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Hybrid Meta-Model Based Design Space Differentiation Method for Expensive Problems**

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ABSTRACT In this work, a hybrid meta-model based design space differentiation (HMDSD) method is proposed for practical problems. In the proposed method, an iteratively reduced promising region is constructed using the expensive points, with two different search strategies respectively applied inside and outside the promising region. Besides, the hybrid meta-model strategy applied in the search process makes it possible to solve the complex practical problems. Tested upon a serial of benchmark math functions, the HMDSD method shows great efficiency and search accuracy. On top of that, a practical lightweight design demonstrates its superior performance.

KEY WORDS hybrid meta-model, design space differentiation, expensive problems, global optimization

I. Introduction

The continuous advance of computer science and numerical techniques makes it possible to replace part of the costly physical experiments with computer analyses and simulations. In engineering, the fidelity simulations, usually the finite element analysis (FEA) or the computational fluid dynamics (CFD), are widely used to predict the performance of the modeled systems. To get the optimal performance of the modeled systems, the engineers also apply the simulations in their design optimizations. However, these computationally expensive problems still present a challenge to the optimization methods, even though the computer techniques have been dramatically improved. Venkataraman and Haftka[1] noted that at least 6 to 8 hours were still required for acceptable accuracy of the simulations throughout the last 30 years. Today one run of a whole car impact simulation still costs more than 10 hours to simulate a 0.1 second impact process, even by using a high performance quad-core computer.

Since the last decade, meta-modeling techniques have attracted many researchers' attention for their efficient response estimation. A meta-model, also called a surrogate model or an approximation model, usually employs analytical functions to predict the responses of costly problems. The commonly used meta-models include quadratic function-QF[2], Kriging[3–5], radial basis functions-RBF[6], multivariate adaptive regression splines-MARS[7], and support vector machine-SVM[8]. Through extensive study, it is found that there does not exist a meta-model that outperforms others in all aspects in theory. Reviews on meta-models can be found in the Refs.[9–14].

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Meta-model based global optimization methods have also been widely studied for their advantages over conventional methods. Many iterative methods have been developed and gained a great success in engineering. Jones et al.[15] employed Kriging in the search process, where one new point would be selected to update the meta-model in the region of interest. Wang et al.[16] used RBF to find the promising area and applied QF to search for the optima. However, the accuracy and efficiency of the meta-model based optimization methods are directly related to the design space. In order to solve the large-scale problems (less than 30 variables), the researchers tried to reduce the design space in the iterative process. Wang et al.[17] applied QF and gradually removed the design space with a given threshold. Balabanov et al.[18] pursued the space of interest through analyses of varying fidelity. Wang and Simpson[19] reduced the design space to a relatively small region with fuzzy c-means. Although some successes have been achieved, the fatal weakness of meta-model based methods of this type that the global optima may be deleted with the removed design space still needs to be addressed.

In the previous work, a so-called hybrid and adaptive meta-modeling method (HAM)[20] was proposed and gained a great success. In this method, three representative meta-models, i.e. Kriging, QF and RBF, were used together in the search process. And in every two iterations, a so-called important region was constructed using the expensive points and the search process was carried out in this region. The hybrid meta-model strategy can expand its range of applications and the important region strategy can increase its efficiency and accuracy. However, the search frequency in the important region can be increased, and its efficiency and accuracy most relied on the search in the important region can be further increased.

In this work, the design space differentiation strategy is proposed and a hybrid meta-model based design space differentiation method (HMDSD) is presented, cooperated with the hybrid meta-model strategy. With the proposed method, the complex practical problems can be successfully solved. The rest of the paper is organized as follows. §II will introduce the HMDSD method with a famous benchmark problem. In §III, several benchmark math functions and a practical problem involving the FEA will be employed to test the performance of the proposed method. The conclusion will be given in §IV.

II. Hybrid Meta-Model Based Design Space Differentiation Method (HMDSD)

In the region elimination methods, the design space is iteratively reduced and the global optimum is captured in a few iterations with the high fidelity meta-models. However, the performance of the methods of this type depends much on the accuracy of the used meta-models. The used coarse meta-models may lead to the deletion of the global optimum with the reduced region. As we all know that the accurate meta-models need a large set of sample points. The scarce sample points provided by engineers to save computation time hinders further applications of the region elimination methods in solving expensive problems.

In this work, instead of removing regions, a design space differentiation strategy is proposed with the combination of the previously developed hybrid meta-model strategy to acquire an efficient global optimization method with high accuracy for the practical expensive problems. In the proposed method, an important region is also constructed using the expensive points; and the search process will be carried out in each iteration instead of every two iterations in the HAM. Several new promising points will be respectively selected from the inside and outside of the important region with different strategies. In this approach, the initial meta-model is coarse and the accuracy of the used meta-model is rapidly increased in the promising area. Since high fidelity meta-models are needed for the trust region based approaches[21,22] and the move-limit optimization strategies[23–25], the coarse meta-model may lead to a local minimum or wrong results.

Procedures of HMDSD method

The proposed HMDSD method is developed especially for expensive problems. Steps of the algorithm are illustrated in Fig.1. A well-known benchmark math function is selected to facilitate the explanation of the HMDSD method[26].

$$f(x_1, x_2) = 100 \times \left[\left(x_2 - x_1^2\right)^2\right] + (1 - x_1)^2 \qquad ((x_1, x_2) \in [-2, 2]) \tag{1}$$

This problem is the well-known Banana function with the global minimum of 0 at (1, 1). Its plot is shown in Fig.2.

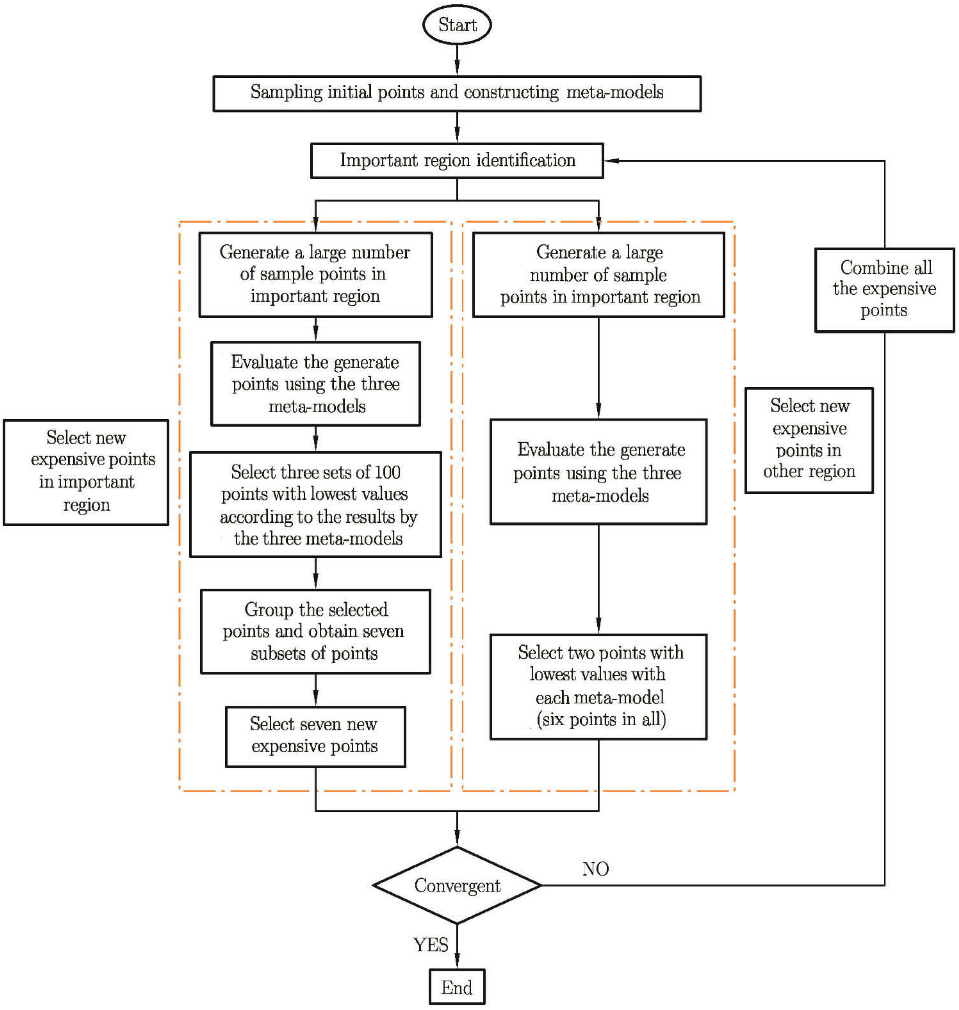

Fig. 1. Procedures of the HMDSD method.

Step 1: Sampling initial points and constructing the meta-model

In engineering, a small number of sample points can be provided by engineers in design optimization considering the production cycle. In each iteration, about thirteen new expensive points are selected and added to the previous expensive points to update the meta-models. To obtain a good balance between accuracy and search efficiency, and to facilitate the definition of the important region in the next step[20], fourteen initial sample points are generated and this number does not increase with an increase in the number of design variables. The performance of the new method with more initial points will be discussed later.

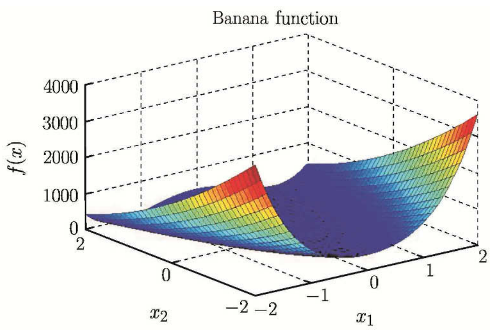

Fig. 2 Plot of the Banana function.

In this work, the Latin hypercube design (LHD) is employed for its space filling characteristic and easy realization[27–29]. Motivated by the previous work[20], three representative meta-models, i.e. Kriging, QF and RBF, are fitted with the fourteen initial points sampled by LHD. Figure 3 shows the three initial meta-models.

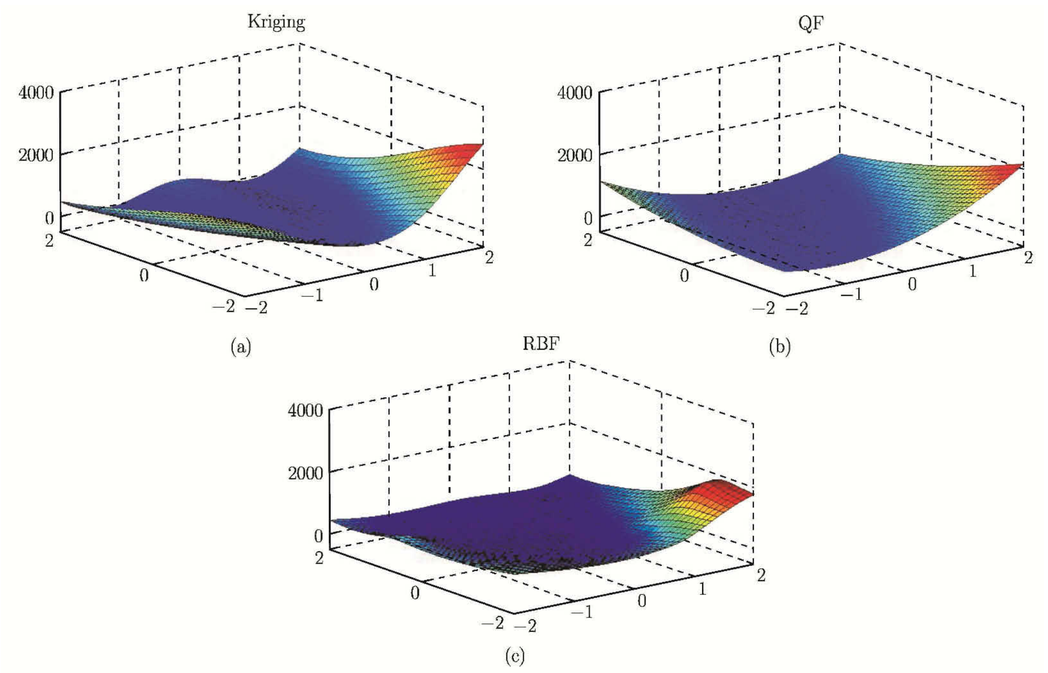

Fig. 3. Initial meta-models.

Step 2: Important region identification

In the meta-model based optimization method, the global optimum will soon be captured when it is contained in a rapidly reduced space. In the HAM method, the iteratively reduced region, called the important region, is constructed using ten expensive points with the lowest function values, which are searched once in every two iterations. To improve the efficiency, the search process in the important region is carried out for each iteration in the proposed method, so it should cover enough space in the first several iterations to avoid local optimum and gradual reduction of high performance. The number of expensive points to construct the important region is defined as follows:

$$n_{\mathrm{e}} = w_i \times m_{\mathrm{e}}, \qquad w_i = [1.0 - 0.15 \times (i-1)] \tag{2}$$

where n_{e} is the number of expensive points to construct the important region; m_{e} is the number of current expensive points; i is the number of iterations. If thirteen new promising points are selected for each iteration (defined in the next section), it can tell from Eq.(2) that the proportion of the points to construct the important region in all expensive points is gradually decreased by 15%. And the numbers of selected points for the first seven iterations are 14, 23, 28, 29, 26, 20 and 9, respectively. To ensure that a proper region is obtained, the number of selected points is changed to 14 since the seventh iteration. And the smallest region to contain the selected points is the important region, as shown in Fig.4. To avoid an improper region without any global optimum included in the important region, the search processes are simultaneously carried out both inside and outside the important region.

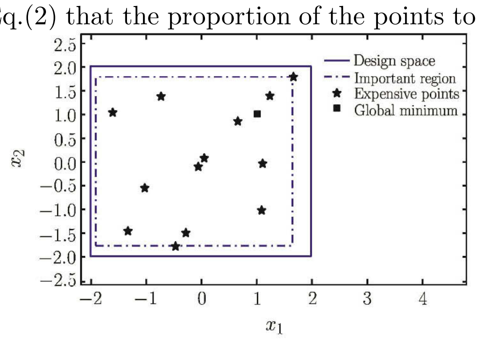

Fig. 4 An illustration of the key region for the first iteration.

Step 3: Selecting and evaluating new expensive points

In the proposed method, a large number of sample points, called cheap points, are generated both inside and outside the important region using LHD. And the generated points are evaluated using the meta-models.

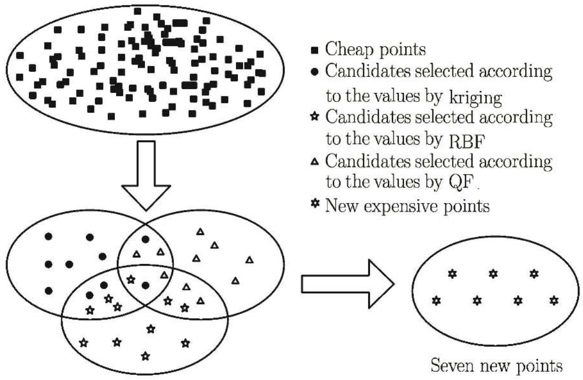

Fig. 5. New expensive points strategy for the points inside the important region.

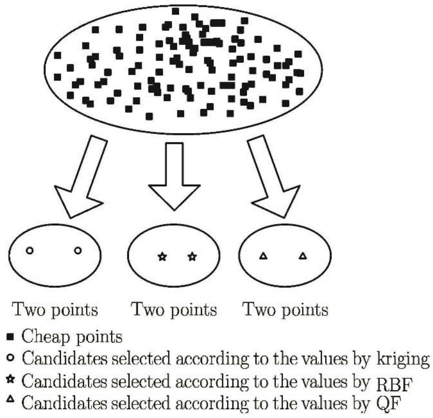

Fig. 6. New expensive points strategy for the points outside the important region.

1. The points generated inside the important region are considered more important and the strategy to the selection of new points is shown as follows:

(a) To evaluate the cheap points using the three meta-models, and to obtain three sets of function values;

(b) To select 100 points with the lowest function values as candidates from the cheap points according to the results gained using Kriging, QF and RBF, and to put them in set A, B and C;

(c) Each set of the candidates selected from the original cheap points may contain identical points and seven subsets are to be obtained using the following Eq.(3).

$$\begin{aligned} &S_1 = A \cap B \cap C, \quad S_2 = A \cap B - S_1, \quad S_3 = A \cap C - S_1, \quad S_4 = B \cap C - S_1 \\ &S_5 = A - S_1 - S_2 - S_3, \quad S_6 = B - S_1 - S_2 - S_4, \quad S_7 = C - S_1 - S_3 - S_4 \end{aligned} \tag{3}$$

If one new expensive point is selected from each set, about seven new expensive points in all will be obtained. Because some of the seven sets may contain more candidates, the number of new expensive points is given in Eq.(4). And the strategy is illustrated in Fig.5.

$$k_i = \mathrm{int}(w_i \times 7), \quad w_i = \frac{m_i \times l_i}{K}, \quad \sum_{i=1}^{k} w_i = 1 \quad (i = 1, 2, \cdots, 7) \tag{4}$$

where k_i is the number of selected points in the ith subset; w_i is the weight of the ith subset; m_i is the number of candidates in the ith subset; and l_i is the factor of the ith subset representing the number of meta-models for candidate selection, with $l_1 = 3$, $l_{2\text{-}4} = 2$ and $l_{5\text{-}7} = 1$.

2. The points generated outside the important region are considered less important and the strategy to the selection of new expensive points is shown below and illustrated in Fig.6.

(a) To evaluate the cheap points outside the important region using the three meta-models and to obtain three sets of function values;

(b) To select two points from the cheap points with the lowest function values according to the results obtained using Kriging, QF and RBF, and to obtain six new expensive points.

In this step, about thirteen new expensive points in all are obtained, which will be combined with the previous points to update the meta-models until the convergent criteria are met. Figure 7 shows the new expensive points selected in the third iteration. In this step, the current minimum is 0.0617 at (0.9797, 0.9845). And the meta-models are shown in Fig.8.

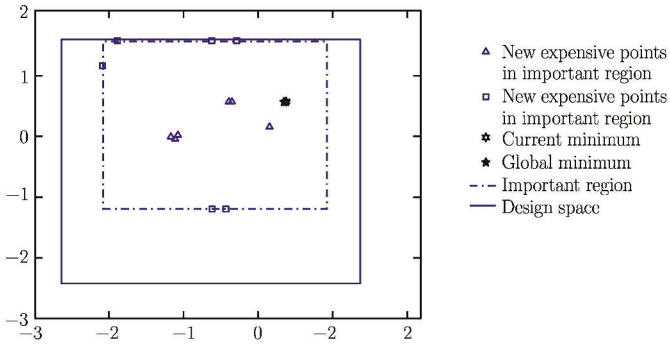

Fig. 7 New expensive points.

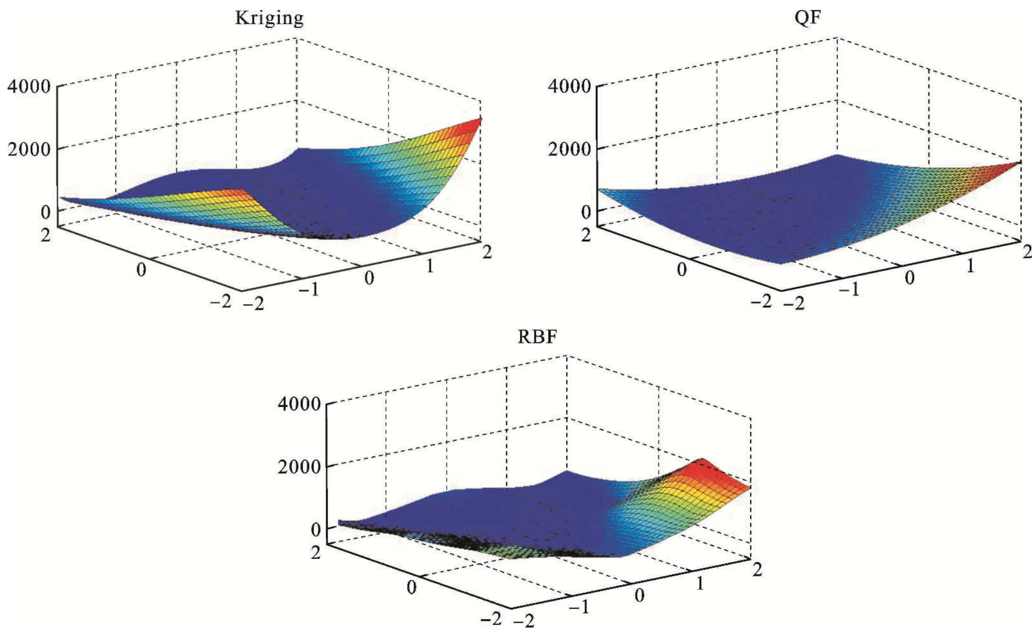

Fig. 8. Meta-models in the third iteration.

Step 4: A repeat of Steps 2 to 3 until the convergence criteria are met

The program will terminate when the mean value of the five lowest function values becomes negligible, as seen in Eqs.(5) and (6). The convergence criterion using five points can avoid premature of the proposed method[20]. The number to construct the important region remains unchanged since the seventh iteration, so the criteria will work since the seventh iteration to avoid premature of the proposed method.

$$|\bar{F}_{i+1} - \bar{F}_i| \leq \varepsilon \tag{5}$$

where a small value ε should be given by the designer, and

$$\bar{F}_i = \frac{\sum\limits_{j=1}^{5} f_j}{5} \tag{6}$$

where f_j is the jth lowest function value.

III. Tests

The proposed HMDSD method will be tested using a series of well-known benchmark math functions and an FEA based vehicle light design problem. The previously developed HAM method and the famous GA are employed for the comparison.

3.1. Math functions

3.1.1. Low-dimensional problems

1. Beak Function (BF)[26]

Beak function is given in Eq.(7) and plotted in Fig.9. This function has two local minima, which are -3.0498 at $(-1.347, 0.205)$, and -0.0649 at $(0.296, 0.32)$; and one global minimum, which is -6.5511 at $(0.228, -1.626)$.

$$\begin{aligned} f(x_1,x_2) = &\ 3\,(1-x_1)^2\,\mathrm{e}^{[-x_1^2-(x_2+1)^2]} - 10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right)\mathrm{e}^{(-x_1^2-x_2^2)} \\ & -\frac{1}{3}\mathrm{e}^{[-(x_1+1)^2-x_2^2]} \qquad ((x_1,x_2)\in[-3,-4;3,4]) \end{aligned} \tag{7}$$

2. Alpine Function (AF)[26]

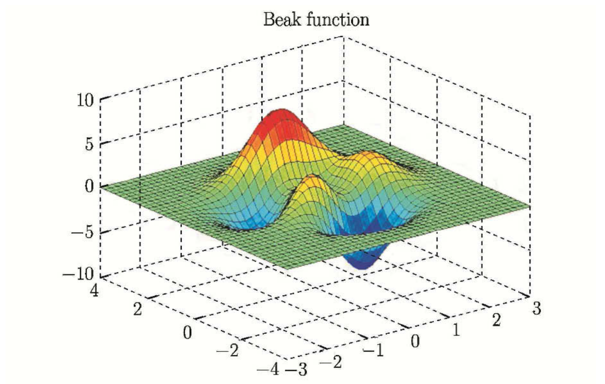

Fig. 9. Plot of Beak function.

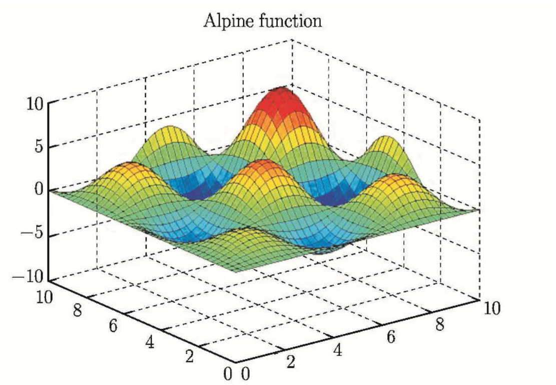

Fig. 10. Plot of the Alpine function.

Alpine function is given in Eq.(8) and plotted in Fig.10. This function has six local minima, which are -2.8542 at $(4.82, 1.84)$ and $(1.84, 4.82)$, -4.8310 at $(7.92, 10)$ and $(10, 7.92)$, and -2.2485 at $(1.84, 10)$ and $(10, 1.84)$; and two global minima, which are -6.1294 at $(7.923, 4.812)$ and $(4.812, 7.923)$.

$$f(x_1, x_2) = \sin(x_1) \times \sin(x_2)\sqrt{x_1 x_2} \qquad ((x_1, x_2) \in [0, 0; 10, 10]) \tag{8}$$

3. Goldstein and Price Function (GP)[16]

GP function is given in Eq.(9) and plotted in Fig.11. This function has four local minima, which are 840 at (1.2, 0.8), 84 at (1.8, 0.2), 30 at (−0.6, −0.4), and 35 at (−0.4, −0.6); and one global minimum, which is 3 at (0, −1).

$$\begin{aligned} f(x_1, x_2) = &\left[1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)\right]\left[(30 + 2x_1 - 3x_2)^2\right. \\ &\left.(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)\right] \qquad ((x_1, x_2) \in [-2, 2]) \end{aligned} \tag{9}$$

4. An exponential function (Ep)[30,31]

This function is given in Eq.(10) and plotted in Fig.12. It has three local minima, which are -0.5008 at $(-2, -2)$, -1.5008 at $(2, -2)$, and -1.0008 at $(-2, 2)$; and one global minimum, which is -2.0008 at (2, 2).

$$\begin{aligned} f(x_x, x_2) = &-0.5\exp\left[\frac{-(x_1+2)^2 - (x_2+2)^2}{2}\right] - \exp\left[\frac{-(x_1+2)^2 - (x_2-2)^2}{2}\right] \\ &-1.5\exp\left[\frac{-(x_1-2)^2 - (x_2+2)^2}{2}\right] - 2\exp\left[\frac{-(x_1-2)^2 - (x_2-2)^2}{2}\right] \\ &((x_1, x_2) \in [-5, 5]) \end{aligned} \tag{10}$$

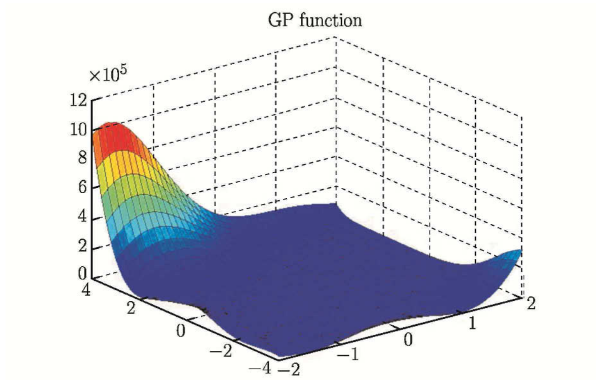

Fig. 11. Plot of GP function.

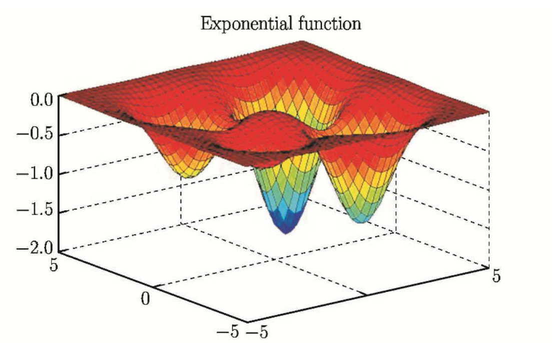

Fig. 12. Plot of Exponential function.

These four problems include a high nonlinear problem-GP, an Exponential problem-EP, a sine problem-AF, and a complex problem-BF. Each of them has several local minima. For each problem, 100 runs are carried out to reduce random variation in the numerical results. The average (arithmetic mean) number of function evaluations, nfe, and the number of iterations, nit, are used to show the efficiency of the proposed method. The mean value of the obtained minimum, min, is also given to show its accuracy. The results are summarized in Table 1.

Table 1. **Test results of low-dimensional problems**

Func.	Analytical minimum	HMDSD			HAM			GA		
		min	nit	nfe	min	nit	nfe	min	nit	nfe
BF	−6.55	−6.27	7.0	102	−5.12	4.9	41	−4.40	51	1040
AF	−6.13	−6.13	7.1	101	−5.53	4.67	40	−0.06	51	1040
GP	3	3.02	10.3	142	3.95	18.9	123	15.91	68.8	1395
EP	−2.00	−2.00	7.0	97	−1.70	4.37	38	−2.00	51	1040

As can be seen from Table 1, the proposed method provides the most accurate results for all the four problems. And the mean values of all the four problems are close to their analytical minima. An average of about seven iterations also shows its great efficiency. For BF, AF and EP, the proposed method used two more iterations than HAM did. When solving GP, HMDSD gained far more accurate results by using about a half of the iterations that HAM did. As to GA, which requires at least an average of 51 iterations and more than 1000 points, its low efficiency hinders its further engineering applications. Moreover, the low accuracy in solving three of the four problems gives it little chance to be selected by engineers.

All the four problems have several local minima; and the low accuracy also shows their low ability to escape from the trap of the local minima, as seen in Table 2.

Table 2. **Results in handling local minima**

Function	Analytical minimum	Algorithm	Number of obtained global minimum
BF	−6.55	HMDSD	92
		HAM	69
		GA	53
AF	-6.13	HMDSD	100
		HAM	74
		GA	0
GP	3	HMDSD	100
		HAM	96
		GA	78
EP	−2.00	HMDSD	100
		HAM	59
		GA	97

As shown in Table 2, the proposed method shows great ability to escape from the trap of the local minima over HAM and GA. Of all the four problems, the proposed method can successfully capture their global minima in solving AF, GP and EP; and of the three problems, it can perfectly escape from the trap of the local minima As to BF, the fact that 92 in 100 runs obtained the global minimum is also acceptable. Both HAM and GA can handle one of the four problems, but may be trapped in the local minima in solving the other three problems.

3.1.2. High-dimensional problems

1. Paviani function with $n = 10$ (PF)[32]

$$f(x) = \sum_{i=1}^{n} \left[\ln^2(x_i - 2.0) + \ln^2(10 - x_i)\right] - \left(\prod_{i=1}^{n} x_i\right)^{0.2} \qquad (x_i \in [2.1, 9.9]) \tag{11}$$

2. Trid Function with $n = 10$ (TF)[33]

$$f(x) = \sum_{i=1}^{n} (x_i - 1)^2 - \sum_{i=1}^{n} x_i x_{i-1} \qquad (x_i \in [-100, 100], \quad (i = 1, 2, \cdots, n)) \tag{12}$$

3. F16 function with n=16 (F16)[16]

$$f(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(x_i^2 + x_i + 1)(x_j^2 + x_j + 1) \qquad (x_i, x_j \in [-5, 5]) \tag{13}$$

where

$$[a_{ij}] = \begin{bmatrix} 1&0&0&1&0&0&1&1&0&0&0&0&0&0&0&1\\ 0&1&1&0&0&0&1&0&0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&1&0&1&1&0&0&0&1&0&0\\ 0&0&0&1&0&0&1&0&0&0&1&0&0&0&1&0\\ 0&0&0&0&1&1&0&0&0&1&0&1&0&0&0&1\\ 0&0&0&0&0&1&0&1&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0&0&0&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&1&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

4. Sum Squares function with $n = 20$ (SSF)[32]

$$f(x) = \sum_{i=1}^{n} i x_i^2 \qquad (x_i \in [-10, 10]) \tag{14}$$

All these four math functions are commonly-used benchmark test functions with the number of design variables ranging from 10 to 20. In these functions, the Paviani function is a low-order problem involving logarithmic function; the F16 function is a high nonlinear problem; and the Sum Squares function and the Trid function are second-order polynomials. Of all the four problems, the Trid function has a large design space, ranging from -100 to 100; and the other three problems have moderate design spaces. For each test problem, 100 runs are carried out. The average number of iterations nit, number of function evaluations nfe and the obtained minimum, min, are summarized in Table 3.

Table 3. **Summary of the results for high-dimensional functions**

Func.	Analytical minimum	HMDSD			HAM			GA		
		min	nit	nfe	min	nit	nfe	min	nit	nfe
PF	−45.8	−45.0	10.1	135	−37.2	13.4	91	−45.6	64	1298
TF	−210	−204.6	20.1	252	−125.4	40.7	297	−208.8	239.8	4816
F16	25.88	35.5	17.9	229	50.3	60.9	423	42.8	132.1	2663
SSF	0	0.78	20.9	260	4.6	47.4	286	10.6	112	2256

Although those high-dimensional problems have no local minima, the optimization method may also have difficulty to capture the global minima due to their large design spaces. The Sum Squares function is a second order polynomial and has no local minima, the global minimum of which is difficult to be captured by the famous GA, even using more than 100 iterations and 2000 function evaluations The proposed HMDSD method provides close accuracy to GA in solving PF and TF with about one tenth of the iterations. In solving F16 and SSF, higher accuracy is provided and about 80% of the computation time is saved when the number of iteration is considered. When considering the number of function evaluations, more than 90% of the computation time can be saved in solving all the four problems. The HAM method can be totally replaced by HMDSD of higher efficiency and accuracy.

In summary, the proposed HMDSD method can strike a good balance between accuracy and search efficiency and has a great potential to be applied in solving practical problems.

3.1.3. Discussion of the number of initial points

In meta-model based optimization methods, the number of initial sample points usually increases with an increase in the number of design variables. According to experience, more accurate meta-models may be provided when fitted with more points. In the proposed method, fourteen initial sample points are used; and this number does not increase with an increase in the number of design variables. It seems to conflict with experience. In this section, the test is carried out to show the performance of HMDSD method with more initial sample points, and the results are shown in Table 4. In this test, GP represents the low-dimensional problems and all the four high-dimensional problems are employed. When solving these problems, the search efficiency and accuracy have the potential to be noticeably improved. In this test, 100 runs are carried; and min, nit and nfe are also presented to show the accuracy and efficiency.

Table 4. **Test results of the HMDSD method with more initial sample points**

Func.	Analytical minimum	14 points			34 points			50 points		
		min	nit	nfe	min	nit	nfe	min	nit	nfe
GP	3	3.02	10.3	143	3.02	9.42	154	3.01	8.6	159
PF	−45.8	−45.0	10.1	135	−45.2	10.5	156	−45.2	10.5	170
TF	−210	−204.6	20.1	252	−209.1	21.6	288	−209.2	21.7	305
F16	25.88	35.5	17.9	229	35.0	18.3	253	34.8	18.4	269
SSF	0	0.78	20.9	260	0.83	21.7	287	0.76	22.0	307

As can be seen from Table 4, the performance of the proposed method has no noticeable improvements, even using fifty initial sample points. Of all the five problems, close accuracy and efficiency of the HMDSD method are obtained with different number of initial sample points. Therefore, fourteen initial sample points are employed in the HMDSD method; and this number does not increase with an increase in the number of design variables.

3.2. Vehicle lightweight design

The lightweight design example involves the finite element analysis and simulation. The performance of the system is evaluated using the Nastran software. This is a real engineering problem and the lightweight design has been done once. To reduce more weight, the optimization process is carried out using the proposed method. The FEA model is shown in Fig.13. Although this is a linear analysis problem, the large design space caused by the large number of design variables makes it difficult to significantly reduce the weight.

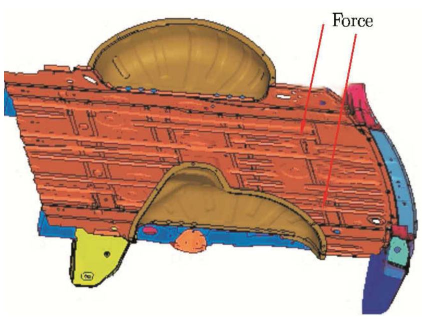

Fig. 13. The FEA model of the rear frame.

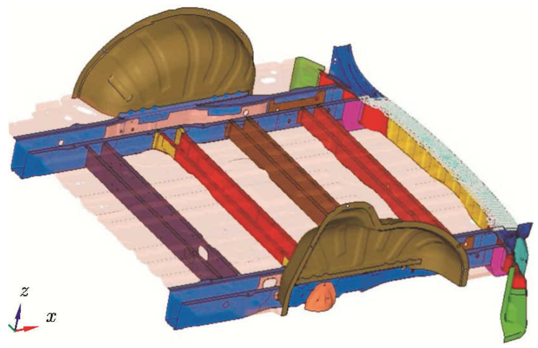

Fig. 14. The parts to be optimized.

Figure 13 shows the FEA model of the rear frame and Fig.14 shows the parts to be optimized. The whole structure contains 43 parts; and its weight is 73.7 kg. The FEA model contains 161656 elements; and one run will cost about 5 minutes using Lenovo T420i. The optimization model is shown as follows:

$$\begin{array}{ll} \min & \text{mass} \\ \text{s.t.} & \text{disp} < 2.0, \quad t_{1\text{-}18} \in [0.6, 2.5] \end{array} \tag{15}$$

where the objective $f(x)$ is the mass (unit: kg); disp denotes the maximum displacement by the load (unit: mm), which is defined as the constraint; $t_{1\text{-}18}$ are the thicknesses of the parts to be optimized (unit: mm), which are defined as variables. The comparison of the displacements is shown in Fig.15; and the detailed results are shown in Table 5. All results retain two decimal places.

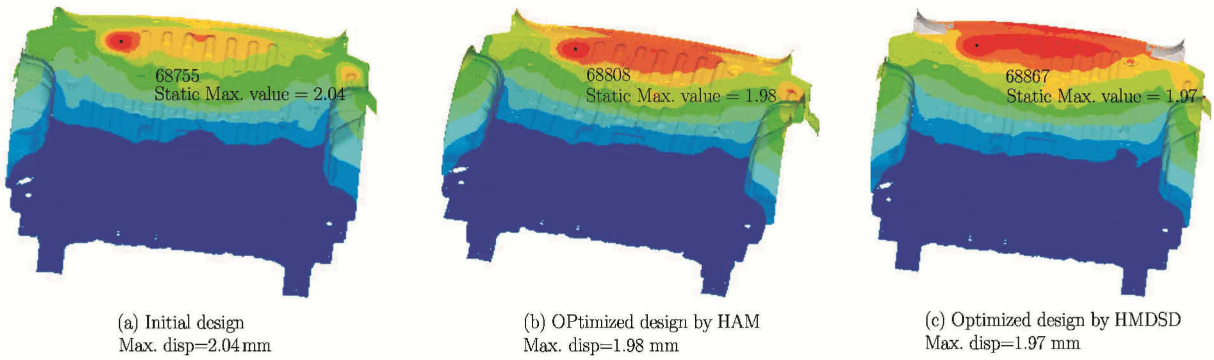

Fig. 15. Comparison of the displacements.

Table 5. **Summary of the lightweight design**

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}
Initial design	0.8	1.0	1.0	1.0	1.0	1.5	1.5	1.0	1.2	0.8	1.5
HAM	1.00	0.92	0.82	1.05	1.69	0.60	1.38	0.78	1.65	1.10	0.70
HMDSD	1.04	0.98	0.69	0.86	1.03	1.12	1.26	0.88	1.09	0.62	0.73
	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	mass	dis	nit	nfe
Initial design	1.2	1.2	1.5	1.0	1.5	2.0	2.0	73.7	2.05		
HAM	1.28	1.00	1.05	0.88	0.63	1.62	1.00	70.5	1.96	35	243
HMDSD	1.29	0.79	0.72	1.63	0.66	0.76	1.45	66.4	1.97	17	231

In the meta-model based optimization method, the new points obtained in each iteration can be evaluated simultaneously; and the number of iterations represents its efficiency. It can be seen from Table 5 that the HMDSD method used 17 iterations; and the mass of the system reduced 7.3 kg, reaching 9.9% of the mass of the system. The maximum displacement by the load reduced to 1.97 mm. The mass of the system reduced 3.2 kg with HAM, using 35 iterations.

IV. Conclusions

In this work, an intuitive method for expensive problems is presented. This approach can be considered as an improved region elimination method, which can overcome the shortcomings resulted from space reduction. Tested upon a series of benchmark math functions and a vehicle lightweight design problem, the HMDSD method shows a great potential for engineering applications Overall, the proposed method has been found to have the following advantages:

1. It is a standalone global optimization method that does not need to call any existing optimization method in the search process.

2. The number of initial sample points and the points selected in each iteration do not increase with an increase in the number of design variables.

3. Although random in nature, the method is robust and the program can work well with the default settings.

4. The newly selected points in each iteration can be evaluated simultaneously and the computation time is controlled by the iterations.

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Appendix: The Used Meta-Modeling Techniques

A. Kriging

Kriging is a commonly used meta-model[34,35]. Its general form can be expressed as follows:

$$\hat{y}(x) = f(x) + Z(x) \tag{16}$$

where $f(x)$ is usually a known polynomial function; and $Z(x)$ is a stochastic process with a mean value of zero and a non-zero covariance $\mathrm{Cov}[Z(x^i), Z(x^j)] = \sigma^2 R[R(x_i, x_j)]$, where σ^2 is the covariance; and R is the correlation matrix.

B. Quadratic Function (QF)

Quadratic function (QF) was first developed and described by Box and Wilson[36,37]. Its form is shown as follows:

$$\hat{y}(x) = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_i \sum_j \beta_{ij} x_i x_j \tag{17}$$

where β_{ij} are parameters calculated with the least squares regression by minimizing the sum of the deviations of predicted function values, $\hat{y}(x)$, from the actual values, $y(x)$:

$$\beta = \left[X'X\right]^{-1} X'y \tag{18}$$

where X is the design matrix constructed by the sample data points; X' is its transpose; and y is a column vector containing the values of the response at each sample point.

C. Radial Basis Functions (RBF)

RBF was introduced by Hardy to fit the irregular topographic contours of geographical data in 1971[6]. A simple form of RBF is shown below:

$$\hat{y} = \phi(x) = \sum_{i=1}^{n} \beta_i ||x - x_i|| \tag{19}$$

where x is the input variable to be estimated; x_i is the variable of the ith evaluated sample point; $||\bullet||$ is the Euclidean norm; and β_i is the coefficients to be estimated.