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CUSUM Techniques for Technical Trading in Financial Markets

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Abstract. It is discovered that the CUSUM techniques often used in the manufacturing industry can be adapted to yield a trading strategy in the financial market. The familiar filter trading strategy in finance is found to be a particular case of CUSUM procedures. A more general form of the CUSUM techniques will yield new trading stategies which have intuitive appeals. Trading characteristics of such strategies will be investigated using CUSUM techniques.

Key words: average run length, CUSUM techniques, filter trading strategy.

1. Introduction

Control chart techniques were first developed in the 1930's. Since then it has become an indispensable tool in the manufacturing industry used heavily in monitoring the quality of the manufactured products. Some commonly used control charts are the Shewhart charts by Shewhart (1939), the cumulative-sum control charts by Page (1954a,b), the moving average control charts and the geometric moving average control charts by Roberts (1959) etc.

On the other hand, technical charts are very popular among people in the financial market. A sizable proportion of traders in financial market are chartists who base their trading strategies on price charts of the financial product. Despite the fact that the financial analysts and quality controllers both rely on charts, they are treated as two different groups of people using completely different techniques for their works. However, it will not take long to see that there is a great similarity between the two charting approaches. People in quality control are concerned with the quality of a product and wish to sound an early alarm when the quality is out of control. When the quality is above (below) a target value, the control chart will generate an upward (downward) shift warning. Upon the generation of a warning signal, the production process has to be stopped and readjusted. For traders in the financial market, what is important is the detection of an upward or downward price trend. Technical charts are used to detect such trends. Whenever an upward (downward) trend is detected, the trader will take long (short) position in the market.

Despite these similarities, linkages between the two have not been discussed in both the quality control and financial literature. This paper will offer a first attempt to link up the two fields of investigation. In particular, we will focus our attention to one popular trading rule in the finance literature, which is the filter trading rule proposed by Alexander (1961). We will show that the filter rule is equivalent to the CUSUM techniques in quality control. In more general terms, any principle adopted in detecting a shift in product quality can also be applied to detect upward or downward shift in prices in the financial market.

In Section 2, we will briefly review the CUSUM techniques and filter trading rule. Then, we will show that the two approaches are equivalent. Along the line that a quality control technique can be adopted to a trading rule in the financial market, we consider, in Section 3, the trading rule corresponding to the general CUSUM procedure and its trading performance. Empirical evidence presented in Section 4 shows that the general CUSUM techniques also work well in the financial market and offers an improvement over the classical filter trading rule.

The performance characteristic of a quality control technique can be described by the average run length of the production process. The average run length has been tabulated by Chiu (1974) for various parameter values. In Section 5, we compute the means and variances of the run length to supplement some existing tables on the control charts literature. When control chart techniques are applied to form a trading rule, the average run length becomes important as it will be tied up with the trading profit. Given that the market is in an upward trend, the longer is the average run length before the generation of a sell signal, the larger is the profit. The variance of the run length will then control the variability of the derived profit. In Section 6, the formulae derived in Section 5 are used to obtain the mean and variance of the profit of a trading strategy. The actual profit obtained empirically can then be used to test whether the obtained profit is statistically significant or not. This could have implications on market efficiency which is a central topic of research in the field of finance. The paper ends with Section 7 which contains a summary and discussion.

2. CUSUM Techniques and Filter Trading Rule

2.1. CUSUM TECHNIQUES

CUSUM techniques were developed in the fifties, see for example Page (1954a,b), Kemp (1961, 1967a,b) and the book by Van Dobben De Bruyn (1968), etc. The CUSUM procedure is designed to detect a shift in the mean value of a measured quantity from a target value. Consider independent observations $y_1, y_2, \ldots, y_n, \ldots$ arising from a manufacturing process with mean level μ. Assume that we are interested in detecting an upward shift in the mean level of the production process. A CUSUM procedure with parameters (k, h) that can signal a warning of an upward shift can be described as follows. Fix a parameter k called the reference value and a parameter h called the threshold value. Take observations $y_1, y_2, \ldots$ and let $x_i = y_i - k$. Quite often, k is set to be μ but, in general, k can be set at any level. The cumulative sum is $S_n = x_1 + x_2 + x_3 + \cdots + x_n$. Define S'_n recursively as follows

$$\begin{cases} S'_0 = 0, \\ S'_n = \max(S'_{n-1} + x_n, 0). \end{cases} \tag{1}$$

Note that $S'_n = 0$ whenever $S_n \leqslant \min_{0 \leqslant i \leqslant n} S_i$. The CUSUM procedure would recommend an action at the first n satisfying $S'_n \geqslant h$. The CUSUM techniques which can signal warning of a downward shift in mean value can be defined in a similar fashion.

2.2. THE FILTER TRADING RULE

The filter trading rule was one of the most investigated trading rules in the finance literature. In the sixties, Alexander (1961, 1964), Fama and Blume (1966) and Dryden (1969) considered a trading rule called the filter rule and empirically showed that after taking transaction costs into account, the filter trading rule cannot outperform the buy-and-hold strategy which simply buys the stock and hold it throughout the time period under consideration. These findings lend considerable support to the market efficiency theory which forms the basis of a wide range of research in the field of finance.

The filter rule is a mechanical trading rule which generates a sequence of buy signals and sell signals alternately according to the following rule. If the daily closing price of a particular stock moves up at least x percent from a low, a buy signal is generated. We then buy and hold the stock until the closing price moves down at least x percent from a subsequent high, at which time a sell signal is generated and we simultaneously sell and go short. Repeat the process so that at the next buy signal we will cover up the short position and go long, etc. Note that price movement of less than x percent (from a low or high) does not generate a signal, x is called the filter size for the trading rule.

2.3. FILTER RULE TREATED AS A CUSUM PROCEDURE

In this subsection, we will show that the filter rule is simply equivalent to the CUSUM procedure. Let p_t denote the closing price of a stock at day $t (t = 0, 1, 2, \ldots)$. Suppose a sell signal has just been generated at time 0 and the filter trading rule is to generate the next buy signal. For each n, let r_n denote how much the price at day n rises from its historical low from time 0 to n. Mathematically,

$$r_n = p_n - \min_{0 \leqslant i \leqslant n} p_i \quad n = 1, 2, \ldots .$$

The filter rule will generate a buy signal at the first n satisfying

$$\frac{r_n}{\min\limits_{0 \leqslant i \leqslant n} p_i} \geqslant x.$$

Here, x is called the filter size of the trading rule.

We will establish below that such a buy-signal generating mechanism can be treated as a CUSUM procedure. Let $q_t = \log p_t$ be the logarithm of the closing prices and let $y_t = q_t - q_{t-1}, t = 1, 2, \ldots$ be the continuously compounded daily return that can be derived from investing in the stock. Consider the CUSUM procedure given in Section 2.1 with parameters $k = 0$ and $h = \log(1 + x)$. Using the notation in Section 2.1,

$$S_n = \sum_{i=1}^{n} y_i = q_n - q_0 \quad \text{and}$$

$$\begin{cases} S'_0 = 0, \\ S'_n = \max(S'_{n-1} + y_n, 0). \end{cases}$$

Note that the graph S_n versus n is simply a plot of the time series of log-prices with q_0 as a reference value. S'_n then measures how much the current log-price rises from a historical low of log-prices, i.e.

$$S'_n = q_n - \min_{0 \leqslant i \leqslant n} q_i.$$

In the CUSUM procedure, a signal is triggered at the first n satisfying $S'_n \geqslant h$. Since

$$\begin{aligned} S'_n \geqslant h &\Leftrightarrow \log p_n - \min_{0 \leqslant i \leqslant n} \log p_i \geqslant h \\ &\Leftrightarrow \frac{p_n}{\min_{0 \leqslant i \leqslant n} p_i} \geqslant e^h \\ &\Leftrightarrow \frac{p_n - \min_{0 \leqslant i \leqslant n} p_i}{\min_{0 \leqslant i \leqslant n} p_i} \geqslant e^h - 1 \\ &\Leftrightarrow \frac{r_n}{\min_{0 \leqslant i \leqslant n} p_i} \geqslant x, \end{aligned}$$

the filter rule generates a buy signal when and only when the CUSUM procedure recommends an action, i.e.,

$$\begin{cases} S_n = \sum_{t=1}^{n} y_t, \\ S'_0 = 0, \\ S'_n = \max(S'_{n-1} + y_n, 0), \\ \text{take action at first } n \text{ satisfying } S'_n \geqslant \log(1 + x). \end{cases}$$

After triggering a buy signal, say at time $t = 0$, the filter rule will generate the next sell signal at the first n satisfying

$$\frac{d_n}{\max\limits_{0 \leqslant i \leqslant n} p_i} \geqslant x,$$

where $d_n = \max\limits_{0 \leqslant i \leqslant n} p_i - p_n$.

Similar argument can easily show that this corresponds to the following CUSUM procedure designed to detect a downward drift in mean level with $x_t = y_t$

$$\begin{cases} S_n = \sum\limits_{t=1}^{n} x_t, \\ S'_0 = 0, \\ S'_n = \min(S'_{n-1} + x_n, 0), \\ \text{take action at first } n \text{ satisfying } S'_n \leqslant -\log(1 + x). \end{cases}$$

3. Generalizing the Filter Trading Rule

3.1. A GENERALIZED FILTER TRADING RULE

The CUSUM procedure defined in Section 2.1 is characterized by two parameters (k, h), where k is the reference value and h is the threshold value. In Section 2, we see that there is a one to one correspondence between a CUSUM procedure with parameters $(0, h)$ and a filter trading rule with filter size $x = e^h - 1$. Note that $x \approx h$ when h and x are close to zero. Obviously, there is no reason why we should restrict ourselves to CUSUM procedure with $k = 0$. We consider a general CUSUM procedure with $k \neq 0$ and $h > 0$. Such CUSUM procedures then give rise to a class of trading rules which is more general than the classical filter trading rule. As far as the authors are aware, such trading rules have not appeared in literature in financial research. The question remains as to whether such generalized filter trading rules make enough investment sense or not.

3.2. RATIONALE BEHIND THE TRADING RULE

The ordinary filter rule is based on a trend following strategy. As Alexander (1961) put it, 'if the stock has moved up $x\%$ (or move down $y\%$), it is likely to move up more than $x\%$ further (or move down more than $y\%$ further) before it moves down by $x\%$ (or moves up by $y\%$)'. This forms the basis for using the filter rule as a practical trading rule in timing the trading of a stock. To interpret the parameter k involved in a general filter rule, consider first the case $h = 0$ and $k > 0$. It is easy to see that such a general filter rule will issue a buy signal whenever the one day return exceeds k. In other words, we will buy a stock at the end of day t whenever

$$y_t = \log p_t - \log p_{t-1} > k,$$

or equivalently,

$$\frac{p_t - p_{t-1}}{p_{t-1}} > e^k - 1.$$

Similarly, this general filter rule will generate a sell signal if

$$((p_t - p_{t-1})/p_{t-1}) < e^{-k} - 1.$$

This rule will be a sensible investment strategy if we believe that a rising trend in the market usually starts with a large single day rise and a downward trend usually starts with a large single day drop.

3.3. SPEED FILTERING AS WELL AS SIZE FILTERING

One drawback of the general filter rule with $h = 0$ and $k > 0$ is that there is no stop-loss mechanism built into the trading rule. Once a sell signal is on, the buy signal will not appear if the market rises gradually for many days, but in each of the days, it rises by not more than $100^*(e^k - 1)\%$. Under such circumstances, the investor will suffer a huge loss. If we allow the parameter h to be non-zero, trading loss is not allowed to accumulate indefinitely. Let $x = e^h - 1$ and $x' = e^k - 1$. We can interpret x as the filter size in magnitude and x' as the filter size in speed. A general filter rule with filter sizes (x', x) has the following property. After a sell signal at day 0, a buy signal at day t will be generated if the percentage change in prices at day t over a span of i days exceeds $ix' + x$ for some i satisfying $1 \leqslant i \leqslant t$, i.e.,

$$\log p_t - \log p_{t-1} \geqslant h + ki \quad \text{for some } 1 \leqslant i \leqslant t,$$

or equivalently if

$$\begin{aligned}
\frac{p_t - p_{t-i}}{p_{t-i}} &\geqslant \exp(h + ki) - 1 \\
&= (1 + x)(1 + x')^i - 1 \\
&\approx (1 + x)(1 + ix') - 1 \\
&\approx x + ix'.
\end{aligned}$$

It is now obvious that x' is a filter for the average daily return and x is a filter for the whole period return.

Similarly, once a buy signal is generated at day 0, a sell signal at day t will be generated if the percentage change in prices at day t over a span of i days is less than or equal $-ix' - x$ for some i satisfying $1 \leqslant i \leqslant t$, i.e.,

$$\begin{aligned}\frac{p_t - p_{t-i}}{p_{t-i}} &\leqslant \exp(-h - ki) - 1 \\ &= (1 + x)^{-1}(1 + x')^{-i} - 1 \\ &\approx (1 - x)(1 - ix') - 1 \\ &\approx -(x + ix') \quad \text{for some } 1 \leqslant i \leqslant t.\end{aligned}$$

4. Empirical Results

In this section, we apply the standard and the generalized filter trading rule to the Hong Kong stock market data using the closing Hang Seng Index as a proxy for prices of a portfolio of stocks. The data used in the analysis cover the period from 24 November 1969 to 6 January 1993. Filter trading rules corresponding to the various parameter values of h and k are tried and the average trading profit per cycle are reported in Table 1 below. Note that a trade can either start with a buy and ends up with a sell (a long cycle) or starts with a sell and then a buy (a short cycle), and profit is measured in percentage changes in prices within a cycle. The mean profit per cycle reported in Table 1 are not directly comparable because each cycle may have different lengths. Together with the mean profit per cycle, Table 1 also reports the daily profit for various parameters values of h and k. The daily profit for each strategy can be compared directly. The higher the daily profit, the better is the trading strategy.

Notice that the profits derived from general filter trading rules are comparable with those from the classical filter trading rule with parameter k equal to zero. For small filter size ($h \leqslant 4\%$), the general filter trading rules with positive k offer an improvement over the classical rule. As is well-known, when the filter size is small, the filter trading rule may overreact to noises. The introduction of the filter k may help to eliminate some of the unsuccessful buy-sell signals. On the other hand, when the filter size increases, the sensitivity for detecting a small upward or downward trend will decrease. In this situation, the trading system may not be sensitive enough for gradual increases or decreases. The introduction of a negative filter k into the process can also help overcome this shortcoming. Therefore, for large filter ($5\% \leqslant h \leqslant 8\%$), the general filter trading rules with negative parameter k perform better than the classical filter trading rule. For very large filter size ($h \geqslant 8\%$), the general filter trading strategy cannot outperform the classical one.

5. Run Length

5.1. AVERAGE RUN LENGTH

In general, we can give an initial value for S'_0 in (1) as $S'_0 = z$. Under this more general setting, let L_z denote the run length which is the number of observations until an action will be taken when the true mean is μ. Denote the average run length (ARL) corresponding to an initial value z by $l(z)$, i.e., $l(z) = E(L_z)$. It can be shown, see Page (1954a), that l satisfies the integral equation

$$l(z) = 1 + l(0)F(-z) + \int_0^h l(x)\,\mathrm{d}F(x - z), \tag{2}$$

where F is the c.d.f. of x_i. Note that $l(0)$ is the ARL when S'_n starts at 0. Integral equation (2) can be solved numerically, see, for example, Chapter 3 of the book Von Dobben De Bruyn (1968) and Goel and Wu (1971).

5.2. VARIANCE OF RUN LENGTH

Crowder (1987) presented a numerical procedure using integral equations for the tabulation of moments of run lengths of exponential weighted moving average charts. Using similar techniques, we can calculate the variance of run length of CUSUM procedure. Notice that, in order to compute the variance of run length, it is sufficient to find the second moment of the run length distribution. Let $g(z) = E(L_z^2)$. We have

$$\begin{aligned} g(z) = & \int_{h-z}^{\infty} f(y)\,\mathrm{d}y + \int_{-z}^{h-z} E[(1 + L_{z+y})^2]f(y)\,\mathrm{d}y \\ & + \int_{-\infty}^{-z} E[(1 + L_0)^2]f(y)\,\mathrm{d}y. \end{aligned}$$

Simplifying and note that $l(z) = E(L_z)$, we can show that $g(z)$ satisfies the integral equation

$$g(z) = A(z) + \int_0^h f(x - z)g(x)\,\mathrm{d}x + g(0)F(-z), \tag{3}$$

where $A(z)$ is given by

$$A(z) = 1 + 2\int_{-z}^{h-z} l(z + y)f(y)\,\mathrm{d}y + 2\int_{-\infty}^{-z} l(0)f(y)\,\mathrm{d}y.$$

To solve for (3), let $g_1(z)$ and $g_2(z)$ be the solutions of the following two integral equations

$$g_1(z) = A(z) + \int_0^h f(x - z)g_1(x)\,\mathrm{d}x \tag{4}$$

$$g_2(z) = F(-z) + \int_0^h f(x-z)g_2(x)\,\mathrm{d}x. \tag{5}$$

It can be easily shown that $g(z) = g_1(z) + g(0)g_2(z)$. Hence,

$$g(0) = \frac{g_1(0)}{1 - g_2(0)}. \tag{6}$$

Since the variance of L_0 is given by

$$\mathrm{Var}(L_0) = g(0) - [l(0)]^2,$$

it can be computed by (2) and (6).

5.3. TABULATED MEAN AND VARIANCE OF RUN LENGTH

In the economic design of CUSUM control charts, one of the major difficulties is to evaluate the mean and variance of run length. We consider the following general situation. Samples of size n are taken from a normal distribution with mean μ and variance σ^2 with values given by $y_1, y_2, \ldots, y_n$. Consider a CUSUM procedure with a reference value k and a threshold value h. Let $x_i = y_i - k$ and define S'_n as in (1). Let L denote the first n to satisfy $S'_n \geqslant h$. The mean and variance of run length L is then a function of the process deviate θ defined by

$$\theta = \frac{(\mu - k)\sqrt{n}}{\sigma}$$

and the standardized decision interval H defined by

$$H = \frac{h\sqrt{n}}{\sigma}.$$

According to Chiu (1974), the economic design often requires evaluations of $E(L)$ for $|\theta|$ in the range (2.0, 3.0) and H in the range (0.2, 2.0). He had constructed a table which has practical importance in the economic approach to the design of CUSUM control charts for the average run length. However, the variance of run length (VRL) was not given in Chiu's table. We now extend Chiu's table to include the variance of run length for the same range (Table 2a and b).

6. Operating Characteristics of a Filter Trading Rule

6.1. DURATION OF LONG POSITION AND THE DURATION OF A SHORT POSITION

We can now borrow the standard CUSUM theory to compute the operating characteristics of a filter trading rule. Let B be the number of days in a run of buy

Table I. Results on generalized filter trading rule.

$h(\%)$	h/k	10	20	30	40	50	∞	−50	−40	−30	−20	−10
1	no. of cycle	557	610	622	630	635	645	664	669	672	685	711
	Mean profit (%)	2.12	2.06	2.04	2.01	1.97	1.97	1.87	1.84	1.85	1.79	1.79
	s.d. profit (%)	6.90	6.66	6.63	6.62	6.57	6.52	6.41	6.39	6.35	6.29	6.16
	Daily profit (%)	−0.0288	−0.0366	−0.0393	−0.0435	−0.0474	−0.0490	−0.0615	−0.0653	−0.0650	−0.0729	−0.0767
2	no. of cycle	295	324	337	343	353	368	384	388	395	411	471
	Mean profit (%)	3.61	3.45	3.37	3.44	3.28	3.20	3.17	3.17	3.25	3.06	2.63
	s.d. profit (%)	9.41	9.01	8.99	9.01	8.81	8.37	8.16	8.09	7.99	7.90	7.43
	Daily profit (%)	0.0626	0.0599	0.0575	0.0626	0.0543	0.0514	0.0520	0.0526	0.0588	0.0482	0.0192
3	no. of cycle	171	206	218	225	231	246	266	272	282	301	356
	Mean profit (%)	5.41	5.07	5.15	4.89	4.66	4.43	4.09	3.95	3.61	3.50	3.36
	s.d. profit (%)	13.91	12.28	11.71	11.26	11.09	10.73	10.41	10.17	9.91	9.71	8.95
	Daily profit (%)	0.0903	0.0967	0.1052	0.0983	0.0916	0.0878	0.0790	0.0740	0.0601	0.0584	0.0600
4	no. of cycle	105	133	154	162	165	187	209	211	217	234	309
	Mean profit (%)	7.47	6.59	5.25	5.00	5.21	5.06	4.58	4.85	4.91	4.85	3.53
	s.d. profit (%)	17.20	15.12	13.76	13.46	12.86	12.87	12.21	12.34	12.20	11.64	9.64
	Daily profit (%)	0.0935	0.0979	0.0771	0.0737	0.0814	0.0875	0.0800	0.0906	0.0955	0.1006	0.0613
5	no. of cycle	73	97	105	111	115	138	158	162	171	193	284
	Mean profit (%)	8.25	8.54	8.49	8.35	8.03	7.04	6.75	6.68	6.41	6.04	3.83
	s.d. profit (%)	20.84	16.87	16.61	14.97	14.71	13.85	13.22	13.00	12.64	12.04	9.98
	Daily profit (%)	0.0750	0.1046	0.1124	0.1159	0.1138	0.1025	0.1207	0.1218	0.1205	0.1234	0.0714

Table I. (continued) Results on generalized filter trading rule.

$h(\%)$	h/k	10	20	30	40	50	∞	−50	−40	−30	−20	−10
6	no. of cycle	43	72	85	90	94	111	136	144	152	171	264
	Mean profit (%)	17.04	9.57	8.71	8.72	8.22	7.91	6.62	6.12	6.56	6.54	4.14
	s.d. profit (%)	29.84	21.90	18.05	16.95	16.67	14.74	13.69	12.54	13.15	12.17	10.07
	Daily profit (%)	0.1106	0.0907	0.0942	0.0996	0.0962	0.1074	0.1007	0.0941	0.1100	0.1244	0.0812
7	no. of cycle	38	51	62	70	72	92	115	126	136	160	260
	Mean profit (%)	10.48	14.70	11.87	10.39	10.56	9.09	8.04	6.91	6.69	6.53	3.91
	s.d. profit (%)	30.65	24.23	24.17	22.46	21.79	18.87	15.49	14.39	13.29	12.96	10.08
	Daily profit (%)	0.0539	0.1101	0.1031	0.0983	0.1032	0.1080	0.1141	0.0998	0.1024	0.1160	0.0692
8	no. of cycle	29	40	49	56	61	79	105	111	124	149	260
	Mean profit (%)	8.40	15.19	13.87	11.05	10.45	9.46	8.24	8.18	7.32	7.15	3.27
	s.d. profit (%)	37.76	28.56	24.29	25.28	24.45	19.68	17.09	16.84	15.06	13.66	10.51
	Daily profit (%)	0.0306	0.0896	0.0987	0.0851	0.0962	0.0980	0.1078	0.1126	0.1071	0.1243	0.0400
9	no. of cycle	23	36	38	42	47	64	96	105	115	144	258
	Mean profit (%)	6.23	7.54	16.52	17.50	15.48	13.07	8.10	7.44	7.52	7.14	3.04
	s.d. profit (%)	46.75	26.75	27.00	28.10	23.24	22.35	19.21	17.92	16.47	13.97	10.38
	Daily profit (%)	0.0155	0.0325	0.0942	0.1114	0.1080	0.1199	0.0961	0.0929	0.1035	0.1198	0.0291
10	no. of cycle	14	29	37	38	39	56	87	93	107	144	254
	Mean profit (%)	6.95	9.14	8.15	13.82	15.52	14.03	8.89	9.03	8.06	6.27	3.23
	s.d. profit (%)	72.36	41.51	27.91	27.72	27.26	23.63	20.41	19.75	17.83	13.52	11.78
	Daily profit (%)	0.0112	0.0343	0.0374	0.0762	0.0899	0.1144	0.0992	0.1082	0.1064	0.098	0.0370

Table IIa. Mean and Variance of run length for $H = 0.02(0.2)1.0$ and $\theta =\ \ 4.0,\ \ 3.0(0.2)3.0, 4.0$

	H 0.20		0.40		0.60		0.80		1.00	
θ	$E(L)$	$\text{Var}(L)$	$E(L)$	$\text{Var}(L)$	$E(L)$	$\text{Var}(L)$	$E(L)$	$\text{Var}(L)$	$E(L)$	$\text{Var}(L)$
−4.00	74930	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$
−3.00	1455	$> 10^5$	2965	$> 10^5$	6273	$> 10^5$	13779	$> 10^5$	31429	$> 10^5$
−2.80	740.5	$> 10^5$	1453	$> 10^5$	2958	$> 10^5$	6251	$> 10^5$	13709	$> 10^5$
−2.60	391.1	$> 10^5$	738.7	$> 10^5$	1447	$> 10^5$	2941	$> 10^5$	6198	$> 10^5$
−2.40	214.3	45709	389.7	$> 10^5$	734.3	$> 10^5$	1434	$> 10^5$	2901	$> 10^5$
−2.20	121.8	14709	213.2	45230	386.1	$> 10^5$	723.7	$> 10^5$	1404	$> 10^5$
−2.00	71.75	5076	120.9	14485	210.3	43996	378	$> 10^5$	702	$> 10^5$
−1.80	43.81	1875	71	4968	118.6	13932	204.1	41431	362.1	$> 10^5$
−1.60	27.71	739.9	43.2	1821	69.2	4714	113.9	12847	192.6	36877
−1.40	18.15	310.9	27.22	712.6	41.81	1701	65.74	4243.7	105.8	11049
−1.20	12.3	138.8	17.76	296.7	26.15	654.5	39.26	1492.9	60.01	3520
−1.00	8.62	65.62	12	131.2	16.95	267.8	24.3	559.4	35.29	1195
−0.80	6.25	32.71	8.388	61.38	11.39	116.4	15.62	223.5	21.59	433.5
−0.60	4.679	17.13	6.07	30.32	7.941	53.68	10.45	95.15	13.79	168.7
−0.40	3.615	9.388	4.544	15.76	5.744	26.19	7.28	43.18	9.221	70.55
−0.20	2.878	5.359	3.515	8.583	4.31	13.49	5.287	20.86	6.466	31.72
0.00	2.359	3.172	2.806	4.881	3.35	7.317	3.997	10.71	4.75	15.32
0.20	1.987	1.938	2.308	2.885	2.691	4.16	3.138	5.824	3.644	7.932
0.40	1.718	1.212	1.952	1.765	2.23	2.47	2.55	3.341	2.908	4.382
0.60	1.52	0.778	1.694	1.112	1.9	1.524	2.137	2.013	2.401	2.57
0.80	1.374	0.506	1.504	0.717	1.66	0.972	1.84	1.266	2.042	1.59
1.00	1.266	0.333	1.364	0.47	1.483	0.636	1.622	0.825	1.78	1.03
1.20	1.187	0.219	1.261	0.312	1.351	0.424	1.46	0.553	1.585	0.692
1.40	1.129	0.144	1.184	0.207	1.253	0.287	1.337	0.378	1.437	0.478
1.60	1.087	0.094	1.127	0.138	1.179	0.194	1.245	0.261	1.324	0.336
1.80	1.058	0.061	1.086	0.091	1.125	0.131	1.175	0.181	1.238	0.238
2.00	1.037	0.038	1.057	0.059	1.085	0.088	1.123	0.124	1.171	0.169
2.20	1.023	0.024	1.037	0.038	1.057	0.057	1.084	0.084	1.121	0.118
2.40	1.014	0.014	1.023	0.023	1.037	0.037	1.056	0.056	1.083	0.0814
2.60	1.008	0.008	1.014	0.014	1.023	0.023	1.037	0.036	1.056	0.055
2.80	1.005	0.004	1.008	0.008	1.014	0.014	1.023	0.023	1.036	0.036
3.00	1.003	0.003	1.005	0.005	1.008	0.008	1.014	0.014	1.023	0.023
4.00	1.000	10^{-4}	1.000	10^{-4}	1.000	10^{-4}	1.001	10^{-4}	1.001	0.001

$E(L)$ = Average Run Length; $\text{Var}(L)$ = Variance of run Length.

signals and S be the number of days in a run of sell signals. From Section 2, $b = E(B) = l(0)$ where $l(z)$ satisfies the integral equation (1) with $F(\bullet)$ being the c.d.f. of $y_t = \log p_t - \log p_{t-1}$. Similarly, $s = E(S) = l(0)$ with $l(z)$ satisfying Equation (1) and $F(\bullet)$ is the c.d.f. of $-y_t = -\log p_t + \log p_{t-1}$. Note that if the return of the stock has a symmetrical distribution about 0, B and S have identical distributions and hence share the same operating characteristics. However, 0 is usually not the point of symmetry for the stock's return, as it is commonly accepted that in the long run, the price of a stock will usually rise. Thus, most likely, B and S have different operating characteristics.

The variance of B can also be computed by solving the integral equations (4) and (5) with F and f being the c.d.f. and p.d.f. of y_t respectively. Then

$$\mathrm{Var}(B) = \frac{g_1(0)}{1 - g_2(0)} - b^2.$$

Replacing F and f by the c.d.f. and p.d.f. of $-y_t$, we can get the variance of S as

$$\mathrm{Var}(S) = \frac{g_1(0)}{1 - g_2(0)} - s^2.$$

6.2. PROPORTION OF TIME IN HOLDING THE STOCK

Let f represent the fraction of days when the filter rule is going long. Mathematically,

$$f = E\left(\frac{B}{B+S}\right),$$

where $B = L_0$ is the run length for a buy and S is the run length for a sell and they are independent random variables.

We are going to compute f by using the approximation formula.

$$\begin{aligned} E[g(B,S)] \;\approx\; & g(E(B), E(S)) + \frac{1}{2}\left[\left(\left.\frac{\partial g}{\partial B}\right|_{E(B),E(S)}\right)^2 \mathrm{Var}(B) \right. \\ & \left. + \left(\left.\frac{\partial g}{\partial S}\right|_{E(B),E(S)}\right)^2 \mathrm{Var}(S)\right]. \end{aligned}$$

Hence,

$$\begin{aligned} E\left(\frac{B}{B+S}\right) \;\approx\; & \frac{E(B)}{E(B)+E(S)} \\ & + \frac{1}{2[E(B)+E(S)]^2}\{[E(S)]^2 \mathrm{Var}(B) + \mathrm{Var}(S)\}. \end{aligned}$$

Table IIb. Mean and Variance of run length for $H = 0.02(0.2)1.0$ and $\theta = -4.0, -3.0(0.2)3.0, 4.0$

	H 1.20		1.40		1.60		1.80		2.00	
θ	$E(L)$	Var(L)	$E(L)$	Var(L)	$E(L)$	Var(L)	$E(L)$	Var(L)	$E(L)$	Var(L)
−4.00	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$
−3.00	74446	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$
−2.80	31210	$> 10^5$	73747	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$
−2.60	13550	$> 10^5$	30724	$> 10^5$	72228	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$	$> 10^5$
−2.40	6083	$> 10^5$	13215	$> 10^5$	29820	$> 10^5$	69133	$> 10^5$	$> 10^5$	$> 10^5$
−2.20	2819	$> 10^5$	5854	$> 10^5$	12557	$> 10^5$	27790	$> 10^5$	63313	$> 10^5$
−2.00	1364	$> 10^5$	2664	$> 10^5$	5432	$> 10^5$	11386	$> 10^5$	24471	$> 10^5$
−1.80	661.6	$> 10^5$	1244	$> 10^5$	2399	$> 10^5$	4740	$> 10^5$	9556	$> 10^5$
−1.60	334.4	$> 10^5$	594.7	$> 10^5$	1081	$> 10^5$	2003	$> 10^5$	3768	$> 10^5$
−1.40	173.9	29991	291.6	84580	497.6	$> 10^5$	860.9	$> 10^5$	1505	$> 10^5$
−1.20	93.29	8559	147.2	21397	235	54736	378.6	$> 10^5$	613.8	$> 10^5$
−1.00	51.84	2602	76.86	5761	114.8	12914	172.1	29177	258.7	66160
−0.80	30.02	848.3	41.9	1669	58.59	3292	81.96	6490	114.5	12749
−0.60	18.22	298.6	24.04	526.4	31.66	922.3	41.54	1603	54.27	2760.9
−0.40	11.65	114.1	14.64	182.3	18.3	287.6	22.72	447.7	28.02	687.5
−0.20	7.866	47.42	9.502	69.66	11.39	100.6	13.53	142.8	15.94	199.4
0.00	5.609	21.45	6.569	29.4	7.626	39.49	8.772	52.08	10	67.52
0.20	4.206	10.53	4.816	13.66	5.466	17.35	6.149	21.62	6.859	26.49
0.40	3.299	5.588	3.716	6.951	4.152	8.462	4.603	10.11	5.063	11.89
0.60	2.688	3.184	2.992	3.843	3.308	4.538	3.632	5.262	3.96	6.011
0.80	2.261	1.934	2.495	2.289	2.738	2.647	2.987	3.006	3.238	3.365
1.00	1.954	1.242	2.14	1.454	2.335	1.661	2.535	1.861	2.738	2.054
1.20	1.725	0.835	1.877	0.976	2.038	1.109	2.206	1.233	2.376	1.348
1.40	1.551	0.582	1.678	0.684	1.814	0.779	1.956	0.866	2.103	0.942
1.60	1.417	0.416	1.523	0.496	1.638	0.571	1.762	0.637	1.891	0.694
1.80	1.313	0.302	1.401	0.367	1.499	0.43	1.606	0.487	1.72	0.534
2.00	1.231	0.22	1.303	0.275	1.387	0.33	1.48	0.381	1.581	0.424
2.20	1.168	0.159	1.226	0.205	1.296	0.254	1.376	0.301	1.465	0.343
2.40	1.119	0.114	1.165	0.152	1.222	0.194	1.29	0.238	1.367	0.28
2.60	1.083	0.079	1.118	0.109	1.163	0.146	1.219	0.186	1.285	0.227
2.80	1.056	0.054	1.082	0.078	1.117	0.107	1.162	0.142	1.217	0.18
3.00	1.036	0.035	1.055	0.053	1.082	0.076	1.116	0.105	1.161	0.139
4.00	1.003	0.003	1.005	0.005	1.008	0.008	1.014	0.137	1.023	0.022

$E(L)$ = Average Run Length; Var(L) = Variance of run Length.

If we specify the exact distribution of the return series, we can compute the mean and variance of run length and we have

$$f = E\left(\frac{B}{B+S}\right) \approx \frac{b}{b+s} + \frac{1}{2[b+s]^2}[s^2\mathrm{Var}(B) + \mathrm{Var}(S)].$$

6.3. PROFIT DERIVED IN A TRADING CYCLE

To an investor, the most important operating characteristic of a trading rule is how much he or she can profit from the trading. Let LP and SP denote respectively the random variables which are continuously compounded return during a long cycle or a short cycle. If the y's are independently and identically distributed, then we can easily express LP and SP in terms of y, B and S as follows

$$LP = y_1 + y_2 + \cdots + y_B$$

$$SP = -y_1 - y_2 - \cdots - y_s.$$

Note that both LP and SP involve the sum of a random number of variables. If the y's are independent, by Wald's equation, the mean of LP and SP can be given by the following formulae

$$E(LP) = b^* E(y) \quad \text{and} \quad E(SP) = -s^* E(y).$$

6.4. DURATION AND PROFIT FOR FILTER TRADING RULE UNDER NORMAL ASSUMPTION

In the previous section, we have derived the mean of LP and SP. If we now assume that y follows a normal distribution with mean μ and variance σ^2, we can compute the mean of LP and SP. We assume $\sigma = 1\%$. Table 3 reports the run length and the variance of run length for buy signal and sell signal. Furthermore, the mean of LP and SP are also reported.

7. Conclusion and Discussion

In this paper, we find that there is a close relationship between the filter trading rule and the CUSUM procedures used in the construction of industrial control charts. Applying CUSUM techniques, we derive the mean and variance of the duration of a long position and short position under filter trading rule. Furthermore, operating characteristics of the filter trading profits have been constructed.

However, the ordinary filter trading rule is just the simplest case of CUSUM techniques. Obviously, we can consider a general filter trading rule using the general version of CUSUM procedures. The generalized filter trading rule is compared with the ordinary filter trading rule. Empirically, we find that the generalized filter trading rules have satisfactory performance. This opens up the possibility of applying QC techniques to derive technical trading rules in the financial market. The empirical results of applying the generalized CUSUM techniques to technical trading are encouraging and further research along this direction is suggested.

Table III. Mean of LP and SP for Filter size = 1%(1%)10% and μ = 0.00%(0.20%)0.1%

μ	Filter size	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
0.00%	$E(B)$	4.749	10.004	17.35	26.679	38.009	51.339	66.669	83.999	103.328	124.657
	s.d.(B)	3.915	8.217	14.198	21.81	31.056	41.937	54.452	68.599	84.381	101.795
	$E(LP)$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	$E(S)$	4.749	10.004	17.35	26.679	38.009	51.339	66.669	83.999	103.328	124.657
	s.d.(S)	3.915	8.217	14.198	21.81	31.056	41.937	54.452	68.599	84.381	101.795
	$E(SP)$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.02%	$E(B)$	4.889	10.437	18.353	28.613	41.334	56.615	74.559	95.278	118.881	145.489
	s.d.(B)	4.053	8.645	15.185	23.711	34.323	47.121	62.206	79.686	99.671	122.281
	$E(LP)$	0.10%	0.21%	0.37%	0.57%	0.83%	1.13%	1.49%	1.91%	2.38%	2.91%
	$E(S)$	4.617	9.597	16.428	24.935	35.071	46.769	59.969	74.612	90.641	108.002
	s.d.(S)	3.782	7.816	13.293	20.101	28.131	37.467	47.903	59.428	71.991	85.536
	$E(SP)$	−0.09%	−0.19%	−0.33%	−0.50%	−0.70%	−0.94%	−1.20%	−1.49%	−1.81%	−2.16%
0.04%	$E(B)$	5.034	10.899	19.448	30.762	45.107	62.728	83.899	108.915	138.097	171.793
	s.d.(B)	4.198	9.102	16.261	25.829	38.043	53.152	71.426	93.158	118.667	148.299
	$E(LP)$	0.20%	0.44%	0.78%	1.23%	1.80%	2.51%	3.36%	4.36%	5.52%	6.87%
	$E(S)$	4.489	9.215	15.578	23.358	32.455	42.793	54.248	66.745	80.203	94.549
	s.d.(S)	3.655	7.441	12.461	18.562	25.641	33.599	42.347	51.798	61.881	72.523
	$E(SP)$	−0.18%	−0.37%	−0.62%	−0.93%	−1.30%	−1.71%	−2.17%	−2.67%	−3.21%	−3.78%

Table III. (continued) Mean of LP and SP for Filter size = 1%(1%)10% and μ = 0.00%(0.20%)0.1%

μ	Filter size	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
0.06%	$E(B)$	5.186	11.392	20.634	33.155	49.401	69.841	95.013	125.519	162.039	205.341
	s.d.(B)	4.349	9.589	17.437	28.194	42.291	60.193	82.438	109.625	142.434	181.631
	$E(LP)$	0.31%	0.68%	1.24%	1.99%	2.96%	4.19%	5.70%	7.53%	9.72%	12.32%
	$E(S)$	4.368	8.856	14.794	21.931	30.147	38.319	49.339	60.109	71.546	83.575
	s.d.(S)	3.533	7.088	11.696	17.174	23.395	30.241	37.611	45.415	53.574	62.022
	$E(SP)$	−0.26%	−0.53%	−0.89%	−1.32%	−1.81%	−2.36%	−2.96%	−3.61%	−4.29%	−5.01%
0.08%	$E(B)$	5.344	11.918	21.932	35.826	54.301	78.151	108.308	145.867	192.111	248.548
	s.d.(B)	4.509	10.112	18.723	30.84	47.151	68.444	95.653	129.869	172.381	224.693
	$E(LP)$	0.43%	0.95%	1.75%	2.87%	4.34%	6.25%	8.66%	11.67%	15.37%	19.88%
	$E(S)$	4.251	8.517	14.069	20.636	28.081	36.273	45.102	54.474	64.308	74.536
	s.d.(S)	3.418	6.757	10.992	15.921	21.403	27.316	33.559	40.046	46.707	53.482
	$E(SP)$	−0.34%	−0.68%	−1.13%	−1.65%	−2.25%	−2.90%	−3.61%	−4.36%	−5.14%	−5.96%
0.10%	$E(B)$	5.511	12.481	23.351	38.811	59.912	87.899	124.296	170.965	230.18	304.721
	s.d.(B)	4.675	10.67	20.129	33.804	52.727	78.144	111.579	154.897	210.376	280.798
	$E(LP)$	0.55%	1.25%	2.34%	3.88%	5.99%	8.79%	12.43%	17.10%	23.02%	30.47%
	$E(S)$	4.139	8.199	13.398	19.457	26.231	33.289	41.427	49.656	58.207	67.019
	s.d.(S)	3.306	6.446	10.343	14.786	19.632	24.761	30.077	35.507	40.991	46.482
	$E(SP)$	−0.41%	−0.82%	−1.34%	−1.95%	−2.62%	−3.33%	−4.14%	−4.97%	−5.82%	−6.70%

$E(B)$ = average run length for buy signal; s.d.(B) = standard deviation of the run length for buy signal; $E(S)$ = average run length for sell signal; s.d.(S) = standard deviation of the run length for sell signal; $E(LP)$ = mean profit for buy signal under normal assumption; $E(SP)$ = mean profit for sell signal under normal assumption.

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