels of human capital.
8. We thank Jess Benhabib for suggestions on this proof.
9. The oscillations must be damped along the entire path of human capital since if ever $H_{t-1} < H$ but $H_{t+1} < H_{t-1}$, then $H_{t+2} > H_t$, $H_{t+3} < H_{t+1}$, and so on so the path thereafter would be explosive.
10. In the case of a CES production function, it can also be shown that the growth rate decreases monotonically. If human capital starts out below its steady-state level, it will increase generation by generation, gradually getting closer to the steady-state level, but the rate of increase will slow over time as the steady-state level is approached. Conversely, if the initial level of human capital is greater than the steady-state level, human capital monotonically decreases, and the growth rate monotonically increases.
11. The extent to which a tradable intermediate good was used in the production of the aggregate consumption good would be determined by the world price of the intermediate good rather than by the quantity produced domestically so that the results in this section apply only to nontradable intermediate goods. We make the assumption that all intermediate goods are nontradable for analytical tractability.
12. A model with a finite but sufficiently large number of intermediate goods yields results similar to those below. We thank Edward Drozd for analyzing this case.
13. The argument is the same as in the previous section: the first-order conditions for the social planner’s problem are the same as the human-capital-investment equilibrium condition in the competitive economy, and the

no-Ponzi-game conditions facing each agent are equivalent to the one facing the social planner.

14. Note that this result is independent of the production functions of sectors other than those being compared. In addition, the result would still hold if the outputs of those other sectors entered differently into the production function of the final consumption good. What is required is that the production function for the consumption good is Cobb-Douglas and symmetric across the inputs being compared and is a concave function of the other inputs.
15. If $f_{12} - f_1 f_2 / f$ equals zero, then after the initial period, human capital will become identical across the sectors.
16. In the case of tradable goods, these dynamics would take place at the level of the world economy, whereas in the case of nontradable goods they will take place within each national economy.
17. This can be shown using the same approach as in the proof of Proposition 2. The first term in the first-order condition in the proof, equation (18), becomes

$$g_1(h_{k,t}, h_{k,t-1}) \cdot \left[\frac{Y_t}{g(h_{k,t}, h_{k,t-1})} \right]^\eta.$$

Thus, the existence of oscillations depends on the derivative of g_1/g^η with respect to $h_{k,t-1}$. That is, it depends on the sign of $g_{11} - \eta g_1 g_2 / g$.

18. The condition for damped oscillations can be met only if $\rho < 1/(1 - \theta)$ —that is, if young and old human capital are sufficiently close substitutes in production of good j . If they are very close complements, then each generation will choose a level of human capital close to that of the previous generation, even if that level is considerably different from the levels of human capital in other sectors.
19. Note too that oscillations are relative to a path determined by the evolution of aggregate human capital. A sector will only experience oscillations in human capital in absolute levels only if the amplitude of the relative oscillations is large enough to offset the general upward trend in the economy's level of human capital.
20. Linearizing the production function about the steady-state value of aggregate human capital gives an expression for deviations of output from its steady-state value in terms of deviations of human capital:

$$\hat{Y}_t = \theta A \bar{H}^{\theta-1} [\alpha \hat{H}_t + \beta \hat{H}_{t-1}] = \theta A B_1 \bar{H}^{\theta-1} [\alpha \Lambda_1 + \beta] \Lambda_1^{t-1}.$$

Hence, if $\Lambda_1 > 0$, then both human capital and output converge monotonically, and if $\Lambda_1 < 0$, then both human capital and output exhibit damped oscillations.

21. If $h_{j,t}$, the human capital in sector j of generation t , exhibits damped oscillations across generations, then total human capital in the sector will also exhibit damped oscillations.
22. Figure 1 assumes the baseline parameter values used in Section 5 with sectoral human capital beginning at its steady-state value and aggregate human capital beginning at one-half of its steady-state value. We have chosen a relatively extreme starting value for sectoral human capital so that the paths of sectoral and human capital can be easily distinguished in the figure. This produces oscillations in sectoral human capital in absolute levels, but, as pointed out in note 18 above, oscillations that occur might only be relative to the path of aggregate human capital. Oscillations in sectoral human capital are more likely to occur in absolute levels for a country close to its steady-state level of aggregate human capital than for a country well below the steady state.
23. One test of our model would be to compare age-wage profiles between developed and developing countries, controlling for education levels. This might also help distinguish between the types of reasons noted in the text for the lack of migration of educated people to poor countries. Steep wage profiles in developing countries would suggest that old, educated workers are reluctant to leave rich countries despite potentially higher wages, whereas flatter wage profiles would suggest a strong country-specific element in old human capital.
24. The argument is the same as for the economy in Section 2. The conditions for the competitive equilibrium are the same as for the social planner's problem, which is concave and thus has a unique solution.
25. The speed of convergence is the percentage of the difference between human capital per efficiency unit of labor Z_t and its steady-state level Z or between income per efficiency unit of labor and its steady-state level—that is, eliminated between periods t and $t + 1$, expressed at an annual rate.
26. Income is defined as gross output less the sum of the rental cost of physical capital and the cost of education during the period.
27. We merely state the results and discuss the intuition here. Details of the derivations are available from the authors.

28. More recent studies by Islam (1995) using a dynamic panel data approach and by Caselli, Esquivel, and Lefort (1996) using a generalized method of moments estimator, find faster rates of convergence.
29. We have considered adjustment costs in a decreasing returns-to-capital case, but in Kremer and Thomson (1992) we examine the case in which there are constant returns to physical and human capital taken together. In this case, adjustment costs lead to a steady-state growth rate rather than a steady-state level of income.
30. The World Bank reports a 20 percent share of education in government expenditure, however.
31. This may overstate the true decline, however, since a large part of the calculated decline is due to increasing unemployment, which was also affected by business cycle factors. Some have suggested that Kenya's education system is ill-suited to the needs of the country, and indeed Kenya has instituted a series of educational reforms to re-orient the system towards technical and agricultural skills. But unemployment is high among graduates of technical schools as well. For example, one informal study cited by Lockhart (1981) found that a year after graduation, 58 percent of graduates of technical secondary schools were still looking for work.

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