Acta Mechanica Solida Sinica, Vol. 27, No. 5, October, 2014 ISSN 0894-9166
Published by AMSS Press, Wuhan, China

FIRST-PASSAGE FAILURE OF PREISACH HYSTERETIC SYSTEMS**

Ming Xu[1] **Xiaoling Jin**[2] **Yong Wang**[2*] **Zhilong Huang**[2]

([1] *Institute of Fluid Mechanics, China Jiliang University, Hangzhou* 310018*, China*)
([2] *Department of Engineering Mechanics, Zhejiang University, Hangzhou* 310027*, China*)

Received 26 October 2012, revision received 28 April 2014

ABSTRACT The first-passage failure of a single-degree-of-freedom hysteretic system with non-local memory is investigated. The hysteretic behavior is described through a Preisach model with excitation selected as Gaussian white noise. First, the equivalent nonlinear non-hysteretic system with amplitude-dependent damping and stiffness coefficients is derived through generalized harmonic balance technique. Then, equivalent damping and stiffness coefficients are expressed as functions of system energy by using the relation of amplitude to system energy. The stochastic averaging of energy envelope is adopted to accept the averaged Itô stochastic differential equation with respect to system energy. The establishing and solving of the associated backward Kolmogorov equation yields the reliability function and probability density of first-passage time. The effects of system parameters on first-passage failure are investigated concisely and validated through Monte Carlo simulation.

KEY WORDS first-passage failure, Preisach hysteretic systems, stochastic averaging of energy envelope, generalized harmonic balance technique

I. INTRODUCTION

In the wake of developments in advanced technology, the amount of devices including non-traditional materials and smart structures incorporating smart materials, such as piezo-ceramics, shape memory alloys and electro-/magneto-rheological fluids, is dramatically increasing. The requirement on precision spurs rigorous description of non-traditional materials and smart materials. Unlike traditional elastic materials, non-traditional materials and smart materials always represent prominent hysteretic behaviors. Hysteretic force depends on both the instantaneous deformation and the past history of deformation, and a hysteretic loop can be created between hysteretic restoring force and displacement in periodic motion. Various analytical models have been proposed for representing the hysteretic constitutive relationship, including the bi-linear model, Ramberg-Osgood model, Iwan's distributed element model, Bouc-Wen model, Ozdemir's model, Masing model, and so on[1–5]. Almost all hysteretic models, however, can only represent hysteresis with local memory. The Preisach model[6], which is extensively used in the field of ferromagnetism, has been adopted in mechanical description of non-traditional materials and smart materials. The main advantage of the Preisach model is its capability of describing

* Corresponding author. E-mail: yongpi.wang@gmail.com
** Project supported by the National Natural Science Foundation of China (Nos. 11025211, 11302064 and 11202181), Zhejiang Provincial Natural Science Foundation of China (No. LQ12A02001) and the special fund for the Doctoral Program of Higher Education of China (Nos. 20110101110050 and 20120101120171).

hysteretic nonlinearity with nonlocal memory and capturing the crossing of minor loops that can arise in many real materials[7].

In structural and mechanical engineering, the dynamic loading to which hysteretic systems are subjected are sometimes random in nature, such as wind, earthquake ground motion and noise[8,9]. The random response of hysteretic systems has been investigated systematically by using several approximate solving techniques, such as the equivalent linearization technique[10] and the stochastic averaging method[11,12]. In the aspect of random response analysis of hysteretic systems described by Preisach model, second-order statistics of random response have been obtained through the covariance and switching probability analysis of a non-local memory hysteretic constitutive model[13,14]. Recently, standard stochastic averaging and stochastic averaging of energy envelope for Preisach hysteretic systems have been established by Spanos et al.[15] and Wang et al.[16]. These approximate techniques provide a complete methodology to evaluate the random responses. Another important research on the random dynamical behaviors of hysteretic systems is the determination of the probability of the structure in the safety domain, which is the so-called the first-passage failure problem. The first-passage failure problem is of crucial significance for reliability analysis, but unfortunately, it is one of the most difficult problems in the field of random vibration. It is only for very special one-dimensional diffusion processes that the exact solution of the first-passage failure problem can be obtained analytically. And so, the approximate solution techniques used in random response analysis of hysteretic systems have been considered to simplify the first-passage failure analysis. In all of the approximate techniques, stochastic averaging is a powerful one for the first-passage failure of hysteretic systems as it is able to reduce the system dimension while retaining the essential behavior. The first-passage failure of hysteretic systems described by Bouc-Wen model has been investigated through stochastic averaging[17]. Research on the first-passage failure of Preisach hysteretic systems is of great significance for the reliability evaluation of devices and structures including non-traditional materials and smart materials owing to the rationality and accuracy of the Preisach model. The complexity of the Preisach hysteretic model, however, limits the corresponding research on Preisach systems.

In this manuscript, the first-passage failure of Preisach hysteretic systems with non-local memory under stationary Gaussian excitation is investigated through the generalized harmonic balance technique and stochastic averaging of energy envelope. First, an equivalent non-hysteretic nonlinear system with amplitude- envelope-dependent damping and stiffness coefficients is obtained by using the generalized harmonic balance technique. Equivalent damping and stiffness coefficients are expressed as functions of system total energy by establishing the relation between amplitude envelope and system total energy. Then, the averaged Itô stochastic differential equation with respect to system total energy, which is a one-dimensional diffusion process, is derived. Establishing and solving the related backward Kolmogorov equation yields the conditional reliability function and the conditional probability density of first-passage time. The approximate solutions are validated by Monte Carlo simulation.

II. PREISACH HYSTERETIC MODEL

The Preisach hysteretic model is expressed in terms of the following integral[7,18]:

$$f(t) = \int\int_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha\beta}(x(t))\, \mathrm{d}\alpha \mathrm{d}\beta \tag{1}$$

where $x(t)$ and $f(t)$ denote the displacement and hysteretic restoring force, respectively. $\mu(\alpha, \beta)$ is a weighting function, called Preisach function, with support on a limiting triangle D of the (α, β)-plane with line $\alpha = \beta$ as the hypotenuse and point $(\alpha_P, \beta_P = -\alpha_P)$ as the vertex. The triangle D in the half-plane $\alpha \geq \beta$ is called Preisach plane as shown in Fig.1. $\mu(\alpha, \beta)$ is equal to zero outside the triangle D. $\gamma_{\alpha\beta}(x)$ is the relay hysteretic operator as shown in Fig.2. It is a two-position relay with only two values $+1$ and -1 corresponding to 'up' and 'down' positions, respectively, and is given by the equation[15,16]

$$\gamma_{\alpha\beta}(x) = \begin{cases} +1 & x > \alpha \quad \text{or} \quad x > \beta \quad \text{and} \quad \text{decreasing} \\ -1 & x < \beta \quad \text{or} \quad x < \alpha \quad \text{and} \quad \text{increasing} \end{cases} \tag{2}$$

The Preisach model, expressed as the superposition of a continuous family of elementary rectangular loops, can be interpreted in terms of the spectral decomposition of a complicated hysteretic constitutive

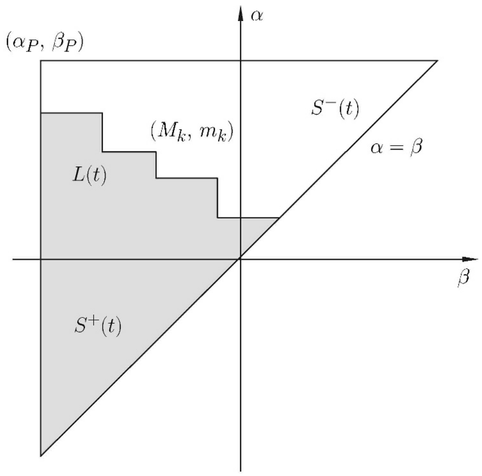

Fig. 1. Preisach plane.

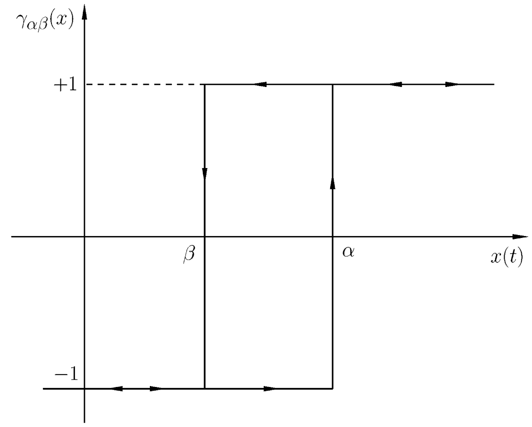

Fig. 2. Relay hysteretic operator.

law that has nonlocal memory in the simplest relay hysteretic operator $\gamma_{\alpha\beta}\,(x)$ with local memory. The Preisach hysteretic behavior is completely characterized by the weighting function $\mu\,(\alpha,\,\beta)$. For an arbitrary input $x(t)$, the hysteretic restoring force $f(t)$ can be determined by the weighting function in conjunction with a staircase line $L(t)$, as shown in Fig.1. And the staircase line divides the Preisach plane into two parts, the domain $S^+\,(t)$ encompassing the set of relay in the $+1$ status, and the domain $S^-\,(t)$ encompassing the set of relay in the -1 status. Each vertex of the staircase line is associated with dominant maxima M_k and minima m_k of the displacement $x(t)$. And the nonlocal selective memory is stored in these vertexes. The hysteretic restoring force $f(t)$ can be equivalently expressed as[15, 16]

$$f\,(t) = \int\int_{S^+(t)} \mu\,(\alpha,\beta)\mathrm{d}\alpha\mathrm{d}\beta - \int\int_{S^-(t)} \mu\,(\alpha,\beta)\mathrm{d}\alpha\mathrm{d}\beta \tag{3}$$

The Preisach function can be determined through the first-order transition curves[7], and for certain classical rheological models the closed form of Preisach function has been derived through this procedure[4]. For the Iwan-Jenkins model, it can be expressed as

$$\mu\,(\alpha,\beta) = \frac{k_J}{2}\left\{\delta\,(\alpha-\beta) - \frac{k_J}{2}\frac{1}{f_{y,\max}-f_{y,\min}}\left[H\left(\alpha-\beta-2\frac{f_{y,\min}}{k_J}\right) - H\left(\alpha-\beta-2\frac{f_{y,\max}}{k_J}\right)\right]\right\} \tag{4}$$

where $\delta\,(\cdot)$ and $H\,(\cdot)$ are the Dirac delta and the Heaviside functions, respectively; k_J represents the linear stiffness of the individual Jenkins element; f_y is the yielding force, and $f_{y,\min} \leq f_y \leq f_{y,\max}$. The weighting function is defined in the domain A shown in Fig.3, and the corresponding hysteretic restoring force can be obtained as follows:

$$f\,(t) = \frac{k_J}{2}\left[\int_{-\alpha_P}^{\alpha_P} \gamma_{\alpha,\alpha}\,(x)\,\mathrm{d}\alpha - \frac{k_J}{2}\frac{1}{f_{y,\max}-f_{y,\min}}\int\int_A \gamma_{\alpha\beta}\,(x)\mathrm{d}\alpha\mathrm{d}\beta\right] \tag{5}$$

Combining Eq.(5) and Eq.(3) yields the following formula of the restoring force

$$f\,(t) = k_J x\,(t) - \frac{k_J^2}{4}\frac{1}{f_{y,\max}-f_{y,\min}}\left(2\int\int_{S_A^+(t)} \mathrm{d}\alpha\mathrm{d}\beta - \int\int_A \mathrm{d}\alpha\mathrm{d}\beta\right) \tag{6}$$

By setting $\alpha_P = -\beta_P = f_{y,\max}/k_J$[15, 16] and $f_{y,\min} = 0$, then the shaded area A is a triangle defined by $\alpha = \alpha_P$, $\beta = \beta_P$ and $\alpha - \beta = 0$. According to Lubarda et al.[4], the domain A is initially divided into two equal parts and thus $f\,(0) = 0$. Introduce a new function $F\,(\alpha_i,\,\beta_i) = (\beta_j - \alpha_j)^2/2$ which represents the area of the triangle within the equations $\alpha = \alpha_i$, $\beta = \beta_i$ and $\alpha - \beta = 0$. The hysteretic

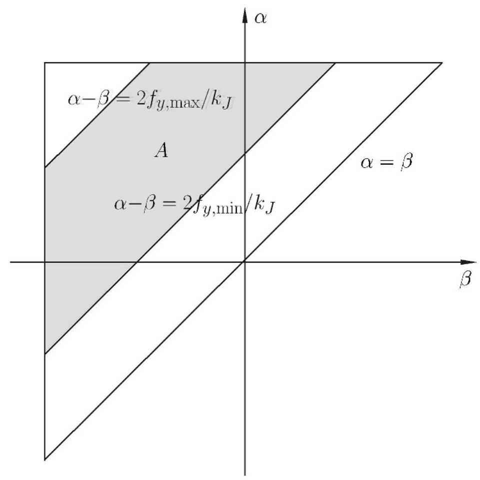

Fig. 3. Domain of the weighing function for the Iwan-Jenkins model.

restoring force for the ascending status is given by[15,16]

$$
\begin{aligned}
f(t) = \ & k_J x(t) - \frac{k_J^2}{4}\frac{1}{f_{y,\max}}\{4[F(0,\beta_P) - F(0,\beta_1)] \\
& +2H[x(t) - \beta_{n-1}]F(x(t),\beta_{n-1}) + 2\sum_{j=2}^{n-1}[F(\alpha_j,\beta_{j-1}) - F(\alpha_j,\beta_j)] - F(\alpha_P,\beta_P)\}
\end{aligned}
\tag{7}
$$

and that for the descending status is described as

$$
\begin{aligned}
f(t) = \ & k_J x(t) - \frac{k_J^2}{4}\frac{1}{f_{y,\max}}\{4[F(0,\beta_P) - F(0,\beta_1)] + 2F(\alpha_n,\beta_{n-1}) \\
& -2H[\alpha_n - x(t)]F(\alpha_n,x(t)) + 2\sum_{j=2}^{n-1}[F(\alpha_j,\beta_{j-1}) - F(\alpha_j,\beta_j)] - F(\alpha_P,\beta_P)\}
\end{aligned}
\tag{8}
$$

In Eqs.(7) and (8), α_i, β_i are dominant maxima and minima of the displacement $x(t)$, respectively. Summation terms in these two equations show the nonlocal memory property of Preisach model.

III. EQUIVALENT NON-HYSTERETIC NONLINEAR STOCHASTIC SYSTEM

Consider a single-degree-of-freedom (SDOF) nonlinear hysteretic system with the hysteretic behavior described by Preisach model. The motion of the system is governed by the following equation:

$$
m\ddot{x}(t) + c\dot{x}(t) + kx(t) + f(t) = w(t) \tag{9}
$$

where $x(t)$ is the system displacement; m, c, and k represent mass, damping and stiffness coefficients, respectively; $w(t)$ is zero-mean Gaussian white noise with spectral density S_w; $f(t)$ denotes the nonlinear hysteretic restoring force governed by the Preisach model Eqs.(7) and (8).

The Preisach hysteretic restoring force contains nonlinear elastic restoring components and nonlinear dissipative components that are intensively coupled. According to the generalized harmonic balance technique, these components can be decoupled and the original Eq.(9) can be replaced by the following equivalent nonlinear non-hysteretic equation (for details, please refer to Refs.[15] and [16]):

$$
\ddot{y}(t) + [2\zeta\bar{\omega} + \psi C(a)]\dot{y}(t) + \left[\bar{\omega}^2 + \psi D(a)\right]y(t) = \hat{w}(t) \tag{10}
$$

where, $\bar{\omega} = \omega_J\sqrt{1+\varphi}$, $\varphi = k/k_J$, $\omega_J = \sqrt{k_J/m}$, $2\zeta\bar{\omega} = c/m$, $y = x\sqrt{2\zeta\bar{\omega}^3 m^2}/\sqrt{\pi S_w}$, $\psi = 2\bar{\omega}^2\sqrt{\pi S_w}/(f_{y,\max}^*\sqrt{2\zeta\bar{\omega}^3})$, $\hat{w}(t) = w(t)\sqrt{2\zeta\bar{\omega}^3}/\sqrt{\pi S_w}$ with $S_{\hat{w}(t)} = 2\zeta\bar{\omega}^3/\pi$. The amplitude-dependent coefficients yielded by the generalized harmonic balance technique are expressed as[15,16]

$$C(a)=\frac{a\bar{\omega}^2}{3\pi\left(1+\varphi\right)^2\omega\left(a\right)} \tag{11}$$

$$D(a)=-\frac{a\bar{\omega}^2}{4(1+\varphi)^2} \tag{12}$$

Equation (10) implies that the response is governed by an equivalent nonlinear non-hysteretic equation with amplitude-dependent damping and stiffness coefficients.

The energy envelope of the system can be expressed as

$$H=\frac{1}{2}\dot{y}^2+\frac{1}{2}\left[\bar{\omega}^2-\psi\frac{a\bar{\omega}^2}{4\left(1+\varphi\right)^2}\right]y^2 \tag{13}$$

So the relation of displacement amplitude a to system energy H, $a=a(H)$, can be obtained by analytically solving the cubic equation $H=\left\{\bar{\omega}^2-\psi a\bar{\omega}^2/[4(1+\varphi)^2]\right\}a^2/2$. Thus, Eq.(10) can be rewritten in the following form:

$$\ddot{y}\left(t\right)+\left[2\zeta\bar{\omega}+\psi\bar{C}\left(H\right)\right]\dot{y}\left(t\right)+\left[\bar{\omega}^2+\psi\bar{D}\left(H\right)\right]y\left(t\right)=\hat{w}\left(t\right) \tag{14}$$

in which, $\bar{C}\left(H\right)=C\left(a\left(H\right)\right)$, $\bar{D}\left(H\right)=D\left(a\left(H\right)\right)$. So far, the original hysteretic Eq.(9) is equivalently replaced by a nonlinear non-hysteretic Eq.(14) with energy-dependent damping and stiffness coefficients, and it is this equivalent replacement that paves the way for the adoption of stochastic averaging of energy envelope.

IV. FIRST-PASSAGE FAILURE

In order to use the stochastic averaging of energy envelope[19], the nonlinear non-hysteretic Eq.(14) are rearranged as state equations

$$\dot{Y}_1=Y_2 \tag{15a}$$

$$\dot{Y}_2=-\left[2\zeta\bar{\omega}+\psi\bar{C}\left(H\right)\right]Y_2-\left[\bar{\omega}^2+\psi\bar{D}\left(H\right)\right]Y_1+\hat{w}(t) \tag{15b}$$

where $Y_1=y$, $Y_2=\dot{y}$ are the system displacement and velocity, respectively, the intensity of excitation $\hat{w}(t)$ is denoted as $2D$. The associated Itô stochastic differential equation is

$$\mathrm{d}Y_1=Y_2\mathrm{d}t \tag{16a}$$

$$\mathrm{d}Y_2=\left\{-\left[2\zeta\bar{\omega}+\psi\bar{C}\left(H\right)\right]Y_2-\left[\bar{\omega}^2+\psi\bar{D}\left(H\right)\right]Y_1\right\}\mathrm{d}t+\sqrt{2D}\mathrm{d}W(t) \tag{16b}$$

in which $W(t)$ is the unit Wiener process.

The system energy is expressed as

$$H=\frac{1}{2}Y_2^2+\frac{1}{2}\left[\bar{\omega}^2+\psi\bar{D}\left(H\right)\right]Y_1^2=H\left(Y_1,Y_2\right) \tag{17}$$

By introducing the following transformation:

$$Y_1=Y_1,\quad H=H\left(Y_1,Y_2\right) \tag{18}$$

Eq.(16) can be transformed into the Itô equations with respect to displacement Y_1 and the energy envelope H via the Itô differential rule

$$\mathrm{d}Y_1=\pm\sqrt{2H-\left[\bar{\omega}^2+\psi\bar{D}\left(H\right)\right]Y_1^2}\mathrm{d}t \tag{19a}$$

$$\begin{aligned}\mathrm{d}H=&\left\{-\left[2\zeta\bar{\omega}+\psi\bar{C}(H)\right]\frac{2H-\left[\bar{\omega}^2+\psi\bar{D}(H)\right]Y_1^2}{1-\psi\bar{D}'Y_1^2/2}+\frac{D}{1-\psi\bar{D}'Y_1^2/2}\right.\\&\left.+\left(\frac{\psi\bar{D}'Y_1^2}{2}\right)^{-3}\frac{D}{2}\psi\bar{D}''Y_1^2\left[2H-\left[\bar{\omega}^2+\psi\bar{D}\left(H\right)\right]Y_1^2\right]\right\}\mathrm{d}t\\&+\sqrt{2D}\frac{\pm\sqrt{2H-\left[\bar{\omega}^2+\psi\bar{D}\left(H\right)\right]Y_1^2}}{1-\psi\bar{D}'Y_1^2/2}\mathrm{d}W(t)\end{aligned} \tag{19b}$$

where, the prime means the derivative with respect to system energy H. For the case in which the difference between energy input by excitation and energy dissipated by damping in one vibrating period is smaller than total system energy, system energy H is a slowly varying process replaceable in the first approximation by a one-dimensional diffusion process. The same symbol H is used to denote this diffusion process for simplicity. The Itô differential equation for this diffusion process is obtained through time averaging to Eq.(19b). Making use of Eq.(19a), time averaging can be replaced by space averaging with respect to system displacement Y_1 under the condition that H remains constant. Then,

$$\mathrm{d}H = U\left(H\right)\mathrm{d}t + V\left(H\right)\mathrm{d}W\left(t\right) \tag{20}$$

where,

$$T\left(H\right) = 2\int_{-a(H)}^{a(H)} \frac{\mathrm{d}Y_1}{\sqrt{2H - \left[\bar{\omega}^2 + \psi\bar{D}\left(H\right)\right]Y_1^2}} \tag{21}$$

$$\begin{aligned} U\left(H\right) = & \frac{1}{T\left(H\right)} 2\int_{a(H)}^{a(H)} \left\{ -\frac{\left[2\zeta\bar{\omega} + \psi\bar{C}\left(H\right)\right]\sqrt{2H - \left[\bar{\omega}^2 + \psi\bar{D}\left(H\right)\right]Y_1^2}}{1 - \psi\bar{D}'Y_1^2/2} \right. \\ & + \frac{D}{\left(1 - \psi\bar{D}'Y_1^2/2\right)\sqrt{2H - \left[\bar{\omega}^2 + \psi\bar{D}\left(H\right)\right]Y_1^2}} \\ & \left. + \frac{D}{2}\frac{\psi\bar{D}''Y_1^2\sqrt{2H - \left[\bar{\omega}^2 + \psi\bar{D}\left(H\right)\right]Y_1^2}}{\left(1 - \psi\bar{D}'Y_1^2/2\right)^3} \right\}\mathrm{d}Y_1 \end{aligned} \tag{22}$$

$$V^2\left(H\right) = \frac{4D}{T\left(H\right)}\int_{-a(H)}^{a(H)} \frac{\sqrt{2H - \left[\bar{\omega}^2 + \psi\bar{D}\left(H\right)\right]Y_1^2}}{\left(1 - \psi\bar{D}'Y_1^2/2\right)^2}\mathrm{d}Y_1 \tag{23}$$

$T(H)$ is the energy-dependent period of system (14) and can be used to determine the circular frequency through the formula $\omega\left(a\left(H\right)\right) = 2\pi/T\left(H\right)$. Once the circular frequency $\omega\left(a\right)$ is expressed through system energy, $\bar{C}\left(H\right)$ and then the drift coefficient $U(H)$ and diffusive coefficient $V^2(H)$ can be derived through Eq.(11) and Eqs.(21)-(23).

So far, the total energy of Preisach hysteretic system has been proved to be a one-dimensional diffusion process and the Itô stochastic differential equation describing this approximate process has also been derived. Based on these conclusions, we will investigate the first-passage failure of Preisach hysteretic system through system energy rather than system state such as displacement and velocity as suggested by conventional opinions.

First, the safety domain Ω_s is chosen via engineering practice, and in one-dimensional system the safety domain is always a given close interval $0 \le H(t) \le H_c$. The conditional reliability function $R\left(t\,|H_0\right)$ is defined as the probability of system energy $H\left(t\right)$ in the safety domain Ω_s within time interval $(0,\ t]$, in which $H\left(0\right) = H_0$ is a given initial energy in the safety domain. The mathematical description of the conditional reliability function is $R\left(t\,|H_0\right) = P\left\{H\left(\tau\right) \in \Omega_s,\ \tau \in (0,\ t]\,|H_0 \in \Omega_s\right\}$.

The governing equation for the conditional reliability function is the well-known backward Kolmogorov equation[20] expressed as follows:

$$\frac{\partial R\left(t|H_0\right)}{\partial t} = U\left(H_0\right)\frac{\partial R\left(t|H_0\right)}{\partial H_0} + \frac{V^2\left(H_0\right)}{2}\frac{\partial^2 R\left(t|H_0\right)}{\partial H_0^2} \tag{24}$$

where $U\left(H_0\right)$ and $V^2\left(H_0\right)$ is similar to the drift coefficient and diffusive coefficient in Eqs.(22) and (23) with H replaced by H_0. The initial condition of backward Kolmogorov Eq.(24) is

$$R\left(0|H_0\right) = 1, \quad H_0 \in \Omega_s \tag{25}$$

and the boundary conditions are

$$\begin{aligned} & R\left(t|H_0\right) = 0, \quad H_0 = H_c \\ & R\left(t|H_0\right) = \text{finity}, \quad H_0 = 0 \end{aligned} \tag{26}$$

The governing Eq.(24) as well as associated initial condition (25) and boundary conditions (26) constitute the complete description with respect to the conditional reliability function. Solving these equations by using numerical technique gives the conditional reliability function.

The conditional probability density of the first-passage time T can be derived from the conditional reliability function as follows:

$$p\left(T|H_0\right) = -\left.\frac{\partial R}{\partial t}\right|_{t=T} \tag{27}$$

Similarly, the mean first-passage time (MFPT) $\mu_1\left(H_0\right)$ of system (20), defined as $\mu_1\left(H_0\right) = \int_0^\infty Tp\left(T|H_0\right)\mathrm{d}T$ can be obtained by solving the following Pontryagin equation:

$$U\left(H_0\right)\frac{\partial\mu_1\left(H_0\right)}{\partial H_0} + \frac{V^2\left(H_0\right)}{2}\frac{\partial^2\mu_1\left(H_0\right)}{\partial H_0^2} = -1 \tag{28}$$

with boundary conditions

$$\mu_1\left(H_c\right) = 0 \tag{29a}$$

$$\mu_1\left(0\right) = \text{finite} \tag{29b}$$

where $U\left(H_0\right)$ and $V^2\left(H_0\right)$ have been illustrated in Eq.(24). Solving Eqs.(28), (29a) and (29b) by using numerical technique gives the mean first-passage time (MFPT) $\mu_1\left(H_0\right)$.

Then, the variance of first-passage time (VFPT) and arbitrary statistic quantity can be directly derived from $p\left(T|H_0\right)$ through integral calculus

$$\mathrm{VFPT}\left(H_0\right) = \int_0^\infty \left[t - \mu_1\left(H_0\right)\right]^2 p\left(t|H_0\right)\mathrm{d}t \tag{30}$$

The MFPT $\mu_1\left(H_0\right)$ describes the mean of failure time, while the VFPT scales the dispersive extent of the failure time. These two indexes give a detailed description of the first-passage failure.

V. NUMERICAL RESULTS AND DISCUSSION

Using the proposed procedure in the last section, the conditional reliability function and the conditional probability density of the first-passage time are calculated numerically. The consistency of the conditional reliability function and the conditional probability density yielded by the present method (denoted by curves) and the Monte Carlo simulation (denoted by symbols •, ▼ and ⋆) proves the effectiveness of the proposed procedure, as shown in each figure. System parameters are selected as $\zeta = 0.01$, $\bar{\omega} = 0.5$ and excitation intensity as $D = 0.2$. The energy boundary is set as $H_c = 2.5$ and the initial energy is $H_0 = 0.0$. The other parameter values used in calculation, such as φ and ψ, are given in figure captions. The numerical results concerning the conditional reliability function $R\left(t|H_0\right)$ and conditional probability density of the first-passage time T for different ψ values and different φ values are shown in Figs.4 and 5, respectively. Larger ψ value represents stronger nonlinearity influence since parameter ψ is an additional factor of equivalent damping and stiffness coefficients $C\left(a\right)$ and $D\left(a\right)$, as shown in Eq.(14). Decreasing φ value induces a decrease of linear damping coefficient $2\zeta\bar{\omega}$ and linear stiffness coefficient $\bar{\omega}^2$ but an increase of nonlinear damping coefficient $\psi C\left(a\right)$ and nonlinear stiffness $\psi D\left(a\right)$ (as shown in Eqs.(11)-(12)). So, a smaller φ value is tantamount to a stronger nonlinearity. It is rational that the present results are more accurate for smaller ψ values and larger φ values. Plotted in Figs.6 and 7 are the effects of the initial value H_0 on the MFPT $\mu_1\left(H_0\right)$. It can be seen that as the initial value H_0 increases, the MFPT $\mu_1\left(H_0\right)$ decreases. The influences of the parameters ψ and φ on the MFPT $\mu_1\left(H_0\right)$ are also compared in Figs.4 and 5. It is obvious that the effects of parameters ψ and φ on the MFPT $\mu_1\left(H_0\right)$ are independent of the initial value H_0.

VI. CONCLUSIONS

The first-passage failure of hysteretic Preisach system with nonlocal memory under stationary Gaussian excitation has been investigated. The equivalent non-hysteretic nonlinear system with amplitude-dependent damping and stiffness coefficients is obtained by employing the generalized harmonic balance

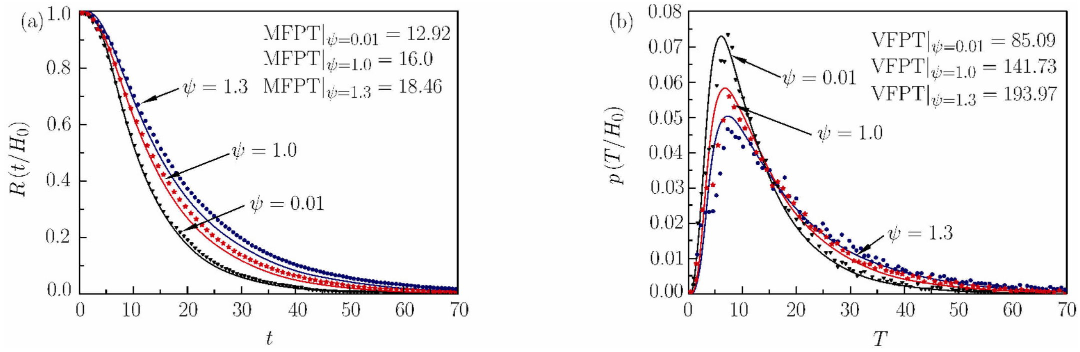

Fig. 4. The conditional reliability function $R(t|H_0)$ and the conditional probability density $p(T|H_0)$of system (9). ($\varphi = 1.0$). (a) Conditional reliability function; (b) Conditional probability density of first-passage time.

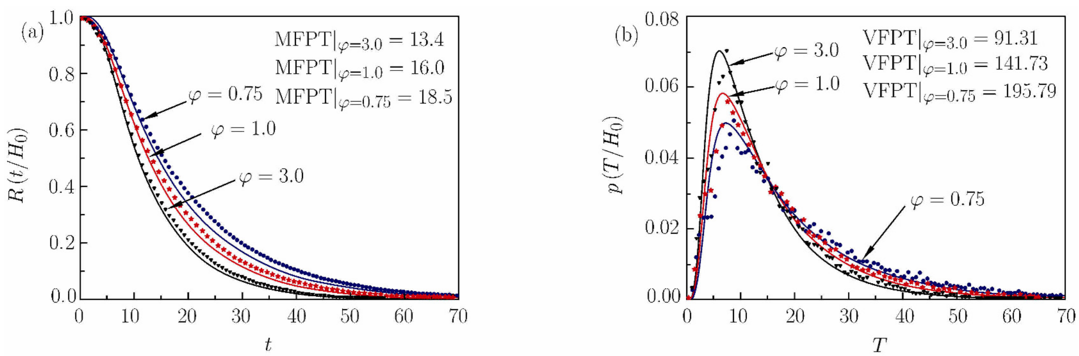

Fig. 5. The conditional reliability function $R(t|H_0)$ and the conditional probability density $p(T|H_0)$ of system (9). ($\psi = 1.0$). (a) Conditional reliability function; (b) Conditional probability density of first-passage time.

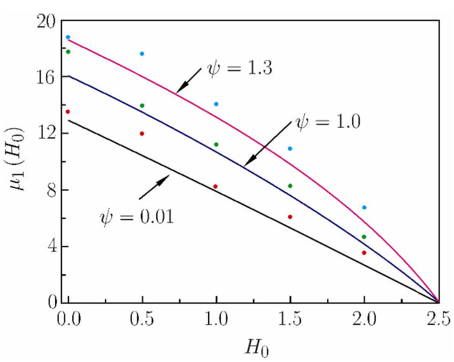

Fig. 6. The dependence of the mean first-passage time $\mu(H_0)$ on the initial value H_0. ($\varphi = 1.0$). Solid lines: the present results; Circles: results from MCS.

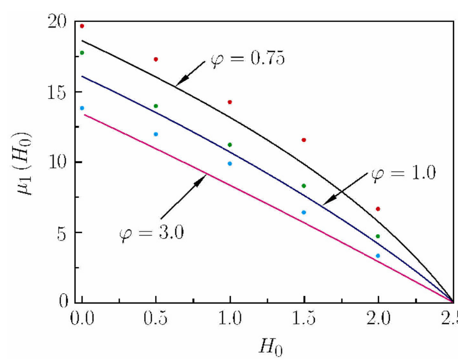

Fig. 7. The dependence of the mean first-passage time $\mu(H_0)$ on the initial value H_0. ($\psi = 1.0$). Solid lines: the present results; Circles: results from MCS.

technique. Then, the relationship between displacement amplitude and system energy is formed with the equivalent damping and stiffness coefficients expressed as functions of system energy. An averaged Itô stochastic differential equation for system energy is derived using stochastic averaging of the energy envelope. The establishing and solving of associated backward Kolmogorov and Pontryagin equations yield the conditional reliability function and mean first-passage time. The effects of hysteresis parameters ψ and φ on the first-passage failure are also investigated.

Revealing the effects of non-dimensional parameters on system reliability, the present results will guide parameter design to enhance the reliability of Preisach hysteretic systems. Another fact that should be mentioned is that traditional reliability analysis is based on the system state such as displacement and velocity, while the present analysis is on the basis of system energy. This is rational since system energy is able to properly evaluate system response and at times more suitable than system state. Different descriptions of system response through system state or system energy can be analogized to strength criterions of material through maximum strain and distortion strain energy.

References

[1] Caughey,T.K., Random excitation of a system with bilinear hysteresis. *ASME Journal of Applied Mechanics*, 1960, 27: 649-652.

[2] Caughey,T.K., Nonlinear theory of random vibrations. *Advances in Applied Mechanics*, 1971, 11: 209-253.

[3] Bouc,R., Forced Vibration of Mechanical Systems with Hysteresis. In: Proceedings of Fourth Conference on Nonlinear Oscillation, Prague, Czechoslovakia, 1967.

[4] Lubarda,V.A., Sumarac,D. and Krajcinovic,D., Preisach model and hysteretic behaviour of ductile materials. *European Journal of Mechanics Series A Solids*, 1993, 12 (4): 445-470.

[5] Ying,Z.G, Zhu,W.Q., Ni,Y.Q. and Ko,J.M., Random response of Duhem hysteretic systems. *Journal Sound Vibration*, 2002, 254: 91-104.

[6] Preisach,F., Uber die magnetische nachwirkung. *Zeitschrift für Physik*, 1935, 94: 277-302.

[7] Mayergoyz,I.D., Mathematical Models of Hysteresis. New York: Springer, 1991.

[8] Simiu,E. and Scanlan,R.H., Wind Effects on Structures: an Introduction to Wind Engineering. New York: John Wiley & Sons, 1986.

[9] Anagnos,T. and Kiremidjian,A.S., A review of earthquake occurrence models for seismic hazard analysis. *Probabilistic Engineering Mechanics*, 1988, 3: 3-11.

[10] Caughey,T.K., Equivalent linearization techniques. *The Journal of the Acoustical Society of America*, 1963, 35: 1706-1711.

[11] Zhu,W.Q. and Yang,Y.Q., Stochastic averaging of quasi non-integrable- Hamiltonian systems. *ASME Journal of Applied Mechanics*, 1997, 64: 157-164.

[12] Khasminskii,R.Z., A limit theorem for the solutions of differential equations with random right-hand sides. *Theory of Probability & Its Applications*, 1966, 11: 390-405.

[13] Ni,Y.Q., Ying,Z.G. and Ko,J.M., Random response analysis of Preisach hysteretic systems with symmetric weight distribution. *Journal of Applied Mechanics*, 2002, 69: 171-178.

[14] Ying,Z.G., Zhu,W.Q., Ni,Y.Q. and Ko,J.M., Random response of Preisach hysteretic systems. *Journal of Sound and Vibration*, 2002, 254(1): 37-49.

[15] Spanos,P.D., Cacciola,P. and Muscolino,G., Stochastic averaging of Preisach hysteretic systems. *Journal of Engineering Mechanics*, 2004, 130(11): 1257-1267.

[16] Wang,Y., Ying,Z.G. and Zhu,W.Q., Stochastic averaging of energy envelope of Preisach hysteretic systems. *Journal of Sound and Vibration*, 2009, 321: 976-993.

[17] Noori,M., First-passage study and stationary response analysis of a BWB hysteresis model using quasi-conservative stochastic averaging method. *Probabilistic Engineering Mechanics*, 1995, 10: 161-170.

[18] Krasnoselski,M.A. and Pokrovskii,A.V., Systems with Hysteresis. Berlin: Springer, 1989.

[19] Zhu,W.Q. and Lin,Y.K., Stochastic averaging of energy envelope. *ASCE Journal of Engineering Mechanics*, 1991, 117: 1890-1905.

[20] Gan,C.B. and Zhu,W.Q., First-passage failure of quasi-non-integrable-Hamiltonian systems. *International Journal of Non-linear Mechanics*, 2001, 36: 209-220.