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A Dynamic Micromechanical Constitutive Model for Frozen Soil under Impact Loading**

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ABSTRACT By taking the frozen soil as a particle-reinforced composite material which consists of clay soil (i.e., the matrix) and ice particles, a micromechanical constitutive model is established to describe the dynamic compressive deformation of frozen soil. The proposed model is constructed by referring to the debonding damage theory of composite materials, and addresses the effects of strain rate and temperature on the dynamic compressive deformation of frozen soil. The proposed model is verified through comparison of the predictions with the corresponding dynamic experimental data of frozen soil obtained from the split Hopkinson pressure bar (SHPB) tests at different high strain rates and temperatures. It is shown that the predictions agree well with the experimental results.

KEY WORDS frozen soil, dynamic, micromechanical model, debonding damage theory, SHPB

I. Introduction

About 24% of the world's land area is permafrost[1]. Recently, more and more engineering projects have been carried out in the cold regions of the earth. Moreover, the method of freezing construction has also been widely used in the metro, tunnels and many other engineering activities. In these activities, the frozen soil was often subjected to impact loading, such as blasting and excavating. Thus, it is necessary to understand the dynamic mechanical properties of frozen soil under the impact loading conditions. The computational models currently used in the dynamic mechanical analysis of frozen soil were developed from the experimental data of ordinary rock and soil materials[2]. However, the mechanical properties of ordinary rocks and soils are different from those of frozen soil, where ice particles exist.

A series of experiments have been conducted on the frozen soil under impact loading conditions by many researchers[3–6], the experimental results of which have shown that the dynamic deformation of frozen soil is anisotropic and dependent on the pressure, strain rate and temperature. However, at present, the complicated mechanical deformation of frozen soil under impact loading conditions has not been reasonably described by a dynamic constitutive model. Although the constitutive model

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developed by Lee[5] described the dynamic stress-strain responses of frozen soil, a lot of experiments were required to determine the parameters used in the model, which limited its applications. Then, Ma[3,6] and Zhang[4] proposed some simple dynamic constitutive models, which, however, could not predict the dynamic mechanical behavior of frozen soil very well.

Since the frozen soil mainly consists of the clay soil and ice particles, it can be taken as one kind of composite materials; and then its constitutive model can be developed by referring to the micro-mechanics of composite materials. Tohgo and Chou[7] and Tohgo and Weng[8] proposed an incremental damage model to describe the mechanical behavior of particulate-reinforced composites by considering the plasticity of the matrix and the progressive debonding damage of the interfaces between the matrix and the particles based on Eshelby's equivalent inclusion method[9] and Mori-Tanaka's mean field concept[10]. Chen[11] further proposed a constitutive model for particulate-reinforced viscoelastic composite materials with debonded micro-voids. However, these models could not be directly used to describe the dynamic mechanical behavior of frozen soil, since they did not consider the effect of test temperature on the ice particles in the frozen soil. To describe the mechanical behavior of frozen soil, Ning and Zhu[12,13] developed a numerical method to predict the equivalent modulus of frozen soil; and Chen[14] conducted a preliminary exploration to the micromechanical constitutive model of frozen soil. However, such explorations did not address the dynamic mechanical behavior of frozen soil.

Therefore, in this work, based on the micromechanical debonding damage theory of particle reinforced composite materials[15,16], a constitutive model is established to describe the dynamic stress-strain responses of frozen soil subjected to impact loading. The proposed model considers the effects of strain rate and temperature on the dynamic mechanical behavior of frozen soil, and is verified through comparison of the predictions with the corresponding dynamic experimental results of frozen soil obtained from the split Hopkinson pressure bar (SHPB) test set-up at various strain rates and temperatures. It is concluded that the model can well predict the dynamic compressive stress-strain curves of frozen soil.

II. Outline of Experimental Results

The dynamic compressive deformation of the artificial frozen soil was observed experimentally by using the SHPB set-up in the previous work of the authors[4], the obtained experimental results of which are outlined in this section in order to keep the integrity of this work. The moisture content of each specimen was 30%; and the test temperatures were prescribed to be -8, -18 and -28°C, respectively. The results are shown in Figs.1 and 2.

It is concluded from Figs.1 and 2 that: (1) the dynamic compressive responses of the frozen soil present an obvious dependence on the applied strain rate, i.e., with the increase of strain rate, the responding peek stress and the dynamic elastic modulus of the frozen soil increase. (2) The dynamic compressive deformation of the frozen soil is also dependent on the test temperature, i.e., the responding peak stress increases with the decrease of test temperature. It should be noted that although the dynamic elastic modulus of the frozen soil increases with the increasing strain rate, its increment decreases with the decreasing test temperature. For example, at -28°C, the dynamic elastic modulus hardly changes with the variation of strain rate.

Therefore, dependence of the dynamic compressive responses of frozen soil on the applied strain rate and test temperature should be reasonably considered in the construction of a new dynamic constitutive model.

III. Dynamic Constitutive Model

3.1. Representative volume element of frozen soil

In this work, the frozen soil is assumed to be a kind of particle reinforced composite materials, and its representative volume element (RVE) is shown in Fig.3, where the reinforcing particle is the ice, and the matrix is the soil. It is seen from Fig.3 that the ice particles are assumed to be spherical with an initial volume fraction of f_{p0} and no void exists in the RVE of initial frozen soil before loading. The volume fraction of soil matrix is f_0. Furthermore, the unfrozen water is assumed to be uniformly distributed in the soil matrix, which can only influence the viscosity of the soil matrix. It means that the soil matrix here is assumed to be an equivalent uniform media. In fact, it is concluded from the

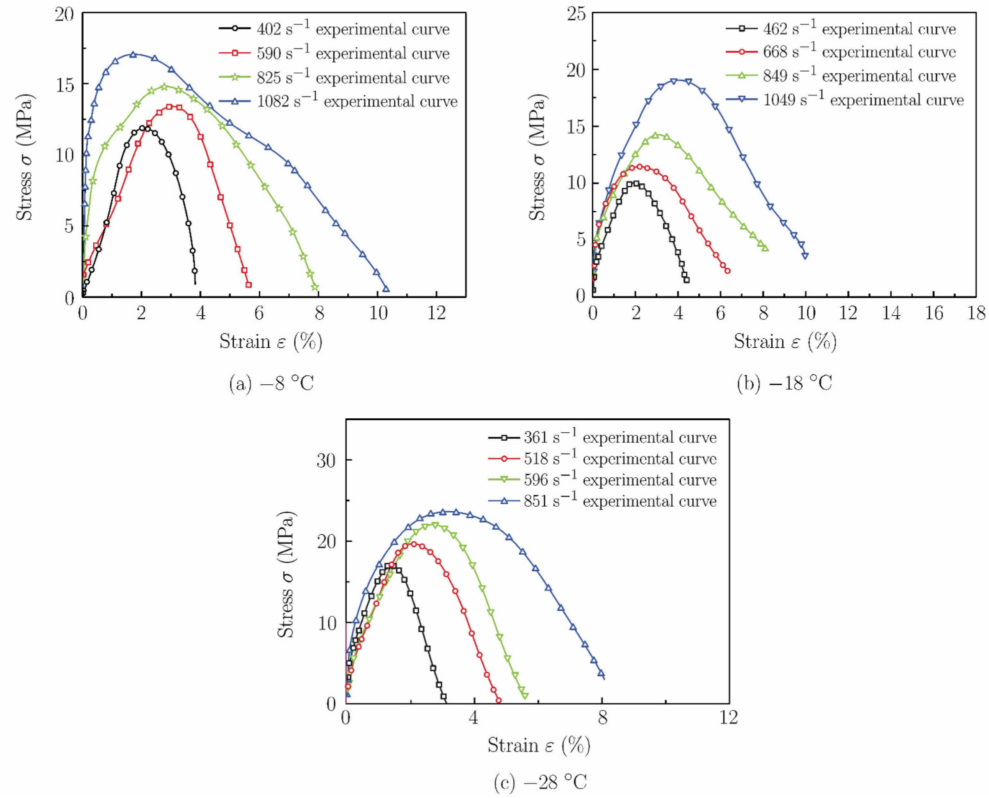

Fig. 1. Experimental dynamic compressive stress-strain curves of frozen soil at different temperatures (from Ref.[4]).

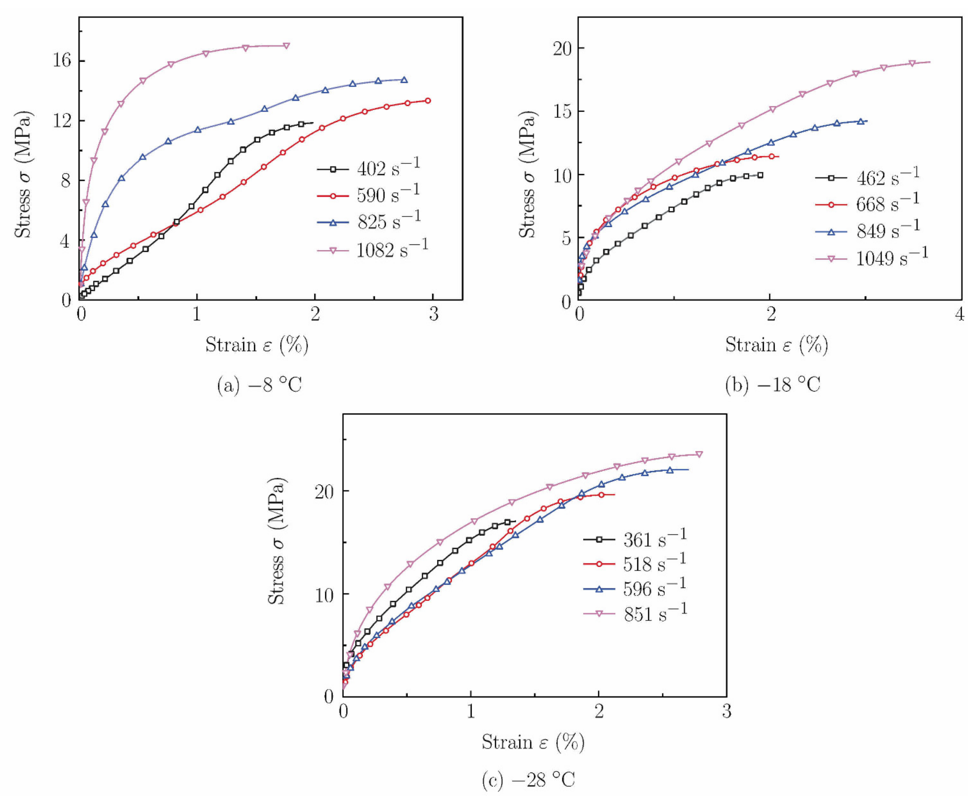

Fig. 2. The first raising stages of experimental curves for the frozen soil at different temperatures.

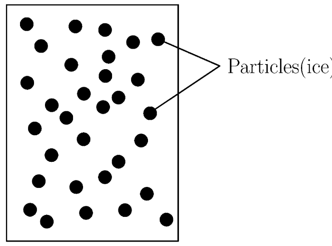

Fig. 3. Representative volume element (RVE) of initial frozen soil.

manufacture process of the frozen soil used in the tests that the content of unfrozen water in the frozen soil is much smaller than that in the clay soil. For simplification, the volume fraction of unfrozen water in the frozen soil is neglected in this work. So, it yields

$$f_0 + f_{p0} = 1 \tag{1}$$

3.2. Debonding damage theory

It is well-known that in the particle reinforced composites the interfaces between the particles and the matrix are the 'weak' parts, which will be firstly damaged and destroyed during the loading process. From this point of view, Tohgo[15] proposed a debonding damage theory to describe the mechanical responses of composites by taking the interfacial failure as a main cause of the damage occurred in the particle reinforced composites. Since the debonding damage theory has the advantages of clear physical meaning and simple form, it is extended in this work to describe the damage and dynamic mechanical response of frozen soil.

Referring to the RVE of frozen soil shown in Fig.3, the process of interfacial debonding damage in the frozen soil under impact loading conditions is concisely illustrated in Fig.4, and stated as follows.

At first, all the ice particles in the frozen soil are assumed to be perfectly bonded with the clay soil matrix before loading. During the deformation of the frozen soil under impact loading conditions, some ice particles will debond from the soil matrix due to the interfacial damage. For simplification, the stress in each particle is assumed equal before the interfacial debonding occurs around the particle. However, if the interfacial debonding damage occurs, the stress in the particle will be gradually released, and then becomes zero when the particle is fully debonded from the soil matrix, as shown in Fig.4(b). After the particle is fully debonded, the site of the particle is regarded as a void. The uniform stress in the fully bonded (i.e., intact) ice particle is denoted as σ_{ij}^p. The effective stress in the particle is simply calculated by $[\sigma_{ij}^p] = k\sigma_{ij}^p$, where the parameter k denotes the extent of interfacial debonding damage, i.e., (1) if $k = 1$, the particle is intact; (2) if $k = 0$, the particle is fully-debonded; (3) if $0 < k < 1$, the particle is partially-debonded.

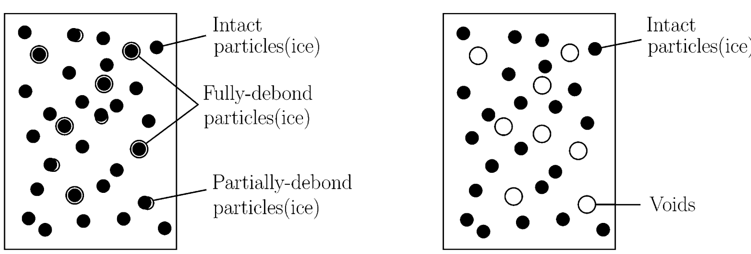

Fig. 4. Process of interfacial debonding damage.

Finally, according to the equivalence principle of micromechanics for particle reinforced composites, the overall stress of frozen soil can be obtained from the following equation[15]:

$$\sigma_{ij} = f_0 \left[\sigma_{ij}^0\right] + \int_0^{f_{p0}} k\sigma_{ij}^p \mathrm{d}f = (1 - f_{p0}) \left[\sigma_{ij}^0\right] + k f_{p0} \sigma_{ij}^p \tag{2}$$

where f_0 is the volume fraction of soil matrix; f_{p0} is the initial volume fraction of ice particles; and $\left[\sigma_{ij}^0\right]$ is the average stress of soil matrix.

3.3. Dynamic constitutive model

From Eq.(2), the overall stress of the composite is determined by the stresses in the ice particles, the soil matrix and the interfaces. For frozen soil, since the yield strength of the ice particle is much higher than those of the soil matrix and the interface, the following assumptions are made:

(1) The ice particle is isotropic and much stronger than the interface and the soil matrix, so it is assumed to be completely linear elastic before fully debonded. It means that the stress-strain relationship of ice particle is adopted as

$$\sigma_{ij}^p = C_{ijkl}^p \varepsilon_{kl}^p \tag{3}$$

where C_{ijkl}^p is the elastic matrix of ice particle.

(2) The strain-rate dependence of the dynamic stress-strain responses of frozen soil is caused by the viscosity of the soil matrix; and the soil matrix is considered homogeneous and isotropic. During the impact loading, the soil matrix exhibits viscoelastic stress-strain response.

(3) In the sense of statistics, the probability distribution of the extent of interfacial debonding damage around the ice particles obeys the Weibull's distribution function at macroscopic scale. So the damage parameter k can be written as

$$k = 1 - \int_0^{\varepsilon_e} \frac{m}{\alpha} x^{m-1} \exp\left[-\left(\frac{\varepsilon_e}{\alpha}\right)^m\right] \mathrm{d}x = \exp\left[-\left(\frac{\varepsilon_e}{\alpha}\right)^m\right] \tag{4}$$

where m and α are the parameters of Weibull's distribution function; and ε_e is the equivalent strain.

In Eq.(2), σ_{ij}^p is the stress in the fully bonded ice particles. As mentioned above, the ice particles and soil matrix are fully bonded if the interfacial debonding damage does not occur. Thus, in this case, the strains of the intact ice particles and soil matrix are the same and equal to the overall strain of frozen soil, even under impact loading conditions, i.e.,

$$\varepsilon_{ij} = \varepsilon_{ij}^0 = \varepsilon_{ij}^p \tag{5}$$

where ε_{ij} is the overall strain of frozen soil; ε_{ij}^0 is the strain of soil matrix; and ε_{ij}^p is the strain of ice particles.

Substituting Eq.(3), Eq.(4), and Eq.(5) into Eq.(2) yields:

$$\sigma_{ij} = (1 - f_{p0})\left[\sigma_{ij}^0\right] + \exp\left[-\left(\frac{\varepsilon}{\alpha}\right)^m\right] f_{p0} C_{ijkl}^p \varepsilon_{kl} \tag{6}$$

The viscoelasticity of a homogeneous and isotropi material can be modeled by a linear elastic element in parallel with the Maxwell bodies. The accuracy of the model can be improved by increasing the number of Maxwell bodies[17,18], which, however, will result in the complexity and difficulty in the numerical implementation of the model. Some researchers pointed out that two Maxwell bodies were sufficient for describing the materials' stress-strain behaviors under impact loading conditions[18]. So we choose the model containing a linear elastic element in parallel with two Maxwell bodies to describe the stress-strain behavior of soil matrix during the impact loading, as shown in Fig.5.

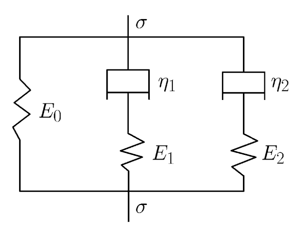

Fig. 5 Viscoelastic model of the soil matrix.

The model can be represented by the following equation:

$$\sigma_s = E_0 \varepsilon + E_1 \int_0^t \dot{\varepsilon} \exp\left(-\frac{t-\tau}{\varphi_1}\right) \mathrm{d}\tau + E_2 \int_0^t \dot{\varepsilon} \exp\left(-\frac{t-\tau}{\varphi_2}\right) \mathrm{d}\tau \tag{7}$$

where σ_s is the uniaxial stress of soil matrix; E_0, E_1, E_2 are elastic constants; $\varphi_1 = \eta_1/E_1$ and $\varphi_2 = \eta_2/E_2$ are relaxation times; and η_1, η_2 are viscosity coefficients. Equations (2)-(6) are three-dimensional equations but Eq.(7) is a one-dimensional equation. In the three-dimensional case, the σ_s,

E_0, E_1, E_2, ε and $\dot{\varepsilon}$ in Eq.(7) need to be replaced with the corresponding equivalent forms. In this study, the frozen soil was tested by uniaxial dynamic compression, so Eq.(6) degrades to the one-dimensional form as

$$\sigma = (1 - f_{p0})\,\sigma_s + \exp\left[-\left(\frac{\varepsilon}{\alpha}\right)^m\right] f_{p0} E_P \varepsilon \tag{8}$$

Since the strain rate is constant, Eq.(7) can be simplified as

$$\sigma = E_0\varepsilon + E_1\varphi_1\dot{\varepsilon}\left[1 - \exp\left(-\frac{\varepsilon}{\varphi_1\dot{\varepsilon}}\right)\right] + E_2\varphi_2\dot{\varepsilon}\left[1 - \exp\left(-\frac{\varepsilon}{\varphi_2\dot{\varepsilon}}\right)\right] \tag{9}$$

Under uniaxial compression, the dynamic constitutive model of frozen soil can be obtained by substituting Eq.(9) into Eq.(8):

$$\begin{aligned}\sigma = &\left\{E_0\varepsilon + E_1\varphi_1\dot{\varepsilon}\left[1 - \exp\left(-\frac{\varepsilon}{\varphi_1\dot{\varepsilon}}\right)\right] + E_2\varphi_2\dot{\varepsilon}\left[1 - \exp\left(-\frac{\varepsilon}{\varphi_2\dot{\varepsilon}}\right)\right]\right\}(1 - f_{p0}) \\ &+ \exp\left[-\left(\frac{\varepsilon}{\alpha}\right)^m\right] f_{p0} E_p \varepsilon\end{aligned} \tag{10}$$

3.4. The effect of temperature

As previously discussed, the test temperature is an important factor influencing the dynamic mechanical properties of frozen soil. The f_{p0} in Eq.(9) is a key parameter, which is obviously dependent on the test temperature. The unfrozen water content W_0 in the frozen soil can be calculated by the following equation[19,20]:

$$W_0 = AT^B \tag{11}$$

where A and B are constants; and T is temperature. So we can obtain the initial volume fraction of ice particles by

$$f_{p0} = \frac{(W - AT^B)G}{\rho_i V} \tag{12}$$

where W is the water content of frozen soil; G and V are the weight and volume of the frozen soil specimen, respectively; and ρ_i is the density of ice particles.

It should be noted that Eqs.(10) and (11) are meaningful only if the test temperature is much lower than 0°C, such as −8, −18, and −28°C used in this work.

IV. Model Validation

To verify the proposed model, the predicted dynamic compressive stress-strain curves of the frozen soil are compared with the corresponding experimental results in this section.

4.1. Determination of material parameters

The material parameters used in the proposed constitutive model are determined using the following method

(1) The initial stress-strain relationship of the frozen soil can be represented by $\sigma = f_{p0}E_p\varepsilon + (1 - f_{p0})\,E_s\varepsilon$. The elastic modulus of ice particle is 6 GPa, and the elastic modulus of soil matrix is 42 MPa, by referring to Ref.[13]. By fitting the initial linear elastic part of the experimental stress-strain curve obtained at a fixed temperature, the initial volume fractions of ice particles at different strain rates can be obtained. Then, the parameter f_{p0} at this temperature can be determined by averaging the initial volume fractions obtained at different strain rates.

(2) Since the parameters $\dot{\varepsilon}$, ε, and E_p are already known, the parameters φ_1, φ_2, E_0, E_1, E_2, m and α can be determined by using the least square method to fit the experimental data obtained at different temperatures and the minimum strain rate.

The values of the material parameters are listed in Table 1. The dynamic stress-strain curves of the frozen soil can then be predicted at other different strain rates.

Table 1. **Parameters of the proposed model**

Temperature	f_{p0}	E_0	E_1	E_2	E_p	φ_1	φ_2	m	α
$-8°C$	0.11	1.0	1.5	1.8	6000	2.8×10^8	2.3×10^9	1.8	0.03
$-18°C$	0.15	1.8	1.6	1.6	6000	4.0×10^9	1.3×10^9	1.4	0.04
$-28°C$	0.28	1.9	1.4	1.5	6000	5.1×10^9	7.1×10^8	2.1	0.03

4.2. Simulation and discussion

The simulation results and the corresponding experimental ones are shown in Fig.6. Since the proposed model fully considers the contribution of ice particles to the carrying capacity of frozen soil and assumes that the deformation of ice particles in the frozen soil is linear elastic with a constant elastic modulus and independent of rate, the model can describe the damage evolution of 'cold' frozen soil, for example, at -18 and -28°C, as shown in Figs.6(b) and 6(c). The damage processes corresponding to the predicted gradual changes in the carrying capacity of frozen soil are in fairly good agreement with the experimental results.

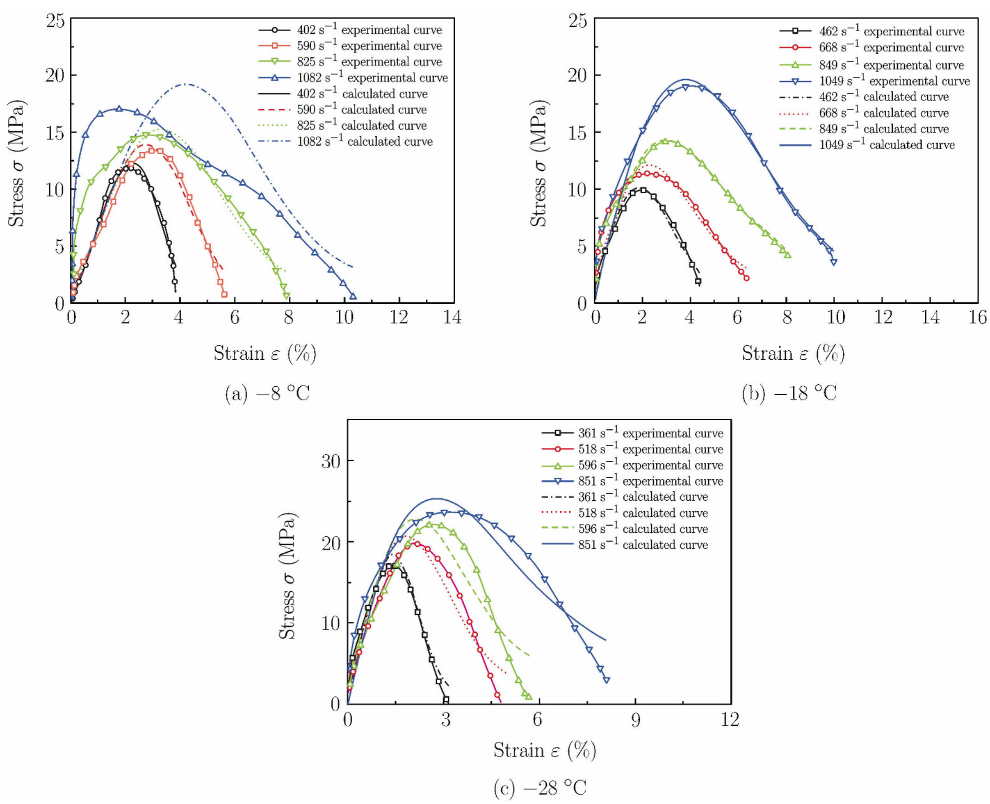

Fig. 6. Comparison of the experimental curves and the calculated ones of the frozen soil at different temperatures.

However, when the test temperature is relatively high, the effect of unfrozen water on the dynamic response of frozen soil can no longer be ignored. It means that the predictions obtained by the proposed model for the 'warm' frozen soil may not be very well. For example, when the test temperature is -8°C, the prediction results are not in very good agreement with the corresponding experimental ones, as shown in Fig.6(a).

However, the critical temperature dividing the 'warm' and 'cold' frozen soils is not absolutely a constant, and is dependent on the initial moisture content of the frozen soil. The lower the initial moisture content of the frozen soil is, the higher the critical temperature will be. As the temperature lowers down, the amount of unfrozen water in the frozen soil decreases. The ice particles become the most important factor influencing the elastic modulus of frozen soil. Since the elastic modulus of ice particle is not so rate-dependent, the dynamic elastic modulus of the frozen soil tends to be unchangeable with the further decreasing temperature, which is also described by the proposed model.

V. Conclusions

Based on the dynamic compressive tests of the frozen soil using the SHPB, a dynamic micromechanical constitutive model is established in this work by referring to the debonding damage theory of composite materials. In the proposed model, the frozen soil is assumed to be a particulate-reinforced composite material, in which the matrix is the clay soil and the reinforcement is the ice particles. The dependence of the dynamic stress-strain responses of the frozen soil on the test temperature and applied strain-rate is also addressed in the proposed model. The effect of test temperature on the dynamic stress-strain responses of frozen soil is reflected in the proposed model by considering its direct influence on the initial volume fraction of ice particles, i.e., the initial volume fraction of ice particles increases with the decrease of test temperature. Through comparison of the predictions with the corresponding dynamic compressive experimental results of the frozen soil, it is concluded that the proposed model can well describe the dynamic compressive stress-strain responses of the frozen soil, and the predictions agree well with the experimental results, especially at lower temperatures. The proposed model has advantages over the traditional geotechnical models in the aspects of physical nature and prediction accuracy.

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