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Cellular automata in nonlinear string vibration

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Cellular automata procedure for the analysis of nonlinear viscously damped transverse string vibrations was defined. Parameters were obtained by comparing the cellular automata defining rules with relations resulting from the discrete form of the mathematical description of the investigated phenomenon. A series of numerical tests that were run confirmed the agreement between the obtained results and the solutions found in the literature. Numerical results demonstrate that cellular automata can constitute a simple and effective tool for the analysis of a range of complex problems which have not been analyzed in this way before.

Keywords: *cellular automata, string, nonlinear vibration, dynamic loading, moving forces*

1. Introduction

Most of mechanics problems are usually mathematically modelled by ordinary differential equations, partial differential equations, differential-integral equations or integral equations. When the phenomenon being described is complex, the equations are solved by numerical methods, except for a very narrow class of problems for which closed analytical solutions can be formulated. The natural desire of researchers is to include in the description of reality the latter's complexity, which results in increasingly more complex mathematical descriptions of reality, difficult to analyze even by advanced numerical methods.

In recent decades descriptions of various physical phenomena by means of simple mathematical models called cellular automata (CA) have been intensively developed. Cellular automata constitute an alternative to describing such phenomena by complex mathematical equations. CA can be regarded as generally simple discrete dynamic systems whose evolution is described by uncomplicated local rules. Using cellular automata one can represent a given phenomenon as a synchronous mutual interaction of a discrete in time and space set of cells. The state of a cell is described by simple rules based on the state of this cell and the states of the cells in its immediate vicinity. One can say that cellular automata enable one to describe global phenomena by means of local rules of evolution and their discrete representation.

The idea and mathematical foundations of cellular automata were formulated by von Neuman and Ulam in the 1940s [1]. Although CA came within the field of interest of scientists, it was as late as in the 1970s, when John Conway presented the famous "game of life" [2], that the interest rapidly grew and has resulted in a huge number of researches on the theory and applications of cellular automata. A broad survey of literature on the description and use of CA can be found in, e.g. [3–8]. S. Wolfram greatly contributed to the theory of cellular automata, collecting his findings in an extensive monograph [9].

In recent years cellular automata have been effectively applied to: traffic flow problems [10], the simulation of brain tumour growth dynamics [11], biochemical phenomena [12], fluid and gas dynamics and particle transport [13–14], ecological modelling [15], biological modelling [16], vegetation dynamics [17], migration problems [18], the modelling of reaction-diffusion systems [19], the modelling of water release and absorption in soils [20], soil erosion by water [21], epidemic modelling [22], forest fire modelling [23], the design of variable-stiffness composite layers [24] and many other problems. CA have also proved to be an effective tool in data compression and encryption [25], the generation of high-quality random numbers [26], cryptographic procedures [27] and the analysis of partial differential equations [28–29]. In mechanics, cellular automata have been used to describe deformations of an elastic body and the frictionless contact between the body and a rigid foundation [30], and to solve nonlinear string vibration problems [31–32], being the subject of this paper. A nonlinear model of transverse string vibration was first formulated by Kirchhoff [33], and Carrier [34–35] included the longitudinal displacement of the string in its nonlinear vibration. The theory formulated in those works has been developed in numerous publications whose survey, with regard to theoretical formulations, numerical methods and experimental tests, can be found in respectively [36–38].

The aim of this paper is to formulate an alternative, modified CA model of damped nonlinear transverse string vibration and to demonstrate that CA can constitute a simple and effective tool for the analysis of a range of complex problems which have not been analyzed in this way before.

Section 2 presents an alternative CA formulation for nonlinear transverse string vibration problems, with a modified description of the viscous damping model. In section 3, tests solutions are presented and the proposed approach is applied to nonlinear statics and dynamics problems of a string with unilateral and bilateral constraints, loaded with stationary and moving forces. In section 4 the obtained results are briefly recapitulated.

2. Cellular automata model of string vibration

Let us assume that the string model is a chain of uniformly distributed point masses (cells) connected by elastic massless elements of the string whose fragment is shown in Figure 1.

The transverse displacement of the i-th string mass over the j-th time, referred to as the cell state, is denoted by $w(i, j)$. As defined in the papers by Kawamura et al. [31–32], the evolution of the string state can be divided into two (identical in their description) stages. In the first stage, for example, balls marked as white in Figure 1 undergo displacement while the black balls remain stationary, whereas in the second stage the opposite happens.

Physically, this way of analysis should be interpreted as the propagation of a transverse disturbance, which in a given time interval covers a distance exactly equal to distance a (resulting from the applied uniform division of the string into n elements) between neighbouring balls, each of which can be a source of transverse vibrations. State ($w(i, j)$) of a black ball after time Δt ($w(i, j + 1)$), during which the transverse disturbance reaches the neighbouring balls and the black ball performs undamped free vibrations, can be calculated from this relation

$$w(i, j + 1) = w(i, j) + 2w_0 \tag{1}$$

where $w_0 = 1/2[w(i - 1, j) + w(i + 1, j)] - w(i, j)$ (2)

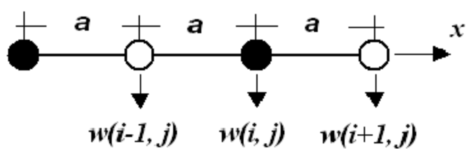

Fig. 1. String model

The relations are represented graphically in Figure 2.

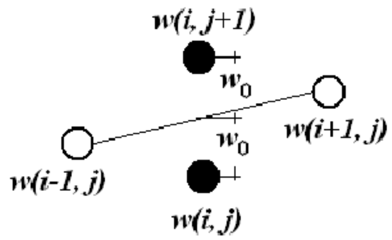

Fig. 2. Graphic representation of cell state updating

If the influence of distributed (transversely to string axis x) excitation forces $q(x, t)$ and that of viscous damping $C(x, t)$ are taken into account, then

$$w(i, j + 1) = \delta[2w_0 + \chi f(i, j)] + w(i, j) \tag{3}$$

where (for the time being undetermined) parameters δ and χ are responsible for respectively vibration damping and the influence of transverse load $f(i, j)$ on the state of displacement $w(i, j + 1)$.

Taking into account relation (2), relation (3) can be written as

$$w(i, j+1) = \delta[w(i-1, j) + w(i+1, j)] + (1-2\delta)w(i, j) + \delta\chi f(i, j), \tag{4}$$

which expresses the state of the i-th cell (its evolution procedure) at time $j + 1$ as a function of: the previous state (before time Δt elapses), the state of the neighbouring cells ($w(i-1, j)$, $w(i+1, j)$), the actual transverse load and damping.

The mathematical Kirchhoff model [33] of nonlinear transverse vibration, neglecting the displacements along the string's axis and averaging tension (N) over its length (l), can be in an elementary way derived from Newton's second law and written as

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + C\frac{\partial w(x,t)}{\partial t} + \left(N_0 + \frac{EA}{2l}\int_0^l \left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx\right)\frac{\partial^2 w(x,t)}{\partial x^2} = q(x,t), \tag{5}$$

where ρ, E, C, N_0 and $q(x, t)$ are respectively the density of the string material, the Young modulus of the latter, the viscous damping parameter, the initial string tension value and the transverse load intensity.

Applying central difference approximation for $\partial^2 w/\partial t^2$, $\partial^2 w/\partial x^2$ and $\partial w/\partial t$ one can write Equation (5), after elementary transformations, in the form presented in the papers by Kawamura et al. [31–32]

$$w(x, t+\Delta t) = \frac{1}{\gamma+1}\left[w(x+\Delta x, t) + w(x-\Delta x, t)\right] + \frac{\gamma-1}{\gamma+1}w(x,t) + \frac{1}{\gamma+1}\frac{(\Delta t)^2}{\rho A}q(x,t) \tag{6}$$

where:

– $\gamma = C\Delta t/\rho A$,

– $1/(\gamma+1)\cdot(\Delta t)^2/\rho A = (\Delta t)^2/ C\Delta t + \rho A)$.

By comparing relations (4) and (6) one can easily determine the still unknown parameters δ and χ from this cellular automaton

$$\begin{aligned} \delta &= \frac{1}{\lambda+1} = \frac{\rho A}{c\Delta t + \rho A} \\ \chi &= \frac{\Delta t}{\rho A} \end{aligned} \tag{7}$$

For the above relations it can be shown that $(1 - 2\delta) = (\gamma - 1)/(\gamma + 1)$, which fully agrees with the relations presented by Kawamura et al. [31–32].

Since work W of load $q(x, t)$ is a linear function of the displacements shown in Figure 2 and energy dissipation function Φ is a homogenous quadratic function of velocity (differentiation over time is marked with a dot),

$$W = 2\int_0^a q(x,t) w \frac{x}{a} dx$$
$$\Phi = \int_0^a C\left(\dot{w}\frac{x}{a}\right)^2 dx \tag{8}$$

the parameter $f(i, j)$ in relation (4) is $f(i, j) = q(x, t)$ and the damping per unit string length (in relation (7)) is $c = {}^2/_3C$. The latter relation is due to the zero velocities of the balls adjoining the ball which moves in one of the string evolution stages.

Relation (4) together with (7) and definitions c and $f(i, j)$ explicitly define the cellular automaton law for the nonlinear transverse vibration of the string.

Velocity v of transverse disturbance propagation is defined as $v = \sqrt{N/\rho A}$, and so in the nonlinear equation considered here it is

$$v(t) = \sqrt{\frac{1}{\rho A}\left(N_0 + \frac{EA}{2l}\int_0^l \left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx\right)}. \tag{9}$$

According to the proposal presented by Kawamura et al. [31–32], time-variable velocity $v(t)$ is the basis for the use of time increment Δt which changes in the course of string evolution (and influences the values of δ and χ). Such a value of Δt is selected that the disturbance wave-front will cover exactly distance $a = l/n$ (constant throughout the analysis) during this time.

3. Numerical analysis

The solutions presented in this section were tested [31–32] for a string with length $l = 0.8$ m, cross section $A = 20\cdot10^{-3}$ m × $0.5\cdot10^{-3}$ m, material density $\rho = 7.8\cdot10^3$ kg/m^3, Young modulus $E = 206$ GPa, damping C = 0.14 Ns/m^2 and initial tension N_0 = 130 N. The string was divided into $n = 100$ segments (cells) each having length $a = 8.0\cdot10^{-3}$ m. For these data, transverse wave propagation velocity $v_0 = \sqrt{N_0/\rho A}$ = 40.8248 m/s and the first natural frequency $f_1 = v_0/2l = 25.5155$ Hz. In some of the solutions presented below it was assumed that load $q(x, t) = q(x) + Q\cos(\omega t)$, where $q(x) = 3.0$ N/m, $Q = 15.0$ N/m ($\omega = 2\pi f$), and the load acts within the segment 0.08 m $\leq x \leq 0.15$ m.

For the above data it is easy to estimate the frequency response function [39], e.g. for the string's midpoint, assuming that the displacement state can be approximated by function $\sin(\pi x/l)$ (for unmovable string ends) and that $q(x) = 0.0$ N/m and $f = 25$ Hz. In Figure 3 this function (the function of fixed string response amplitude variation amp measured in meters) is marked with a solid line and dots mark the solutions obtained using the linearized (neglecting changes in tension force $N(t) = N_0$) cellular automaton defined in the previous section.

The maximum difference between the solutions for the excitation frequency adopted in the considered example amounts to 13% of the exact solution and is much better than in [32]. This demonstrates the effectiveness of a well calibrated CA in such an analysis.

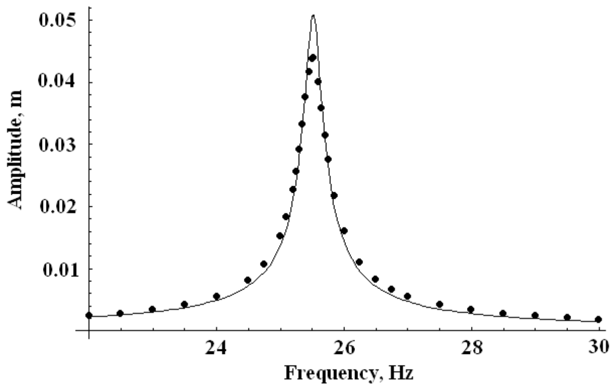

Fig. 3. Frequency response function for linear problem

Figure 4 shows selected schematics of the analyzed strings. The nonlinear transverse string vibration model was used in each of the problems.

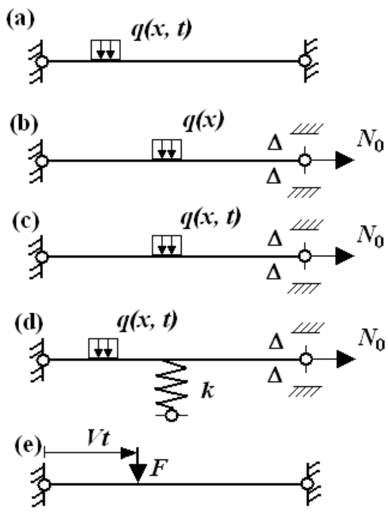

Fig. 4. Schematics of analyzed strings

Figure 5 shows a typical frequency response function at the midpoint of the string whose schematic is shown in Figure 4a, for $q(x)$ = 0.0 N/m and Q = 30 N/m in frequency range 20 Hz ≤ f ≤ 50Hz.

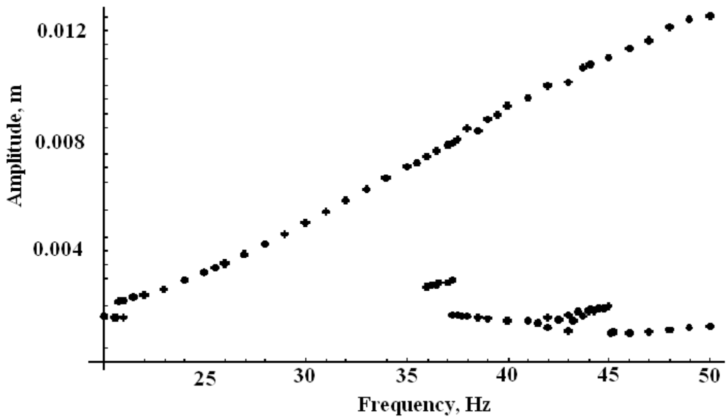

Fig. 5. Frequency response function for nonlinear problem

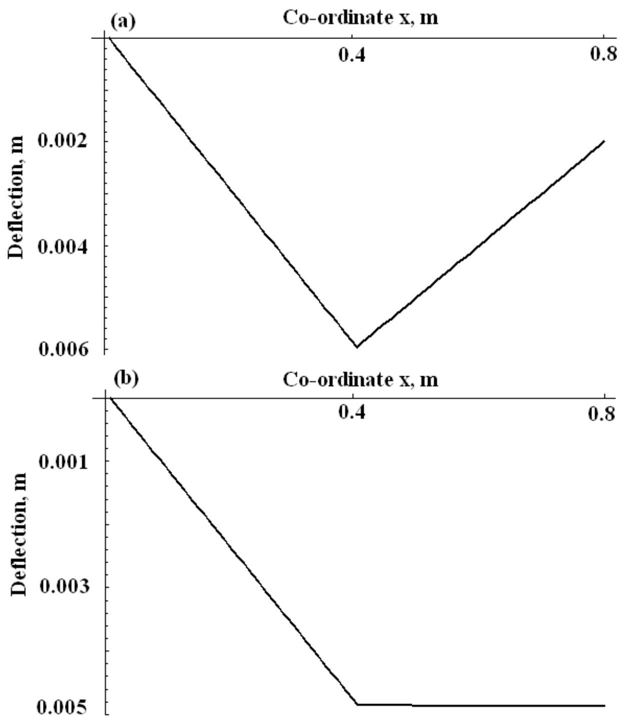

Fig. 6. Static deflections of string Δ = 0.002 m (a) and Δ = 0.005 (b)

Different string response branches can be obtained by, for example, controlling excitation function frequency f in the initial time of the analysis, after which f reaches a constant target value. The analysis of string vibration amplitude at different excitation frequencies requires (in certain ranges of the frequencies) arduous searches for possible steady string vibration amplitude values.

Interesting responses of the vibrating string are obtained under a static load when, for instance, the string's right end can freely move transversely in the parameter Δ determined range which describes a unilateral stiff constraint (Figure 4b).

Figure 6 shows lines of static string deflections under load $q(x)$ = 1250 N/m applied to segment $a = l/n$ precisely at the string's midpoint for respectively Δ = 0.002 m (Figure 6a) and Δ = 0.005 m (Figure 6b). The acting load $q(x)$ corresponds to concentrated force F = 10 N applied in the middle of the string span. As it can be easily verified (checking the current tension of the string), the static equilibrium conditions, in the form of the sum of forces projections onto the vertical axis, are satisfied here.

Under dynamic load $q(x, t) = 1250 + 1250\cos(2\pi ft)$ applied to the middle segment (a) of the string (Figure 4c) and at Δ = 0.002 m, f = 10 Hz and C = 0.6 Ns/m^2, the state of displacement of the string's midpoint and its right end over time t is shown in respectively Figure 7a and 7b.

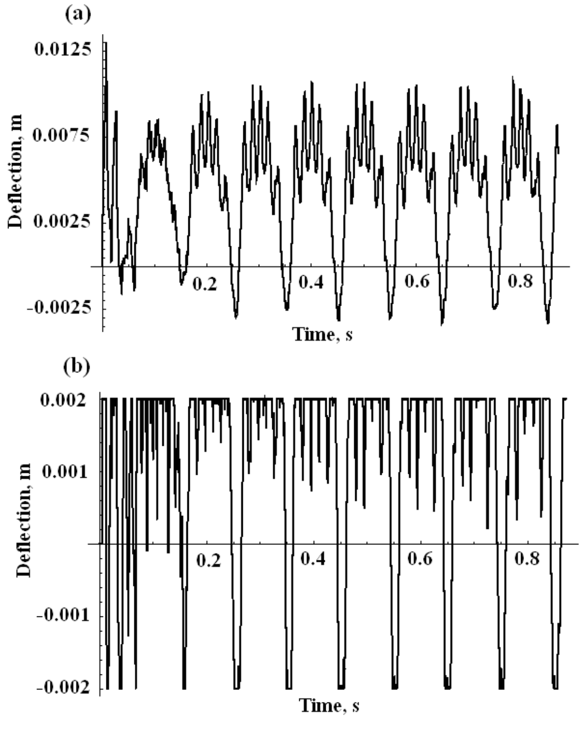

Fig. 7. Dynamic deflections of string's midpoint (a) and right end (b)

The CA algorithm allows one to easily impose arbitrary constraints on any string point. This has been confirmed by several numerical tests (not presented here). The CA algorithm can be easily generalized to cover cases with any number of bilateral elastic constraints with any (also nonlinear or time-dependent) characteristic of their stiffness (k). Bilateral constraints are taken into account by modifying node load $f(i, j)$ by adding an elastic reaction proportional to the node displacement multiplied by a function characterizing the stiffness of the added constraint.

As an example, Figures 8 and 9 show phase portraits of the vibration of points at $x = {}^3/_4\ l$ and $x = l$ of a string whose displacements are confined by a bilateral elastic constraint (with stiffness k = 400 N/m) in the middle of the string and a stiff unilateral constraint at the string's end at Δ = 0.005 m (Figure 4d). Vibration excitation load $q(x, t)$ = 1250 + 1250cos($2\pi f t$) was applied to one segment a at $x = {}^1/_4\ l$ and f = 175 Hz (Figure 8) and f = 179.9 Hz (Figure 9). It was assumed that C = 0.45 Ns/m^2 and initial tension N_0 = 2600 N.

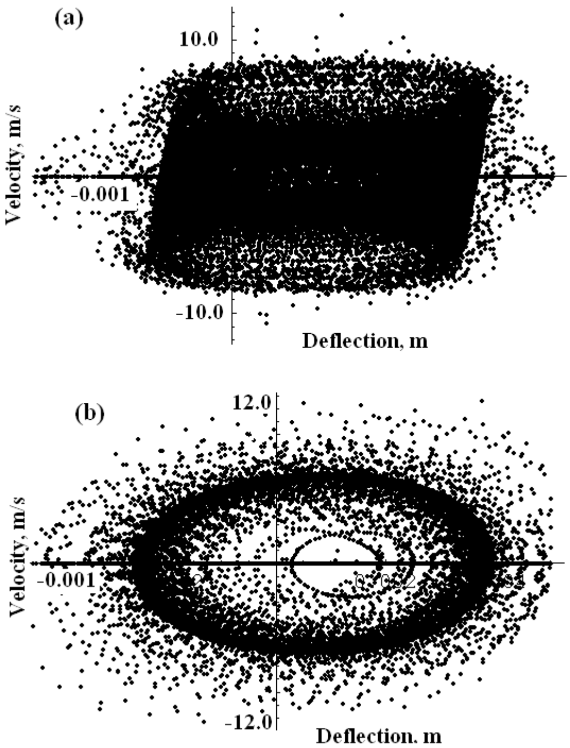

Fig. 8. Phase portraits of vibration of string points at x = 3/4 l (a) and $x = l$ (b) for f = 175 Hz

Figure 10 shows the Fourier transform $P(f)$ of string midpoint displacements at the above frequencies: f = 175 Hz (Figure 10a) and f = 179.9 Hz (Figure 10b).

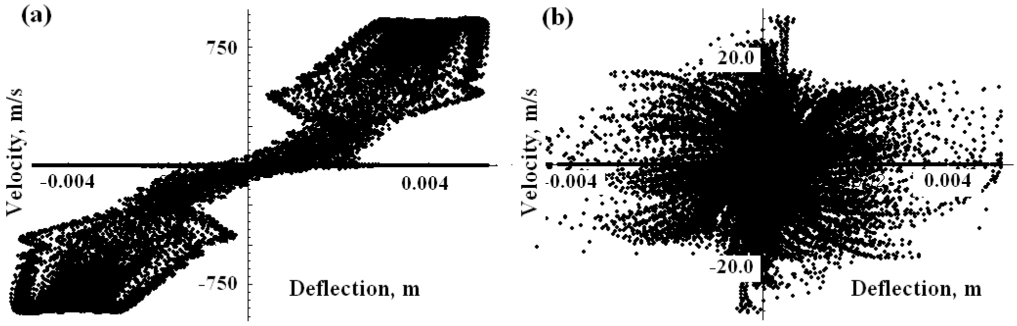

Fig. 9. Phase portraits of vibration of string points at $x = 3/4\ l$ (a) and $x = l$ (b) for $f = 179.9$ Hz

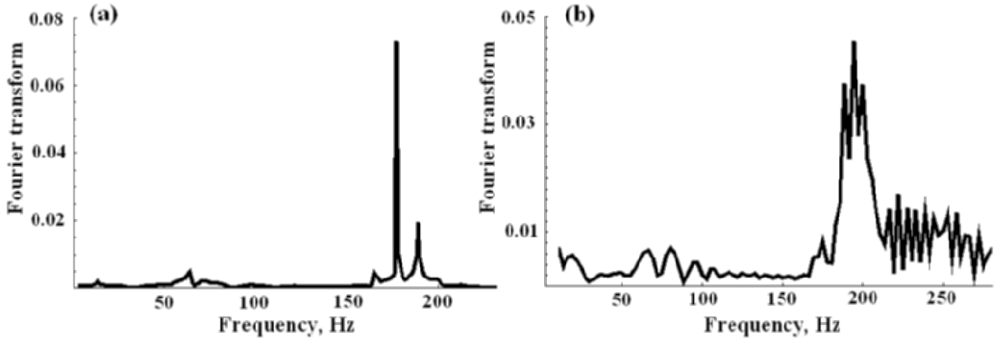

Fig. 10. Fourier transform of string response at $f = 175$ Hz (a) and $f = 179.9$ Hz (b)

The string response at frequency $f = 179.9$ Hz has the features of chaotic motion.

Ratio β of elastic wave velocity at current tension $N(t)$ to its velocity at initial tension N_0, which can be regarded as a certain measure of problem nonlinearity, was controlled in each of the solutions. Figure 11 shows the variation of β in the initial stage of analysis of the string (Figure 4d) at respectively $f = 175$ Hz (Figure 11a) and $f = 179.9$ Hz (Figure 11b).

The CA algorithm presented in section 2 has proved effective also in the analysis of the string under moving static and dynamic loads. A moving load can be realized on a string with elastic and inelastic constraints. If it is assumed that force F moves on the string with, for example, uniform motion with velocity V (this problem can be generalized to cover a case of the force moving with an arbitrary acceleration), interesting solutions can be obtained at different velocity values, particularly at critical velocities (close to elastic wave propagation velocity v). Figure 12 shows the state of displacement (a snapshot) of the string under force $F = 10$ N moving at velocity $V = 2$ m/s (Figure 4e). Figure 12a shows the deflection of the string over time $t = 0.26$ s when the force acted directly on the string. Figure 12b shows string deflection over time $t = 0.58$ s when the string's free damped vibration after the travel of force F.

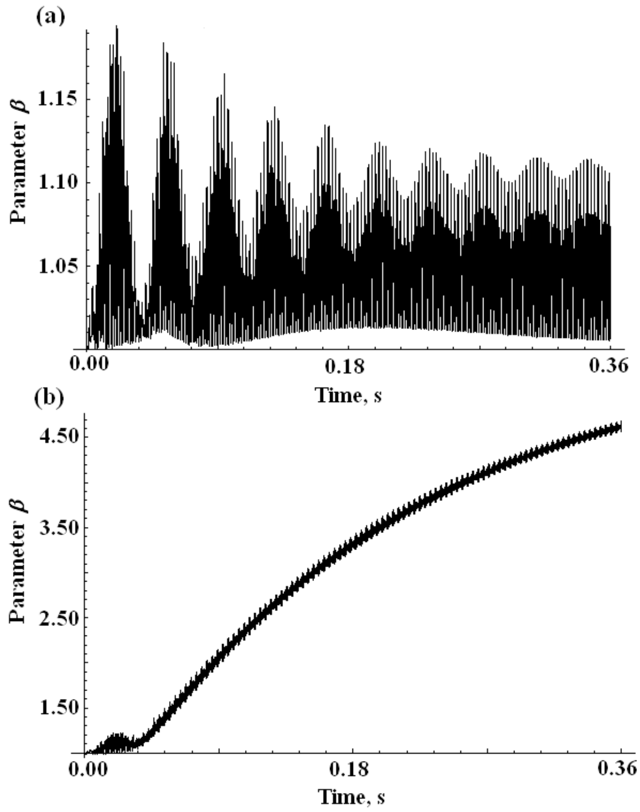

Fig. 11. Variation of parameter β at f = 175 Hz (a) and f = 179.9 Hz (b)

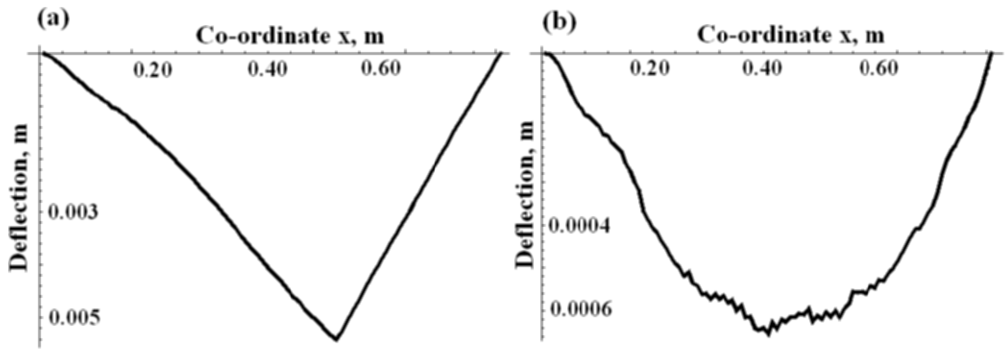

Fig. 12. String displacement state under moving load (V = 2 m/s) over t = 0.26 s (a) and t = 0.58 s (b)

The displacement state at subcritical speed V = 40 m/s as the force acted directly on the string (t = 0.009 s) and after its travel (t = 0.023 s) is shown in respectively in

Figure 13a and 13b. In the analysis it was assumed that C = 3.0 Ns/m^2. In Figure 13a the disturbance front preceding the force has not reached the end of the string. Figure 13b shows the wave reflected from the string's end during its free vibration.

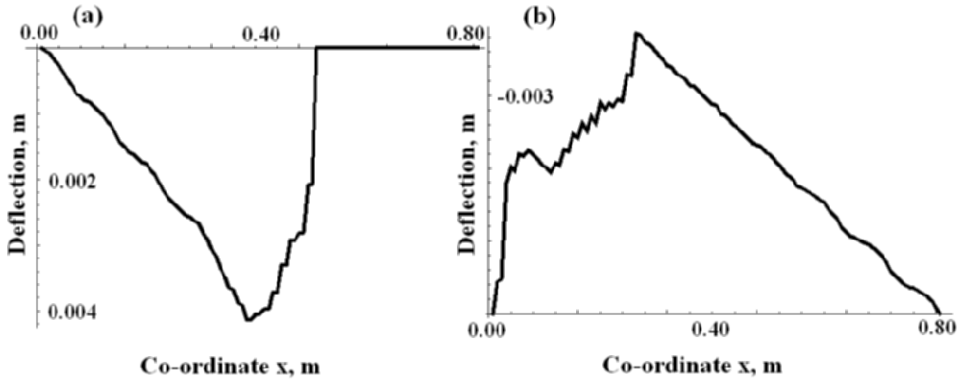

Fig. 13. String displacement state under moving load (V = 40 m/s) over t = 0.009 s (a) and t = 0.023 s (b)

Similarly as in Figure 13, the string responses in the linearized problem ($N(t) = N_0$) are shown for t = 0.011 s and t = 0.027 s in respectively Figures 14a and 14b.

CA were also used to good effect in problems with given kinematic excitation of an arbitrary string point and in problems with forced time-variable string tension.

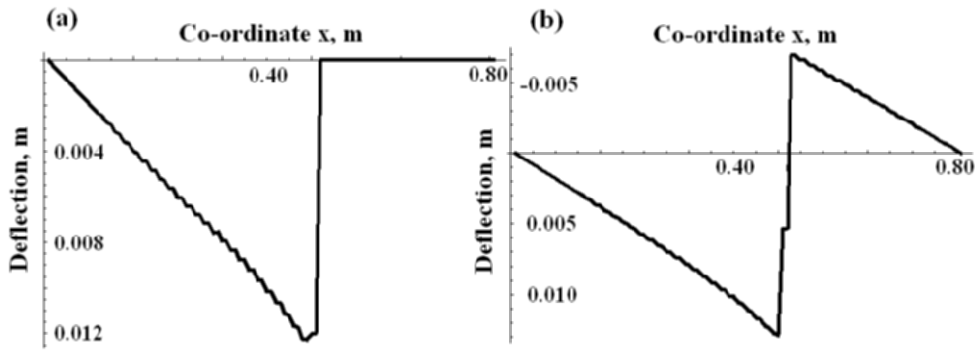

Fig. 14. Linearized string displacement state under moving load (V = 40 m/s) over t = 0.011 s (a) and t = 0.027 s (b)

4. Conclusion

Cellular automata, intensively developed in recent years, can constitute an effective alternative to complex mathematical descriptions of many physical phenomena. The latter are described, as a rule, by simple, repeatable, CA-evolution-expressing formulas used in the discrete representations of the space of the phenomena. The calibration of CA's parameters is of critical importance for their effectiveness. As a rule, when CA are used, it is easier to obtain a qualitatively correct form of the sought solutions than their quantitatively correct form.

In this paper a modified CA procedure for the analysis of nonlinear viscously damped transverse string vibrations was defined. CA parameters were obtained by comparing the CA defining rules with relations resulting from the discrete form of the mathematical description of the investigated phenomenon. A series of numerical tests that were run confirmed the agreement between the obtained results and the solutions found in the literature. The CA model calibrated in this way was used in the analysis of string vibrations under arbitrary static and dynamic loads at different unilateral and bilateral constraints imposed on the motion of an arbitrary string point.

The numerical tests have shown that:

– cellular automata are simple and effective tools for the analysis of linear and nonlinear string vibrations,

– owing to the fact that the CA algorithm is easily modifiable, it can be used (usually after minor changes in its structure) to analyze various complex problems,

– in order to build a properly functioning CA, one must carefully calibrate its parameters which determine the qualitative and quantitative correctness of the results.

References

[1] Neumann J. V.: *The theory of self-reproducing automata*, A. W. Burks (ed), Univ. of Illinois Press, Urbana and London, 1966.

[2] Gardner M.: *The fantastic combinations of John Conway's new solitaire game "Life"*, Sci. Am., Vol. 223, 1970, pp. 120–123.

[3] Vollmar T.: *Cellular space and parallel algorithms: an introductory survey*, Parallel Computation-Parallel Mathematics, M. Feilmeier (ed), North Holland Publishing Co, 1977, pp. 49–58.

[4] Toffoli T., Margolus N.: *Invertible cellular automata: a review*, Physica D, Vol. 66, 1994, pp. 1–24.

[5] Mitchell M., Crutchfield J., Das R.: *Evolving cellular automata with genetic algorithms: a review of recent work*, First International Conference on Evolutionary Computation and its Applications, 1996.

[6] Chaudhuri P.Pal, Chowdhury A.R., Nandi S., Chatterjee S.: *Additive cellular automata – theory and applications*, IEEE Computer Society Press, CA, USA, Vol. 1, 1997.

[7] Sarkar P.: *A brief history of cellular automata*, ACM Computing Systems, Vol. 32, 2000, pp. 80–107.

[8] Kari J.: *Theory of cellular automata: a survey*, Theoretical Computer Science, Vol. 334, 2005, pp. 3–33.

[9] Wolfram S.: *A new kind of science*, Wolfram Media, Inc., 2002.

[10] Maerivoet S., Moor De B.: *Cellular automata of road traffic*, Physics Reports, Vol. 419, 2005, pp. 1–64.
[11] Kansal A.R., Torqato S., Harsh IV G.R., Chiocca E.A., Deisboeck T.S.: *Simulated brain tumour growth dynamics using tree-dimensional cellular automata*, J. Theor. Biol., Vol. 203, 2000, pp. 367–382.
[12] Kier L.B., Cheng Chao-Kun, Testa B., *Cellular automata models of biochemical phenomena*, Future Generation Computer Systems, Vol. 16, 1999, pp. 273–289.
[13] Chopard B., Masselot A.: *Cellular automata and lattice Boltzmann methods: a new approach to computational fluid dynamics and particle transport*, Future Generation Computer Systems, Vol. 16, 1999, pp. 249–257.
[14] Boghosian B. M.: *Lattice gases and cellular automata*, Future Generation Computer Systems, Vol. 16, 1999, pp. 171–185.
[15] Hogeweg P.: *Cellular automata as a paradigm for ecological modelling*, Applied Mathematics and Computation, Vol. 27, 1988, pp. 81–100.
[16] Bard G., Edelstein-Keshet L.: *Cellular automata approaches to biological modelling*, J. Theor. Biol., Vol. 160, 1993, pp. 97–133.
[17] Balzter H., Braun P.W., Kohler W.: *Cellular automata models for vegetation dynamics*, Ecological Modelling, Vol. 107, 1998, pp. 113–125.
[18] Schonfisch B., Lacoursiere C.: *Migration in cellular automata*, Physica D, Vol. 103, 1997, pp. 537–553.
[19] Weimar J. R.: *Cellular automata for reaction-diffusion systems*, Parallel Computing, Vol. 23, 1997, pp. 1699–1715.
[20] Townend J.: *Modelling water release and absorption in soils using cellular automata*, Journal of Hydrology, Vol. 220, 1999, pp. 104–112.
[21] Ambrosi D.D., Di Gregorio S., Gabriele S., Gaudio R.: *A cellular automata model for soil erosion by water*, Phys. Chem. Earth (B), Vol. 26, 2001, pp. 33–39.
[22] White S.H., del Rey A.M., Sanches G.R.: *Modelling epidemics using cellular automata*, Applied Mathematics and Computation, Vol. 186, 2007, pp. 193–202.
[23] Barros F.J., Mendes M.T.: *Forest fire modelling and simulation in DELTA environment*, Simulation Practice and Theory, Vol. 5, 1997, pp. 185–197.
[24] Setoodeh S., Gurdal Z., Watson L.T.: *Design of variable-stiffness composite layers using cellular automata*, Comput. Methods Appl. Mech. Engrg., Vol. 195, 2006, pp. 836–851.
[25] Lafe O.: *Data compression and encryption using cellular automata transforms*, Engng. Applic. Artif. Intell., Vol. 10, 1997, pp. 581–591.
[26] Tomassini M., Sipper M., Zolla M., Perrenoud M.: *Generating high-quality random numbers in parallel by cellular automata*, Future Generation Computer Systems, Vol. 16, 1999, pp. 291–305.
[27] del Rey A.M., Mateus J.P., Sanches G.R.: *A secret sharing scheme based on cellular automata*, Applied Mathematics and Computation, Vol. 170, 2005, pp. 1356–1364.
[28] Toffoli T.: *Cellular automata as an alternative to (rather than an approximation of) differential equations in modelling physics*, Physica 10D, Vol. 195, 1984, pp. 117–127.
[29] Doeschl A., Davison M., Rasmussen H., Reid G., *Assessing cellular automata based models using partial differential equations*, Mathematical and Computer Modelling, Vol. 40, 2004, pp. 977–994.
[30] Abdellaoui M., El Jai A., Shillor M.: *Cellular automata model for contact problem,* Mathematical and Computer Modelling , Vol. 36, 2002, pp. 1099–1114.

[31] Kawamura S., Shirashige M., Iwatsubo T.: *Simulation of the nonlinear vibration of a string using the cellular automation method*, Applied Acoustics, Vol. 66, 2005, pp. 77–87.
[32] Kawamura S., Yoshida T., Minamoto H., Hossain Z.: *Simulation of the nonlinear vibration of a string using cellular automata based on the reflection rule*, Applied Acoustics, Vol. 67, 2006, pp. 93–105.
[33] Kirchhoff G.: *Vorlesungen uber Mathematische Physik: Mechanik*, Druck und Verlag von B.G. Teubner, Leipzig, 1867.
[34] Carrier G.F.: *On the nonlinear vibration problem of the elastic string*, Q. J. Appl. Math., Vol. 3, 1945, pp. 157–165.
[35] Carrier G.F.: *A note on the vibrating string*, Q. J. Appl. Math., Vol. 7, 1949, pp. 97–101.
[36] Chen Li-Quen, Ding Hu: *Two nonlinear models of a transversely vibrating string*, Arch. Appl. Mech., Vol. 78, 2008, pp. 321–328.
[37] Bilbao S.: *Conservative numerical methods for nonlinear strings,* J. Acoust. Soc. Am., Vol. 118, 2005, pp. 3316–3327.
[38] Molteno T.C., Tufillaro N.B.: *An experimental investigation into the dynamics of a string*, Am. J. Phys., Vol. 72, 2004, pp. 1157–1169.
[39] Inman D.J.: *Vibration: with control, measurement, and stability*, Prentice-Hall, Inc., 1989.

Automaty komórkowe w nieliniowych drganiach struny

W artykule zdefiniowano procedurę automatów komórkowych, którą przystosowano do analizy nieliniowych, wiskotycznie tłumionych drgań poprzecznych struny. Parametry automatów komórkowych otrzymano porównując reguły definiujące ewolucję CA ze związkami wprost wynikającymi z dyskretnej postaci matematycznego opisu zjawiska. Przeprowadzono szereg testów numerycznych potwierdzających jakościową i ilościową zgodność otrzymywanych wyników z rozwiązaniami znanymi z literatury. Wykonane testy numeryczne pokazują, że automaty komórkowe mogą być prostym i skutecznym narzędziem analizy szeregu złożonych zagadnień dotychczas tym sposobem nie analizowanych.