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Survival probability for brittle isotropic foams under multiaxial loading

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The cell-strut modulus of rupture for brittle isotropic foams is not a constant, depending on the volume and Weibull modulus of solid cell struts. In this paper, the existing model describing the failure stresses for isotropic foams under multiaxial loading is modified to take into account the effect of variability in the cell-strut modulus of rupture. As a result, the failure envelopes of brittle foams with different prescribed survival probability and Weibull modulus can be generated and presented. Results suggest that the effects of cell size, Weibull modulus and prescribed survival probability on the failure envelopes of brittle foams are significant. © *2000 Kluwer Academic Publishers*

1. Introduction

Ceramic foams are increasingly being used as lightweight cores in sandwich structures, especially for load-bearing components, because of their high melting temperature and low thermal conductivity. However, pre-existing flaws within ceramic foams reduce their loading capacity and cause strength variability in both tension and compression. Ceramic foams in many engineering applications are subjected to multiaxial loading. Various failure mechanisms might occur for ceramic foams under multiaxial loading, depending on the properties of solid cell struts. Accounting for the cell-strut strength variability, ceramic foams could have different failure mechanisms and resulting failure envelopes. The existing model for the failure envelopes of foams assumes that the cell struts have a constant modulus of rupture. Here, we modify the model to account for cell struts of variable strength. The analysis is based on the results on survival probability for brittle honeycombs under in-plane biaxial loading by Huang and Chou [1]. The variability in the cell-strut modulus of rupture is accounted for using a Weibull analysis in a manner similar to that used for brittle honeycombs.

Gibson and Ashby [2] verified that bending moment dominates cell-strut deformation in foams and thus proposed a cell-strut-bending model to analyze the mechanical properties of foams. For foams in uniaxial compression, cell struts might fail either by elastic buckling or by crushing [3–6]. The uniaxial compressive crushing strength of foams was described well by the cell-strut-bending model [2] if the cell-strut modulus of rupture is assumed to be constant. For foams in uniaxial tension, the tensile fracture strength is much smaller than the compressive crushing strength, caused by the propagation of pre-existing cracks. Maiti *et al.* [7] found that the tensile fracture strength of foams depends on cell size, relative density, and cell-strut modulus of rupture. Morgan *et al.* [8] observed that the tensile fracture strength of glass foams increases with the square root of cell size. Meanwhile, Brezny and Green [9] experimentally measured the cell size effect on the fracture toughness of reticulated carbon foams.

By assuming a constant cell-strut modulus of rupture, Gibson *et al.* [10] developed equations describing the failure surfaces for foams under multiaxial loads and Huang and Lin [11] analyzed the mixed mode fracture criterion for foams. In practice, the cell-strut modulus of rupture of brittle foams depends on the volume and Weibull modulus of solid cell struts. The strength variability in the cell-strut modulus of rupture should be taken into account in developing the failure envelopes for brittle foams under multiaxial loading. The survival probability of a brittle solid subjected to a non-uniform tensile stress can be calculated from a Weibull statistic analysis[12–14]. Huang and Gibson [15] verified that the cell-strut modulus of rupture of brittle reticulated vitreous carbon foams was described well by the Weibull statistic analysis. In the previous paper [1], we reanalyzed the failure envelopes of brittle honeycombs assuming that the cell-wall modulus of rupture followed a Weibull distribution. We now apply similar ideas to the analysis of failure envelopes of brittle isotropic foams under multiaxial loading.

2. Strength variability of brittle cell struts

A three-dimensional, open cubic cell of isotropic foams having square cell struts with a cross-sectional area of t^2 and a cell length ℓ is shown in Fig. 1. When the foam is subjected to remote multiaxial stresses σ_1^*, σ_2^* and σ_3^*, solid cell struts mainly suffer axial force or bending moment. For a brittle solid cell strut with a volume of V subjected to a non-uniform tensile stress, the failure probability of the cell strut can be calculated from the Weibull statistic analysis [12]:

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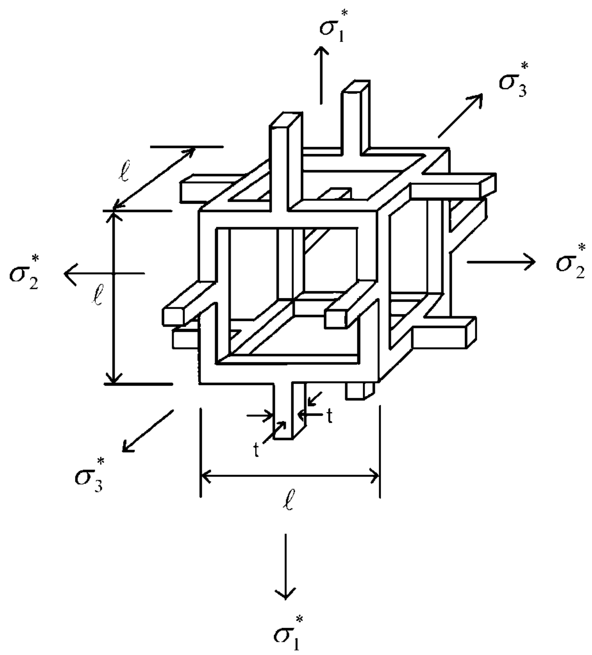

Figure 1 A three-dimensional, open cubic cell of isotropic foams with a cell strut thickness t and a cell length ℓ.

$$P_\mathrm{f} = 1 - P_\mathrm{s} = 1 - \exp\left[-\int_V \left(\frac{\sigma_\mathrm{s}}{\sigma_0}\right)^m \frac{\mathrm{d}V}{V_0}\right] \quad (1)$$

Here P_s is the survival probability, V_0 is the unit volume, σ_s is the tensile stress acting at any point within the cell strut, σ_0 is a scale parameter and m is the Weibull modulus. Note that only tensile stresses within the cell strut are taken into account in calculating the failure probability.

Owing to the nature of microstructure in foams, the loading configuration of individual cell strut is very complicated and its corresponding stress distribution is difficult to analyze. Nevertheless, the tensile stress at any point can be expressed in terms of the maximum tensile stress within the solid cell strut, σ_max:

$$\sigma_\mathrm{s} = H\sigma_\mathrm{max} \quad (2)$$

Here H depends on the loading configuration of the cell strut. The survival probability of the solid cell-strut beam can be obtained by substituting Equation 2 into 1, giving:

$$\begin{aligned} P_\mathrm{s} &= \exp\left[-\int_V \left(\frac{\sigma_\mathrm{s}}{\sigma_0}\right)^m \frac{\mathrm{d}V}{V_0}\right] \\ &= \exp\left[-\int_V \left(\frac{H\sigma_\mathrm{max}}{\sigma_0}\right)^m \frac{\mathrm{d}V}{V_0}\right] \\ &= \exp\left[-\left(\frac{\sigma_\mathrm{max}}{\sigma_0}\right)^m \int_V H^m \frac{\mathrm{d}V}{V_0}\right] \end{aligned} \quad (3)$$

When the maximum tensile stress reaches the cell-strut modulus of rupture, σ_fs, failure occurs. By setting $\sigma_\mathrm{max} = \sigma_\mathrm{fs}$ in equation (3), the survival probability of the solid cell-strut beam becomes:

$$P_\mathrm{s} = \exp\left[-\left(\frac{\sigma_\mathrm{fs}}{\sigma_0}\right)^m \int_V H^m \frac{\mathrm{d}V}{V_0}\right] \quad (4)$$

Therefore, the corresponding cell-strut modulus of rupture for a prescribed survival probability is found to be:

$$\begin{aligned} \sigma_\mathrm{fs} &= \left[\log\left(\frac{1}{P_\mathrm{s}}\right)\right]^{1/m} \left(\int_V H^m \frac{\mathrm{d}V}{V_0}\right)^{-1/m} \sigma_0 \\ &= \left[\log\left(\frac{1}{P_\mathrm{s}}\right)\right]^{1/m} \left(\frac{V_0}{V}\right)^{1/m} f(m)\sigma_0 \end{aligned} \quad (5)$$

Here $f(m)$ is a function of the Weibull modulus. From Equation 5, it is clear that the cell-strut modulus of rupture in brittle foams is not a constant, depending on the cell geometry, loading condition and material parameters of solid cell struts, and the prescribed survival probability.

The mean modulus of rupture of solid cell struts can be calculated from Equation 4:

$$\begin{aligned} \overline{\sigma_\mathrm{fs}} &= \int_0^\infty P_\mathrm{s}\,\mathrm{d}\sigma_\mathrm{fs} \\ &= \int_0^\infty \exp\left[-\left(\frac{\sigma_\mathrm{fs}}{\sigma_0}\right)^m \int_V H^m \frac{\mathrm{d}V}{V_0}\right] \mathrm{d}\sigma_\mathrm{fs} \\ &= \sigma_0 \left(\int_V H^m \frac{\mathrm{d}V}{V_0}\right)^{-1/m} \Gamma\left(1 + \frac{1}{m}\right) \end{aligned} \quad (6)$$

Here $\Gamma(1 + m^{-1})$ is the gamma function. Again, the mean cell- strut modulus of rupture depends on cell geometry and loading condition of solid cell struts. The mean cell-strut modulus of rupture can be further expressed as:

$$\overline{\sigma_\mathrm{fs}} = \sigma_0 \left(\frac{V_0}{V}\right)^{1/m} \Gamma\left(1 + \frac{1}{m}\right) f(m) \quad (7)$$

At the same time, the ratio of the cell-strut modulus of rupture for a prescribed survival probability and the mean cell-strut modulus of rupture can be obtained from Equations 5 and 7, regardless of the cell geometry and loading condition within solid cell struts:

$$\frac{\sigma_\mathrm{fs}}{\overline{\sigma_\mathrm{fs}}} = \frac{\left[\log\left(\frac{1}{P_\mathrm{s}}\right)\right]^{1/m}}{\Gamma\left(1 + \frac{1}{m}\right)} \quad (8)$$

It is obvious that the ratio depends on the prescribed survival probability and the Weibull modulus of solid cell struts.

3. Survival probability of brittle isotropic foams

The existing model for describing the failure envelopes of foams under multiaxial loading [10] will be modified to take into account the effect of strength variability in brittle cell struts. For brittle isotropic foams under multiaxial loading, brittle crushing, fast brittle fracture and elastic buckling are the three possible failure modes and will be considered here. Since the cell-strut modulus of rupture is not constant, the prescribed survival probability and the Weibull modulus of solid cell struts, presumably, will change the failure envelopes of brittle foams.

3.1. Brittle crushing

When a brittle isotropic foam subjected to a remote uniaxial compressive stress, σ_1^*, the induced bending moment exerted on an individual solid cell strut is found to be $M_s \propto \sigma_1^* \ell^3$ by using dimensional argument analysis. Brittle crushing occurs when the critical skin stress of the cell strut, which is $\sigma_s \propto M_s/t^3$ from the elementary mechanics of materials, reaches the cell-strut modulus of rupture. Since the relative density of the foam (the density of the foam, ρ^*, divided by that of the solid from which it is made, ρ_s) is proportional to t^2/ℓ^2, the uniaxial crushing strength can be expressed as:

$$\sigma_{cr}^* \propto \frac{t^3}{\ell^3}\sigma_{fs} = C_1\left(\frac{\rho^*}{\rho_s}\right)^{3/2}\sigma_{fs} \qquad (9)$$

Where C_1 is a microstructure coefficient and was found to be 0.2 by Gibson *et al.* [10]. The uniaxial crushing strength of brittle foams for a prescribed survival probability is thus obtained by substituting Equation 8 into 9:

$$\sigma_{cr}^* = C_1\left(\frac{\rho^*}{\rho_s}\right)^{3/2}\overline{\sigma_{fs}}\frac{\left[\log\left(\frac{1}{P_s}\right)\right]^{1/m}}{\Gamma\left(1+\frac{1}{m}\right)} \qquad (10)$$

For brittle foams under multiaxial loading, the induced axial force and bending moment are taken into account in calculating the critical skin stress within solid cell struts; the effect of shear force is negligible. Brittle crushing occurs when the critical skin tensile stress exceeds the cell-strut modulus of rupture for a given survival probability. Hence, the maximum tensile stress resulting from bending moment is:

$$\sigma_{fs} - \sigma_{as} = \frac{M_s t/2}{I_s} = \frac{6M_s}{t^3} \qquad (11)$$

Here σ_{as} is the axial stress and I_s is the second moment of area of individual solid cell strut. Note that the bending moment can be either positive or negative. Multiaxial loads with the three principal stresses of σ_1^*, σ_2^* and σ_3^*, can be decomposed into a hydrostatic stress state with $\sigma_m^* = (\sigma_1^* + \sigma_2^* + \sigma_3^*)/3$ and a deviatoric stress state with $\sigma_d^* = \sqrt{1/2[(\sigma_1^*-\sigma_2^*)^2 + (\sigma_2^*-\sigma_3^*)^2 + (\sigma_3^*-\sigma_1^*)^2]}$. Solid cell struts within foams deform axially under a hydrostatic stress state while they suffer bending under a deviatoric stress state.

At first, we consider a hydrostatic stress state of $\sigma_1^* = \sigma_2^* = \sigma_3^*$. The resulting axial stress exerted on individual solid cell strut can be expressed as:

$$\sigma_{as} = \frac{3\sigma_m^*}{\rho^*/\rho_s} \qquad (12)$$

Only one-third of solid cell struts carry axial force applied in any one direction, giving a constant 3 in the above equation. For brittle foams under a deviatoric stress state, the induced bending moment can be found using dimensional argument analysis:

$$M_s \propto \ell^3\sigma_d^* \propto \ell^3 \times \sqrt{\frac{1}{2}\left[\left(\sigma_1^*-\sigma_2^*\right)^2 + \left(\sigma_2^*-\sigma_3^*\right)^2 + \left(\sigma_3^*-\sigma_1^*\right)^2\right]} \qquad (13)$$

From Equations 11–13, the failure stresses for brittle crushing are:

$$\frac{\sigma_d^*}{\sigma_{fs}} = \pm\gamma\left(\frac{\rho^*}{\rho_s}\right)^{3/2}\left[1 - \frac{3\sigma_m^*}{\sigma_{fs}(\rho^*/\rho_s)}\right] \qquad (14)$$

The constant γ can be determined from a simple uniaxial compression by setting $\sigma_m^* = \sigma_1^*/3$ and $\sigma_d^* = \sigma_1^*$ in the above equation; γ is roughly equal to 0.2 if the relative density of foams is less than 0.25. Therefore, the failure stresses for brittle crushing become:

$$\frac{\sigma_d^*}{\sigma_{fs}} = \pm 0.2\left(\frac{\rho^*}{\rho_s}\right)^{3/2}\left[1 - \frac{3\sigma_m^*}{\sigma_{fs}(\rho^*/\rho_s)}\right] \qquad (15)$$

Since the cell-strut modulus of rupture is not a constant, the failure stresses for brittle crushing can be expressed in terms of the mean cell-strut modulus of rupture, the Weibull modulus and the prescribed survival probability from Equation 8:

$$\frac{\sigma_d^*}{\overline{\sigma_{fs}}}\frac{\Gamma\left(1+\frac{1}{m}\right)}{\left[\log\left(\frac{1}{P_s}\right)\right]^{1/m}} = \pm 0.2\left(\frac{\rho^*}{\rho_s}\right)^{3/2} \times \left[1 - \frac{3\sigma_m^*}{\overline{\sigma_{fs}}\left(\frac{\rho^*}{\rho_s}\right)}\frac{\Gamma\left(1+\frac{1}{m}\right)}{\left[\log\left(\frac{1}{P_s}\right)\right]^{1/m}}\right] \qquad (16)$$

From Equation 9, the failure stresses can be further expressed in terms of the mean uniaxial crushing strength, $\overline{\sigma_{cr}^*}$:

$$\pm\frac{\sigma_d^*}{\overline{\sigma_{fs}}} \pm 0.6\left(\frac{\rho^*}{\rho_s}\right)^{1/2}\frac{\sigma_m^*}{\overline{\sigma_{cr}}}\frac{\Gamma\left(1+\frac{1}{m}\right)}{\left[\log\left(\frac{1}{P_s}\right)\right]^{1/m}} = 1 \qquad (17)$$

It is clear that the failure stresses for brittle crushing depend on the relative density of foams, the Weibull modulus of solid cell struts, and the prescribed survival probability.

3.2. Fast brittle fracture

Figure 2 illustrates a brittle isotropic foam with a central macro-crack of length c, subjected to multiaxial loads. Using dimensional argument analysis, Maiti *et al.* [7] were able to derive the expression for mode I fracture toughness of foams as a function of cell size, relative

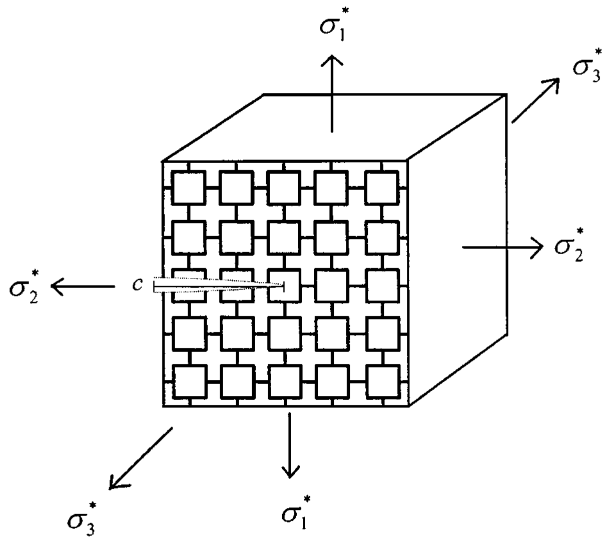

Figure 2 A brittle foam with a central macrocrack *c* under remote triaxial loading.

density and the modulus of rupture of solid cell struts:

$$K_{\mathrm{IC}}^* = C_2 \sigma_{\mathrm{fs}} \sqrt{\pi \ell} \left(\frac{\rho^*}{\rho_{\mathrm{s}}}\right)^{3/2} \tag{18}$$

Again, the cell-strut modulus of rupture is assumed to be constant and the microstructure coefficient $C_2 = 0.2$ suggested by Gibson *et al.* [10].

The tensile fracture strength of the brittle foam will be much lower than its brittle crushing strength due to the stress concentration effect around the macrocrack tip. When the critical skin stress within the first unbroken solid cell strut exceeds the cell-strut modulus of rupture for a prescribed survival probability, the macrocrack advances, giving the tensile fracture strength of the foam, σ_{fr}^*:

$$\begin{aligned} \sigma_{\mathrm{fr}}^* &= \frac{K_{\mathrm{IC}}^*}{\sqrt{\pi c}} = 0.2\sqrt{\frac{\ell}{c}} \left(\frac{\rho^*}{\rho_{\mathrm{s}}}\right)^{3/2} \sigma_{\mathrm{fs}} \\ &= 0.2\sqrt{\frac{\ell}{c}} \left(\frac{\rho^*}{\rho_{\mathrm{s}}}\right)^{3/2} \overline{\sigma_{\mathrm{fs}}} \frac{\left[\log\left(\frac{1}{P_{\mathrm{s}}}\right)\right]^{1/m}}{\Gamma\left(1+\frac{1}{m}\right)} \end{aligned} \tag{19}$$

Equations 8 and 18 have been used in the above equation. The tensile fracture strength depends on the Weibull modulus, cell size, relative density of the foam, and the prescribed survival probability. Meanwhile, the tensile fracture strength can be further expressed in terms of the mean brittle crushing strength from Equation 9:

$$\sigma_{\mathrm{fr}}^* = \sqrt{\frac{\ell}{c}} \sigma_{\mathrm{cr}}^* = \sqrt{\frac{\ell}{c}} \overline{\sigma_{\mathrm{cr}}^*} \frac{\left[\log\left(\frac{1}{P_{\mathrm{s}}}\right)\right]^{1/m}}{\Gamma\left(1+\frac{1}{m}\right)} \tag{20}$$

Under a multiaxial stress state of σ_1^*, σ_2^* and σ_3^*, fast brittle fracture occurs when the maximum tensile stress exceeds the tensile fracture strength of the brittle foam:

$$\mathrm{Max}\left(\sigma_1^*, \sigma_2^*, \sigma_3^*\right) = \sigma_{\mathrm{fr}}^* = \sqrt{\frac{\ell}{c}} \overline{\sigma_{\mathrm{cr}}^*} \frac{\left[\log\left(\frac{1}{P_{\mathrm{s}}}\right)\right]^{1/m}}{\Gamma\left(1+\frac{1}{m}\right)} \tag{21}$$

The failure stresses for fast brittle fracture depend on the Weibull modulus, cell size, macrocrack length and mean brittle crushing strength of brittle foams, and the prescribed survival probability.

3.3. Elastic buckling

Brittle foams under either uniaxial or multiaxial compressive loading might fail due to the bucking of one set of cell struts loaded axially up to their Euler buckling load. The remote triaxial stresses produce an axial load on individual cell strut. When the axial load reaches the Euler buckling load, elastic buckling occurs, giving the elastic buckling strength of the foams:

$$\sigma_{\mathrm{el}}^* = \frac{n^2 \pi^2 E_{\mathrm{s}} I_{\mathrm{s}}}{\ell^4} = \frac{n^2 \pi^2 E_{\mathrm{s}} t^4}{12 \ell^4} = \frac{n^2 \pi^2 E_{\mathrm{s}}}{12} \left(\frac{\rho^*}{\rho_{\mathrm{s}}}\right)^2 \tag{22}$$

Here E_{s} is the elastic modulus of solid cell struts. End constraint factor n^2 depends on stress state and buckling mode; Gibson *et al.* [10] presented a full analysis for a possible buckling mode and the corresponding end constraint factor for various multiaxial stress states.

Since the Euler buckling load depends only on the elastic modulus and slenderness of individual cell strut, the elastic buckling strength of brittle foams will not be affected by the Weibull modulus of solid cell struts and the prescribed survival probability. The end constraint factors for brittle foams under multiaxial loading suggested by Gibson *et al.* [10] are listed in Table I and will be utilized to construct the failure envelopes for brittle foams.

4. Discussion

From Equation 5, it is known that the cell-strut modulus of rupture in brittle foams depends on the cell geometry, the Weibull modulus of solid cell struts, and the prescribed survival probability. Consider two brittle foams made from the same solid material but with different cell size, relative density and prescribed survival probability; ℓ_1, $\rho_1^*/\rho_{\mathrm{s}}$ and $P_{\mathrm{s},1}$ for foam 1 while ℓ_2, $\rho_2^*/\rho_{\mathrm{s}}$ and $P_{\mathrm{s},2}$ for foam 2. From Equation 5, the ratio of the cell-strut moduli of rupture for the two foams is found to be:

$$\begin{aligned} \frac{\sigma_{\mathrm{fs},1}}{\sigma_{\mathrm{fs},2}} &= \left[\left(\frac{\ell_2 t_2^2}{\ell_1 t_1^2}\right) \log\left(\frac{P_{\mathrm{s},2}}{P_{\mathrm{s},1}}\right)\right]^{1/m} \\ &= \left[\left(\frac{\ell_2}{\ell_1}\right)^3 \left(\frac{\rho_2^*/\rho_{\mathrm{s}}}{\rho_1^*/\rho_{\mathrm{s}}}\right) \log\left(\frac{P_{\mathrm{s},2}}{P_{\mathrm{s},1}}\right)\right]^{1/m} \end{aligned} \tag{23}$$

TABLE I End constraint factor n^2 for elastic buckling of foams under multiaxial loads [10]

Load condition	n^2	$\sigma^*/\sigma_{\mathrm{el}}^*$
Uniaxial compression, $\sigma_1^* = \sigma^*$, $\sigma_2^* = \sigma_3^* = 0$	0.41	1.00
Biaxial compression, $\sigma_1^* = 0$, $\sigma_2^* = \sigma_3^* = \sigma^*$	0.36	0.88
Hydrostatic compression, $\sigma_1^* = \sigma_2^* = \sigma_3^* = \sigma^*$	0.34	0.83
$\sigma_1^* = \sigma^*$, $\sigma_2^* = \sigma_3^* = -\sigma^*/8$	0.42	1.02
$\sigma_1^* = -\sigma^*/2$, $\sigma_2^* = \sigma_3^* = \sigma^*$	0.37	0.90

It is noted that the cell-strut modulus of rupture increases with decreasing cell size, relative density and prescribed survival probability. The ratio in Equation 23 becomes smaller for a larger Weibull modulus. Also, Equation 23 indicates that the cell-strut modulus of rupture is a constant regardless of prescribed survival probability, cell size and relative density as the Weibull modulus approaching infinity for ductile foams.

From Equations 17, 21 and 22, it is found that the failure stresses for brittle crushing and fast brittle fracture in brittle foams are affected by cell size, relative density, Weibull modulus and prescribed survival probability while those for elastic buckling are only influenced by relative density. To investigate the effect of prescribed survival probability on the failure surfaces for brittle foams under multiaxial loading, the Weibull modulus is assumed to be constant and three different prescribed survival probabilities of 0.2, 0.5 and 0.8 are considered here. The resulting failure envelopes for axisymmetric loading of $\sigma_2^* = \sigma_3^*$ are plotted in Figs 3–6 for brittle foams with a Weibull modulus of 3, 6, 9 and 100, respectively. In the figures, the material parameters of brittle foams are assumed to be: $c/\ell = 4$, $\rho^*/\rho_s = 0.1$ and $\overline{\sigma_{fs}}/E_s = 0.01$. From Figs 3–5, it is seen that the area contained within the failure envelope for brittle foams decreases with increasing prescribed survival probability. For brittle foams with a higher prescribed survival probability, the cell-strut modulus of rupture will be smaller. As a result of that, the brittle foams will be more likely to fail, giving a smaller area contained within the failure envelope.

From Figs 3–6, it is also seen that the difference between the areas contained within the failure envelopes for various survival probabilities becomes smaller as the Weibull modulus increases. That is, the failure stresses for foams with same cell size and relative density but with a smaller Weibull modulus will scatter more widely than those with a larger Weibull modulus. It is expected that the failure envelopes will come close to a set of intersecting lines when the Weibull modulus becomes much larger as shown in Fig. 6.

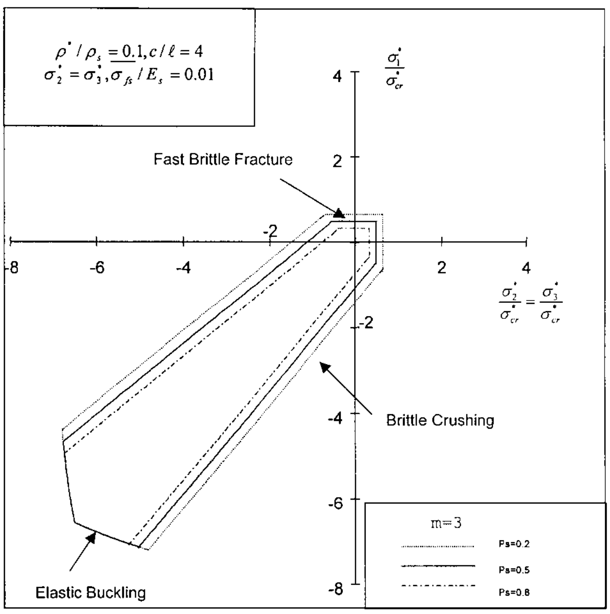

Figure 3 Failure envelopes for brittle foams with $m = 3$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8.

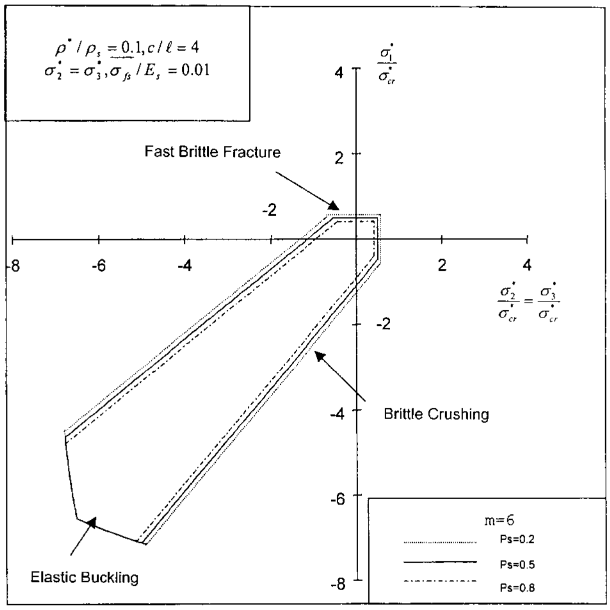

Figure 4 Failure envelopes for brittle foams with $m = 6$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8.

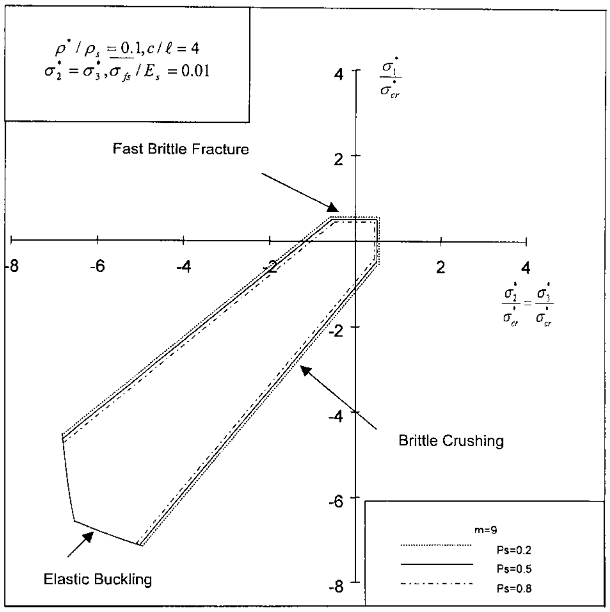

Figure 5 Failure envelopes for brittle foams with $m = 9$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8.

Since the cell-strut modulus of rupture depends on its volume, the failure stresses are different for brittle foams with various cell sizes even though they have same cell geometry and relative density. Consider two brittle foams having the same relative density, Weibull modulus and prescribed survival probability but different cell size; for instance, $\rho_1^*/\rho_s = \rho_2^*/\rho_s = \rho^*/\rho_s$, $m_1 = m_2 = m$ and $P_{s,1} = P_{s,2}$ but $\ell_1 \neq \ell_2$. The failure stresses for brittle crushing of foam 1 can be expressed in terms of the uniaxial brittle crushing strength of foam 2:

$$\pm\frac{\sigma_{d,1}}{\sigma_{cr,2}^*} + 0.6\left(\frac{\rho^*}{\rho_s}\right)^{1/2}\frac{\sigma_{m,1}}{\sigma_{cr,2}^*} = \left(\frac{\ell_1}{\ell_2}\right)^{-3/m} \qquad (24)$$

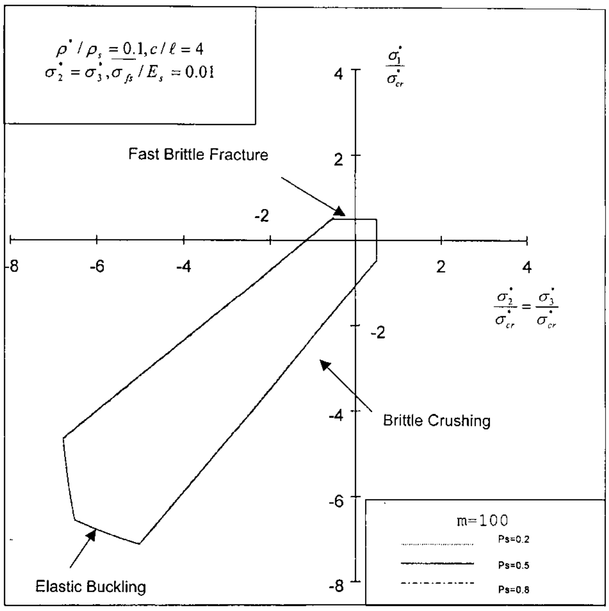

Figure 6 Failure envelopes for brittle foams with $m = 100$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8. The failure envelopes for various survival probabilities come closer to a set of intersecting lines as suggested by the existing model [10].

Meanwhile, if $c_1 = c_2 = c$ and $c/\ell_2 = c/\ell$, the failure stresses for fast brittle fracture of foam 1 can also be rewritten as:

$$\mathrm{Max}\left(\sigma_1^*, \sigma_2^*, \sigma_3^*\right) = \sigma_{\mathrm{fr},1}^* = \left(\frac{\ell_1}{\ell_2}\right)^{\frac{1}{2}-\frac{3}{m}} \sigma_{\mathrm{fr},2}^*$$
$$= \left(\frac{\ell_1}{\ell_2}\right)^{\frac{1}{2}-\frac{3}{m}} \sqrt{\frac{\ell}{c}}\sigma_{\mathrm{cr},2}^* \qquad (25)$$

The Euler buckling load of solid cell struts depends only on the relative density instead of the cell size of brittle foams. As a result, the failure stresses for elastic buckling of brittle foams exhibit no cell size effect.

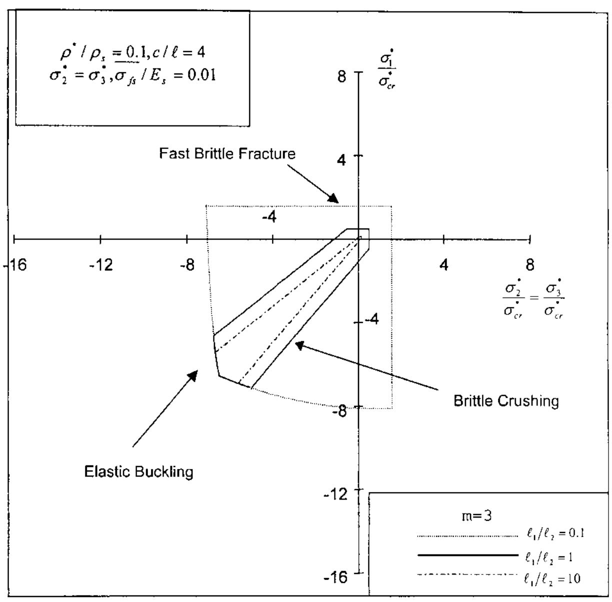

Figure 7 Failure envelopes for brittle foams with $m = 3$ and different cell sizes. The failure stresses for fast brittle fracture and brittle crushing are higher for brittle foams with a smaller cell size.

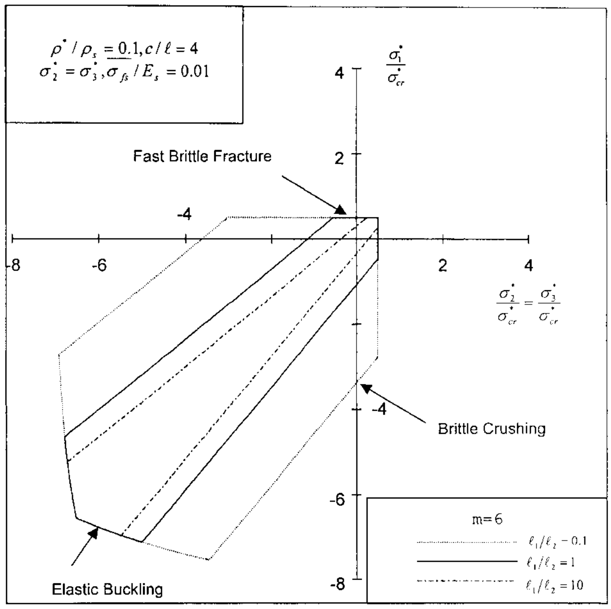

Figure 8 Failure envelopes for brittle foams with $m = 6$ and different cell sizes. Brittle foams with a smaller cell size have higher failure stresses for brittle crushing. There is no cell size effect for both elastic buckling and fast brittle fracture.

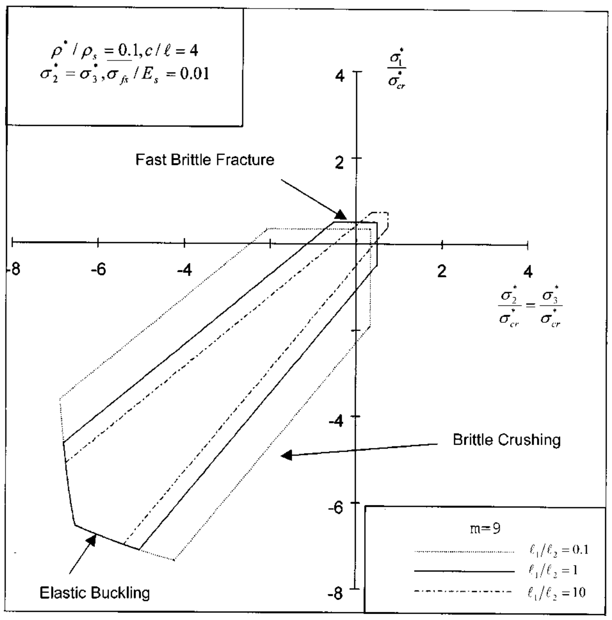

Figure 9 Failure envelopes for brittle foams with $m = 9$ and different cell sizes. Brittle foams with a smaller cell size have higher failure stresses for brittle crushing but lower failure stresses for fast brittle fracture. There is no cell size effect for elastic buckling.

The failure surfaces for brittle foams with different cell sizes are plotted in Figs 7–10 for various Weibull moduli. In the figures, the cell size ratio ℓ_1/ℓ_2 is set to be 0.1, 1.0 and 10, and the cell geometry and material properties of the two foams are assumed to be $c/\ell = 4$, $\rho^*/\rho_s = 0.1$ and $\overline{\sigma_{fs}}/E_s = 0.01$. It is seen that the failure stresses for brittle crushing in foams with a smaller cell size is higher than those with a larger cell size. However, the effect of cell size on the failure surfaces of brittle foams decreases as Weibull modulus increases. Cell size effect is insignificant if Weibull modulus becomes much larger, for instance $m = 100$ in Fig. 10, as expected for ductile foams.

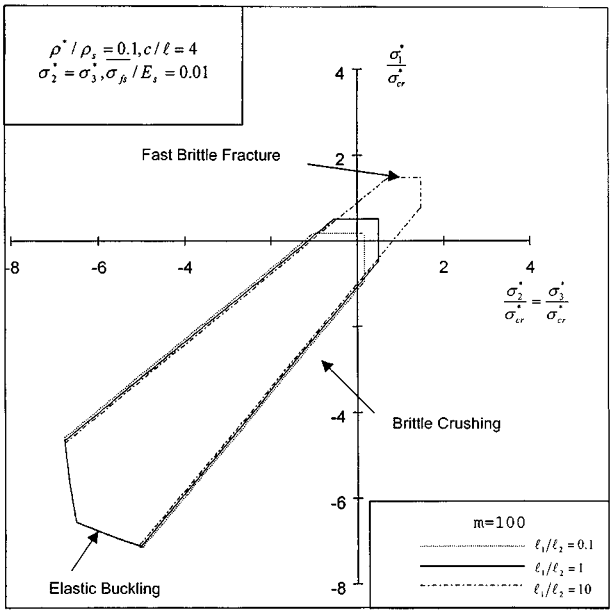

Figure 10 Failure envelopes for brittle foams with $m = 100$ and different cell sizes. The cell size effect on brittle crushing and elastic buckling is negligible.

From Equation 25, it is found that the failure surfaces for fast brittle fracture in brittle foams depend on the Weibull modulus of solid cell struts. The failure stresses for fast brittle fracture increase with decreasing cell size if m is smaller than 6; Fig. 7 shows the trend for $m = 3$. When $m = 6$, there is no cell size effect on the failure surfaces for fast brittle fracture as shown in Fig. 8. But, the failure stresses for fast brittle fracture increases with increasing cell size for brittle foams with $m > 6$ as shown in Fig. 9 for $m = 9$. It is expected that the failure stresses for fast brittle fracture in ductile foams increases with the square root of their cell size as illustrated in Fig. 10 for $m = 100$.

5. Conclusions

The Weibull analysis suggests that the cell-strut modulus of rupture depends on the volume and Weibull modulus of solid cell struts, and the prescribed survival probability. The cell-strut modulus of rupture increases with decreasing prescribed survival probability, relative density and cell size. In addition, the failure envelopes for brittle foams under multiaxial loading are presented for various prescribed survival probabilities and Weibull moduli. It is found that the areas contained within the failure envelopes for brittle foam increase with decreasing prescribed survival probability. The failure stresses for brittle foams with a smaller Weibull modulus will scatter more widely than those with a larger Weibull modulus. Meanwhile, cell size effect is significant for brittle crushing and fast brittle fracture in brittle foams. The failure stresses for brittle crushing increase with decreasing cell size. The failure stresses for fast brittle fracture decrease with increasing cell size if the Weibull modulus, m, of the cell strut material is less than 6; if $m = 6$, there is no cell size effect; and if m is larger than 6, the failure stresses increase with increasing cell size.

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