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Review

Oval concrete domes

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ARTICLE INFO

Article history:

Received 10 August 2016

Accepted 27 November 2016

Available online 18 January 2017

Keywords:

Concrete oval domes

Structural design

Forming techniques

ABSTRACT

The paper presents a review and update of the design and construction of oval concrete domes with a particular emphasis on the rectangular plane, with rounded corners, of the RC dome constructed in Wrocław, Poland, which is one of the few shells of this type realized in the world. The roots of oval domes lie in mediaeval times, when they were made as masonry. The best known dome examples of this period, as well as the development outline of thin oval concrete structures and the data on the geometry of such domes were given. One of the major challenges in the realization of such non-standardized concrete domes is applying novel, cost-effective forming techniques in their construction. A review of innovative low cost aesthetic forming systems of thin concrete shells was made.

Nearly a half of the paper makes a description of the design and construction of Wrocław's oval dome (23.7 m × 18.20 m × 4.70 m) of thickness 80 mm, which covers the rectangular nave of the church building. The RC dome is supported by two longitudinal and two transversal beams (200 mm × 2600 mm), which rest in the four corners of the building's masonry walls. The geometry and equations of the section curves as well as the results of a static analysis with the use of FEM as well as reinforcement drawings of the dome and its support beams were given. The particular phases of the dome construction illustrated by means of drawings were presented.

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1. Introduction

Oval concrete domes, which serve primarily as roof structures, are built in rounded forms. In everyday language, “oval” most often means the shape of an ellipse. The golden age of ovality was the Baroque period, in which this form occurred in architectural solutions of buildings as well their supported structures and covers with masonry domes (Fig. 1) [1].

The largest dome of this kind in the world spans the roof of the Sanctuary of Vicoforte in Italy, with an elliptical plane and the dimensions of axes 37.15 m and 24.80 m. The dome in

Vicoforte is the fourth largest in the world, after Saint Peter, S. Maria del Fiore and the Gol Gumbaz Mausoleum in India) [7]. The dimensions of the dome are much larger than those of the other elliptical domes in Rome – of Andrea del Quirinale (1658) by Bernini, S. Carlo by Borromini 1638, (Fig. 2), S. Giacomo by Volterra (1592), S. Hermenegildo in Córdoba (1616) and Convento de las Bernardas in Alcalá [5].

The world's first reinforced concrete oval dome on the plan of an ellipse was realized in Wrocław, Poland, in the years 1912–1913, designed by Hans Poelzig, in the object called Four Domes Pavilion [8], nearby the monolithic reinforced concrete ribbed dome of Centennial Hall from 1913 of Max Berg,

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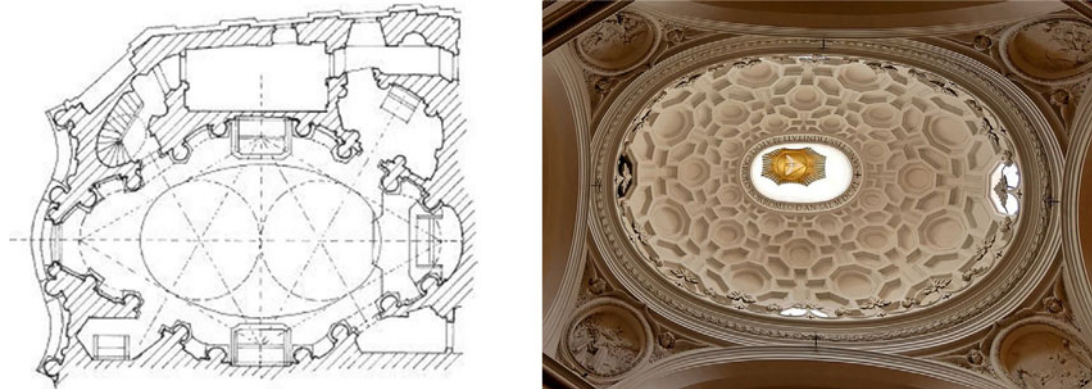


Fig. 1 – Horizontal section of the church building [2] and internal view of the dome [3]: San Carlo alle Quattro Fontane, designed by Francesco Borromini (1638) [4].

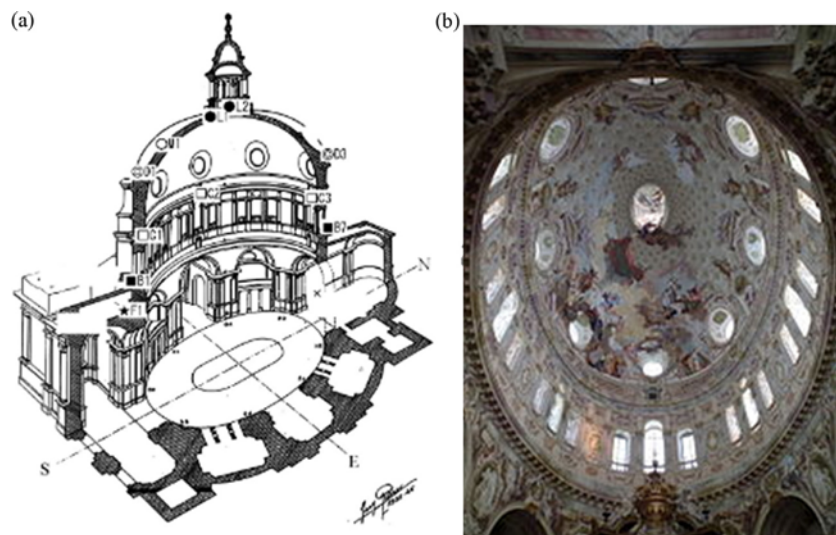


Fig. 2 – The elliptical masonry dome of the sanctuary of Vicoforte, Italy: (a) general view of the Sanctuary [5], (b) internal view of the dome [6].

spanning 65 m, which exceeded the span of the Pantheon in Rome. In 1957, in Jerusalem, a synagogue building shaped like a dome was constructed on a square plan, with rounded corners [9]. Heinz Isler, between 1956 and 1985, at the time of increasing labour costs in Europe, provide in for 749 shells standard sized so called bubble shells on rectangular plan construction in large scale re-use of formwork [10]. The largest spans of Isler's "bubble" shell had the size 54.6 m × 58.8 m [11]. The largest, until that time, solid (without ribs) concrete elongated oval domes in the form of an ellipsoid (93 m × 52 m × 22 m) was constructed in Chiasso, Switzerland [12]. In the city of Wrocław, in 1979, the construction of an oval concrete dome of a rectangular plan with rounded corners above the church nave¹ was started. The construction of the dome was completed in December 1987. This paper describes the design and construction of all the above mentioned domes and, broadly, the dome structure in Wrocław.

¹ Structural design: Sylwester Kobiela. Architecture: Wacław Kamocki.

2. Review of oval dome structures

2.1. Geometric configuration of oval domes and bases of structural design

An oval dome may be defined as a dome whose plan or profile (or both) has an oval form. The word "oval" comes from the Latin "ovum", i.e. "egg" [13]. Thus, an oval dome is egg-shaped. As the oval takes a curve made up of circular arcs or rounded forms approximate an ellipse of the same axes [13]. The ground plan of oval domes belongs to the family of curves ranging between an ellipse and a rectangle (Fig. 3). Shapes of curves configuration plan are expressed by the equation of the horizontally long super-ellipse.

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1 \quad (1)$$

When the $n=2$ figure is the ordinary ellipse, for values $n > 2$, we receive a super-ellipse, in particular, a rectangle for $n = \infty$, and a rectangle with rounded corners for $n = 9.9$.

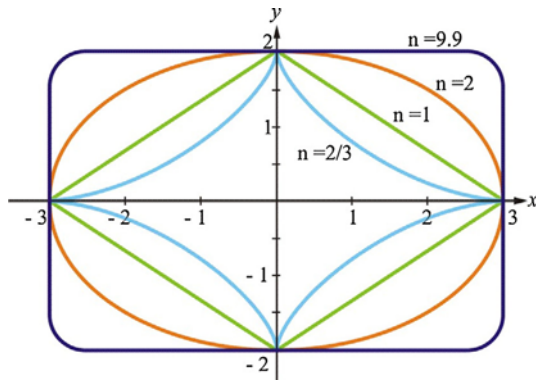


Fig. 3 – Graphic construction of super-ellipse [14].

Roof examples of plans with an ellipse or a super-ellipse equation are shown in Fig. 4a and b, respectively.

The ellipsoid dome of National Grand Theatre is given by Eq. (2) of axis lengths (a , b) 106 m and 72 m, respectively, and is 46 m high.

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1 \quad (2)$$

2.2. Elliptic dome of Hans Poelzig at Four Domes Pavilion in Wrocław, Poland

The axis lengths of the ellipse are 18.0 m and 13.6 m. The height of the dome is 5.1 m. The thickness of the reinforced concrete dome varies; in the keystone, it is 12 cm. The dome rests on a massive reinforced concrete ring of the cross section

dimensions 0.9 m × 1.3 m, supported by 22 pillars, which, in turn, stand on lower structural elements. The reinforced concrete dome is reinforced only meridionally by means of 12 mm diameter smooth steel rods of spacing 150–210 mm. The dome is also reinforced with ribs, hidden in its thickness, constructed of two steel angles L100 × 100 × 10 (Fig. 5).

2.3. Synagogue Dome, Jerusalem

The synagogue is maximum 3.74 m high, constructed from exposed concrete with an exterior oval hemisphere shape on eight arches [18] (Fig. 6).

2.4. Heinz Isler's bubble domes

Bubble shape domes were introduced by Swiss Heinz Isler, who, at the end of the 1950s, used inventive reusable formwork in their construction. During the time of 1956 and 1985, a total of 749 domes of the bubble type, arranged in groups, were constructed [20]. Heinz Isler managed to design his formwork is such a clever way (using, among other things, prefabricated curved wooden segments) that he was able to re-use it numerous times (Fig. 7). Because of the rigid strong curved corner ribs, bubble domes transfer about 90% of their total load directly onto the four corner supports. The two largest bubble shells of span 54.6 m × 58.8 m, with the maximum thickness of 150 mm in the critical areas and the span of 54 m × 54 m, were constructed in 1960. One dome serves as a distribution facility for railway wagons [10] with the centre rise of 9 m, with 17 circular roof lights supported by six intermediate supports and 17 circular roof lights, and the other dome covers Nekrasov (Maltzev) Market in St. Petersburg (Fig. 8).

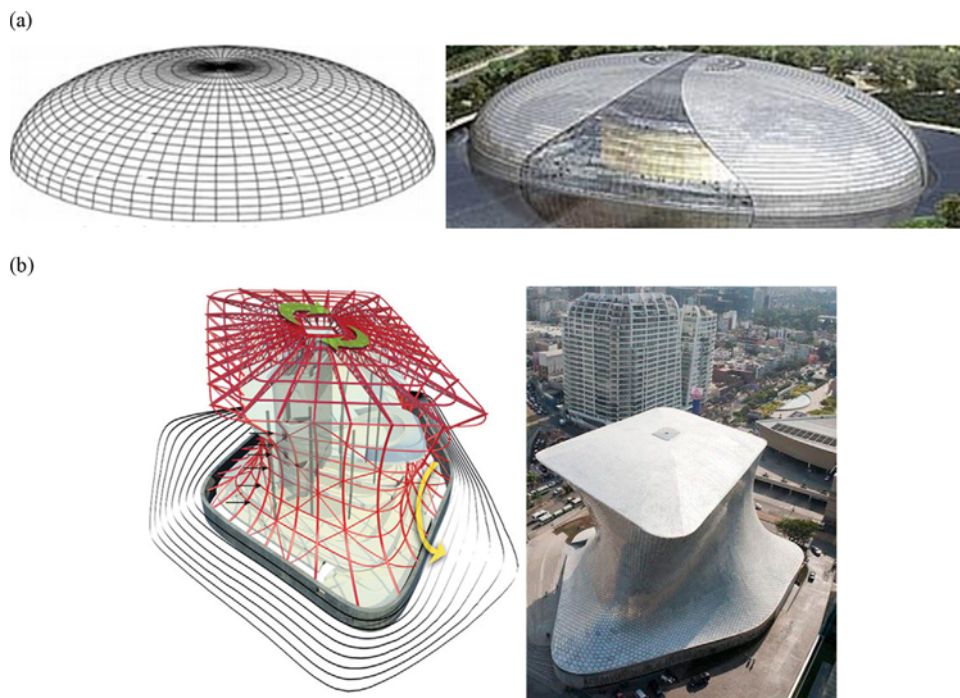


Fig. 4 – Buildings with an oval roof plan and profile: (a) structure and view (photo: Paul Andreu) of glass and titanium areas of the ellipsoid dome of National Grand Theater, Beijing. The ellipsoid dome designed by Paul Andreu, completed in 2007, (b) super-ellipse roof plan of the structure and view of Museum Soumaya by the architect Fernando Romero, Mexico 2011 [15,16].

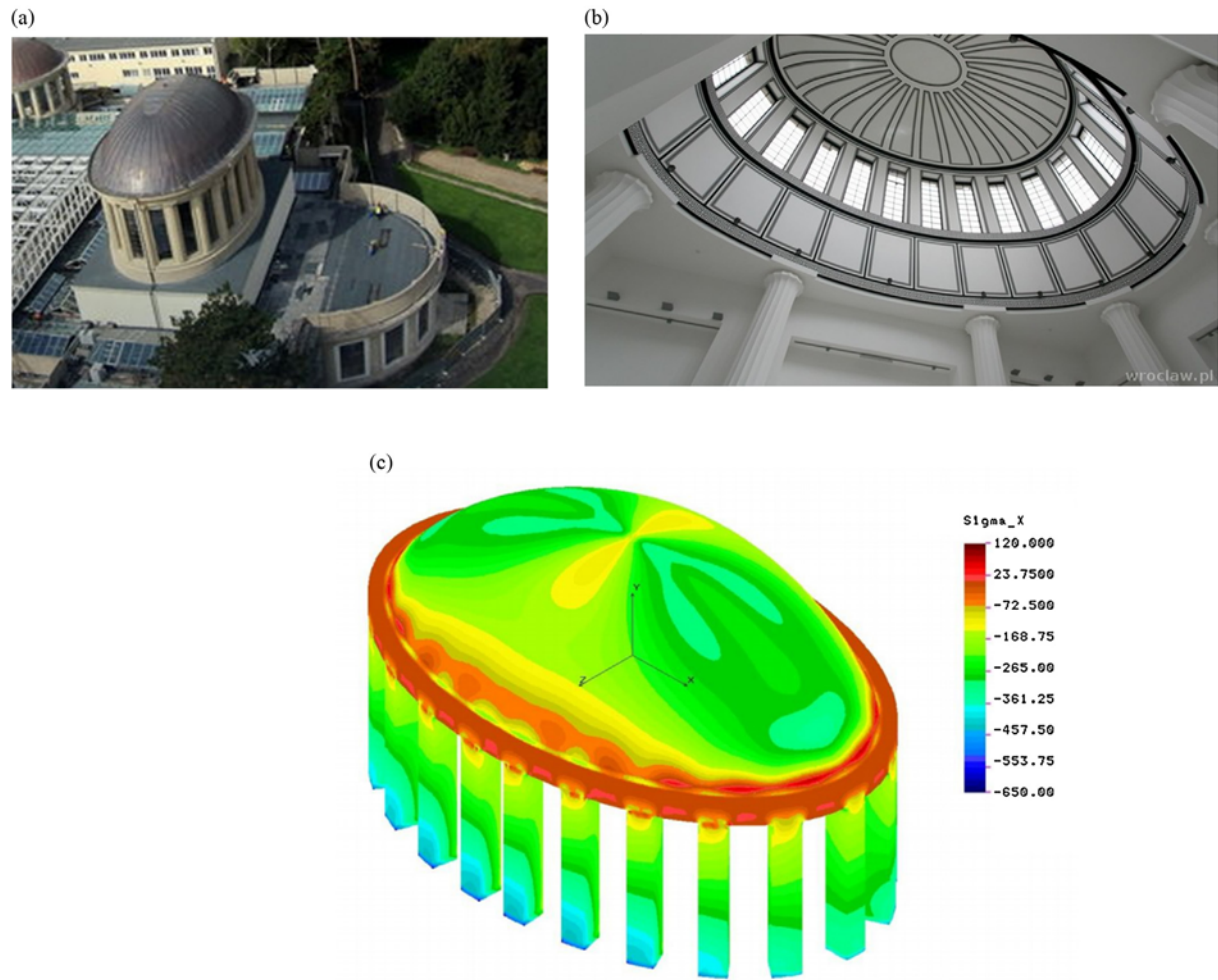


Fig. 5 – Elliptic oval dome designed by Hans Poelzig: (a) view of the dome after refurbishment and reconstruction with a glass cover of the internal courtyard as an exhibition place, completed in 2015 (photo: Michalak M./Agencja Gazeta), (b) internal view of the dome [17], (c) stress distribution in σ_x [kPa] in the dome structure [8].

2.5. Elliptical dome of Sree Kanteerava Indoor Stadium in Bangalore, India

The elliptical dome with long and short axes, 119 m and 91 m in length, respectively, which covers the Stadium is con-



Fig. 6 – The Rabbi Dr. Goldstein Synagogue of the capacity of 100 worshippers is located on the campus of the Hebrew University, designed by Heinz Rau and David Resnick, completed in 1957 [19].

structed of 120 folded concrete precast plates, spanning about 40 m between two rings (lower and upper) of a varying cross-section (average 2.0 m), with the plate thickness of 40 mm and a series of interconnected ribs. The lower external elliptical ring at the 8 m level is supported by 24 arch columns. The upper, also elliptical, ring with the axes size of 16 m and 8 m is at the level of 29 m. An elliptic paraboloidal in situ dome is constructed on the top of the upper ring [24–26]. This dome is probably the largest concrete oval dome in the world (Fig. 9).

2.6. Centro Ovale Shopping Centre in Chiasso, Switzerland

In Chiasso, Switzerland, a concrete dome was designed for covering a mall in the form of an ellipsoid of 92.8 m (major axis) \times 51.8 m (minor axis) \times 22.5 m (high), with the thickness varying between 100 and 120 mm. The thickness of 100 mm fulfilled the requirements of the reinforcement cover and the adequate safety against buckling. The structure of the concrete dome was designed by Muttoni A., Lurati F. and Fernandez and the architecture was made by Ostinelli E. It was completed in 2010 (Fig. 10).

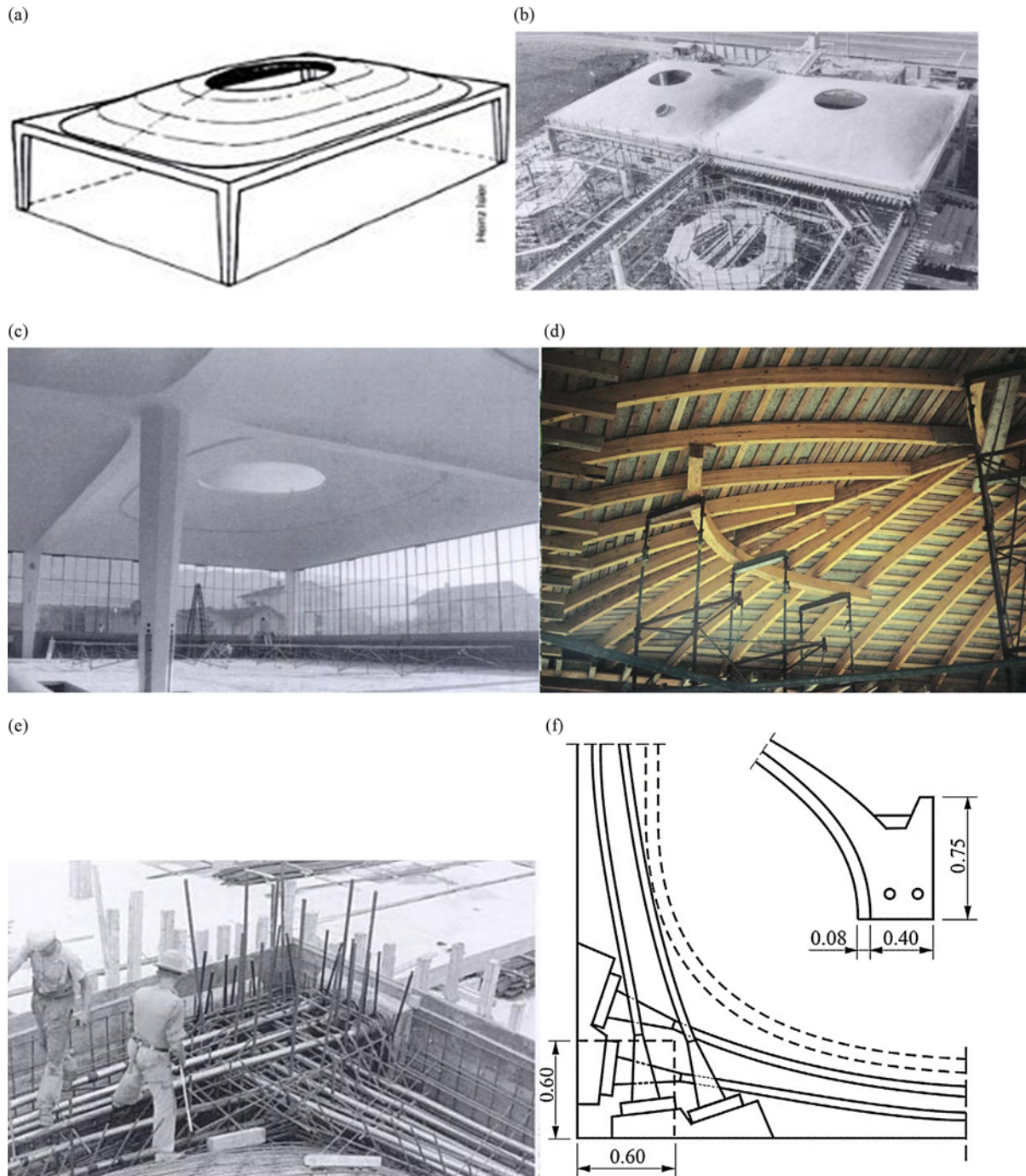


Fig. 7 – Isler's bubble domes [10]: (a) standard bubble shell type with only corner supports [10], (b) construction view of bubbles domes [10], (c) forms of edge beams and columns for the internal support of the multi-bay building for Eschmann, Thun [10], (d) formwork system by Heinz Isler [21,22], (e) the prestressing cables are crossed over and under each other at the corners of the shell [10], (f) plan and section of typical prestressing cables anchorage in the bubble shell [10].

2.7. The Ankara CSO Concert Hall

The egg-shaped dome of CSO Concert Hall was built in 1966–2000 with the use of the Monolithic Dome technology, with a vaulted triangular foyer (Fig. 11).

3. Construction methods of double curved shells

Concrete shells are generally made with the use of the following methods [11,31–33]:

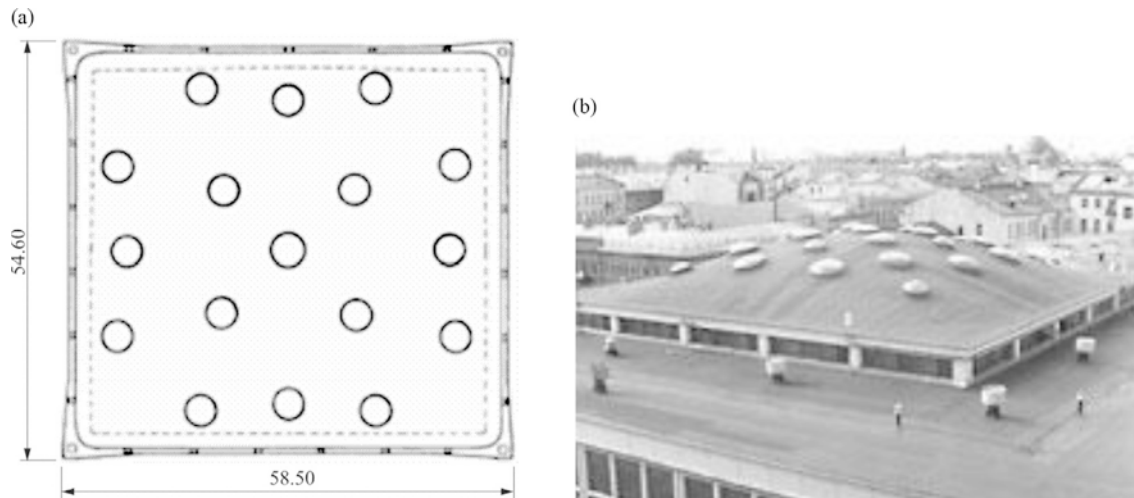


Fig. 8 – The largest bubble shells of Heinz Isler: (a) the shell plan with 17 circular roof lights [10], (b) reinforced concrete dome under Nekrasov (Maltzev) Market in St. Petersburg [23].



Fig. 9 – Elliptical Sree Kanteerava Indoor Stadium in Bangalore. Capacity 5000, built: 1995, architecture design: Sundaram Consultants (a) Stadium view [27] (photo: Sanyam Bahga), (b) erection of roof elements [28].

- conventional formwork system consisting of formwork (board) fix framework with cast-in-place concrete on a formwork,
- pneumatic air-supported formwork systems and
- cast in fabric formworks.

Cast-in-place shells are suitable for unique and complicated shapes, especially where the geometry of the shell is not easy in partition of the shell surface into precast elements. Fig. 12 shows the reinforced concrete formwork of a spherical dome with the diameter of 20.13 m as the house of a school's planetarium [34]. If air-supported forming is used, a wide variety of shell shapes can be built, also with a complex geometry. Fig. 13 presents the frequently applied air-supported pneumatic oval forms.

An important development for a fast and more economical shell construction was the concept of air-inflated membranes as the formwork for concrete shells. The best-known concepts include:

- the Bini Shell of Dante Bini [35] developed and patented in 1986 [36], which an inflated airform has place after placing the reinforcement and concrete horizontally, on a pre-shaped, neoprene-coated, nylon membrane.
- shotcrete domes (Fig. 14),
- and the free-form, without formwork and falsework (centering), pneumatic wedge method (Fig. 15), which is followed by a transformation process from the initially flat plates to a dome [37].

Other construction techniques for shell concrete domes include:

- fabric formwork (Fig. 16),
- innovative industrial method called the TaylorCrete (Fig. 17),
- metallic framework (Fig. 18).

West [38], Veenendaal et al. [39], Brennan et al. [40] and Pedreschi [41], taking into account the benefits of textile

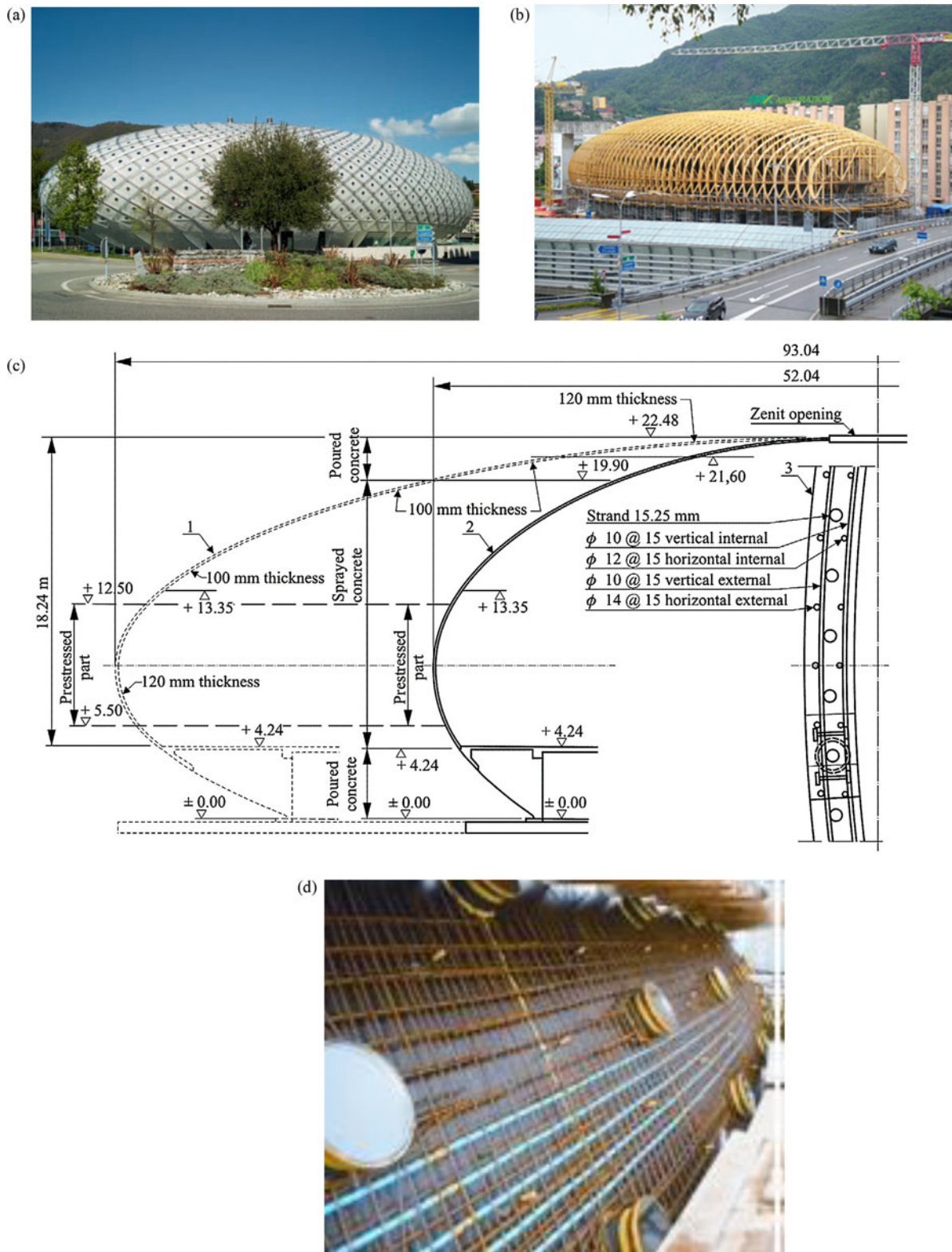


Fig. 10 – Centro Ovale Shopping Centre in Chiasso [12]: (a) general view, (b) temporary falsework photo: Aurelio Muttoni, (c) shell sections: 1 – longitudinal section, 2 – cross section, 3 – reinforcement in the equator region with prestressing tendons and transverse reinforcement equal to 0.5% of the concrete surface, (d) view of prestressing tendons at the level from +5.50 to +12.50 m.



Fig. 11 – Reinforced concrete ellipsoid dome of CSO Concert Hall of up to 2000 concert-goers, Ankara, Turkey [29,30].

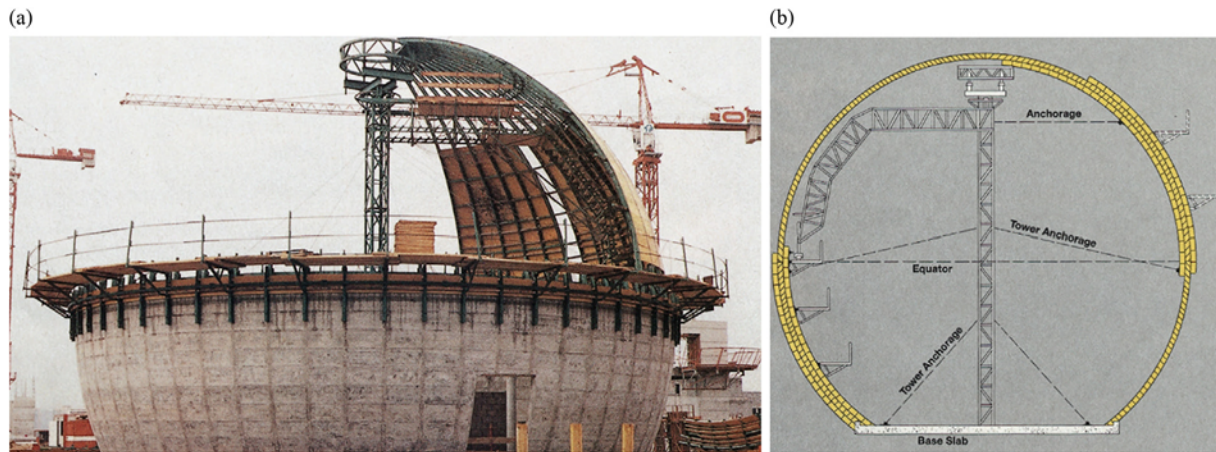


Fig. 12 – Construction of a concrete sphere house planetarium [34]: (a) placing the interior formwork for the upper half of the sphere above the equator, (b) schematic view of the equipment with a central tower and a rotating framework supporting the access ladder for the workers.

formwork, which is lighter and easier to construct, as well as a wide variety of new and exciting architectural forms and finishes (see Fig. 16), concluded that formworks with textile is a rapidly evolving technology, which challenges the conventional approach to the production of concrete structures.

Another construction method is called the TaylorCrete, developed by Danish Technological Institute (DTI) [42], which makes it possible to create concrete structures of complex shapes using industrialized approaches based on a digital architecture design, robotic fabrication and advanced concrete technologies. The Turkish architectural firm SuperPool [43] designed the structure called Demonstrator (Fig. 17) using the

technology developed in the TaylorCrete research project of DTI.

A metallic framework (Fig. 18) for the dome erection is based on the implementation of a steel prefabricated framework and then its support precast slabs. The slabs are grouted and covered with a thin concrete layer, cast in place [48].

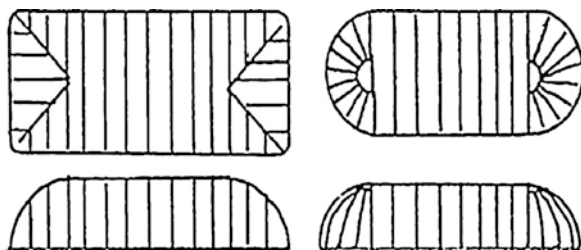


Fig. 13 – Elongated air-supported domes.

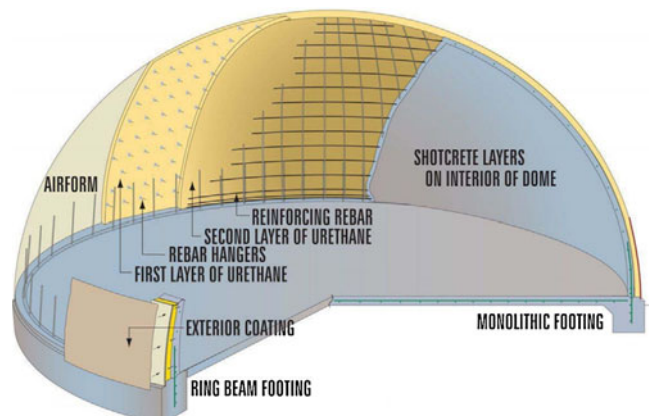


Fig. 14 – Monolithic dome concept [29].

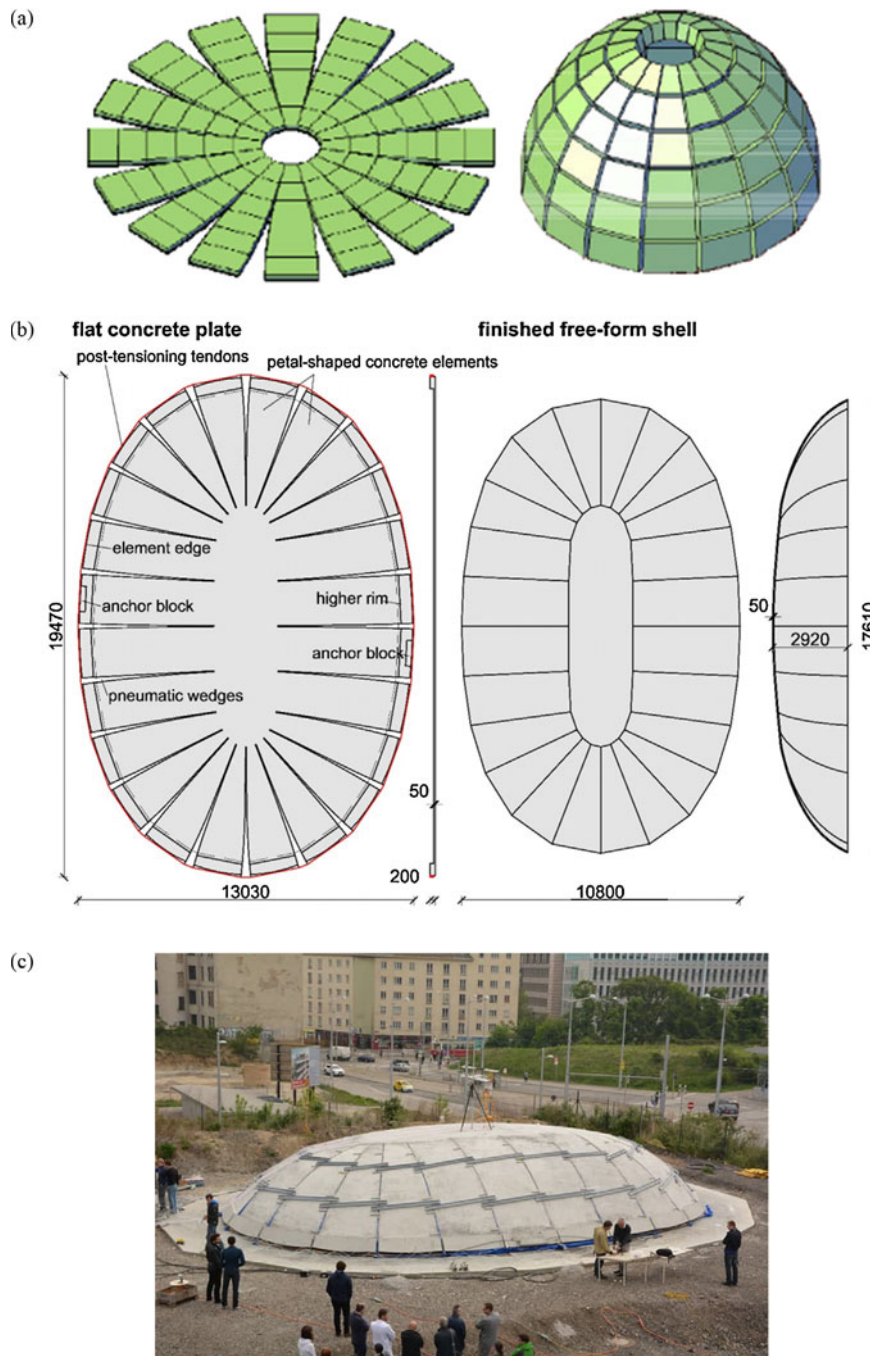


Fig. 15 – Erection of an oval concrete dome with the free-form pneumatic wedge method [37]: (a) transformation process from the initially flat plate to a double-curvature shell [44], (b) erection project of a free-form concrete shell prototype of dimensions $17.6 \text{ m} \times 10.8 \text{ m} \times 2.9 \text{ m}$ [37], (c) final stage of erection [45].

4. Design and construction of a reinforced concrete oval dome over a rectangular nave of the church building in Wrocław, Poland

4.1. General characteristics of the reinforced concrete dome structure

For covers on rectangular plans, which is characterized by the optimal use of the space underneath vs. the flat load-carrying

structure as one, two, three or four way truss arrangement belongs the oval elliptical reinforced concrete domes. The paper describes the design and construction of the RC dome cover over a rectangular nave of the church building (Fig. 19). Two frontal triangle naves are adherent to this nave, forming a building of a parallelogram plane. In the frontal naves, there are mezzanines covered with a flat rib-and-slab floor.

The dome is supported on two longitudinal beams B1 and two transversal beams B2 with the cross-section of $200 \text{ mm} \times 2600 \text{ mm}$ (Fig. 20). At the supports, these beams

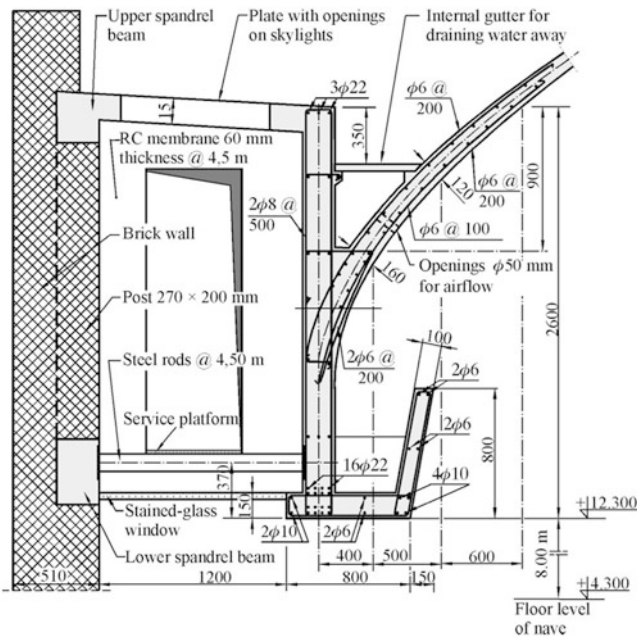


Fig. 20 – Reinforcement of the abutment beam and the dome in the support zone.

were “undercut” by 280 mm, and their width, on account of the operation of high shear forces, increased from 200 mm to 400 mm (Fig. 21). These beams were undercut because of the need of hiding them behind the stained-glass window. Considering the use of the internal gutter for draining water away, 50 mm openings were allowed to enable the flow of warm air and, consequently, to ensure snow melting under winter conditions.

The shell thickness equals 80 mm. The dome weight is transferred onto a brick wall of thickness 510 mm in the four corners of the building, in the areas of mutual penetration of beams B1 and B2 (Figs. 21 and 29) by reinforced concrete pads.

The longitudinal beams B1 and the transversal beams B2 are situated 1.40 m from the external combined masonry-concrete walls. The dimensions of the rectangular nave inside the brickwork are 27.52 m × 21.00 m. The axial spacing of the longitudinal and transversal beams is 18.2 m and 23.7 m, respectively. The dome is composed of two types of elliptical surfaces (zones). Zones I are formed by an elliptical cylindrical shell, whereas zones II constitute four wedges of an ellipsoidal

shell situated in each corner of the dome. The remaining flat roof strip between the external brick wall and beams B1 and B2 is covered with a reinforced concrete plate. In this strip of the RC plate, around the dome, there are 55 square openings covered with skylights (Figs. 19 and 20) through which daylight penetrates the inside of the nave.

These plates, being at the upper level of the beams together with the Viereendel trusses formed from the lower “channel” part of beam B1 and B2 and from spandrel beams in the walls connected by means of reinforced concrete vertical diaphragms, given every 2.25 m near supports and 4.50 m in span, constitute a bearing system transferring the horizontal component of the reaction of the dome. In the beam span, alternately with the RC diaphragms, horizontal posts were given (Fig. 21). The mentioned diaphragms and posts were applied for the purpose of stability protection of beam B1 and B2. These elements were also used to support the steel service platform. In the beams as well as in the diaphragms, openings are made and after the platform has been laid (Fig. 20), it is possible for electricians to move in the outside and inside space of the dome (nave). At the lower level of the beams beneath the service platform, a stained-glass window has been given.

4.2. Geometry and structural analysis of the dome

An approximate structural analysis of the dome was carried out for a substitutional bar space system (Fig. 22) with loads imposed into joints with the use of the existing programme intended for space frames. The coordinates of the joints and other geometrical data for bars necessary for the analytic purpose were established by way of using the equations of elliptical curves of the dome sections and the equations of the parallels describing the dome surface in zones I and II.

4.2.1. Equations of the section curves

Assuming the dome geometry and notations according to Fig. 23, the equations of the section curves are as follows:

Zone I:

$$0 \leq \frac{y}{x} \leq \frac{c}{d}$$

$$z^2 = a^2 \left(1 - \frac{x^2}{d^2} \right), \text{ does not depend from } y \tag{3}$$

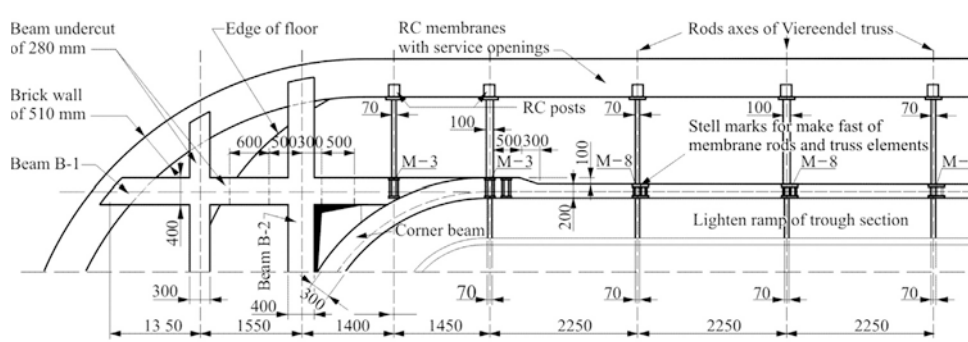


Fig. 21 – Mutual penetration of beams B1 and B2 (400 mm wide) at the support (see also Fig. 29).

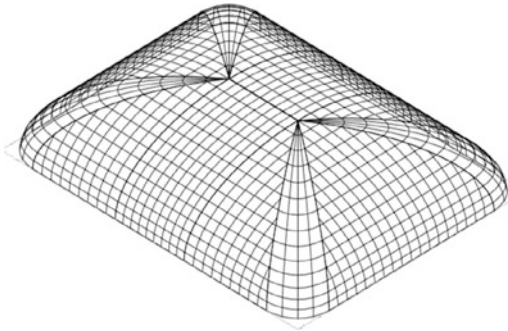


Fig. 22 – Model of the finite elements of the dome.

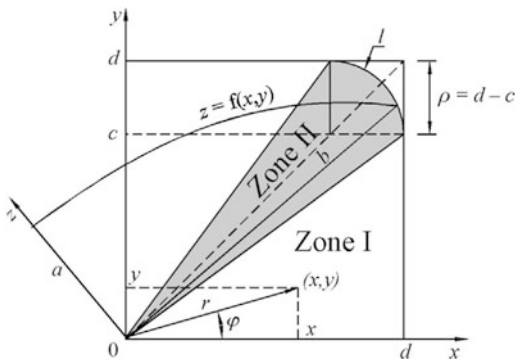


Fig. 23 – Geometry and notations of the dome.

Zone II:

$$\frac{c}{d} \leq \frac{x}{y} \leq 1$$

$$z^2 = a^2 \left(1 - \frac{r^2}{b^2} \right) \tag{4}$$

where

$$r = \sqrt{x^2 + y^2} \tag{5}$$

The circle l ($x_0 = d$) is expressed by the equation:

$$(x-c)^2 + (y-c)^2 = \rho^2$$

By introducing the polar coordinates: $\varphi = \arctg \frac{y}{x}$, $x = r \cos \varphi$ and $y = r \sin \varphi$ into Eq. (5), we obtain:

$$r^2 = 2cr(\cos \varphi + \sin \varphi) + 2c^2 - \rho^2 = 0$$

and

$$r = c \left(t + \sqrt{t^2 + p} \right) \tag{5a}$$

where:

$$p = \frac{\rho^2}{c^2} - 2 = \left(\frac{d-c}{c} \right)^2 - 2$$

$$t = \frac{x+y}{r}$$

According to Eq. (5a):

$$b^2 = 2c^2 \left(t^2 + t \sqrt{t^2 + p} + \frac{p}{2} \right)$$

4.2.2. Equations of the parallels

The equations of the parallels corresponding to the initial value of x_0 (Fig. 24)

In Zone I:

$$0 \leq \frac{y}{x} \leq \frac{c}{d}$$

$$\text{for } z = \sqrt{a^2 \left(1 - \frac{x_0^2}{d^2} \right)} \tag{6}$$

the equation of the parallel is a straight line $x = x_0$

In Zone II:

$$\frac{c}{d} \leq \frac{x}{y} \leq 1$$

the equation of the parallel is a circle with the centre (c', c') and the radius ρ'

$$\left(x - \frac{x_0 c}{d} \right)^2 + \left(y - \frac{x_0 c}{d} \right)^2 = \left(\frac{x_0 \rho}{d} \right)^2 \tag{7}$$

where:

$$c' = \frac{x_0}{d} c$$

$$\rho' = \frac{x_0}{d} \rho$$

4.3. Reinforcement and construction of the dome

The reinforcement of the abutment beams B1 and B2 (Fig. 20) and the dome (Fig. 25) was determined based on the results of the analysis of the substitutional bar system.

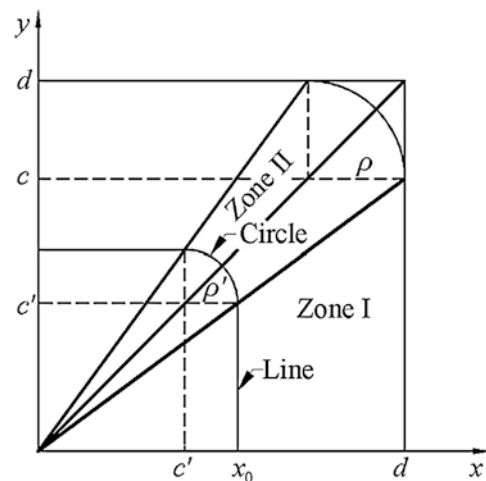


Fig. 24 – System of parallels.

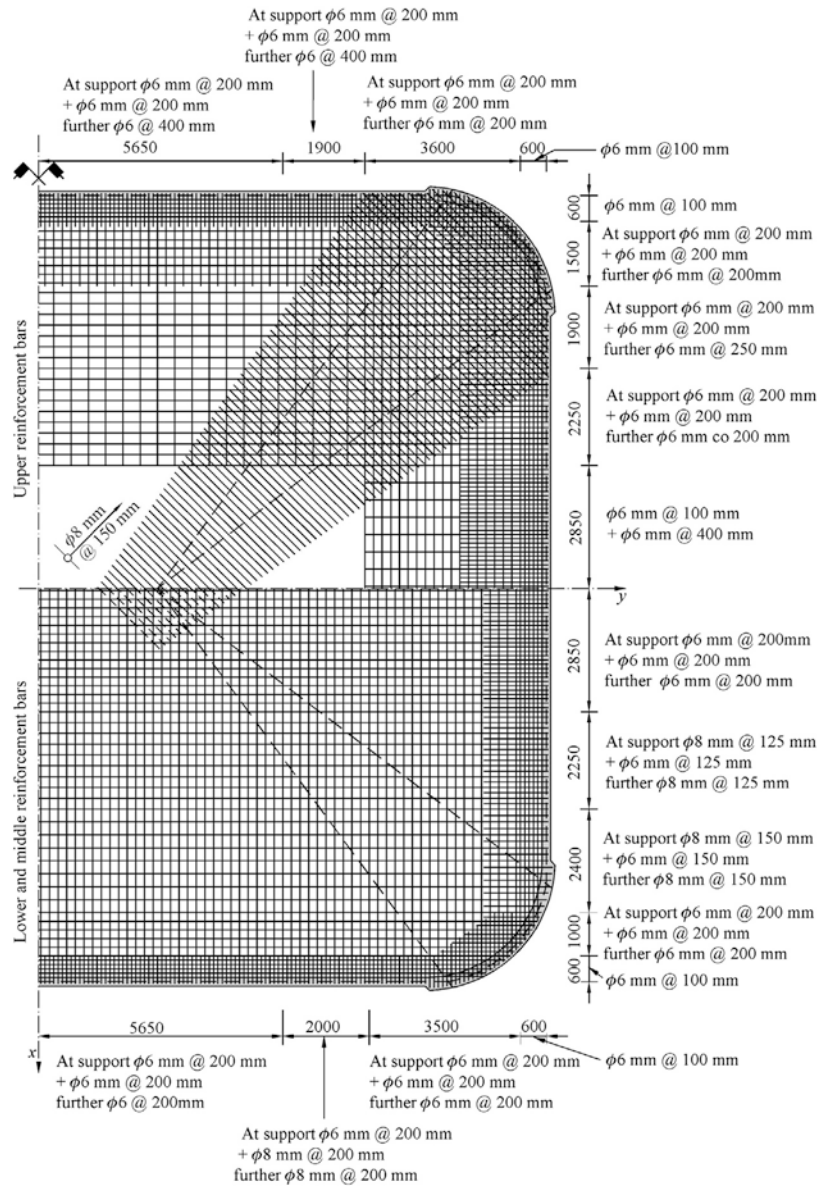


Fig. 25 – Reinforcement of dome.

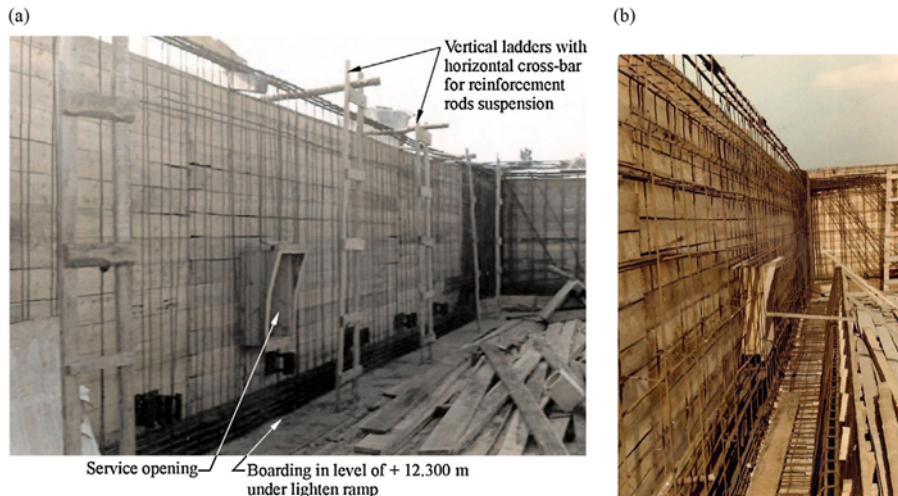


Fig. 26 – View of the scaffolding of the vertical beam and the vertical ladders with longitudinal bars on the top and suspended stirrups: (a) and (b) with and without reinforcement and a formwork channel lighten ramp.

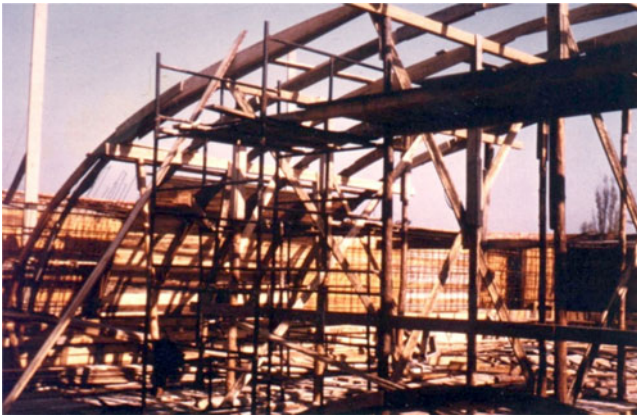


Fig. 27 – View of the assembled wooden centerings and their shoring. Behind them, visible reinforcement of the abutment beam B1 and its formwork.

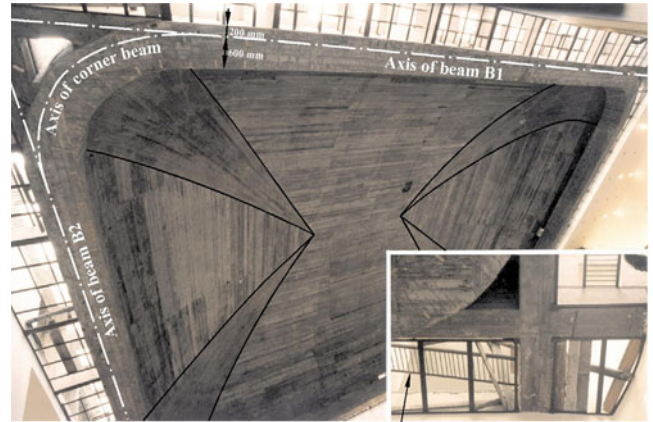


Fig. 29 – Picture of the dome from below after the removal of the boarding, with visible board tracks.

After the erection of the outside brick walls to the level of the upper spandrel beam (see Fig. 20), with leaving cavities for posts with steel rods and the execution of floors over the triangle naves, the realization of the dome began, which was progressing in the following sequence:

1. Setting up of a wooden platform (boarding) under the abutment beams B1 and B2 at their lower level, i.e. +12.300 m (Fig. 20): platform supported by wooden shores on the floor of the nave (+4.00 m).
2. Assembly of the formwork of the vertical face of the abutment beams B1 and B2 (Fig. 26a and b) and the circular

- corner beams as well as the formwork and reinforcement of these beams in the support zones (so called undercut beams, Fig. 21) with leaving service openings (see Fig. 26).
3. Assembly of reinforcement bars with the use of vertical ladders, starting from the placement of longitudinal bars on cross-bars and the suspension of stirrups in the respective spacing, and then the “distribution” of longitudinal bars along the height of the beam and their tying; at this stage, steel marks were nailed to the formwork to which steel rods and reinforcement bars of the diaphragms were welded. The state of the construction work after this stage is illustrated in Fig. 26a.

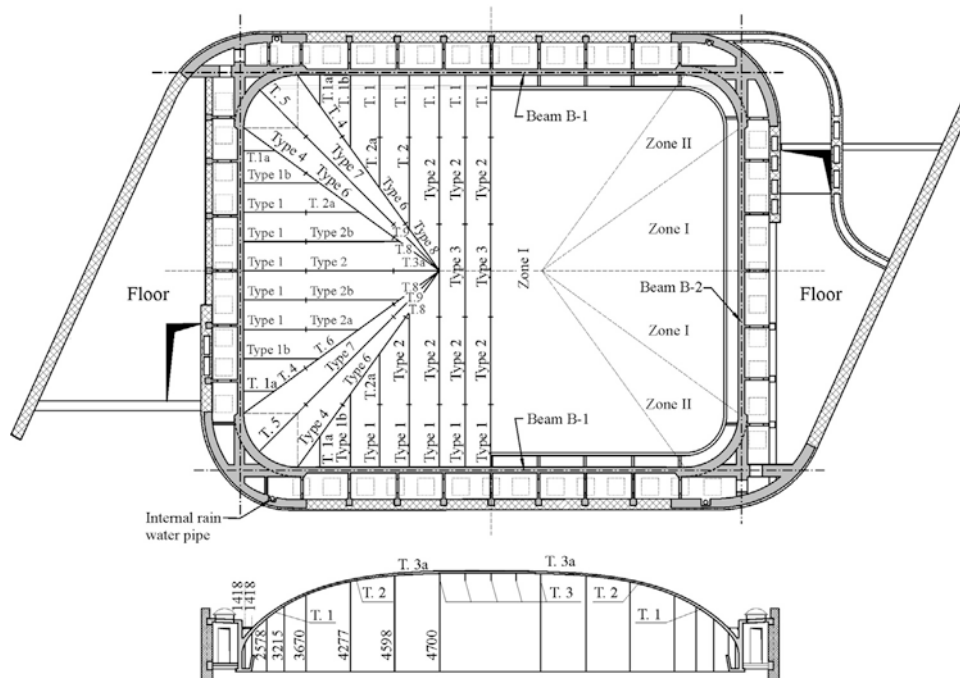


Fig. 28 – System of centerings used for dome scaffolding.



Fig. 30 – Placing of a gypsum finishing coat on the concrete dome surface.

4. Carrying out the formwork and reinforcement of the channel fulfilling the function of a lightening ramp (Fig. 26b) and the formwork of the circular corner beam.
5. Assembly of wooden centerings and their shoring (Fig. 27) and boarding, after which steel bars were arranged according to Fig. 20 and next concrete was placed; the particular types of centering were laid-out according to Fig. 28. The centerings were made from boards on a special platform after the shape of each particular centering was previously drawn on it, and next the boards were laid-out respectively to the centering shape, followed by their nailing.
6. Disassembly of the boarding; Fig. 29 shows the view of the concrete dome from below after the removal of the boarding; the accuracy of ± 10 mm of the lower dome surface was obtained.
7. From below, a gypsum finishing coat on a plastic screen was made (Fig. 30), after the previous clearcoating of the concrete base.

4.4. Concluding remarks

The work presented a review and update of the design of oval domes with a particular emphasis on the rectangular plane concrete dome constructed in Wrocław, Poland, which is one of the few shells of this type constructed in the world. The roofs on the orthogonal plane of the oval elliptical concrete dome described above can be used to cover large spaces and they are characterized by the optimal use underneath the space, compared with the steel long span planar supporting systems, such as one-directional, bidirectional or three way truss and lattice grids. The oval roof shape on a rectangular plane gives the designer more architectural freedom. Also, less effort must be invested to ensure structure stability as well as the constraints of packing rooms together, flexibility of dimensioning allowed by rectangular arrangements [49].

The important issues in the case of oval covers are the construction of excessively complicated and expensive formwork systems, especially with the use of conventional construction methods, and the design of the structure of such concrete shells, which is more complex than for the surface of

revolution, due to second-order effects and edge forces, as well as the design of the buckling, which is not suitably covered by codes of practice [12]. Some advantages to conventional erection method give presented here new construction methods: the TailorCrete of Danish Technological Institute and the free-form with the use of a pneumatic formwork invented at Vienna University.

Acknowledgements

Authors would like to thank Dr. A. Klimek and Dr. R. Tatko for developing a FEM.

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