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Electro-osmotic flow and heat transfer of a non-Newtonian fluid in a hydrophobic microchannel with Navier slip*

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Abstract: In this paper, we investigate the implications of electro-osmosis on electrohydrodynamic transport of a non-Newtonian fluid on a hydrophobic micro-channel by developing a suitable analytical method. Velocity-slip and temperature-jump conditions are paid due attention. An attempt has been made to examine the effects of rheological and electro-osmotic parameters on the kinematics of the fluid. The nonlinear Poisson-Boltzmann equation governing the formation of the electrical double layer and the body force that is generated by the applied potential are accounted for in the study. Perturbation solutions are presented. In order to exhibit the applicability of the analysis, the problem of electro-osmotic flow and heat transfer of blood in an arteriole has been taken up as an illustrative example of a real-life problem. An intensive quantitative study has been made through numerical computation of the physical variables involved in the analysis, which are of special interest in the study. The computational results are presented graphically. The study reveals that the temperature of blood can be controlled by increasing/decreasing the Joule heating parameter.

Key words: electro-osmotic flow, hydrophobic microchannel, electrical double layer, viscoelastic fluid, velocity-slip, temperature-jump

Introduction

Microchannels refer to flow passages having hydraulic diameter lying between 10 μm and 200 μm. When the smallest channel dimension is 0.1 μm or less, the channel is called a nano-channel. In microtechnological devices like MEMS and lab-on-a-chip, the micro-channels constitute the basic structure. Micro-channels also exist in the micro-circulatory system of the human body. Remarkable characteristics of these channels are that all the energy is carried away when a fluid flows through them and that their heat dissipation rate is very high. While studying the liquid flow and heat transfer through a micro-channel between two parallel plates, scientists have observed that for micro-channel flows, diffusion dominates over the momentum transport and that the mass-flow rate cannot be properly estimated on the basis of the conventional Navier-Stokes equation. In this case, in the Navier-Stokes equation, the convective velocity should be replaced by the total velocity. In studies pertaining to slip flow, if the velocity-slip and temperature-jump are accounted for, one can use the Navier-Stokes equation for estimating liquid flow in micro-channels.

A hydrophobic material is one, which possesses the physical property of repelling a mass of water. The formation of water drops on the hydrophobic surface of leaves is an example of hydrophobicity. Hydrophobic materials are usually neutral electrically. In contrast, a hydrophilic material is one that has affinity towards water. Such a material normally carries charge due to which water is attracted towards it. In chemical separation processes, hydrophobic materials are extensively used for removal of non-polar material content from polar compounds and also oil from water. In available scientific literatures, it has been mentioned that when micro-structured, the hydrophobic property of a surface gets enhanced. It is known that super-hydrophobic micro-nano structured surfaces possess self-cleaning property. In recent years several super-

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hydrophobic materials have been developed, which are very useful for the treatment of physiological fluids and various other fluids. Ressine et al.[1] pointed out that patterned surfaces with sufficiently high degree of hydophobicity can play a significant role in bringing about radical improvement in surface based bioanalysis. They also bear the promise of useful applications in lab-on-a-chip and different microfluidic devices.

In a recent communication by Misra et al.[2], it has been mentioned that when a solid surface comes in contact with an aqueous solution of an electrolyte, it is observed that a structure comprising a layer of charges of one polarity on the solid side and a second layer of charges of opposite polarity on the liquid side of the solid-liquid interface are formed. This observation is described by saying that an electrical double layer (EDL) has been in existence. There is experimental evidence that when an EDL is formed, an internal third layer of charges, called the “Stern layer” is formed in the immediate vicinity of the wall and that the ions that comprise this layer are bound quite strongly. It is worthwhile to mention that the typical thickness of Stern layer is nearly equal to the diameter of an ion and that the nature of charges of this layer is opposite to that of the wall. This is why the ions of the Stern layer are attracted towards the wall with very strong forces, which are electrostatic in nature. However, the ions in the outer layer, which is of diffuse type are less associated. If and when the system is subject to the action of an externally applied electric field, the ions belonging to the outer layer experience a force, resulting in a bulk motion of the liquid. The flow of a liquid generated in this manner is termed as electro-osmotic flow (cf Ref.[2]).

Electrokinetic transport provides a very effective technique of flow of liquids, which is actuated by favorable interfacial phenomena at small scales in a controlled manner. Usually the electrokinetic phenomenon essentially depends on the formation of a locally non-neutral layer of liquid particles adjacent to the charged interface. In the close proximity of the said layer, a diffuse cloud of oppositely charged ions screens the surface charge in an effective manner. Under the action of an externally applied electric field a force is likely to be exerted on the ionic species located within the charged diffuse layer, mentioned above. This may be transmitted to the incipient fluid due to viscosity effects. As a result, there occurs a motion of the liquid relative to the solid surface. The flow of the liquid (the so-called electro-osmotic flow) relative to the stationary solid surfaces has its origin in the preferential transport of mobile counter-ions in the diffuse portion of the electrical double layer, when acted on by an applied electric field. This type of flow behaviour bears the promise of wide applications in analytical chemistry and life sciences, as well. A systematic discussion on different types of fluid flows having relevance to different problems of physiological fluid dynamics has been made by Misra et al.[2].

Electro-osmotic flows of Newtonian fluids in micro-channels have been the subject of several investigations in the recent past, because of their relevance to various industrial problems, including those of biomedical engineering and technology. Reports are available for studies pertaining to the effect of the surface potential on liquid transport through ultrafine capillary slits, by using the Debye-Huckel linear approximation for the electric potential, when the liquid flow is subject to an imposed electric field. Similar studies have also be carried out for a narrow cylindrical capillary. Successful attempts were also made to obtain analytical solutions for the velocity distribution, mass flow rate, pressure gradient, wall shear stress, and vorticity in mixed electro-osmotic/pressure driven time-periodic flows in two-dimensional straight channels, considering symmetry of the electrical double layers and assuming that these are small (but finite). These studies have, however, limited applications, since they are restricted only to cases, where the distance between the two walls of a microfluidic device is about 1-3 orders of magnitude larger than the thickness of the electrical double layers. Wang et al.[3] analyzed a two-dimensional model for the electro-osmotic flow in a rectangular micro-channel. Another flow problem in the case of periodical electro-osmosis in a rectangular micro-channel was also discussed by them, by using a semi-analytical approach. This study was performed by using Poisson-Boltzmann and Navier-Stokes equations. Solution of a problem of heat transfer during electro-osmotic flow of a Newtonian fluid under combined pressure in planar micro-channels, in available in the literature. Solution of problems concerning two-dimensional, time-dependent/time-independent electro-osmotic flows that are driven by a uniform electric fields with non-uniform zeta potential distributions along the walls of a conduit has also been reported. Electro-osmotic flow in capillaries filled with symmetric electrolyte has also been discussed. Analytical solution for problem concerning electro-osmotic flow and chaotic stirring in rectangular cavities is also available. Wang et al.[4] studied numerically the characteristics of electro-osmotic flow for varying zeta potential and dimension. Some other numerical/experimental models on electro-kinetic flow were developed in the past. All these studies were also limited to simple Newtonian fluids only.

The flow behavior of non-Newtonian fluids are of greater interest in the study of various problems in different branches of science and technology. It is known that most physiological fluids such as blood, saliva and also DNA solutions are viscoelastic in nature. Existing scientific literature indicates that changes due to various diseases or surgical intervention can be readily identified, considering blood viscoelasticity as a useful clinical parameter. Several investigators carried out the study of viscous fluid flow over a stretching sheet, where the fluids obey non-Newtonian viscoelastic (cf. Ref.[5]-[9]) constitutive equations. Misra et al.[10,11] also carried out a series of investigations to explore a variety of important information on the characteristics of blood flow in arteries in normal physiological state/pathological condition, such as arteriosclerosis. Keeping in view the enormous application potential of studies on electro-osmotic flow of non-Newtonian fluids in many industrial problems and in physiological fluid dynamics, very recently Misra et al.[2] investigated channel flow characteristics of electro-osmotic flow of a viscoelastic fluid. The analysis presented by them was applied successfully to put forward useful numerical estimates of some important characteristics of electro-osmotic flow of physiological fluids. Afonso et al.[12] analytically studied the mixed elctroosmotic/pressure driven flows of viscoelastic fluids in micro-channels. Ng and Qi[13] analytically solved the problem of an elctroosmotic flow of a viscopastic material through a slit channel with walls of arbitrary zeta potential. Some other authors presented a solution for a viscoelastic fluid model using Phan-Thien-Tanner model. Influence of viscosity index and electro-kinetic effect on the velocity of a third-grade fluid between micro-parallel plates has also been reported in scientific literatures. Reports on the effect of dynamic viscosity on the velocity of the electro-osmotic flow of power law fluids are also available.

It is known that one of the important factors responsible for actuating electro-osmotic flows is the so-called "velocity-slip" of the fluid particles. Bao and Lin[14] analytically studied the gas flow and heat transfer in micro-channels by considering higher order slip boundary conditions. The effects of second-order slip conditions on flow and heat transfer in micro Couette fluid flow has been investigated by Bao et al.[15]. Studies on peristaltic flows of different types of fluids in circular cylindrical tubes under the velocity-slip condition have also been reported previously.

Keeping in view the fact that most fluids used in different industries are far from being Newtonian, a theoretical investigation has been made in the present paper to explore a variety of information regarding the characteristics of electro-osmotic flow of a non-Newtonian fluid, -a viscoelastic fluid, in particular. This study is motivated towards throwing some light on the electrokinetic transport of physiological fluids, which are prominently viscoelastic in nature, in situations when the fluid transport takes place through lab-on-a-chip based micro-system subject to an externally applied electric field. For this purpose, a model has been developed, which takes into account the velocity-slip of the viscoelastic fluid during electro-osmotic flow on a hydrophobic microchannel. The model has been analyzed by using suitable constitutive equations and by employing appropriate analytical techniques. Solution of the problem has been achieved by developing a suitable perturbation scheme, with an aim to investigate the influence of fluid viscoelasticity on the ionized motion of the fluid, under the action of electro-kinetic forces. As an illustration of the applicability of the analysis, the derived analytical expressions have been computed for the electrokinemetically actuated flow of blood that possesses prominent viscoelastic properties, as evidenced by experimental observations reported in existing scientific literatures. The numerical estimates have been presented in graphical form.

The study along with the numerical data presented here will be immensely useful for validating the results of future theoretical studies for more realistic models, which may necessitate the use of numerical models to handle the complexity of the problem. Experimentalists will also find the results valuable, because they will be able to validate their experimental observations by using the results of the present study.

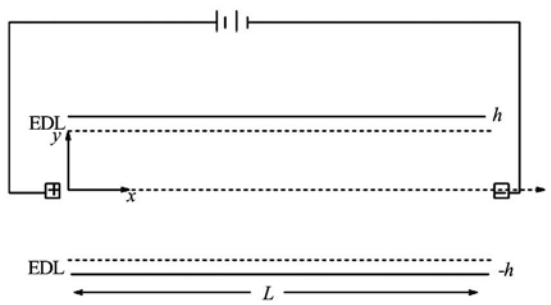

Fig.1 Physical sketch of the problem

1. The model

Let us consider the steady incompressible two-dimensional electro-osmotic flow of a viscoelastic fluid in a microchannel bounded by two parallel plates. The x-axis is taken along the centre line of the channel and the y-axis in the transverse direction. The flow is taken to be symmetric about x-axis (cf. Fig.1). The plates are supposed to be under the action of an electric field, which is responsible for the flow to be electro-kinetic. The distance between the plates is denoted by $2h$ and the length of each plate by L. Considering symmetry in the electric potential and velocity fields, the flow domain to be studied is reduced to one half, if the electrical potential and pressure gradients are assumed to be applied along the x-axis of the channel.

Assuming incompressible laminar flow of the fluid, the equations that govern its motion are the equations of continuity, linear momentum and energy transfer, which are respectively given by

$$\operatorname{div}(V)=0 \tag{1}$$

$$\rho\frac{\mathrm{d}V}{\mathrm{d}t}=\operatorname{div}T+F_{EK} \tag{2}$$

and

$$\rho\frac{\mathrm{d}\xi_1}{\mathrm{d}t}=T\cdot\operatorname{grad}(V)-\operatorname{div}(q)+\sigma E_x^2 \tag{3}$$

with

$$F_{EK}=\rho_e E \tag{4}$$

In the above-written equations, V is the velocity vector, ρ the density of the fluid and T the stress tensor. F_{EK} stands for the electrokinetic body force, ρ_e for the density of the net electric charge and E for the resultant electric field. ξ_1 is the specific internal energy and q represents the heat flux vector. Heat flux vector is given by the Fourier equation, thermal radiation being neglected. σ denotes the permittivity of the electric field. The term σE_x^2 in the right hand side of Eq.(3) represents Joule heating. The electric field is assumed to act in x-direction only. The particular type of fluid to be considered in the present study is a second order fluid. For such a fluid, the Cauchy stress tensor T for an incompressible homogeneous thermodynamically compatible may be put in the form

$$T=pI+\mu A_1+\alpha_1 A_2+\alpha_2 A_1^2 \tag{5}$$

where p is the pressure and μ the coefficient of viscosity, α_1 and α_2 are the normal stress moduli and A_1 and A_2 are defined as

$$A_1=(\operatorname{grad}V)+(\operatorname{grad}V)^{\mathrm{T}} \tag{6}$$

and

$$A_2=\frac{\mathrm{d}A_1}{\mathrm{d}t}+A_1\cdot\operatorname{grad}(V)+\operatorname{grad}(V)^{\mathrm{T}}A_1 \tag{7}$$

$\mathrm{d}/\mathrm{d}t$ being the material time derivative. The assumptions concerning the sign of α_1 in the fluid model represented by Eq.(5) together with Eqs.(6) and (7) are mentioned in the next section in course of the theoretical analysis of the problem.

Considering the hydrophobic nature of the channel surface, the boundary condition can be given by the Navier slip model described as

$$V=b\frac{\partial V}{\partial n},\ \text{at the boundary} \tag{8}$$

where b denotes the slip length (see Fig.2).

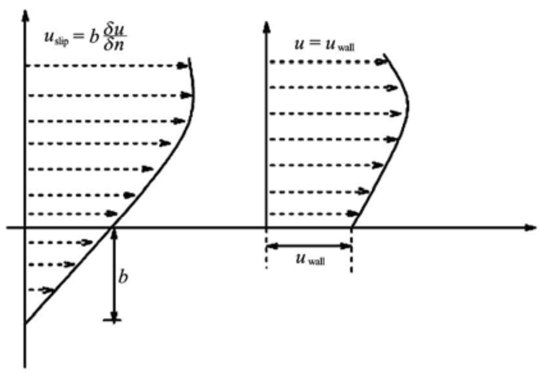

Fig.2 Schematic illustration of the paradigm of velocity slip

2. Analysis

From the laws of thermodynamics, it follows that the material constant α_1 has to be positive (cf. Ref.[7]). Since the viscoelastic fluid for the study is being represented by the second order fluid model given by Eq.(5), it needs to be compatible with the principles of thermodynamics. So it has to satisfy the Clausius-Duhem inequality for all motions. The analysis that follows is based on the assumption that the specific Helmholtz free energy of the fluid is a minimum, when it is locally at rest.

Then $\mu\ge 0$, $\alpha_1\ge 0$ and $\alpha_1+\alpha_2=0$.

The constitutive equation given by Eq.(5) represents a non-Newtonian fluid with viscoelastic behaviour ($\mu>0$, $\alpha_1>0$).

For a steady flow with no pressure gradient, Eq.(2) reduces to

$$\rho V\cdot\operatorname{grad}(V)=\operatorname{div}(\mu A_1+\alpha_1 A_2+\alpha_2 A_1^2)+F_{EK} \tag{9}$$

For a symmetric $(z:z)$ electrolyte, the density of the total ionic charge, ρ_e is given by

$$\rho_e=\mathrm{e}z(n^+-n^-) \tag{10}$$

in which n^+ and n^- are the number densities of cations and anions, respectively, and are given by the Boltzmann distribution (considering no EDL overlap)

$$n^{\pm} = n_0 \exp\left(\mp \frac{\mathrm{e}z\psi^*}{k_B T_{ab}} \right) \tag{11}$$

In Eq.(11), n_0 represents the concentration of ions at the bulk, e stands for the charge of a proton, z is the valence of ions, k_B the Boltzmann constant, and T_{ab} the absolute temperature.

To express the body force, we use the Poisson-Boltzmann equation and the Debye-Huckel approximation. The Poisson-Boltzmann equation is related to the potential distribution ψ^* within the EDL. In the vertical direction, it takes the form

$$\frac{\mathrm{d}^2\psi^*}{\mathrm{d}y^2} = -\frac{\rho_e}{\varepsilon} \tag{12}$$

where ε is the dielectric constant (or the permittivity) of the fluid. Using Eq.(12) and taking help of Eqs.(10) and (11), we obtain the Poisson-Boltzmann equation in the form

$$\frac{\mathrm{d}^2\psi^*}{\mathrm{d}y^2} = \frac{2n_0\mathrm{e}z}{\varepsilon}\sinh\left(\frac{\mathrm{e}z\psi^*}{k_B T_{ab}} \right) \tag{13}$$

Let us now introduce a normalized electro-osmotic potential function ψ with zeta potential ζ of the medium in the form

$$\psi = \frac{\psi^*}{\zeta} \tag{14}$$

Henceforward we shall use the non-dimensional coordinates

$$\xi = \frac{x}{h}, \quad \eta = \frac{y}{h} \tag{15}$$

In terms of the non-dimensional variables defined in Eqs.(14) and (15), Eq.(13) now reads

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\eta^2} = \frac{h^2}{\lambda_D^2 \alpha}\sinh(\alpha\psi) \tag{16a}$$

Since the potential function on the boundary should be equal to unity, we have

$$\psi = 1 \quad \text{on} \quad \eta = 1 \tag{16b}$$

In Eq.(16a), $\alpha = \mathrm{e}z\zeta / k_B T_{ab}$ is the ionic energy parameter and $\lambda_D = (\mathrm{e}z)^{-1}(\varepsilon k_B T_{ab} / 2n_0)^{1/2}$ is the Debye length.

Now considering that the gradient of the EDL potential acts only in the normal direction of the boundary, using the boundary condition (16b) and assuming that $\psi \to 0$ and $\mathrm{d}\psi / \mathrm{d}\eta = 0$, at points far away from the EDL, we obtain the solution of Eq.(16a) in the form interfere

$$\psi(\eta) = \left(\frac{4}{\alpha} \right) \tanh^{-1}\left[\mathrm{e}^{m\eta} \tanh\left(\frac{\alpha}{4} \right) \right] \tag{17}$$

where $m = h / \lambda_D$ is the electo-osmotic parameter. The solution (17) is valid for all situations, where the EDLs formed on different surfaces do not interfere with each other. For low values of ζ potential, the Debye-Huckel linearization solution (17) reduces to the form

$$\psi(\eta) = \frac{\cosh(m\eta)}{\cosh(m)} \tag{18}$$

Assuming that the fluid is thermodynamically compatible $(\alpha_1 \geq 0)$, we consider the flow of an incompressible second order fluid through two parallel impermeable sheets $y = \pm h$. Considering the flow to be symmetric, we can confine our study to the region $0 \leq y \leq h$ for $x \in [0, L]$. In a steady state and in absence of any pressure gradient, the continuity Eq.(1) and momentum Eq.(8) can be written in the form that

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{19}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2} \right) - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3} \right] + \frac{\rho_e E_x}{\rho} \tag{20}$$

where u and v denote the fluid velocity components along x- and y-directions respectively and ν represents kinematic viscosity coefficient.

In the Cartesian coordinate system, the boundary conditions for the flow problem concerning a second-order fluid, which is being studied here are

$$u = b\frac{\partial u}{\partial y}, \quad v = 0 \quad \text{at} \quad y = h \tag{21}$$

$$\frac{\partial u}{\partial y} = 0, \quad v = 0 \quad \text{at} \quad y = 0 \tag{22}$$

In the analysis that follows we shall make use of the following transformations

$$u=\frac{U_{HS}}{h}xf'(\eta)\quad\text{and}\quad v=-U_{HS}f(\eta) \tag{23}$$

where U_{HS} stands for the Helmholtz-Smoluchowski velocity given by

$$U_{HS}=-ME_x=\frac{-\zeta\varepsilon E_x}{\mu} \tag{24}$$

in which M represents the mobility of the medium.

Using the non-dimensional coordinate defined in Eq.(15) and non-dimensional variables defined in Eq.(23), the non-dimensional form of the momentum Eq.(20) is obtained in the form

$$f'''+K(2f'f'''-f''^2-ff^{\text{iv}})+\lambda\frac{\mathrm{d}^2\psi}{\mathrm{d}\eta^2}=0 \tag{25}$$

$K=\alpha_1U_{HS}/h\mu$ being the viscoelastic parameter and $\lambda=h/L$ the aspect ratio. Dashes denote differentiation with respect to η. This equation has been derived by neglecting the inertia term and is valid for low Reynolds number flow.

The boundary conditions in terms of non-dimensional variables read

$$\psi(\eta)=1,\quad f'(\eta)=S_f f''(\eta),\quad f(\eta)=0\quad\text{at}\quad\eta=1 \tag{26}$$

$$\psi(\eta)\to 0,\quad \frac{\mathrm{d}\psi}{\mathrm{d}\eta}=0,\quad f''(\eta)=f(\eta)=0\quad\text{at}\quad\eta=0 \tag{27}$$

In Eq.(26), $S_f=b/h$ is the non-dimensional velocity slip factor.

Considering K to be small, a perturbation expansion of f is taken in the form

$$f(\eta)=f_0(\eta)+Kf_1(\eta)+K^2f_2(\eta)+\cdots \tag{28}$$

Substituting Eq.(28) into Eq.(25), equating like powers of K and neglecting second and higher powers of K, we obtain

$$f_{0'''}=-\lambda\frac{\mathrm{d}^2\psi}{\mathrm{d}\eta^2} \tag{29}$$

and

$$f_{1'''}+2f_{0'}f_{0'''}-f_{0''}^2-f_0f_0^{iv}=0 \tag{30}$$

From the Eqs.(26) and (27), it now follows that the boundary conditions for ψ and f can be listed as

$$\psi(1)=1,\quad f_{0'}(1)=S_f f_{0''}(1),\quad f_{1'}(1)=S_f f_{1''}(1),$$

$$f_0(1)=f_1(1)=0 \tag{31}$$

$$\psi(0)\to 0,\quad \psi'(0)=0,$$

$$f_{0''}(0)=f_{1''}(0)=f_0(0)=f_1(0)=0 \tag{32}$$

Solving the third-order differential Eqs.(29) and (30) subject to the boundary conditions (31) and (32), the expressions for f_0 and f_1 are found in the form

$$f_0(\eta)=\eta(\lambda-S_1)-\frac{\lambda\sinh(m\eta)}{m\cosh(m)} \tag{33}$$

and

$$f_1(\eta)=\frac{5\lambda(\lambda-S_1)\sinh(m\eta)}{m\cosh(m)}-\frac{\lambda\eta(\lambda-S_1)\cosh(m\eta)}{\cosh(m)}+\frac{\lambda^2m^2}{\cosh^2(m)}\left(\eta+\frac{\eta^3}{3}\right)+S_2\eta \tag{34}$$

where

$$S_1=\frac{S_f\lambda m\sinh(m)}{\cosh(m)},\quad S_2=\frac{\lambda(\lambda-S_1)S_3}{m\cosh(m)},$$

$$S_3=5m^2\sinh(m)-(4+m^2)m\cosh(m)$$

From the Eqs.(33) and (34), we further have

$$f_{0'}(\eta)=(\lambda-S_1)-\frac{\lambda\cosh(m\eta)}{\cosh(m)} \tag{35}$$

and

$$f_{1'}(\eta)=\frac{4\lambda(\lambda-S_1)\cosh(m\eta)}{\cosh(m)}-\frac{\lambda\eta(\lambda-S_1)m\sinh(m\eta)}{\cosh(m)}+\frac{\lambda^2m^2}{\cosh^2(m)}(1+\eta^2)+S_2 \tag{36}$$

The volumetric flow rate is then given by

$$V_m=2\int_0^1 f'(\eta)\mathrm{d}\eta=2(\lambda-S_1)-\frac{2\lambda\tanh(m)}{m}+2KV_{lm} \tag{37}$$

in which

$$V_{lm} = \frac{5\lambda(\lambda - S_1)}{m}\tanh(m) + \frac{4\lambda^2 m^2}{3\cosh^2(m)} - \lambda(\lambda - S_1) + S_2$$

The energy equation is given by

$$\alpha\frac{\partial^2 T^*}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \alpha_1\frac{\partial u}{\partial y}\left(u\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + v\frac{\partial^2 u}{\partial y^2}\right) + E_x^2\sigma = 0 \tag{38}$$

where T^* represents the temperature of the fluid, α and σ denote respectively the thermal conductivity of the fluid and the electrical permittivity, while the term σE_x^2 represents Joule heating.

The thermal boundary conditions are given by:

$$T^* = T_w + k_0\frac{\partial T^*}{\partial y} \quad \text{at} \quad y = h \tag{39}$$

$$\frac{\partial T^*}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{40}$$

where T_w denotes the surface temperature and k_0 is the temperature jump factor.

Let us now introduce the non-dimensional temperature variable defined by

$$\theta(\xi, \eta) = \frac{T^*}{T_w} = \theta_0(\eta) + \xi^2\theta_1(\eta) \tag{41}$$

Thus by using the non-dimensional coordinates as well as the non-dimensional variables and then equating the coefficients of ξ^0 and ξ^2, we get the following ordinary differential equations:

$$\theta_{0''} + \nu_1 = 0 \tag{42}$$

and

$$\theta_{1''} + \nu_2 f''^2 + K\nu_2(f'f''^2 - ff''f''') = 0 \tag{43}$$

in which $\nu_1 = \sigma E_x^2 h^2/\alpha T_w$ is a non-dimensional parameter related to Joule effect and $\nu_2 = \alpha_1 U_{HS}/h\mu$ is Brinkmann number.

The non-dimensional forms of the thermal boundary conditions (39) and (40) are

$$\theta_0(1) = 1 + S_t\theta_{0'}(1), \quad \theta_1(1) = S_t\theta_{1'}(1) \tag{44}$$

and

$$\theta_{0'}(0) = \theta_{1'}(0) = 0 \tag{45}$$

where $S_t = k_0/h$ is the non-dimensionalized temperature jump factor.

Solving the Eqs.(42) and (43) subject to the boundary conditions (44) and (45), we obtain the analytical expressions for θ_0 and θ_1 given respectively by

$$\theta_0 = 1 + \frac{\nu_1}{2}(1 - \eta^2 - 2S_t) \tag{46}$$

and

$$\theta_1 = \frac{\lambda^2\nu_2}{8\cosh^2(m)}[S_4\cosh(2m) + (1 + 6\lambda K - 6S_1K)\cosh(2m\eta) - 64\lambda K\frac{\cosh(m\eta)}{\cosh(m)} + mS_5\sinh(2m) + 32\lambda Km\eta\frac{\sinh(m\eta)}{\cosh(m)} + Km(S_1 - \lambda)\eta\sinh(2m\eta) + 2S_6 - S_t] \tag{47}$$

where

$S_4 = 2K(\lambda - S_1)[m^2(\eta - 1) - 3] + 2S_t - 1$,

$S_5 = 2 - 2\eta - K(\lambda - S_1)(11\eta - 12) - 2S_t$,

$S_6 = m^2(5S_1K - 1)(\eta - 1)^2 + \lambda K[32 - m^2(\eta - 1)(5\eta + 11)] + 16\lambda Km(\eta - 2)\tanh(m)$

3. Computation of electro-osmotic flow and heat transfer variables in the micro-circulatory system

In this section, we shall illustrate the applicability of the model developed and analyzed in the preceding sections. For this purpose, let us consider the electro-osmotic flow of blood and heat transfer in the micro-circulatory system subject to the action of an applied electric field. The derived expressions for the velocity profile, temperature profile and volumetric flow rate are numerically computed by taking into account the velocity-slip and temperature-jump at the wall of an arteriole. In order to have a proper insight into the flow behaviour of blood through an arteriole under the influence of electro-osmosis, the variation of f', θ_0 and V_m have been estimated and the computational results have been presented in graphical form. The values (or ranges of values) of the physical parameters involved in the model study conform to experimental data for blood flow during micro-circulation, reported in scientific literatures.

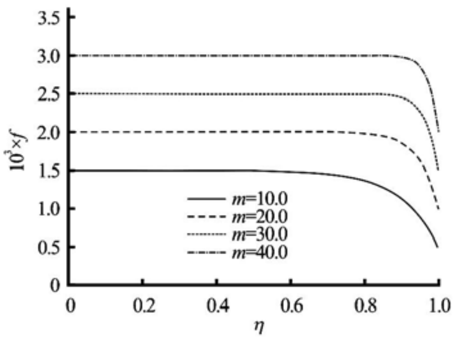

Fig.3 Velocity distribution of blood in the micro-circulatory system for different values of electro-osmotic parameter m

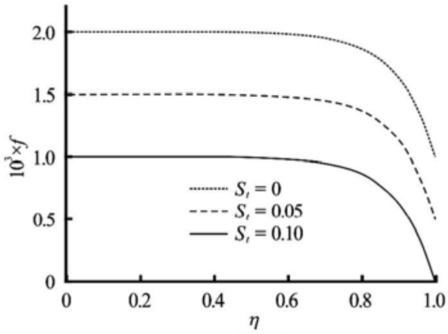

Fig.4 Distribution of blood velocity for different values of velocity-slip factor S_f, when $K = 0.001$, electro-osmotic parameter $M = 10.0$ and $\lambda = 0.001$

Figure 3 depicts the variation of blood velocity for different values of the electro-osmotic parameter m when values of other parameters are kept constant. It should be noted that velocity is minimum at the wall of the arteriole. It is further seen that velocity is enhanced with an increase in the electro-osmotic parameter. Since m is the ratio of height of the channel and the Debye thickness λ_D, it is indicated that an increase in the height of the channel causes a rise in axial velocity throughout the channel for a viscoelastic fluid.

Figure 4 reveals that velocity-slip at the wall of the arteriole bears the potential to alter the velocity distribution to a significant extent. This figure shows that during electro-osmotic flow, blood velocity increases as the slip factor increases. It may be noted that the velocity maintains a constant value in the vicinity of the axis of the channel, but in the proximity of the wall the velocity distribution is strongly influenced by the velocity-slip factor. It is revealed that the wall-slip effects bear the potential to arrest the velocity gradients in the interfacial region and also that such effects promote the advective transport of mobile ions in the EDL. This coupling between the wall slip and EDL transport plays the most critical role in determining the streaming potential field and in turn strongly influences the consequent variations in the energy transfer efficiency.

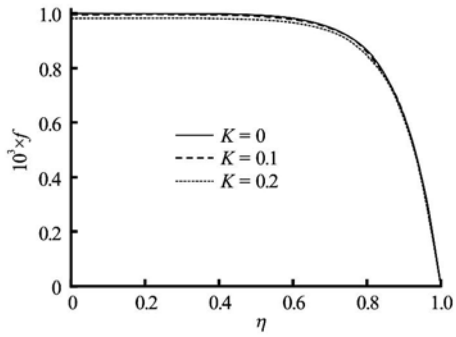

Fig.5 Velocity distribution of blood for different values of K in the absence of velocity-slip in the micro-circular system, when $m = 10.0$ and $\lambda = 0.001$

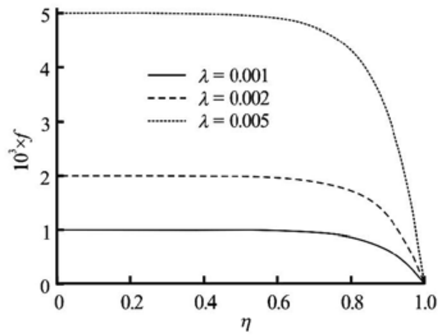

Fig.6 Change in the velocity distribution with change in the various value of λ in the absence of velocity-slip, for $M = 10.0$ and $K = 0.001$

Nature of velocity distribution for different values of the viscoelastic parameter K under no-slip condition is depicted in Fig.5. It may be noted that for a Newtonian fluid $(K = 0)$, blood velocity is greater than that of a non-Newtonian viscoelastic fluid $(K > 0)$. It is also seen from this figure that the velocity is minimum at the endothelium of the arteriole.

Figure 6 illustrates the change in velocity distribution for different values of the aspect ratio λ in the absence of wall-slip. This figure reveals that the change in aspect ratio does not bring about any appreciable change in the velocity distribution qualitatively, however, an increase in aspect ratio enhances the magnitude of the velocity.

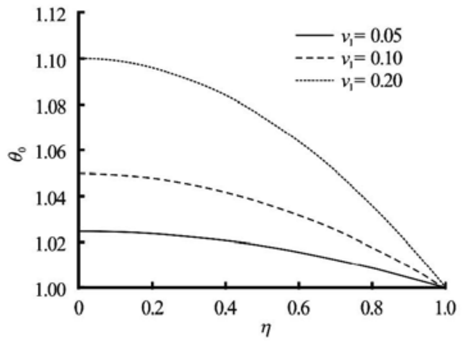

Fig.7 Temperature distribution for different values of the Joule heating parameter v_1 when temperature-jump factor $S_t = 0$

Figure 7 gives temperature profiles $\theta_0(\eta)$ for different values of Joule heating parameter v_1 in the absence of temperature-jump. One may note from this figure that temperature rises as the value of the Joule heating parameter is increased. This effect is more pronounced at the axis of the arteriole.

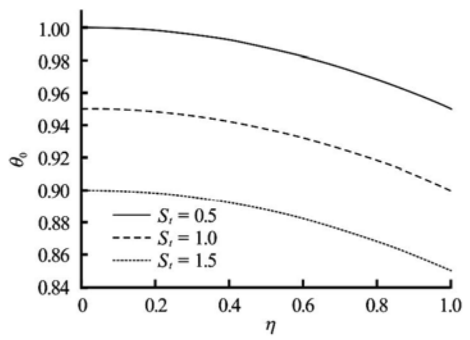

Fig.8 Change in temperature distribution with change in the value of S_t, when $v_1 = 0.1$

One can have an idea of the temperature profiles $\theta_0(\eta)$ for various values of the temperature-jump factor from Fig.8, when Joule heating parameter v_1 is equal to 0.1. This figure indicates that temperature decreases as the temperature-jump factor or the vertical distance increases.

Figure 9 illustrates the extent of variation in the volumetric flow rate, as blood viscoelasticity changes corresponding to different values of the velocity-slip factor $S_f = -0.1, -0.2, -0.3$ when electro-osmotic parameter $m = 20.0$ and aspect ratio $\lambda = 0.002$. The plots presented in this figure reveal that volumetric flow rate increases with a rise in the value of the parameter S_f. It is important to note from this figure that the volumetric flow rate is inversely proportional to the viscoelastic parameter K.

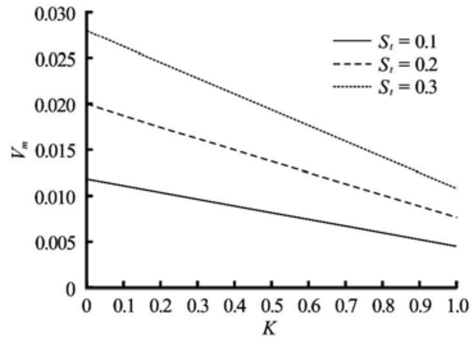

Fig.9 Variation in volumetric flow rate V_m with K for different values of S_f, in the case when $m = 20.0$ and $\lambda =$ 0.002

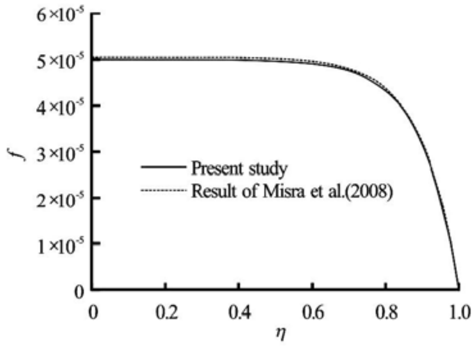

Fig.10 Comparison of velocity distribution between the results of the present study and those of a previous study[6]

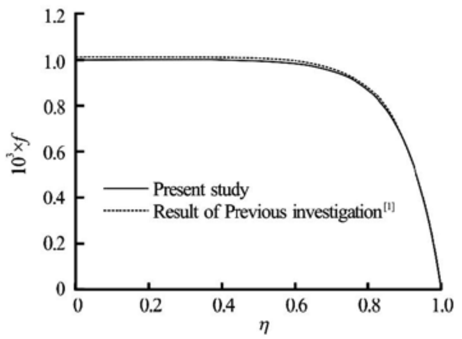

Fig.11 Comparison of velocity distribution between the results of the present study and those of another previous study on electro-osmosis[2]

4. Comparison of the results of the present study with those of previous investigations

In Fig.10, results of a previous investigation[6] have been presented along with the results of the present study. For the purpose of comparison, both the studies have been naturally brought to the same platform, by considering a vanishing value for the electric potential and also by disregarding the slip effects for the present study, while for the previous study[6], wall stretching and magnetic field effects have been considered to be zero.

Figure 11 gives a comparison of the results of the present study in the absence of slip factors with those of another previous study[2] on electro-osmotic flow, in the case when wall stretching is negligibly small.

One may observe from these figures that the results of the present study are in good agreement with those of both the previous studies.

5. Concluding remarks

Analytical solution for the electro-osmotic flow and heat transfer of a non-Newtonian viscoelastic fluid has been derived by taking into account the Navier slip and temperature-jump. Symmetric boundary conditions with equal zeta potential at the walls have been considered. A non-linear Poisson-Boltzmann equation governing the electrical double-layer field and a body force generated by the applied electric potential field have been incorporated in the Navier-Stokes equations.

The central objective of the study has been to investigate the distributions of the electric potential, as well as the variation of velocity, temperature and volumetric flow rate for the non-Newtonian viscoelastic fluid under the influence of electro-kinetic forces.

It may be mentioned that the references of many of the previous publications relevant to the present work, had to be ommited in the final version of the manuscript owing to the restriction in the number of references stipulated for this journal. However, some of the previous papers published recently, in which the state-of-the art of the area has been duly reviewed along with references, have been included in the Introduction Section of this paper and their references have been given in the Bibliography.

The study serves as a first step towards understanding the role of electro-osmosis during blood flow in the micro-circulatory system. It also bears the potential of having useful applications in nano-scale biosensor technologies as well as in electro-mechanical transducers. A strong merit of the present study lies in the fact the associated problem of heat transfer has also been elucidated here. This serves as an important step towards understanding the effect of Joule heating on blood flow in the micro-circulatory system. On the basis of the present study, it may be conjectured that Joule heating has a dominant role to play in controlling the temperature of blood in micro-vessels of the circulatory system.

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References

[1] RESSINE A., MARCO-VARGA G. and LAURELL T. Porous silicon protein microarray technology and ultra-/superhydrophobic states for improved bioanalytical read out[J]. **Biotechnology Annual Review,** 2007, 13(7): 149-200.

[2] MISRA J.C., SHIT G. C. and CHANDRA S. et al. Electro-osmotic flow of a viscoelastic fluid in a channel: Application to physiological fluid mechanics[J]. **Applied Mathematics and Computation,** 2011, 217(20): 7932-7939.

[3] WANG C., WONG T. and YANG C. et al. Characterization of electroosmotic flow in rectangular microchannels[J]. **International Journal of Heat and Mass Transfer,** 2007, 50(15-16): 3115-3121.

[4] WANG R., LIN J. and LI Z. Analysis of electro-osmotic flow characteristics at joint of capillaries with step change in ζ-potential and dimension[J]. **Biomedical Microdevices,** 2005, 7(2): 131-135.

[5] DATTI P. S., PRASAD K. V. and ABEL M. S. et al. MHD viscoelastic fluid flow over a non-isothermal stretching sheet[J]. **International Journal of Engineering Sciences,** 2004, 42: 935-946.

[6] CORTELL R. A note on flow and heat transfer of a viscoelastic fluid over a stretching sheet[J]. **International Journal of Non-linear Mechanics,** 2006, 41(1): 78-85.

[7] PRASAD K. V., PAL D. and UMESH V. et al. The effect of variable viscosity on MHD viscoelastic fluid flow and heat transfer over a stretching sheet[J]. **Communications in Nonlinear Science and Numerical Simulation,** 2010, 15(2): 331-344.

[8] ABEL M. S. Sanjayanand, Nandeppanavar MM, viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations[J]. **Communications in Nonlinear Science and Numerical Simulation,** 2008, 13(9): 1808-1821.

[9] BATALLER R. C. Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of non-uniform heat source, viscous dissipation and thermal radiation[J]. **Internaional Journal of Heat and Mass Transfer,** 2007, 50(15-16): 3152-3162.

[10] MISRA J. C., SINHA A. and SHIT G. C. A numerical model for the magnetohydrodynamic flow of blood in a porous channel[J]. **Journal of Mechanics in Medicine and Biology,** 2011, 11(3): 547-562.

[11] MISRA J. C., SINHA A. and SHIT G. C. Mathematical modeling of blood flow in a porous vessel having double stenoses in the presence of an external magnetic field[J]. **International Journal of Biomathematics,** 2011, 4(2): 207-225.

[12] AFONSO A. M., ALVES M. A. and PINHO F. T. Analytical solution of mixed electro-osmotic/pressure driven flows of viscoelastic fluids in micro-channels[J]. **Journal of Non-Newtonian Fluid Mechanics,** 2009,

159: 50-63.

[13] NG C. O., QI C. Electroosmotic flow of a viscoplastic material through a slit channel with walls of arbitrary zeta potential[J]. **Physics of Fluids,** 2013, 25(10): 103101.

[14] BAO F., LIN J. Burnett simulation of gas flow and heat transfer in micro Poiseuille flow[J]. **International Journal of Heat and Mass Transfer,** 2008, 51(15-16): 4139-4144.

[15] BAO F., LIN J. and SHI X. Burnett simulations of flow and heat transfer in micro Couette flow using second-order slip conditions[J]. **Heat and Mass Transfer,** 2007, 43(6): 559-566.