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Testing Gaussianity and Linearity of Japanese Stock Returns

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Abstract. In this article, we first investigate the Gaussianity of Japanese stock return time series (214 daily, 18 weekly) by the Gaussianity test proposed by Kariya, Tsay, Terui and Li (1994) comprehensively and consistently. And it is observed that all the series are not Gaussian when the 6th order moment structures are taken into account. Up to the 4th order moments there are some series which are compatible with the Gaussianity. Secondly, we apply five well-known nonlinearity tests for stationary time series to the data set and examine the specific nonlinearity of the series. Some series strongly exhibit the specific types of nonlinearity. Typically the Nikkei daily index shows the TAR (Threshold Autoregressive) type nonlinearity. Comparing daily return series with weekly series, it is also shown that a central limit effect is working on the weekly stock returns, where daily information is accumulated over a week, in the sense that weekly returns are relatively closer to Gaussian.

Key words: Gaussianity, Hermitian transformation, nonlinearity test, stationary time series, stock return.

1. Introduction

In economic time series analysis, Gaussianity is often assumed for modeling and for constructing asymptotic tests or tabulating the critical values. In particular, in financial time series, the Gaussianity or equivalently the Brownianity in the case of continuous time plays an important role in modeling security prices. For example, in option theory, as is well known, the Black-Sholes stock option theory assumes that the log prices follow a Brownian process or equivalently that the stock returns follow a Gaussian process. Also the Vasicek interest rate model assumes the Brownianity or equivalently Gaussianity in discrete time. A third example is the well-known CAPM (Capital Asset Pricing Market) model, in which normality is assumed when the theory is derived. On the other hand, in deriving asymptotic tests based on likelihood such as Johansen's cointegration test and in tabulating the critical values for most asymptotic tests usually the Gaussianity is assumed or needed. Also in nonlinear time series models such as GARCH, the conditional Gaussianity gives an important conceptual vehicle for developing nonlinearity. However, when these models or tests or tables are applied, it is often the case that the Gaussianity (Brownianity) or conditional Gaussianity is not tested.

As a matter of fact, so far, there are only a few omnibus tests that directly detect the Gaussianity in time series. In the literature, the skewness and kurtosis tests are often applied as tests for Gaussianity. However, these tests assume the *i.i.d.* ness for time series and hence they are not valid as they stand when time series structure is involved. In addition, they are partial tests which check the consistency of the third and fourth moments of normality separately. The tests recently proposed by Kariya, Tsay, Terui and Li (1994), which is referred to as the KTTL test below, will be a most omnibus test for Gaussianity among others. It tests the consistency of all the moment structures of a time series with those of Gaussianity. As is well-known, Gaussianity implies linearity and hence nonlinearity implies non-Gaussianity. Therefore we may test the linearity of a series from the beginning. However, in testing linearity, we need a specification of the nonlinearity as an alternative hypothesis. Hence, the test depends on the specified alternative and in the tabulation of critical points Gaussianity is again needed. In fact, it is very difficult to construct an omnibus test for linearity. In addition, as has been pointed out, testing Gaussianity is important in applications as it stands.

In this paper, we first apply the KTTL test and investigate the Gaussianity of Japanese stock return series (214 daily, 18 weekly) comprehensively and consistently. And it is observed that all the series are not Gaussian when the 6th order moment structures are taken into account. Up to the 4th order moments there are some series which are compatible with the Gaussianity. Secondly, we apply to the data set the nonlinearity tests proposed by Tsay (1986, 1988, and 1989), Luukkonenn, Saikkonen and Teräsvirta (1988), Petrucceli and Davis (1986) and examine the specific nonlineary of the series. Some series strongly exhibit the specific types of nonlinearity. Typically the Nikkei daily index shows the TAR (Threshold Autoregressive) type nonlinearity. And we observe by the KTTL and nonlinearity tests that Toho Rayon Co. Ltd.'s return is very close to a Gaussian process.

In Section 2, the KTTL test is described. Our empirical results on the Gaussianity of Japanese stock returns are presented in Section 3. In Section 4, we apply the five nonlinearity tests and present some empirical observations.

2. The KTTL Test for Gaussianity

In this section, we describe the KTTL test of Gaussianity. To do so, we first summarize the KTTL test of multinormality, from which the KTTL test of Gaussianity follows.

Let $\mathbf{Y} = (y_1, y_2, \ldots, y_n)'$ follow multivariate normal distribution with mean vector $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)'$ and covariance matrix $\Sigma = (\sigma_{ij})$, and denote the standardized variate of y_i as $z_i = (y_i - \mu_i)/\sqrt{\sigma_{ii}}$. Define $w_j^{(p)} = h^{(p)}(z_j)$ as the pth order Hermitian polynomial of z_j for $p = 1, \ldots, P$ and $j = 1, \ldots, n$. For example, $w_j^{(1)} = z_j, w^{(2)} = (z_j^2 - 1)/\sqrt{2}, w_j^{(3)} = (z_j^3 - 3z_j)/\sqrt{6}, w_j^{(4)} = (z_j^4 - 6z_j^2 + 3)/\sqrt{24}$ and so on.

Let

$$\psi_{ij}^{(p,q)} = \mathrm{Cov}(w_i^{(p)}, w_j^{(q)}). \tag{1}$$

Then, under the null hypothesis of multinormality of $\mathbf{Y}$, it holds that for every $i, j = 1, \ldots, n$ and $p, q = 1, \ldots, P$, $E(w_i^{(p)}) = 0$ and $\psi_{ij}^{(pq)} = \phi_{ij}^{(pq)}$ with

$$\phi_{ij}^{(p,q)} = \begin{cases} \phi_{ij}^p & p = q, \\ 0 & p \neq q, \end{cases} \tag{2}$$

where $\phi_{ij} = \mathrm{Cov}(w_i^{(1)}, w_j^{(1)})$. This characterization is shown in Kendall and Stuart (1964; p. 600). Granger and Newbold (1986; p. 308) discussed its application to time series as a technique of instantaneous transformation.

Let $\mathbf{Y}_t, t = 1, \ldots, T$, be the vectors of observation of $\mathbf{Y}$ and define the sample version of z_i as $u_{it} = (y_{it} - \overline{y}_i)/\sqrt{s_{ii}}$, where $\overline{y}_i = (1/T)\Sigma_{t=1}^{T} y_{it}$ and $s_{ij} = (1/T)\Sigma_{t=1}^{T}(y_{it} - \overline{y}_i.)(y_{jt} - \overline{y}_j.)$. And denote the corresponding transformed variates of u_{it} via Hermitian polynomials as $v_{it}^{(p)} = h^{(p)}(u_{it})$ for $i = 1, \ldots, n$ and $p = 1, \ldots, P$. Under the assumption of the existence of the $2P$th order moments of $y_1, y_2, \ldots, y_n$, where P is the maximum order of the Hermite transformation under consideration and fixed in advance, two kinds of estimator for the covariance between $w_i^{(p)}$ and $w_j^{(q)}$ are used to construct an asymptotic test of multinormality. One is a consistent estimator only under the null hypothesis of multinormality, and the other is a consistent estimator under any arbitrary distribution. The former is given by

$$\widehat{\phi}_{ij}^{(p,q)} = \begin{cases} (\widehat{\phi}_{ij})^p & p = q, \\ 0 & p \neq q, \end{cases} \tag{3}$$

where $\widehat{\phi}_{ij}$ is the sample correlation coefficient between $z_i = w_i^{(1)}$ and $z_j = w_j^{(1)}$, and the latter is the conventional samples covariance estimate of $\psi_{ij}^{(p,q)}$;

$$r_{ij}^{(pq)} = \frac{1}{T}\sum_{t=1}^{T} v_{it}^{(p)} v_{jt}^{(q)}. \tag{4}$$

The proposed test employs the difference between (3) and (4) as a test statistic.

Define a $Pn \times Pn$ symmetric matrix $\mathfrak{R}$ by

$$\mathfrak{R} = \begin{bmatrix} \mathbf{R}^{(1,1)} & \mathbf{R}^{(1,2)} & \ldots & \mathbf{R}^{(1,P)} \\ \mathbf{R}^{(2,1)} & \mathbf{R}^{(2,2)} & \ldots & \mathbf{R}^{(2,P)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \mathbf{R}^{(P,1)} & \mathbf{R}^{(P,2)} & \ldots & \mathbf{R}^{(P,P)} \end{bmatrix}, \tag{5}$$

where the $\mathbf{R}^{(p,q)}$ is an $n \times n$ matrix whose (i,j) element is $r_{ij}^{(p,q)}$. Now compose $Pn(Pn+1)/2$ dimensional vector $\mathbf{r}_P$ as follows

$$\mathbf{r}^{(p,p)} = \mathrm{Vech}(\mathbf{R}^{(p,p)}): n(n+1)/2 \times 1, \tag{6}$$

$$\mathbf{r}^{(p,q)} = \mathrm{Vec}(\mathbf{R}^{(p,q)}): n^2 \times 1, \tag{7}$$

$$\mathbf{r}_{P1} = (\mathbf{r}^{(1,1)\prime}, \mathbf{r}^{(2,2)\prime}, \ldots, \mathbf{r}^{(P,P)\prime})': f_1 = Pn(n+1)/2 \times 1, \tag{8}$$

$$\begin{aligned} \mathbf{r}_{P2} &= (\mathbf{r}^{(1,2)\prime}, \mathbf{r}^{(1,3)\prime}, \ldots, \mathbf{r}^{(2,3)\prime}, \ldots, \mathbf{r}^{(P-1,P)\prime})': f_2 \\ &= P(P-1)n^2/2 \times 1, \end{aligned} \tag{9}$$

$$\mathbf{r}_P = (\mathbf{r}'_{P1}, \mathbf{r}'_{P2})': \quad f = f_1 + f_2 = Pn(Pn+1)/2 \times 1, \tag{10}$$

Here for any $n \times n$ symmetric matrix $\mathbf{A} = (a_{ij})$,

$$\mathrm{Vech}(\mathbf{A}) = (a_{11}, a_{12}, \ldots, a_{1n}; a_{22}, a_{23}, \ldots, a_{2n}; \ldots; a_{nn})$$

and for any $n \times n$ matrix $\mathbf{B} = (b_{ij})$,

$$\mathrm{Vec}(\mathbf{B}) = (b_{11}, b_{12}, \ldots, b_{1n}; b_{21}, b_{22}, \ldots, b_{2n}; \ldots; b_{n1}, b_{n2}, \ldots, b_{nn}).$$

Correspondently, by replacing $r_{ij}^{(p,q)}$ by $\psi_{ij}^{(p,q)}$, we can derive an $f \times 1$ parameter vector $\boldsymbol{\psi}_P$ under the alternative hypothesis. Also under the null hypothesis, in the same way we define, by replacing $r_{ij}^{(p,q)}$ by $\widehat{\phi}_{ij}^{(p,q)}$ and $\phi_{ij}^{(p,q)}$, we can define

$$\widehat{\boldsymbol{\phi}}_P = (\widehat{\boldsymbol{\phi}}'_{P1}, \mathbf{O}')' \quad \text{and} \quad \phi_P = (\phi'_{P1}, \mathbf{O}')', \tag{11}$$

which correspond to $\mathbf{r}_P$ and $\boldsymbol{\psi}_P$ respectively. Then, assuming the $2P$th order moment of y_{it}, $\mathbf{r}_P$ follows the asymptotic multivariate normal distribution,

$$\sqrt{T}(\mathbf{r}_P - \boldsymbol{\psi}_P) \sim N_f(\mathbf{O}, \boldsymbol{\Lambda}) \quad (\boldsymbol{\Lambda} \text{ is the function of } \boldsymbol{\psi}_P). \tag{12}$$

Similarly it follows from (3) under the null hypothesis that

$$\sqrt{T}(\widehat{\phi}_p - \phi_P) \sim N_f(\mathbf{O}, \boldsymbol{\Lambda}^\star) \quad (\boldsymbol{\Lambda}^\star \text{ is a function of } \phi_{ij}^{(1,1)}). \tag{13}$$

The KTL test detects the equivalence between $\boldsymbol{\psi}_P$ and $\boldsymbol{\phi}_P$ under the null hypothesis based on the difference of $\mathbf{r}_P$ and $\widehat{\boldsymbol{\phi}}_P = (\widehat{\boldsymbol{\phi}}'_{P1}, \mathbf{O}')$. As we have $\{n(n+1)/2\}$ equivalent relationships; $\widehat{\phi}_{ij}^{(1,1)} \equiv r_{ij}^{(1,1)}$, the test is $g = \{f - n(n+1)/2\}$ dimensional vector

$$\mathbf{C}_{P0} = \mathbf{C}_{P0}(\mathbf{r}_P) = (\mathbf{C}'_{P1}, \mathbf{C}'_{P2})', \tag{14}$$

where

$$\mathbf{C}_{P1} = (\mathbf{c}^{(2,2)\prime}, \ldots, \mathbf{c}^{(P,P)\prime})', \tag{15}$$

$$\mathbf{C}_{P2} = (\mathbf{c}^{(1,2)\prime}, \ldots, \mathbf{c}^{(1,P)\prime}; \ldots; \mathbf{c}^{(P-1,P)\prime})' \quad \text{and} \tag{16}$$

$$\mathbf{c}^{(p,q)} = \begin{cases} \mathbf{r}^{(p,p)} - \widehat{\boldsymbol{\phi}}^{(p,p)} & p = q, \\ \mathbf{r}^{(p,q)}, \quad p \neq q. \end{cases} \tag{17}$$

In fact, $\mathbf{C}_{P0}$ is expressed as

$$\mathbf{C}_{P0} = [\mathbf{O}\ \mathbf{I}](\mathbf{r}_p - \widehat{\phi}_P), \tag{18}$$

where $\mathbf{O}$ is the $(f - n(n+1)n/2) \times n(n+1)/2$ zero matrix, and $\mathbf{I}$ is the $(f - n(n+1)n/2)$ dimensional identity matrix. Under the null hypothesis, where $\psi_P = \phi_P$, it holds from (12) and (13) that

$$\sqrt{T}\mathbf{C}_{P0} \sim N_g(\mathbf{O}, \mathbf{J}(\phi_P)\boldsymbol{\Lambda}\mathbf{J}(\phi_P)'), \tag{19}$$

where the $(f - n(n+1)/2) \times f$ matrix $\mathbf{J}(\boldsymbol{\phi}_P)$ is

$$\begin{aligned} \mathbf{J}(\boldsymbol{\phi}_P) &= \left.\frac{\partial \mathbf{C}_{P0}}{\partial \mathbf{r}_P}\right|_{\mathbf{r}_P = \boldsymbol{\phi}_P} \\ &= \left.\left(\frac{\partial \mathbf{c}^{(2,2)}}{\partial \mathbf{r}_P}, \ldots, \frac{\partial \mathbf{c}^{(P,P)}}{\partial \mathbf{r}_P}; \frac{\partial \mathbf{c}^{(1,2)}}{\partial \mathbf{r}_P}, \ldots, \frac{\partial \mathbf{c}^{(P-1,P)}}{\partial \mathbf{r}_P}\right)\right|_{\mathbf{r}_P = \boldsymbol{\phi}_P} \end{aligned} \tag{20}$$

and the matrix differential is given by

$$\frac{\partial \mathbf{c}^{(p,q)}}{\partial \mathbf{r}_P} = \left(\frac{\partial c_{ij}^{(p,q)}}{\partial r_{kl}^{(a,b)}}\right). \tag{21}$$

The KTTL test statistic for multinormality is a Wald type chi-square test defined by

$$W_0 = T\mathbf{C}_{P0}'[\widehat{\mathbf{J}}\widehat{\boldsymbol{\Lambda}}\widehat{\mathbf{J}}']^{-1}\mathbf{C}_{P0}, \tag{22}$$

where $\widehat{\mathbf{J}} = J(\widehat{\boldsymbol{\phi}}_P)$ and $\widehat{\boldsymbol{\Lambda}} = (\widehat{\lambda}_{ij,kl}^{(pq,ab)})$ with

$$\widehat{\lambda}_{ij,kl}^{(pq,ab)} = \frac{1}{T}\sum_{i=1}^{T}(v_{it}^{(p)}v_{jt}^{(q)} - r_{ij}^{(pq)})(v_{kt}^{(a)}v_{lt}^{(b)} - r_{kl}^{(ab)}). \tag{23}$$

The KTTL test of Gaussianity for univariate series is a modification of the above test. Let $\{x_t\}$ be a univariate stationary process with $E(x_t) = \mu$ and

$\mathrm{Cov}(x_t, x_{t-k}) = \gamma_k$ and assume the mixing condition $\Sigma_{k=-\infty}^{\infty}|k||\gamma_k| < \infty$. Then the following two methods for constructing a test are proposed.

[I] Overlapping method

Set, for $i = 1, \ldots, n$.

$$y_{it} = x_{t-i+1}, \tag{24}$$

and corresponding to *i.i.d.* case, we define $z_{it} = (y_t - \mu)/\sqrt{\gamma_0} = (x_{t-i+1} - \mu)/\sqrt{\gamma_0}$.

[II] Non-overlapping method

For a given positive integer n and the realizations $\{x_1, \ldots, x_N\}$, set n-dimensional non-overlapping random vectors $\{y_t = (y_{1t}, \ldots, y_{nt})'\}$ with

$$y_{it} = x_{n(t-1)+i}, i = 1, \ldots, n; t = 1, \ldots, T, \tag{25}$$

where $T = [N/n]$ is the integer part of N/n. Then we define $z_{it} = (x_{n(t-1)+i} - \mu)/\sqrt{\gamma_0}$.

For both [I] and [II], it follows that $\mathrm{Cov}(z_{it}, z_{jt}) = \gamma_{i-j}/\gamma_0 = \rho_{i-j} = \phi_{ij}$. Defining $w_{it}^{(p)} = h^{(p)}(z_{it})$, just like *i.i.d.* case, under the Gaussianity of x_t, it holds that $\psi_{ij}^{(p,q)} = \phi_{ij}^{(p,q)}$ with

$$\phi_{ij}^{(pq)} = \begin{cases} (\phi_{ij})^p & p = q, \\ 0 & p \neq q. \end{cases} \tag{26}$$

The sample variates of z_{it} and $x_{it}^{(p)}$ are defined as $u_{it} = (y_{it} - \bar{y}_i)/\sqrt{s_{ii}}, v_{it}^{(p)} = h^{(p)}(u_{it})$ for $i = 1, \ldots, n$ and $p = 1, \ldots, P$, where $\bar{y}_i = (1/T)\Sigma_{t=1}^{T} y_{it}$ and $s_{ij} = (1/T)\Sigma_{t=\max(i,j)}^{T}(y_{it} - \bar{y}_i)(y_{jt} - \bar{y}_j)$. For time series data, the estimates corresponding to (3) and (4) should be respectively modified as

$$r_{ij}^{(pq)} = \frac{1}{T}\sum_{t=\max(i,j)}^{T} v_{it}^{(p)} v_{jt}^{(q)} \tag{27}$$

and

$$\hat{\phi}_{ij}^{(pq)} = \begin{cases} \hat{\rho}_{i-j}^{p} & p = q, \\ 0 & p \neq q, \end{cases} \tag{28}$$

for $i, j = 1, \ldots, n; p, q = 1, \ldots, P$. Under the Gaussianity of $\{x_t\}$ with the mixing condition above, Keenan (1983) proved the asymptotic normality that $\sqrt{T}(\mathbf{r}_P - \phi_P) \to N(\mathbf{O}, \mathbf{J}\Lambda\mathbf{J}')$. Therefore, exactly following the argument of the *i.i.d.* case, we obtain the asymptotically χ^2 test statistic (22) with d.f. $\{f - n(n+1)/2\}$ for testing the Gaussianity of a stationary time series.

The test in (22) is an omnibus test which detects departures from Gaussianity, and it can be decomposed into two parts. The first part tests departures from the even moment structure when $p = q$ and the second part tests departures from the odd moment structure when $p \neq q$. It is useful to have a separate test for each part when we are interested in the symmetry and the tail behavior of the underlying distribution separately. Corresponding to the dimensions of $\mathbf{C}_{P1}$ and $\mathbf{C}_{P2}$, the appropriate decomposition of $\boldsymbol{A}$ and $\mathbf{J}$ produces the following test statistics

$$W_1 = T\mathbf{C}'_{P1}[\widehat{\mathbf{J}}_{11}\widehat{\boldsymbol{A}}_{11}\widehat{\mathbf{J}}'_{11}]^{-1}\mathbf{C}_{P1}, \text{ and} \tag{29}$$

$$W_2 = T\mathbf{C}'_{P2}[\widehat{\boldsymbol{A}}_{22}]^{-1}\mathbf{C}_{P2}. \tag{30}$$

Under the null hypothesis of Gaussianity, the asymptotic distributions of these two test statistics are χ^2 with degrees of freedom $f_1 - n(n+1)/2$ and f_2 respectively. In fact, setting $\mathbf{A} = [\widehat{\mathbf{J}}\widehat{\boldsymbol{A}}\widehat{\mathbf{J}}']^{-1}$, and decomposing $\mathbf{A}$ appropriately according to the dimension of $\mathbf{C}_{P1}$ and $\mathbf{C}_{P2}$, then we have the following relationship among W_0, W_1 and W_2;

$$\begin{aligned} W_0 &= T\mathbf{C}'_{P0}[\widehat{\mathbf{J}}\widehat{\boldsymbol{A}}\widehat{\mathbf{J}}']^{-1}\mathbf{C}_{P0} \\ &\equiv T(\mathbf{C}'_{P1}, \mathbf{C}'_{P2}) \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{12} & \mathbf{A}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{C}_{P1} \\ \mathbf{C}_{P2} \end{pmatrix} \\ &= T\mathbf{C}'_{P1}\mathbf{A}_{11}\mathbf{C}_{P1} + T\mathbf{C}'_{P2}\mathbf{A}_{22}\mathbf{C}_{P2} + 2T\mathbf{C}'_{P1}\mathbf{A}_{12}\mathbf{C}_{P2} \\ &= W_1 + W_2 + 2\mathbf{C}_{12}, \end{aligned} \tag{31}$$

where $\mathbf{C}_{12} = T\mathbf{C}'_{P_1}\mathbf{A}_{12}\mathbf{C}_{P2}$. Now considering here $\mathbf{C}'_{P1}\mathbf{A}_{12}\mathbf{C}_{P2}/(\sqrt{W_1}\sqrt{W_2})$ may be regarded as a kind of correlation between $\mathbf{C}_{P1}$ and $\mathbf{C}_{P2}$. In fact, in the case where both $\mathbf{C}_{P1}$ and $\mathbf{C}_{P2}$ are one dimensional vector, $C_{12}/(\sqrt{W_1}\sqrt{W_2})$ is shown to be equivalent to negative correlation between C_1 and C_2.

3. Gaussianity of Japanese Stock Returns

In this section, we test the Gaussianity of Japanese stock return series by applying the KTTL tests to the following data sets

(A) 214 daily stock returns out of Nikkei 225 series in the Tokyo market (August 16, 1988 – October 1, 1993) and

(B) 18 daily and weekly stock returns (January 6, 1988 – October 31, 1989 for daily and July 19, 1988 – October 29, 1989 for weekly data). (See Table IX for specific names.)

Data set (A) is used for analyzing overall characteristics of Japanese stock returns and data set (B) is for the examination of effect of the change of observational frequency on time series structure of stock returns.

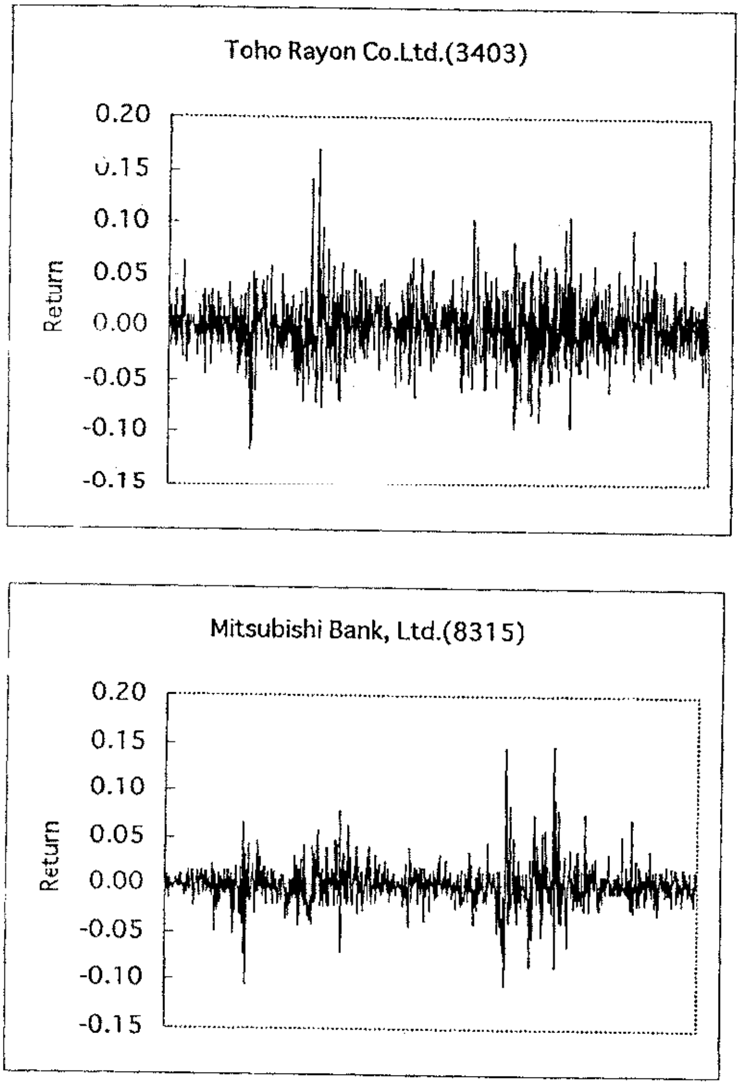

Figure 1. Time series plot of Japanese stock returns (89/8/16–93/10/1).

We define a stock return x_t at time t as

$$x_t = \log S_t - \log S_{t-1},$$

where S_t is a stock price at t. Figure 1 shows the time series plots of two stock return series from data set (A), which carry some specific features we will discuss.

3.1. NIKKEI 225 STOCK RETURNS

We use 214 stocks in the Tokyo market from August 16, 1989 to October 1, 1993 (1020 prices). The values of P (maximum degree of Hermitian transformation) and n (dimension of vector) are needed to be fixed in advance. We set all of the combinations for $P = 2, 3$ and $n = 1, 2, 3, 4$. The test with $P = 2$ compares the moment structure of $\{x_t\}$ up to the 4th order with that of Gaussianity. Therefore the odd order moment test W_1 and the even order moment test W_2 are respectively the counterparts of a skewness test and a kurtosis test in time series structure. The W_0 test deals with the both moment structures jointly. Similarly, the KTTL test with $P = 3$ compares the moment stucture up to the 6th order moments with that of Gaussianity of $\{x_t\}$ in the framework of hypothesis testing.

Table I shows the results of the tests, where $P = 2$ is fixed and $n = 1, 2, 3, 4$. The figures in the table denote the numbers and percentages of stock returns which are significant with 5% and 1%. Note that in case of $n = 1$, the overlapping method is equivalent to non-overlapping method by definition. We observed in case of $P = 3$ that the Gaussianity of all the stock returns were strongly rejected for every $n = 1, 2, 3, 4$, where it is not listed here for an obvious reason.

In the followings, we enumerate the empirical findings from overall and individual results of the KTTL tests.

Observations from Overall Testing Results

(1) The W_0, W_1 and W_2 tests with $P = 3$ reject the Gaussianity of all the stock returns significantly (less than 1%). Hence, the moment structures of all the stock returns including up to the 6th order are not consistent with those of the Gaussianity.

(2) We observe the followings from the results with $P = 2$.

(a) For any stock return series, the Gaussianity tends to be rejected more significantly as the dimension n increases.
(b) All the p-values of the W_1 test with more than $n = 1$ are less than 1%. Hence, the even order moment structures of all the stock returns are not consistent with those of a Gaussian process, even if we confine the order of the moments to the 4th order.
(c) As for the W_2 test, in case of $n = 1$, some returns are not incompatible with Gaussianity, although the number of the series decreases greatly when $n \geqslant 2$.
(d) The results of the W_0 test has a similar tendency with those of the W_2 test. However, the rate of the decrease is not so large as the case of the W_2 test.

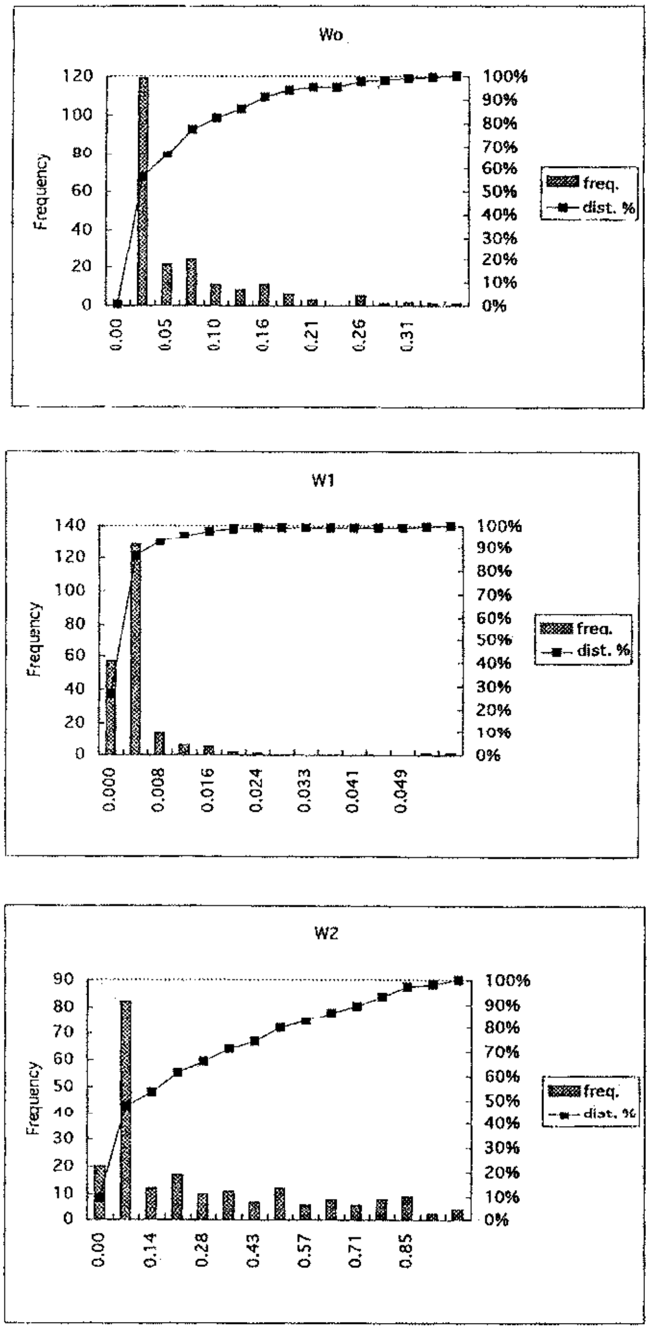

Figure 2. The distribution of P-values of KTTL tests (O-21).

Table I. KTTL Test ($P = 2; n = 1, 2, 3, 4$): Nikkei 225

$P = 2$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
		W_0 Test(5%)		
Overlapping	75(35.0%)	31(14.5%)	15(7.0%)	14(6.5%)
Non-overlapping	75(35.0%)	74(34.6%)	45(21.0%)	34(15.9%)
		W_1 Test(5%)		
Overlapping	2(0.9%)	0(0%)	0(0%)	0(0%)
Non-overlapping	2(0.9%)	0(0%)	0(0%)	0(0%)
		W_2 Test(5%)		
Overlapping	119(55.6%)	69(32.2%)	22(10.3%)	8(3.7%)
Non-overlapping	119(55.6%)	53(24.8%)	6(2.8%)	1(0.5%)

Each figure means the number of stock returns whose p-values are greater than 5%. The ratio is written in parenthesis.

$P = 2$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
		W_0 Test(1%)		
Overlapping	131(61.2%)	80(37.4%)	52(24.3%)	36(16.8%)
Non-overlapping	131(61.2%)	148(69.2%)	121(56.5%)	74(34.6%)
		W_1 Test(1%)		
Overlapping	13(6.1%)	0(0%)	0(0%)	0(0%)
Non-overlapping	1(0.5%)	0(0%)	0(0%)	0(0%)
		W_2 Test(1%)		
Overlapping	144(67.3%)	97(45.3%)	35(16.4%)	11(5.1%)
Non-overlapping	144(67.3%)	70(32.7%)	19(8.9%)	1(0.5%)

Each figure means the number of stock returns whose p-values are greater than 5%. The ratio is written in parenthesis.

Observations from Individual Testing Results

$\langle P = 2, n = 1 \rangle$

(1) Judging from the results of the W_0 test, the Gaussianity for many stock returns are rejected. However there still remain some return series which are consistent with Gaussianity. The 5 largest p-values are, 0.362 (No. 4001 (Mitsui Toatsu Chemicals, Inc.)), 0.329 (No. 1925 (Daiwa House Co. Ltd.)), 0.307 (No.

Table II. Stock return with P-value greater than 5%(O-24 and N-24)

			O-24				
Code No.	Code Name	W_0	$P(W_0)$	Code No.	Code Name	W_2	$P(W_2)$
9062	Nippon Express, Co. Ltd.	24.873	0.526	2602	Nissin Oil Mills, Ltd.	18.126	0.317
1331	Nichiro Corp.	29.514	0.288	8053	Sumitomo Corp.	18.998	0.269
3861	Oji Paper, Co. Ltd.	31.804	0.200	7205	Hino Motors, Ltd.	20.672	0.191
2801	Kikkomann Corp.	32.599	0.174	8583	Nippon Shinpan Co. Ltd.	21.189	0.171
7004	Hitati Zosen Corp.	34.348	0.126	2502	Asahi Breweries, Ltd.	21.965	0.144
4061	Denkikagaku Kogyo K.K.	34.661	0.119	8313	Bank of Tokyo, Ltd.	23.796	0.094
7012	Kawasaki Heavy Industry, Ltd.	34.830	0.115	5981	Tokyo Rope Mfg, Co. Ltd.	24.704	0.075
5802	Sumitomo Electric Industry, Ltd.	35.425	0.103	3865	Hokuetu Paper Mills, Ltd.	24.884	0.072
5701	Nippon Light Metal, Co. Ltd.	36.062	0.091	—	—	—	—
8511	Japan Security Finance, Co. Ltd.	36.287	0.087	—	—	—	—
4063	Shin-Etu Chemical Co. Ltd.	36.369	0.085	—	—	—	—
3101	Toyobo, Co. Ltd.	37.220	0.071	—	—	—	—
7202	Isuzu Motors, Ltd.	37.925	0.062	—	—	—	—
4902	Konica Corp.	38.648	0.053	—	—	—	—
			N-24				
Code No.	Code Name	W_0	$P(W_0)$	Code No.	Code Name	W_2	$P(W_2)$
2536	Sanraku, Ltd.	18.074	0.873	2533	Goudou Liquor, Co. Ltd.	25.702	0.058
5715	Furukawa, Co. Ltd.	23.321	0.615	—	—	—	—
2801	Kikkoman Corp.	23.842	0.585	—	—	—	—
2108	Nippon Beet Sugar Mfg. Co. Ltd.	25.362	0.499	—	—	—	—
3201	Nihon Woolen Co. Ltd.	28.258	0.346	—	—	—	—
5479	Nippon Metal Industry, Co. Ltd.	28.316	0.343	—	—	—	—
8511	Japan Security Finance Co. Ltd.	29.200	0.302	—	—	—	
8751	Tokyo Marine & Fire Insurance Co. Ltd.	31.076	0.225	—	—	—	—

Table II. (contd.) Stock return with P-value greater than 5% (O-24 and N-24)

6474	Nachi-Fujikoshi Corp.	31.157	0.222	—	—	—	—
6503	Mitubishi Electric Corp.	31.470	0.211	—	—	—	—
3403	Toho Rayon Co.Ltd.	32.274	0.184	—	—	—	—
6103	Okuma Machinery Works, Ltd.	33.761	0.141	—	—	—	—
6752	Matushita Electric Industrial Co. Ltd.	33.879	0.138	—	—	—	—
3104	Fuji Spinning Co. Ltd.	33.894	0.138	—	—	—	—
5706	Mitsui Mining & Smelting Co. Ltd.	34.035	0.134	—	—	—	—
8604	Nomura Securities, Co. Ltd.	34.489	0.123	—	—	—	—
6472	NTN Corp.	34.888	0.114	—	—	—	—
8231	Mitukoshi Ltd.	35.247	0.106	—	—	—	—
5108	Bridgestone Corp.	35.656	0.098	—	—	—	—
3101	Toyobo Co. Ltd.	35.793	0.096	—	—	—	—
8031	Mitsui & Co. Ltd.	36.387	0.085	—	—	—	—
8058	Mitsubishi Corp.	36.389	0.085	—	—	—	—
4401	Asahi Denka Kogyo K.K.	36.534	0.082	—	—	—	—
4061	Denki Kagaku Kogyo K.K.	37.132	0.073	—	—	—	—
6501	Hitati Ltd.	37.268	0.071	—	—	—	—
9009	Keisei Electric Railway Co. Ltd.	37.326	0.070	—	—	—	—
6703	Oki Electric Industry Co. Ltd.	37.418	0.069	—	—	—	—
3404	Mitubishi Rayon Co. Ltd.	37.712	0.064	—	—	—	—
6508	Meidensha Corp.	37.834	0.063	—	—	—	—
1331	Nichiro Corp.	38.077	0.060	—	—	—	—
5632	Mitsubishi Steel Mfg. Co. Ltd.	38.647	0.053	—	—	—	—
6758	SONY Corp.	38.818	0.051	—	—	—	—
5233	Onoda Cement Co. Ltd.	38.880	0.050	—	—	—	—
6701	NEC Corp.	38.919	0.050	—	—	—	—

Table III. Specific Cases: W_0 is not significant and $W_i (i = 1, 2)$ are significant

Code No.	Code Name	W_0	$P(W_0)$	W_1	$P(W_1)$	W_2	$P(W_2)$	$2C_{12}$
4001	Mitsui Toatsu Chemicals, Inc.	2.030	0.362	37.214	0.000	37.335	0.000	−72.518
1925	Daiwa House Industry Co. Ltd.	2.226	0.329	33.213	0.000	23.670	0.000	−54.657
3402	Toray Industries, Ltd.	2.423	0.298	25.554	0.000	23.746	0.000	−46.877
6758	Sony Corp.	2.773	0.250	21.721	0.000	19.762	0.000	−38.709
4063	Shin-Etsu Chemical Co. Ltd.	2.898	0.235	24.092	0.000	17.748	0.000	−38.941
5201	Asahi Glass Co. Ltd.	2.901	0.234	24.150	0.000	13.168	0.000	−34.417
6702	Fujitsu, Ltd.	3.378	0.185	17.444	0.000	23.753	0.000	−37.820
9532	Osaka Gas Co. Ltd	3.506	0.173	18.294	0.000	12.178	0.000	−26.967
2201	Morinaga & Co. Ltd.	3.714	0.156	20.862	0.000	14.899	0.000	−32.047
3103	Unitika, Ltd.	3.783	0.151	25.601	0.000	20.763	0.000	−42.581
7202	Isuzu Motors, Ltd.	4.021	0.134	28.761	0.000	25.979	0.000	−50.719
7751	Cannon Inc.	4.202	0.122	49.380	0.000	62.943	0.000	−108.120
8031	Mitsui & Co. Ltd.	4.403	0.111	14.731	0.000	14.123	0.000	−24.451

Table IV. Correlation between test statistics

KTTL test	(W_0, W_1)	(W_0, W_2)	(W_1, W_2)
O-21	0.219	−0.312	0.723
O-22	−0.106	−0.402	0.854
O-23	−0.185	−0.374	0.906
O-24	−0.238	−0.380	0.925
N-22	−0.198	−0.295	0.957
N-23	0.013	−0.094	0.943
N-24	0.185	0.106	0.973

Table V. KTTL test for squared stock return

OS-21			
Code No.	Code Name	W_0	$P(W_0)$
4001	Mutsui Toatsu Chemicals, Inc.	3.252	0.197
5202	Nippon Sheet Glass, Co. Ltd.	4.115	0.128
9202	All Nippon Airways, Co. Ltd.	4.607	0.100
5005	Tonen Corp.	4.762	0.092
2802	Ajinomoto, Co. Ltd.	4.937	0.085
7751	Canon, Inc.	5.327	0.070
5002	Showa Shell Oil, Co. Ltd.	5.328	0.070
5901	Toyo Seikan Kaisha, Ltd.	5.352	0.069
3101	Toyobo, Co. Ltd.	5.704	0.058
4063	Shin-Etsu Chemical, Co. Ltd.	5.971	0.051
7752	Richo, Co. Ltd.	5.976	0.050

OS-22			
Code No.	Code Name	W_0	$P(W_0)$
4061	Denkikagaku Kogyo K.K.	10.961	0.140
4208	Ube Industries, Ltd.	12.401	0.088
8058	Mitsubishi Corp.	13.495	0.061
5631	Japan Steel Works, Ltd.	13.510	0.061

NS-22			
Code No.	Code Name	W_0	$P(W_0)$
2802	Ajinomoto, Co. Inc.	10.654	0.154

5802 (Sumitomo Electric Industries, Ltd.)), 0.298 (No. 3402 (TORAY Industries, Inc.)) and 0.281 (No. 9602 (Nippon Expresses Co. Ltd.)), where the number in the first parenthesis is the stock code number in the Tokyo stock market and the name of the company appears in the second parenthesis.

(2) The W_1 tests reject the Gaussianity of most of stock returns. The p-values greater than 5% are, 0.057 (No. 5805 (Showa Electric Wire & Cable Co. Ltd.)) and 0.050 (No. 7912 (Dainippon Printing Co. Ltd.)).

(3) The W_2 tests can not reject the Gaussianity for many stock returns. The 5 largest p-values are 0.977 (No. 4021 (Nissan Chemical Industrial Co. Ltd.)), 0.995 (No. 2502 (Asahi Breweries, Ltd.)), 0.985 (No. 5351 (Shinagawa Refractories Co. Ltd.)), 0.982 (No. 5701 (Nippon Light Metal Co. Ltd.)) and 0.903 (No. 5001 (Nippon Oil Co. Ltd.)).

Figure 2 shows the histograms and the empirical distribution functions for the p-values of these tests.

$\langle P = 2, n = 2, 3, 4\rangle$

For notation, we define O-ij as the KTTL test with $P = i$ and $n = j$ by overlapping method and N-ij as the corresponding test by the non-overlapping method. Note that O-21 is equivalent to N-21 by definition.

The results of the omnibus test W_0 and the marginal moment tests W_1 and W_2 are of the same features as those of the O-21, irrespective of overlapping or non-overlapping methods. That is, as P and n increase, the null hypothesis of Gaussianity tends to be rejected more significantly. Table II shows the names of the stocks with the p-values which are greater than 5%. The W_1 test rejects all the series when $n = 2$. This does not change for the overlapping and non-overlapping methods.

From Table II, we can observe that the two methods of the KTTL test do not always produce the same results. However, the stock series, whose Gaussianities are not rejected by the both methods for the W_0 test, are No. 1331 (Nichiro Corp.), No. 2801 (Kikkoman Corp.), No. 4061 (Denki Kagaku Kogyo K.K.), No. 8511 (Japan Securities Finance Co. Ltd.) and No. 3101 (Toyobo Co. Ltd.).

$\langle P = 3\rangle$

All the tests with $P = 3$ have p-values less than 1% for the both methods.

Specific Cases: W_0 is not Significant and $W_i's (i = 1, 2)$ are Significant

There are some cases where the marginal tests W_1 and W_2 reject the null hypothesis of Gaussianity but the omnibus test W_0 does not reject it. In Table III, for O-21, we tabulate the name of the stocks with the p-values which satisfy

$$P(W_0) \geqslant 0,1 \quad \text{and} \quad \{P(W_i) < 10^{-4}, i = 1, 2\},$$

where $P(W_i), i = 0, 1, 2$ is the p-value of the W_i test. From (31), these cases can happen when $2C_{12}$ takes a large negative value.

Next we examine the relationship between the test statistics. Table IV shows pairwise correlations among W_0, W_1 and W_2 in the cases of $P = 2$ and $n = 1, 2, 3, 4$ for overlapping and non-overlapping methods.

From Table IV, we observe

(1) high positive correlations between the marginal moment tests W_1 and W_2, and
(2) many negative correlations between the omnibus test and marginal moment tests.

These observations accommodate the existence of specific cases pointed out above and in such cases the omnibus test performs rather poorly. However we can improve it by increasing the order P of Hermitian transformation.

Squared Process of Stock Returns $\{x_t^2\}$

Next, we apply the KTTL test to the squared process of stock returns $\{x_t^2\}$. When we regard x_t^2 as a volatility, it turns out to be a test for the Gaussianity of the volatilities. From the result of $P = 2, n = 1$ in Table V, we observe the following

(1) Both the W_1 and W_2 tests reject the Gaussianity strongly for all squared series. In fact, the p-values are less than 10^{-4}. The minimum value of W_1 is 67.8 (No. 8236 (Maruzen Co. Ltd.)) and the minimum value of W_2 is 149.3 (No. 8314 (Mitsui Taiyo Kobe Bank, Ltd.)).
(2) Comparing the W_1 test with the W_2 test, the even order moment test W_1 rejects the Gaussianity more significantly. This is the same result as $\{x_t\}$ process.
(3) In case of the W_0 test, the number of statistically significant squared series decreases drastically. That is, the number of tests with p-values greater than 5% is 76 for $\{x_t\}$ process and 11 for $\{x_t^2\}$.

Next we summarize the results when P and n get larger for the two methods. The OS-ij and NS-ij means the KTTL test for squared process with overlapping method and non-overlapping method for $P = i, n = j$ respectively. In the first, as for the test with $P, n \geqslant 2$, all the tests, W_1, W_2 and W_0 reject the null hypothesis strongly. In fact, there are no case with the p-value greater than 5%. Table V tabulates the name of squared series whose p-values are greater than 5%.

Judging from the results here as a whole, the Gaussianity for all of squared series are strongly rejected, as P and n get larger, and the number of series increases more rapidly than $\{x_t\}$ process.

3.2. DAILY AND WEEKLY STOCK RETURNS

In this subsection, we examine the effect of observation interval (frequency) on the Gaussianity of stock return series. We use 18 stocks in the Tokyo stock market from January 6, 1988 to October 31, 1989 for daily data (480 prices) and from July

19, 1987 to October 29, 1989 (120 price of Monday) for weekly data.

Figure 3 shows the time series plot of stock prices and returns for TOPIX and Nikkei 225 stock index.

In table VI, the results of the KTTL omnibus test W_0 applied to these data are listed.

As in this table, the Gaussianity of the following 4 stock return series are not rejected at 5% level even for $n = 4$, where these 4 series are those with the 4 largest averaged p-values over $n = 1, 2, 3, 4$.

(1) Weekly series of 'Mitsubishi Metal Corp.' However, in case of $P = 3$, the strong non-Gaussianity: Maximum of p-value is 0.000054 for $n = 1$.
(2) Daily series of 'Toray Industries, Inc.' However, in case of $P = 3$, the strong non-Gaussianity: Maximum of p-value is 0.000616 for $n = 1$.
(3) Weekly series of 'Kajima Corp.' However, in case of $P = 3$, non-Gaussianity: p-values is 0.01434 for $n = 1$ and 0.000007 for $n = 2$.
(4) Weekly series of 'Kirin Brewery Co. Ltd.' However, in case of $P = 3$, non-Gaussianity: p-value is 0.000153 for $n = 1$.

As a whole, we can safely conclude that, given P, the hypothesis of Gaussianity tends to be rejected more strongly as n gets larger, and in case of $P = 3$, the Gaussianity is completely rejected for all the series, especially in case of $n \geqslant 2$.

Table VII shows the number of the stocks which are not rejected with 5% and 1%. For example, in case of $P = 2, n = 1$, the number of the stocks which have the p-value greater than 5% is 10 out of 18, and 16 returns out of 18 have the p-values greater than 1%.

Comparing the results between daily and weekly series, we find the followings

(1) $P = 2$: For 1% level, the number of the weekly series which are not rejected is greater than that of the daily series. The same observation follows for 5% level except for $n = 4$.
(2) $P = 3$: The results with $n \geqslant 2$ indicate the non-Gaussianity for both of the daily and weekly returns. In case of $n = 1$, we can see the same tendency as (1).

From these observations, even for our tests we can reach the same conventional conclusion that, probably due to a central limit effect, a weekly series is more compatible with the Gaussianity relative to its daily series. However, our results demonstrate the non-Gaussianity of a weekly series when the 6th order moments are taken into accounts.

4. Some Nonlinearity Tests

In this section, we apply some nonlinearity tests to data sets used in Section 3.

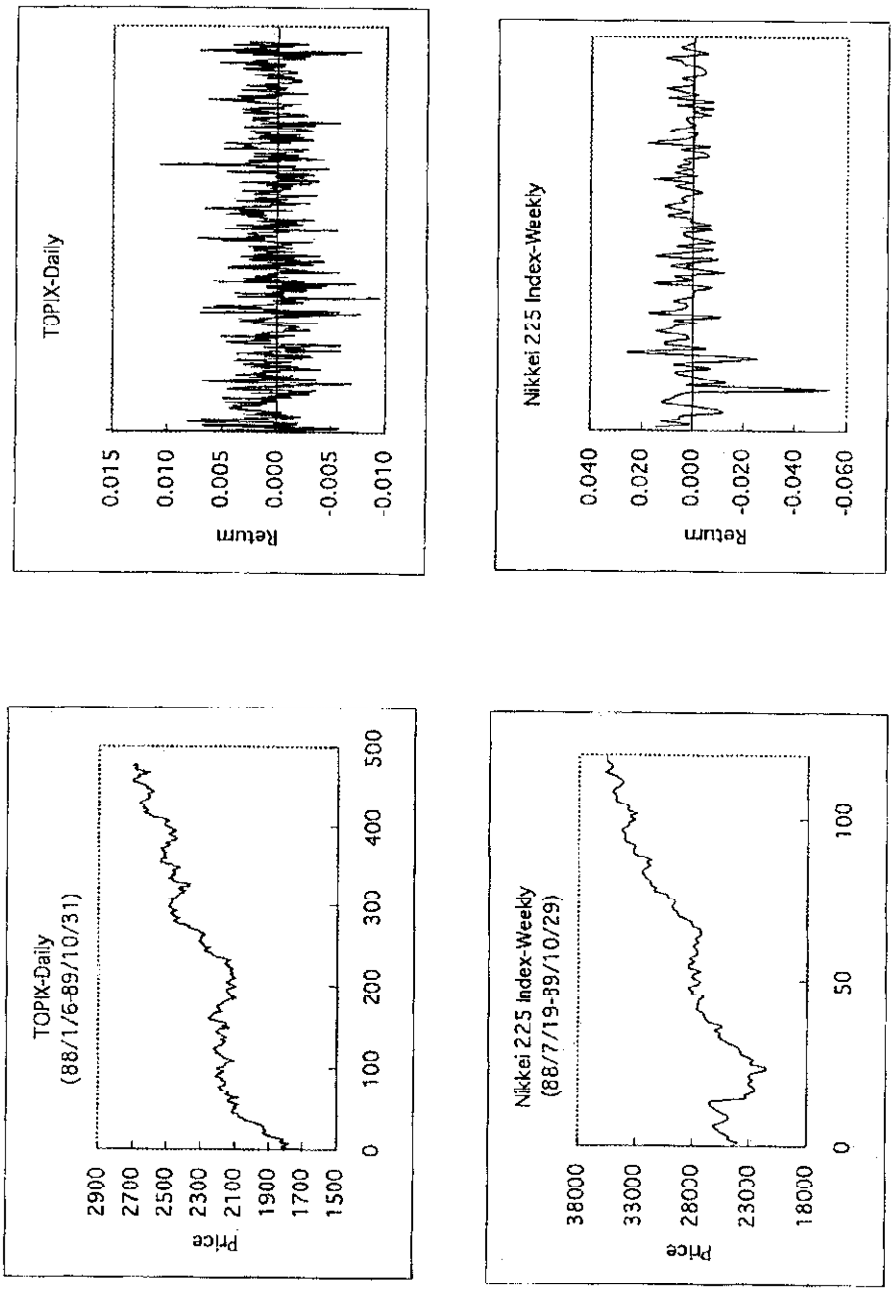

Figure 3. Time series plot of price and return.

Table VI. KTTL test ($P = 2; n = 1, \ldots, 4$): daily and weekly data

		Daily data ($P = 2$)							
		$n = 1$(d.f.2)		$n = 2$(d.f.7)		$n = 3$(d.f.15)		$n = 4$(d.f.26)	
No.	Company Name	W_0	$P(W_0)$	W_0	$P(W_0)$	W_0	$P(W_0)$	W_0	$P(W_0)$
1	Topix	1.622	0.444426	14.826	0.038299	34.194	0.003199	52.686	0.001482
2	Nikkei 225 Stock Index	3.468	0.176594	16.320	0.022351	30.914	0.009019	50.199	0.002977
3	Kajima Corp.	4.835	0.089161	11.092	0.134669	20.497	0.153684	41.973	0.024706
4	Kirin Brewery Co. Ltd.	5.656	0.059117	15.052	0.035344	25.256	0.046609	34.814	0.115740
5	Toray industries, Inc.	4.143	0.125982	9.629	0.210574	15.535	0.413626	26.267	0.448532
6	Oji Paper Co. Ltd.	7.109	0.028602	18.367	0.010342	30.763	0.009449	41.882	0.025245
7	Takeda Chemical Industries, Ltd.	7.771	0.020533	18.939	0.008381	32.492	0.005515	59.972	0.000169
8	Nippon Oil Co. Ltd.	9.345	0.009348	20.251	0.005052	31.264	0.008097	52.534	0.001547
9	Asahi Glass Co. Ltd.	6.852	0.032519	21.369	0.003261	33.931	0.003482	49.512	0.003593
10	Nippon Steel Corp.	5.303	0.070552	14.036	0.050543	30.922	0.008999	54.616	0.000849
11	Mitsubishi Metal Corp.	6.054	0.048470	12.729	0.079002	20.316	0.160154	37.300	0.070231
12	Komatsu, Ltd.	10.487	0.005282	20.714	0.004217	30.829	0.009259	52.842	0.001417
13	Hitati, Ltd.	6.703	0.035038	14.330	0.045608	24.272	0.060621	38.142	0.058748
14	NEC Corp.	5.602	0.060747	16.318	0.022367	25.356	0.045362	43.386	0.017580
15	Matsushita Electric Industrial Co. Ltd.	11.566	0.003079	26.357	0.000435	51.450	0.000007	74.321	0.000002
16	Mitsubishi Heavy Industries, Ltd.	3.925	0.140472	13.685	0.057067	25.220	0.047070	38.001	0.060543
17	Toyota Motor Corp.	4.296	0.116728	11.102	0.134244	22.519	0.094894	43.275	0.018067
18	Dainippon Printing Co. Ltd.	4.437	0.108792	13.695	0.056888	28.031	0.021373	44.184	0.014441

Table VI. (contd.) KTTL test ($P = 2; n = 1, \ldots, 4$): daily and weekly data

		Weekly data ($P = 2$)							
		$n = 1$(d.f.2)		$n = 2$(d.f.7)		$n = 3$(d.f.15)		$n = 4$(d.f.26)	
No.	Company Name	W_0	$P(W_0)$	W_0	$P(W_0)$	W_0	$P(W_0)$	W_0	$P(W_0)$
1	Topix	1.516	0.468546	8.514	0.289465	24.514	0.056868	66.633	0.000020
2	Nikkei 225 Stock Index	1.541	0.462859	9.015	0.251561	35.539	0.002059	89.709	0.0–
3	Kajima Corp.	2.351	0.308705	6.518	0.480780	14.715	0.472114	35.288	0.105563
4	Kirin Brewery Co. Ltd.	4.697	0.095510	9.420	0.223889	20.402	0.157053	37.169	0.072181
5	Toray industries, Inc.	5.856	0.053492	14.744	0.039419	36.968	0.001279	106.451	0.0–
6	Oji Paper Co. Ltd.	5.598	0.060882	12.135	0.096201	25.074	0.048957	50.274	0.002915
7	Takeda Chemical Industries, Ltd.	6.212	0.044789	13.702	0.056741	29.895	0.012304	56.209	0.000530
8	Nippon Oil Co. Ltd.	4.637	0.098409	13.182	0.067788	29.101	0.015608	49.012	0.004117
9	Asahi Glass Co. Ltd.	4.199	0.122532	8.916	0.258769	32.932	0.004796	93.831	0.0–
10	Nippon Steel Corp.	2.159	0.339702	10.059	0.185280	21.863	0.111460	40.718	0.033118
11	Mitsubishi Metal Corp.	3.702	0.157082	9.736	0.204064	19.492	0.192287	42.461	0.021994
12	Komatsu, Ltd.	5.284	0.071220	15.458	0.030561	28.071	0.021131	55.263	0.000720
13	Hitati, Ltd.	5.523	0.063195	15.822	0.026797	27.140	0.027619	70.252	0.000006
14	NEC Corp.	4.530	0.103829	15.684	0.028161	33.766	0.003673	177.818	0.0–
15	Matsushita Electric Industrial Co. Ltd.	6.164	0.045866	23.028	0.001685	66.970	0.0–	138.566	0.0–
16	Mitsubishi Heavy Industries, Ltd.	0.589	0.744901	4.283	0.746616	7.895	0.927917	19.232	0.826488
17	Toyota Motor Corp.	6.834	0.032804	32.710	0.000030	63.895	0.0–	118.441	0.0–
18	Dainippon Printing Co. Ltd.	5.516	0.063411	14.165	0.048318	29.427	0.014165	79.521	0.0–

Table VII. Difference between daily and weekly data

	Daily							
	$n=1$		$n=2$		$n=3$		$n=4$	
	5%	1%	5%	1%	5%	1%	5%	1%
$P=2$	10	16	7	11	5	7	8	3
$P=3$	1	2	0	0	0	0	0	0
	Weekly							
$P=2$	15	16	11	14	6	10	3	5
$P=3$	4	6	0	0	0	0	0	0

The figure means the number of not significant with 5% out of 18 returns.

Let

$$X_t = h(X_{t-1}, X_{t-2}, \dots, X_{t-p}) + e_t \tag{32}$$

be an autoregressive nonlinear time series model, where $\{e_t\}$ is *i.i.d.* with mean zero. If we assume the innovation e_t as Gaussian, nonlinearity test is equivalent to a Gaussianity test with a specific alternative hypothesis.

Here we use the five well-known nonlinearity tests

(i) Ori-F test by Tsay (1986);
(ii) Aug-F test by Luukkonenn, Saikkonen and Teräsvirta (1988);
(iii) CUSUM test by Petruccelli and Davis (1986);
(iv) TAR-F test by Tsay (1989);
(v) New-F test by Tsay (1988).

All of these test set up, as a null hypothesis, a linear process. Based on the Volterra expansion of (32) around $\mathbf{O} = (0, 0, \dots)'$

$$x_t = \mu + \sum_{u=1}^{\infty} \psi_u x_{t-u} + \sum_{u,v=1}^{\infty} \psi_{uv} x_{t-u} x_{t-v} + \sum_{u,v,w=1}^{\infty} \psi_{uvw} x_{t-u} x_{t-v} x_{t-w} + \cdots + e_t, \tag{33}$$

where

$$\mu = h(\mathbf{O}), \qquad \phi_u = \left.\frac{\partial h}{\partial x_{t-u}}\right|_O, \qquad \phi_{uv} = \left.\frac{\partial^2 h}{\partial x_{t-u} \partial x_{t-v}}\right|_O,$$

$$\phi_{uvw} = \left.\frac{\partial^3 h}{\partial x_{t-u} \partial x_{t-v} \partial x_{t-w}}\right|_O,$$

the Ori-F and Aug-F tests detect against the nonlinearity of the second and third order polynomials respectively. The CUSUM, TAR-F and New-F tests assume the threshold type nonlinear alternatives;

$$x_t = \beta_0^{(j)} + \sum_{i=1}^{p} \beta_i^{(j)} x_{t-i} + a_i^{(j)} \quad (j = 1, 2), \tag{34}$$

where $\{a_t^{(j)}\}$ is the innovation of mean zero and variance σ_j^2. The New-F test covers the most extensive alternatives of nonlinearity, including ExpAR (Exponential Autoregressive) and bilinear models in addition to (34). The detailed procedures and distributional properties regarding these tests are found in Granger and Teräsvirta (1993).

In order to implement these tests, the order p of autoregression for all the tests and the value of delay parameter d for the tests (iii), (iv), and (v) need to be specified. We set the maximum of p as 10 and let d run from 1 to 10. Each of nonlinearity tests with different set of (p, d) brings out different results. We employ the most significant result of the test among all the combinations of (p, d).

4.1. NIKKEI 225 SERIES

Figure 4 shows the histograms and the empirical distribution functions of p-values of nonlinearity tests applied to 214 stock returns from Nikkei 225 (Data set A).

From Figure 4, we observe that

(1) many stock returns are not consistent with linearity,
(2) the New-F test rejects linearity most frequently and the CUSUM test rejects it least frequently,
(3) the results of the Ori-F and Aug-F tests are similar,
(4) the p-values of the CUSUM test have the largest variance,
(5) the average of p-values of the Ori-F and CUSUM tests is over ten times as those of other tests.

The observation (3) can be interpreted by the similarity of the both tests, because the Ori-F test includes the 2nd order polynomial terms for nonlinearity and the Aug-F test includes up to the 3rd order polynomials. With respect to (4), the p-values of the CUSUM test have a different shape of distribution function compared with those of the four other tests. The simulation studies by Tsay (1988, 1989) indicate the low power of CUSUM test and our results accommodate it as an empirical evidence of it by the observation (4). Therefore we leave out the results of CUSUM test below.

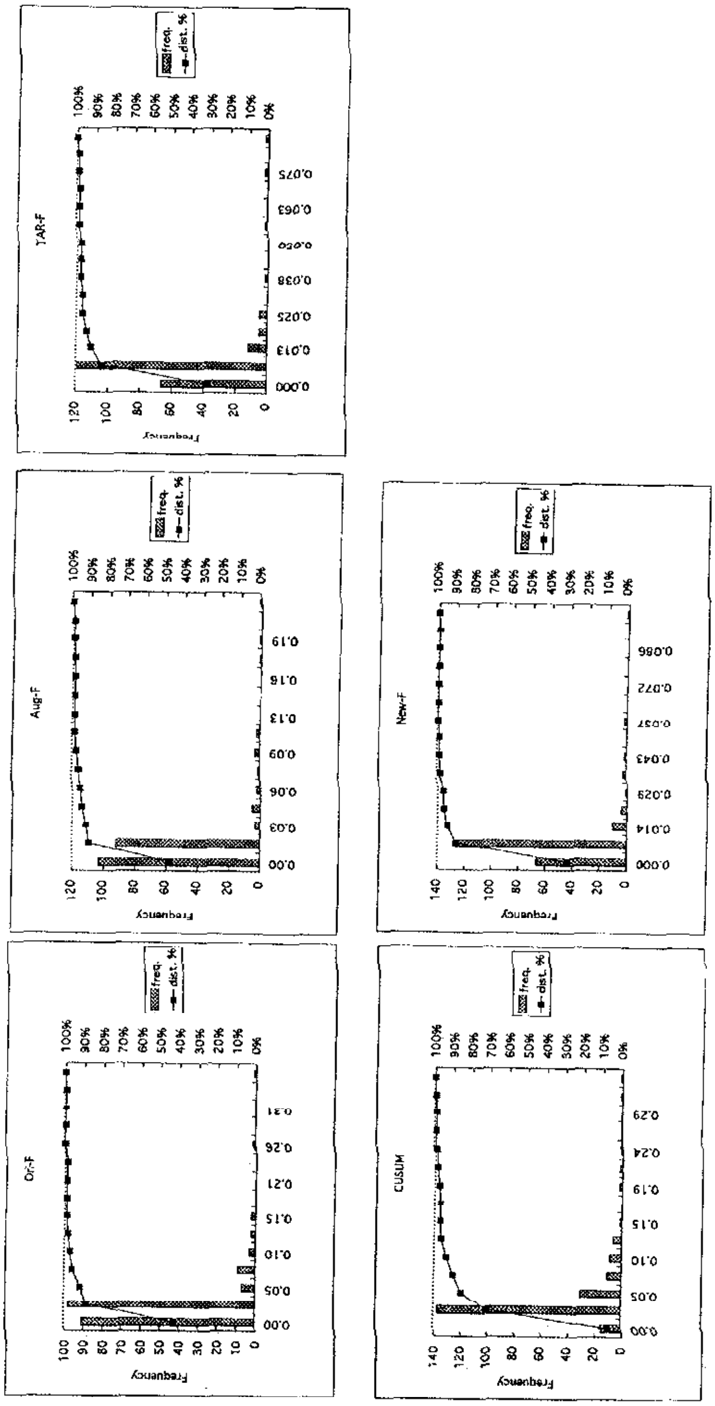

Figure 4. Distribution of P-values of non-linear tests.

Table VIII. Stock returns with P-value greater than 5 percent by nonlinearity tests

Ori-F		Aug-F		TAR-F		CUSUM		New-F	
Name	P-value	Name	P-value	Name	P-value	Name	P-value	Name	P-value
Sanyo Electric	0.35994	Meidensya	0.22129	Asahi Brewery	0.08781	Mitsui Mining	0.33950	Toho Rayon	0.10021
Kyowa Hakko	0.25477	Kyowa Hakko	0.17488	Nec	0.08468	Sankyo	0.30215	Richo	0.05456
Hino Motors	0.13884	Sanyo Electric	0.15942	Dainippon Pharmaceutical	0.07478	Toyobo	0.24170	Yokogawa Electric	0.05428
Sharp	0.13643	Nec	0.10196	Toho Rayon	0.07375	Fuji Photo	0.21658	—	—
Dainippon Pharmaceutical	0.12108	Ajinomoto	0.09939	Mitsubishi Oil	0.05067	Honen	0.21596	—	—
Noritake	0.10306	Mitsubishi Estate	0.08750	—	—	Toho Rayon	0.18455	—	—
Meidensha	0.10037	Sharp Electric	0.08529	—	—	Nisshinbo	0.18118	—	—
Nec	0.08364	Toho Rayon	0.08229	—	—	Shotiku	0.13579	—	—
Toho Rayon	0.08329	Kawasaki Steel	0.07382	—	—	Nihon Woolen	0.12123	—	—
Mitsubishi Estate	0.07335	—	—	—	—	Asahi Brewery	0.11678	—	—
Ajinomoto	0.06832	—	—	—	—	Nippon Yusen	0.11243	—	—
Kawasaki Steel	0.06471	—	—	—	—	Odakyu Railway	0.10994	—	—
Showa Shell Oil	0.06391	—	—	—	—	Maruzen	0.10977	—	—
Sumitomo Electric	0.06110	—	—	—	—	Asahi Denka	0.10634	—	—
Nippon Steel	0.05944	—	—	—	—	Fujita	0.09531	—	—
Asahi Glass	0.05943	—	—	—	—	Meidensya	0.09133	—	—
Honda	0.05514	—	—	—	—	Tokyo Rope Mfg.	0.08738	—	—
Mazda	0.05407	—	—	—	—	Goudou Liquor	0.08545	—	—
—	—	—	—	—	—	Toa	0.08393	—	—
—	—	—	—	—	—	Asahi Chemical	0.07990	—	—
—	—	—	—	—	—	Dai-ichi Kangyo Bank	0.07866	—	—
—	—	—	—	—	—	Toyo Seikan	0.07565	—	—
—	—	—	—	—	—	Mitsubishi Paper	0.07186	—	—
—	—	—	—	—	—	Matsushita	0.07008	—	—
—	—	—	—	—	—	Nihon Cemento	0.06542	—	—
—	—	—	—	—	—	Shimura Chemical	0.06438	—	—
—	—	—	—	—	—	Nachi-Fujikoshi	0.06290	—	—
—	—	—	—	—	—	Mazda	0.06120	—	—
—	—	—	—	—	—	Toto	0.05767	—	—
—	—	—	—	—	—	Chiyoda	0.05079	—	—

Table IX. Nonlinearity tests: daily and weekly data

Name	Daily or Weekly	Ori-F	Aug-F	TAR-F	CUSUM	New-F
	Daily	0.00642	0.00880	0.00017	0.00001	0.00091
Topix	Weekly	0.00131	0.00306	0.00007	0.16963	0.00620
	(Daily/Weekly)	(0.49008)	(2.87582)	(2.42857)	(0.00006)	(0.14677)
	Daily	0.08307	0.10981	0.00389	0.00014	0.01353
Nikkei 225 Stock Index	Weekly	0.26519	0.03832	0.00013	0.07146	0.01326
	(Daily/Weekly)	(0.31325)	(2.86561)	(29.92308)	(0.00196)	(1.02036)
	Daily	0.15787	0.23474	0.02045	0.04603	0.09003
Kajima Corp.	Weekly	0.63553	0.66189	0.03716	0.19604	0.04306
	(Daily/Weekly)	(0.24841)	(0.35465)	(0.55032)	(0.23480)	(2.09080)
	Daily	0.10208	0.07327	0.06391	0.16063	0.04488
Kirin Brewery Co. Ltd.	Weekly	0.49845	0.42720	0.00150	0.32130	0.14435
	(Daily/Weekly)	(0.20479)	(0.17151)	(42.60667)	(0.49994)	(0.31091)
	Daily	0.00049	0.00232	0.03172	0.25533	0.00261
Toray Industries, Inc.	Weekly	0.11609	0.12579	0.01771	0.22401	0.03563
	(Daily/Weekly)	(0.00422)	(0.01844)	(1.79108)	(1.13982)	(0.07325)
	Daily	0.03343	0.06552	0.01373	0.11561	0.01953
Oji Paper Co. Ltd.	Weekly	0.60157	0.63649	0.25787	0.30523	0.37330
	(Daily/Weekly)	(0.05557)	(0.10294)	(0.05313)	(0.37876)	(0.05232)
	Daily	0.00079	0.00018	0.00003	0.13246	0.00001
Takeda Chemical Industries, Ltd.	Weekly	0.43635	0.72443	0.61053	0.46088	0.01145
	(Daily/Weekly)	(0.00181)	(0.00025)	(0.00025)	(0.28741)	(0.00087)
	Daily	0.03751	0.01891	0.07467	0.10775	0.07326
Nippon Oil Co. Inc.	Weekly	0.40865	0.60010	0.04215	0.12737	0.22216
	(Daily/Weekly)	(0.09179)	(0.03151)	(1.77153)	(0.84596)	(0.32976)
	Daily	0.30220	0.21841	0.04987	0.09894	0.06300
Asahi Glass Co. Ltd.	Weekly	0.12765	0.03027	0.02628	0.52262	0.14135
	(Daily/Weekly)	(2.36741)	(7.21539)	(1.89764)	(0.18932)	(0.44570)

Table IX. (contd.) Nonlinearity tests: daily and weekly data

Name	Daily or Weekly	Ori-F	Aug-F	TAR-F	CUSUM	New-F
	Daily	0.00000	0.00000	0.00019	0.00000	0.00042
Nippon Steel Corp.	Weekly	0.07988	0.03709	0.00233	0.10011	0.01675
	(Daily/Weekly)	(0.00000)	(0.00000)	(0.08155)	(0.00000)	(0.02507)
	Daily	0.00001	0.00000	0.00009	0.02232	0.00023
Mitsubishi Metal Corp.	Weekly	0.11648	0.05452	0.07714	0.09771	0.45161
	(Daily/Weekly)	(0.00009)	(0.00000)	(0.00117)	(0.22843)	(0.00051)
	Daily	0.00008	0.00019	0.00151	0.00759	0.00344
Komatsu, Ltd.	Weekly	0.04074	0.00640	0.01247	0.18179	0.00734
	(Daily/Weekly)	(0.00196)	(0.29688)	(0.12109)	(0.04175)	(0.46866)
	Daily	*0.00947*	*0.01055*	*0.00552*	*0.09114*	0.04093
Hitati, Ltd.	Weekly	0.01440	0.05511	0.01376	0.17012	0.00623
	(Daily/Weekly)	(0.65764)	(0.19144)	(0.40116)	(0.53574)	(6.56982)
	Daily	0.04877	0.01034	0.07593	0.51542	0.07988
Nec Corp.	Weekly	0.02677	0.14570	0.00047	0.11649	0.03322
	(Daily/Weekly)	(1.82182)	(0.07097)	(161.55319)	(4.42459)	(2.40458)
	Daily	0.04092	0.05757	0.00354	0.15153	0.00802
Matsushita Electric Industrial Co. Ltd.	Weekly	0.02905	0.08615	0.00853	0.03587	0.00790
	(Daily/Weekly)	(1.40861)	(0.66825)	(0.41501)	(4.22442)	(1.01519)
	Daily	0.01896	0.00096	0.00100	0.00031	0.00046
Mitsubishi Heavy Industries, Ltd.	Weekly	0.99780	0.98561	0.66861	0.54252	0.56793
	(Daily/Weekly)	(0.01900)	(0.00097)	(0.00150)	(0.00057)	(0.00081)
	Daily	0.00054	0.00106	0.01201	0.00088	0.00046
Toyota Motor Corp.	Weekly	0.00421	0.00763	0.00004	0.15635	0.01684
	(Daily/Weekly)	(0.12827)	(0.13893)	(300.25000)	(0.005653)	(0.02732)
	Daily	0.14750	0.05001	0.14058	0.07205	0.03347
Dainippon Printing Co. Ltd.	Weekly	0.00010	0.00003	0.00738	0.32797	0.00909
	(Daily/Weekly)	*(1475.00000)*	*(1667.00000)*	*(19.04878)*	(0.21968)	(3.68207)

Table VIII shows the stocks which are not rejected with 5% significance level, from which we observe

(1) Each test produces different results, but the linearity of 'Toho Rayon Co. Ltd.' return is not rejected by all the tests. The results of the KTTL test with $P = 2$ in Section 3 support this results as 'Toho Rayon Co. Ltd.' return is very close to a Gaussian process.
(2) All the stock returns not rejected by the Aug-F test are not rejected by the Ori-F test either. This is consistent with the structural relationship between the Ori-F and Aug-F tests, where the latter includes the former.
(3) 'NEC Corp.' is rejected by the New-F test but this is not rejected by the Ori-F, Aug-F and TAR-F tests. This suggests that it could have neither polynomial type nor threshold type nonlinearity but could have an ExpAR or bilinear type nonlinearity.
(4) 'Ashi Brewery Co. Ltd.', 'Dainippon Pharmaceutical Co. Ltd.' and 'Mitsubishi Oil Co. Ltd.' are not rejected by the TAR-F test and not rejected by the New-F test and it is possible for these returns to have an ExpAR or bilinear type nonlinearity.

4.2. DAILY AND WEEKLY SERIES

Next we compare daily and weekly returns in data set (B) by nonlinearity tests. With regard to the choice of p and d, the procedure explained in previous subsection 4.1 is used. Table IX tabulates the p-values of the nonlinearity tests for 18 daily and weekly stock returns and the ratios of the p-values. The results are summarized as follows.

Observations from Overall Testing Results

The null hypotheses of linearity are rejected for many daily returns. The case with p-value ratio less than 1.0 shows that the weekly series is closer to Gaussianity than the daily series. 56 cases out of 80 have the ratio less than 1.0 and again we may point out a central limit effect on weekly series.

Observations from Individual Testing Results

⟨ Daily Return ⟩

(1) 'Nikkei 225 stock index' and 'Kajima Corp.' are not rejected by the Ori-F and Aug-F tests and rejected by the TAR-F test, implying a possibility of a threshold type nonlinearity for these returns.
(2) 'Nippon Steel Corp.' and 'Mitsubishi Metal Corp.' are rejected strongly for all tests (the p-values are less than 10^{-4}) except for the CUSUM test, implying strong nonlinearities.

(3) 'Nippon Oil Co. Ltd.' is not rejected by the TAR-F and New-F tests and rejected by the Ori-F and Aug-F tests with 5% level. This might suggest that a polynomial type nonlinearity exists for this return.

⟨ Weekly Return ⟩

(1) 'Kajima Corp.' and 'Toray Industries, Inc.' are not rejected by the Ori-F and Aug-F tests and rejected by the TAR-F and New-F tests. Therefore it is possible for these returns to have a threshold type nonlinearity.
(2) 'Takeda Chemical Industries, Ltd.' is rejected solely by the New-F test, implying a possibility for an ExpAR or bilinear type nonlinearity.
(3) 'Asahi Glass Co. Ltd.' is rejected by the Ori-F and New-F tests and not rejected by the Aug-F and TAR-F tests, which may indicate a possibility of a polynomial or threshold type nonlinearity for these returns.
(4) 'Oji Paper Co. Ltd.' and 'Mitsubishi Heavy Industries, Ltd.' are not rejected by any of the tests.

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