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RESEARCH ARTICLE

Cross-Efficiency Evaluation Method with Performance Level as a Management Objective in Consideration of Bounded Rationality

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Abstract

According to management by objectives (MBO) theory, the significance of management objectives must be considered as a reference point in a performance evaluation. Cross efficiency evaluation has always been considered to be one of the important performance evaluation methods. However, few studies to date have considered the impact of management objectives on cross efficiency. According to prospect theory, the choice of reference point will cause irrational psychology in decision makers. A management objective is a natural reference point, which will cause a 'gain and loss' psychology in enterprises and may create irrational psychology. Performance level is an important index by which to evaluate resource allocation, which in turn can be regarded as an important enterprise management objective. This paper proposes a cross efficiency evaluation method based on performance level. Cross efficiency evaluation models are constructed, based on the irrational psychology that occurs under organization objectives, personal objectives and composite objectives. This method not only considers the bounded rational behavior of enterprises, but is also more flexible. A numerical example is given to illustrate the application of the bounded rational cross efficiency evaluation method in data envelopment analysis (DEA) ranking.

Keywords Data envelopment analysis (DEA) · Cross efficiency evaluation · Prospect theory · Bounded rationality · Performance level

1 Introduction

Data envelopment analysis (DEA), first proposed by Charnes et al. [1], is a non-parametric method used to evaluate decision-making unit (DMU) performance based on input and output data. In this method, the weights of the input and output indexes do not need to be given subjectively; they are objectively obtained through model optimization. The advantages of the DEA method have been generally recognized by scholars [9, 11, 20]. A traditional DEA model, such as the CCR model, will have multiple solutions of weights, leading to the problem of the full ranking of DMU performance. In this regard, Sexton et al. [15] proposed cross-efficiency. The study regarded CCR efficiency as the decision-making unit based on self-evaluation efficiency, and achieved peer evaluation by introducing secondary objectives to optimize input and output weights. Doyle and Green [7] further proposed aggressive and benevolent cross-efficiency methods under three secondary objectives. As a performance evaluation tool, cross-efficiency evaluation has been widely studied and applied worldwide [5, 6]

Cross-efficiency evaluation is, in fact, a relative efficiency evaluation method with reference points that may be either efficient or inefficient frontiers [8, 12, 17, 21], or a DMU, such as ideal DMU (IDMU) or anti-ideal DMU (ADMU) [13, 18, 19], Shi et al., 2020), or interval reference points, which take the best production state and the worst production state as reference states (Huang, et al., [10]). In performance evaluation, management objectives serve as an important measure for the degree of achievement of decision-making units. As a significant performance evaluation method, the cross-evaluation method should consider management objectives as a reference point. Many existing

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studies have also considered the reference point in cross-evaluation, but few have taken management objectives as the reference point.

According to management by objectives (MBO) as proposed by Drucker, management objectives are diverse, but regardless of the type of management objectives, they should abide by the SMART (specific, measurable, achievable, relevant and time-bound) principle. To be clear, “SMART” means that management objectives must be specific (this feature requires management objectives to be clear and easy to understand), measurable (this feature requires that the reference standards used in the evaluation are consistent with the management objectives), attainable (this feature refers to management objectives not being too low or too high, and that the objectives can be achieved through hard work), relevant (this feature requires that the sub-objectives are related to the enterprise’s overall objectives) and time-based (this feature requires that the completion of the objective is time-limited). The reference points in existing research do not meet the principles of management objectives. For example, a positive IDMU or an ADMU is used as a reference point; the point will vary with the changes of members of the evaluated DMUs, and can only reflect the best and worst states of the evaluation criteria. Obviously, this type of reference point is not clear, measurable and achievable; the efficient and inefficient frontiers also have these limitations. This study proposes to use the relevant performance level as a reference point. The performance level is defined according to the application requirements (see definition), and is not limited to the optimal performance level. Thus, this approach conforms to the SMART principle of management objectives.

The theory of MBO was further developed by Odiorne in [14], and holds that management objectives should not only focus on the relationship between performance evaluation and management objectives, but should also consider the preferences (for example, attitude, rationality, irrationality, etc.) of decision makers. Prospect theory was put forward by Kahneman and Tversky in [14], and maintains that people will have bounded rational psychology when facing a reference point. When the performance is better than the reference point, the degree of excellence will be underestimated. Conversely, when the performance is worse than the reference point, the degree of poor performance will be enlarged (Shi et al., 2020). Judgments of value vary due to changes in the reference points; similarly, performance will vary due to changes in management objectives. Therefore, performance evaluation should not only consider the significance of management objectives as reference points, but also consider the bounded rationality of the evaluated object when facing management objectives.

In conclusion, traditional cross-efficiency evaluation methods often use specific DMU or virtual DMU as the reference point for performance evaluation, rarely consider the role and significance of performance level as the reference point, and ignore the influence of bounded rationality on the behavior of decision-making units. In reality, decision makers are often not entirely rational, and their decision-making processes are influenced by cognitive limitations, information asymmetry, and other factors. Therefore, it is essential to consider the bounded rationality of decision makers and adopt performance level as the management objective in cross-efficiency evaluation methods, which holds significant theoretical and practical value.

Performance evaluation is the ranking of evaluated DMUs based on performance level. High level performance is the pursuit of all DMUs, so a high performance level is a natural management objective. The performance level value under the DEA method is between 0 and 1, and the value can be solved by a DEA model. When the performance of the DMU is good enough, the value can reach the highest level of 1, which conforms to the three principles of specificity, measurability and reachability. Obviously, for the cross efficiency evaluation model, the performance level is the ideal management objective. This paper considers the bounded rationality psychology of DMUs when performance level is taken as the management objective, constructs a cross efficiency model, realizes the full ranking of each DMU, and provides effective technical means for decision-makers to make scientific decisions.

2 Theory Premise

2.1 Prospect Theory

Prospect theory is a theory describing the decision-making process of decision-makers in the face of risk. This is inconsistent with traditional expectation value theory and expected utility theory. Risk preference on the part of decision-makers is inconsistent in the face of loss and profit; they become risk-seeking in the face of loss, while risk-averse in the face of profit. The choice and change of a reference point will affect the decision-maker’s perception of revenue and loss, and then affect the decision-making results. The value function of prospect theory is as follows:

$$v(\Delta z) = \begin{cases} \Delta z^{\alpha} \Delta z \geq 0 \\ -\lambda(-\Delta z)^{\beta} \Delta z < 0 \end{cases} \tag{1}$$

where

$$\Delta z = \begin{cases} u_0 - u \text{ when } u \text{ is cost} \\ u - u_0 \text{ when } u \text{ is profit} \end{cases}$$

when $\Delta z \geq 0$, there is a gain; when $\Delta z < 0$, there is a loss; α and β represent the concavity and convexity of the regional value function of return and loss, respectively. Loss aversion coefficient $\lambda \geq 1$ indicates that the loss area is steeper than the income area. In this theory, there is an inflection point in the value function; that is, there is a so-called "reference point".

As shown in Fig. 1, the reference point is the inflection point of the value curve. The value curve is divided into the profit area and loss area. Obviously, the loss curve is steeper than the income curve, which shows that decision makers are more sensitive to loss than income. On the other hand, the outlook value curve is concave in the income domain and convex in the loss domain. This reflects the decision makers' risk aversion tendency to the income and their risk seeking tendency to the loss.

2.2 Efficiency Evaluation

2.2.1 Self-Evaluation

Suppose there are n DMUs to be evaluated with m inputs and s outputs. Denote by $x_{ij}(i = 1, \ldots, s)$ and $y_{rj}(r = 1, \ldots, s)$ the input and output values of DMU_j $(j = 1, \ldots, n)$. Consider a DMU, say, DMU_k, $k \in \{1, \ldots, n\}$, whose efficiency relative to the other DMUs can be measured by the following CCR model [1]:

$$\begin{aligned} &\text{Maximize } \theta_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk} \\ &\text{Subject to } \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ &\sum_{r=1}^{s} u_{rk} y_{rk} - \sum_{i=1}^{m} v_{ik} x_{ik} \leq 0, \quad j = 1, \ldots, n, \\ &u_{rk} \geq \varepsilon, \ r = 1, \ldots, s, \\ &v_{ik} \geq \varepsilon, \ i = 1, \ldots, m. \end{aligned} \tag{2}$$

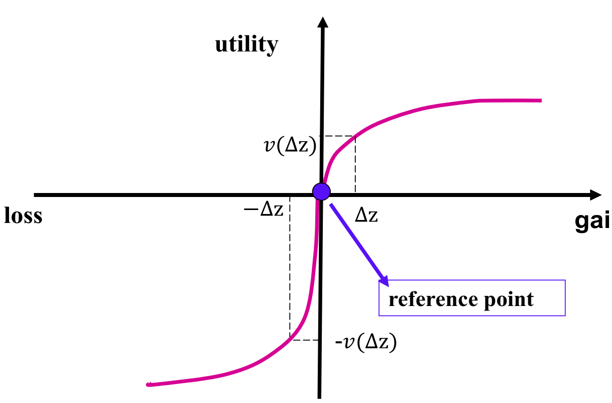


Fig. 1 Value function curve

where ε is an infinitesimal. Let u^*_{rk} $(r = 1, \ldots, s)$ and v^*_{ik} $(r = 1, \ldots, m)$ be the optimal solution to Model (2). Then, $\theta^*_{kk} = \sum_{r=1}^{s} u^*_{rk} y_{rk}$ is referred to as the CCR-efficiency of DMU_k, which also reflects the self-evaluated efficiency of DMU_k. As such, $\theta_{jk} = \sum_{r=1}^{s} u^*_{rk} y_{rj} / \sum_{i=1}^{m} v^*_{ik} x_{ik}$ is referred to as a cross-efficiency of DMU_j and reflects the peer evaluation of DMU_k to DMU_j $(j = 1, ..., n; j \neq k)$.

2.2.2 Peer Evaluation

Scholars have proposed a variety of cross-evaluation models for different application problems, considering either the preference or the psychology of DMUs (Chen, [2–4]). However, few studies to date have considered the irrational psychology of DMUs. A study by Liu et al. [13] proposed one of the few cross-evaluation methods that consider the irrational psychology of DMUs. The cross-evaluation method considers the risk aversion of decision makers with the best and worst performing decision units as reference points. The best performing decision unit (IDMU) and the worst performing decision unit (ADMU) are defined, as can be seen in Definition 1 and Definition 2. Then, cross-evaluation models are constructed, taking IDMU and ADMU as reference points, and regarding both IDMU and ADMU as reference points.

Definition 1: (IDMU): Assume a virtual DMU is the ideal DMU, which can use the least input $x_i^{\min}$ to generate the most outputs $y_r^{\max}$; where $x_i^{\min} = \min_j \{x_{ij}\}, j = 1, 2, ..., n$, $y_r^{\max} = \max_j \{y_{rj}\}, j = 1, 2, ..., n$, denoted by IDMU.

Definition 2: (ADMU)**:** Assume a virtual DMU is the anti-ideal DMU, which would use the most input $x_i^{\max}$ to generate the least outputs $y_r^{\min}$; where $x_i^{\max} = \max_j \{x_{ij}\}, i = 1, 2, ..., n$, $y_r^{\min} = \min_j \{y_{ij}\}, i = 1, 2, ..., n$, denoted by ADMU.

Shi et al. [16] further expanded ADMU and IDMU into interval DMU, which is defined in Definition 3.

Definition 3: Assume that there is a production possibility set which can use m interval inputs $\left[a_i x_i^{\min}, b_i x_i^{\max}\right]$ $(i = 1, 2, ..., m)$, to generate s interval outputs $\left[c_r y_r^{\min}, d_r y_r^{\max}\right]$ $(r = 1, 2, ..., s)$, where $a = \left(a_1, a_2, ..., a_m\right)$, $b = \left(b_1, b_2, ..., b_m\right)$, $c = \left(c_1, c_2, ..., c_s\right)$ and $d = \left(d_1, d_2, ..., d_s\right)$ are all vectors, and $1 \leq a_i \leq \frac{x_i^{\max}}{x_i^{\min}}$, $\frac{x_i^{\min}}{x_i^{\max}} \leq b_i \leq 1, i = 1, 2, ..., m$, $1 \leq c_r \leq \frac{y_r^{\max}}{y_r^{\min}}$, $\frac{y_r^{\min}}{y_r^{\max}} \leq d_r \leq 1, r = 1, 2, ..., s$, and $x_i^{\min} > 0$, $x_i^{\max} > 0, y_r^{\max} > 0$, $y_r^{\min} > 0$. This is called an interval-DMU.

The cross-evaluation model taking interval-DMU as a reference point is shown as follows:

$$\text{Maximize } W_k^{\text{inerval}} = \sum_{i=1}^{m} v_{ik} w_{ik}^{\text{interval - in}} + \sum_{r=1}^{s} u_{rk} w_{rk}^{\text{interval - out}}$$
$$\text{Subject to } \sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0$$
$$\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ j = 1, \dots, n,$$
$$u_{rk} \ge \varepsilon\ r = 1, \dots, s,$$
$$v_{ik} \ge \varepsilon\ i = 1, \dots, m. \tag{3}$$

where

$$w_{ik}^{\text{inerval - in}} = \begin{cases} -\lambda\left(-b_i x_i^{\max} + x_{ik}\right)^{\beta} & x_{ik} > b_i x_i^{\max} \\ -\lambda\left(-a_i x_i^{\min} + x_{ik}\right)^{\beta} + \left(b_i x_i^{\max} - x_{ik}\right)^{\alpha} & a_i x_i^{\min} \le x_{ik} \le b_i x_i^{\max} \\ \left(a_i x_i^{\min} - x_{ik}\right)^{\alpha} & x_{ik} < a_i x_i^{\min} \end{cases}$$
$$w_{rk}^{\text{inerval - out}} = \begin{cases} -\lambda\left(-y_{rk} + c_r y_r^{\min}\right)^{\beta} & y_{rk} < c_r y_r^{\min} \\ -\lambda\left(-y_{rk} + d_r y_r^{\max}\right)^{\beta} + \left(y_{rk} - c_r y_r^{\min}\right)^{\alpha} & c_r y_r^{\min} \le y_{rk} \le d_r y_r^{\max} \\ \left(y_{rk} - d_r y_r^{\max}\right)^{\alpha} & y_{rk} > d_r y_r^{\max} \end{cases},$$

and θ_{kk}^{*} is the self-evaluation efficiency of DMU_k obtained using Model (2). Also, u_{rk} $(r = 1, \dots, s)$ and v_{ik} $(i = 1, \dots, m)$ are decision variables. The objective function is the prospect value of $DMU_{k,}$ based on interval-DMU. The model unifies multiple data types of reference points, but the model is too complex.

3 Cross-Efficiency Evaluation Models Based on Management Objectives

The DEA method is used to evaluate the performance of DMUs. According to the modeling ideas and meaning of the DEA method, each inefficient DMU can achieve improved efficiency through the adjustment of inputs and outputs. The DMU's efficiency reflects the reasonable degree of resource allocation of the DMU, and DEA efficiency ranges from 0 to 1. That is, when the DEA efficiency is 1, the resource allocation of the DMU is optimal. Obviously, the DEA efficiency, as a management objective, is attainable, specific and measurable; therefore, this study uses DEA efficiency as the performance management objective.

In practical applications, management objectives have multiple levels, multiple angles and diversity. As a reference point for DMUs, an objective may be for all DMUs or a single DMU. When the management objective is the performance evaluation criterion of all DMUs, that objective is an organizational objective, which arises from the needs of either the industry or society. When the management objective is the evaluation criterion of a DMU, that objective is a personal objective, which is subjectively formulated by the DMU according to the status quo and needs. In fact, the performance evaluation of a DMU often has to simultaneously face both personal objectives and organizational objectives. We regard this management objective system (i.e., one that considers both organizational objectives and personal objectives) as a composite objective. The cross-efficiency evaluation models based on personal objectives, organizational objectives and composite objectives are constructed in consideration of bounded rationality.

Assumption: (1) Organizational objectives and personal objectives are consistent; that is, both organizational objectives and personal objectives urge the DMU to try its best to improve resource allocation (that is, to increase efficiency). (2) All DEA models in this study assume constant returns to scale.

3.1 Profit and Loss Model Under Management Objectives

Objectives management is divided into the objective setting stage and the performance evaluation stage. This study focuses on performance evaluation, with management objectives as the reference points, rather than the formulation of management objectives. In the management objective setting stage, the management objective should be higher than the performance level of most DMUs. However, in the performance evaluation stage, the performance of some DMUs may meet or even exceed the management objective. For a DMU, say DMU_j ($j = 1, 2, \dots, n$), the management objective is θ_{MO} ($0 < \theta_{MO} < 1$), and its self-evaluation efficiency (CCR efficiency) is θ_j ($j = 1, 2, \dots, n$) in the performance evaluation stage. DMU_k, being the evaluated DMU, takes θ_{MO} as a reference level to evaluate peer DMU_j. There is redundancy in inputs and deficiencies in output, which are all losses for DMU_j ($j = 1, 2, \dots, n$). The prospect value of DMU_j can be defined by the redundancy of inputs and the deficiencies of outputs as described below.

Definition 4: There will be an equation [see Formula (4)], when DMU_k is evaluated with θ_{MO} as a reference point to evaluate its peer DMU_j ($j = 1, 2, \dots, n$) and $\theta_{MO} > \theta_{jj}$:

$$\frac{\sum_{r=1}^{s} u_{rj}y_{rj} + \Delta y_j}{\sum_{i=1}^{m} v_{ij}x_{ij} - \Delta x_j} = \theta_{\mathrm{MO}},\ 0 \le \Delta y_j,\ 0 \le \Delta x_j,\ j = 1, 2, \ldots, n \tag{4}$$

where $-\Delta x_j$ and $-\Delta y_j$ represent the redundancies in inputs and deficiencies in output, respectively, such that $S_{j1}^{k} = -\lambda\left(-\left(-\Delta y_j\right)\right)^{\beta} - \lambda\left(-\left(-\Delta x_j\right)\right)^{\beta}$ is defined as the loss of $\mathrm{DMU}_{j,}$ taking θ_{MO} as the reference point.

Definition 2 There is an equation, shown as Formula (5), when $\theta_{\mathrm{MO}} \le \theta_{jj}$

$$\frac{\sum_{r=1}^{s} u_{rj}y_{rj} - \Delta y_j}{\sum_{i=1}^{m} v_{ij}x_{ij} + \Delta x_j} = \theta_{\mathrm{MO}}, 0 \le \Delta y_j, 0 \le \Delta x_j, j = 1, 2, ..., n \tag{5}$$

where Δx_j and Δy_j represent the savings in inputs and profits in output, respectively, such that $S_{j2}^{\mathrm{k}} = \left(\Delta y_j\right)^{\alpha} + \left(\Delta x_j\right)^{\alpha}$ is defined as the gain of $\mathrm{DMU}_{j,}$ taking θ_{MO} as the reference point.

Models (6)–(7), which take S_{j1}^{k} and S_{j2}^{k} as objective functions, are all nonlinear programming, and there may be multiple optimal solutions, which will destroy the availability of the models. To ensure that the objective functions of these models have unique optimal solutions, the propositions shown below are given and proved.

Proposition 1: *There is a function $S = (\Delta y)^{\alpha} + (\Delta x)^{\alpha}$, where $\alpha(0 < \alpha < 1)$ is a constant, and $\Delta x \ge 0,\ \Delta y \ge 0$. Then, the function S is monotonically increasing and will have the unique extreme values.*

Proof: Let $\mathrm{B}(x, y)$ and $\mathrm{B}_1\left(x_1, y_1\right)$ be two points in two dimensional space, and let l be the direction of the vector $\overrightarrow{\mathrm{BB}_1} = (\Delta x, \Delta y)\left(\Delta x = x_1 - x,\ \Delta y = y_1 - y\right)$, and $(\Delta x \ge 0,\ \Delta y \ge 0)$.

A unit vector in the same direction as l is denoted as e_j

$$e_j = \left(\frac{\Delta x}{\sqrt{(\Delta y)^2 + (\Delta x)^2}}, \frac{\Delta y}{\sqrt{(\Delta y)^2 + (\Delta x)^2}}\right), \text{ where } \Delta x = x_1 - x,\ \Delta y = y_1 - y.$$

Partial derivative of function S, $\frac{\partial S}{\partial \Delta x} = \alpha(\Delta x)^{\alpha - 1}$, $\frac{\partial S}{\partial \Delta y} = \alpha(\Delta y)^{\alpha - 1}$.

The derivative of S in the direction of l is

$$\frac{\partial S}{\partial l} = \alpha(\Delta x)^{\alpha - 1}\frac{\Delta x}{\sqrt{(\Delta y)^2 + (\Delta x)^2}} + \alpha(\Delta y)^{\alpha - 1}\frac{\Delta y}{\sqrt{(\Delta y)^2 + (\Delta x)^2}}$$

$x_1 > x, y_1 > y$, then $\Delta x > 0,\ \Delta y > 0$

$\frac{\partial S}{\partial l} > 0$, when $0 < \alpha < 1$;

$x_1 = x, y_1 > y$ or $x_1 > x, y_1 = y$, then $\Delta x = 0,\ \Delta y > 0$ or $\Delta x > 0,\ \Delta y = 0$

$\frac{\partial S}{\partial l} > 0$, when $0 < \alpha < 1$;

$x_1 = x, y_1 = y$, then $x_1 = x, y_1 = y$

$\frac{\partial S}{\partial l} = 0$;

such that S is monotonically increasing in the direction of $\overrightarrow{\mathrm{BB}_1}$ and has the unique extreme values.

Proposition 2 *There is a function $S = -\lambda(\Delta y)^{\beta} - \lambda(\Delta x)^{\beta}$, where $\beta(0 < \beta < 1)$ is a constant, and $\Delta x \ge 0,\ \Delta y \ge 0$. Then, the function S is monotonically decreasing and will have the unique extreme values.*

Proof: The proof process is similar to Proposition 1.

3.2 Cross-Efficiency Evaluation Model Under Organizational Objectives

Organizational objectives, as one of the management objectives, are the reference points for all DMUs. Based on self-interest, when a DMU faces organizational objectives, the DMU will pay more attention to the prospect value of peers with organizational objectives as the reference point. This enlarges the disadvantages and degrades the advantages of peers to achieve an increase in a DMU's ranking.

According to prospect theory, the prospect value is divided into gain domain (advantage) and loss domain (disadvantage) (see Fig. 1); the value is positive in the gain domain and negative in the loss domain. In other words, minimizing the prospect value of peers means minimizing the gains of peers and maximizing the losses of peers. Therefore, when a DMU, say, DMU_k, k {1,…, n} faces organizational objectives $\left(\theta^{\mathrm{OO}}\right)$ as a reference point and has the opportunity to evaluate each peer (DMU_j), the DMU will choose a set of weights from the optimal self-evaluation weights to obtain a high self-evaluation score. This set of weights is obtained by minimizing the prospect value $\left(S_{j1}^{k}\right)$ and maximizing the prospect value $\left(S_{j2}^{k}\right)$ of DMU_j.

When the organizational objective $\left(\theta^{OO}\right)$ is less than the self-evaluation efficiency $\left(\theta^{*}_{jj}\right)$ of DMU$_j$, the cross-evaluation model with θ^{OO} as reference point, taking the S^k_{j1} as the objective function, is shown as Model (6):

$$\text{Minimize } S^k_{j1} = \left(\Delta y_{j1}\right)^{\alpha} + \left(\Delta x_{j1}\right)^{\alpha}$$
$$\text{Subject to } \sum_{r=1}^{s} u_{rk} y_{rj} - \Delta y_{j1} = \theta^{OO}\left(\sum_{i=1}^{m} v_{ik} x_{ij} + \Delta x_{j1}\right)$$
$$\sum_{r=1}^{s} u_{rk} y_{rk} - \theta^{*}_{kk} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \tag{6}$$
$$\sum_{r=1}^{s} u_{rk} y_{rt} - \sum_{i=1}^{m} v_{ik} x_{it} \le 0, t = 1, \ldots, n; t \ne k$$
$$u_{rk}, v_{ik} \ge \varepsilon; r = 1, \ldots, s; i = 1, \ldots, m,$$
$$0 \le \Delta y_j, 0 \le \Delta x_j$$

where Δx_{j1} and Δy_{j1} are, respectively, the savings in inputs and profits in outputs of DMU$_j$, with the θ^{OO} as the reference point, when DMU$_k$ evaluates its peer, DMU$_j$.

When the organizational objective $\left(\theta^{OO}\right)$ is greater than the self-evaluation efficiency $\left(\theta^{*}_{jj}\right)$ of DMU$_j$ (peer of DMU$_k$), the cross-evaluation model, taking the prospect value $\left(S^k_{j2}\right)$ of DMU$_j$ as the objective function, is shown as Model (7):

$$\text{Maximize } S^k_{j2} = -\lambda\left(-\left(-\Delta y_{j2}\right)\right)^{\beta} - \lambda\left(-\left(-\Delta x_{j2}\right)\right)^{\beta}$$
$$\text{Subject to } \sum_{r=1}^{s} u_{rk} y_{rj} + \Delta y_{j2} = \theta^{OO}\left(\sum_{i=1}^{m} v_{ik} x_{ij} - \Delta x_{j2}\right)$$
$$\sum_{r=1}^{s} u_{rk} y_{rk} - \theta^{*}_{kk} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \tag{7}$$
$$\sum_{r=1}^{s} u_{rk} y_{rt} - \sum_{i=1}^{m} v_{ik} x_{it} \le 0, t = 1, \ldots, n; t \ne k$$
$$u_{rk}, v_{ik} \ge \varepsilon; r = 1, \ldots, s; i = 1, \ldots, m,$$
$$0 \le \Delta y_j, 0 \le \Delta x_j$$

where $-\Delta x_{j2}$ and $-\Delta y_{j2}$ are, respectively, redundancies in the inputs and deficiencies in the outputs of DMU$_j$, with the θ^{OO} as the reference point.

3.3 Cross-Efficiency Evaluation Model Under Personal Objectives

The DMU pays more attention to its own interests, rather than the interests of peers, when faced with personal objectives. This occurs to such an extent that the DMU often takes personal objectives as a reference point to maximize its own prospect values by maximizing gains and minimizing losses. Therefore, this study's idea of cross-efficiency modeling under personal objectives is that each DMU will try to select a set of input and output weights to maximize its own prospect value, and this set of weights is also used to evaluate the DMU's peers.

When personal objective θ^{PO} is less than the self-evaluation efficiency $\left(\theta^{*}_{kk}\right)$ of DMU$_k$, there is a savings in inputs and profits in output, which are all gains for DMU$_k$. The prospect value $\left(S^k_{k1}\right)$ can be defined by the gains of DMU$_k$. For DMU$_k$, k $\{1,\ldots, n\}$, DMU$_k$ will try to choose a group of weights from multiple self-evaluation weights of itself, specifically weights which can maximize the prospect value of DMU$_k$ when facing its own personal objectives $\left(\theta^{PO}\right)$. The cross-evaluation model with θ^{PO} as the reference point takes S^I_k as the objective function; this is shown as Model (8):

$$\text{Maximize } S^k_{k1} = \left(\Delta y_{k1}\right)^{\alpha} + \left(\Delta x_{k1}\right)^{\alpha}$$
$$\text{Subject to } \sum_{r=1}^{s} u_{rk} y_{rk} - \Delta y_{k1} = \theta^{PO}\left(\sum_{i=1}^{m} v_{ik} x_{ik} + \Delta x_{k1}\right)$$
$$\sum_{r=1}^{s} u_{rk} y_{rk} - \theta^{*}_{kk} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \tag{8}$$
$$\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, j = 1, \ldots, n; j \ne k$$
$$u_{rk}, v_{ik} \ge \varepsilon; r = 1, \ldots, s; i = 1, \ldots, m,$$
$$0 \le \Delta y_k, 0 \le \Delta x_k$$

where Δx_{k1} and Δy_{k1} are, respectively, savings in the inputs and profits in the outputs of DMU$_k$, with the θ^{PO} as the reference point.

Conversely, when the personal objective θ^{PO} is greater than the self-evaluation efficiency $\left(\theta^{*}_{kk}\right)$ of DMU$_k$, there are redundancies in inputs and deficiencies in outputs, which are all losses for DMU$_k$. The prospect value $\left(S^k_{k2}\right)$ is defined by the losses of DMU$_k$. The cross-efficiency evaluation model minimizing the prospect value $\left(S^k_{k2}\right)$ is shown as Model (9):

$$\text{Minimize } S_{k2} = -\lambda\left(-\left(-\Delta y_{k2}\right)\right)^{\beta} - \lambda\left(-\left(-\Delta x_{k2}\right)\right)^{\beta}$$
$$\text{Subject to } \sum_{r=1}^{s} u_{rk} y_{rk} + \Delta y_{k2} = \theta^{PO}\left(\sum_{i=1}^{m} v_{ik} x_{ik} - \Delta x_{k2}\right)$$
$$\sum_{r=1}^{s} u_{rk} y_{rk} - \theta^{*}_{kk} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \tag{9}$$
$$\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, j = 1, \ldots, n; j \ne k$$
$$u_{rk}, v_{ik} \ge \varepsilon; r = 1, \ldots, s; i = 1, \ldots, m,$$
$$0 \le \Delta y_k, 0 \le \Delta x_k$$

where $-\Delta x_{k2}$ and $-\Delta y_{k2}$ are, respectively, redundancies in the inputs and deficiencies in the outputs of DMU$_k$, with the θ^{PO} as the reference point.

3.4 Calculation of Cross Efficiency

Parameter λ, α and β (see Sect. 2.1) reflects the bounded rationality of DMUs, which is determined in line with people's psychology through empirical research (Rieger et al., 2017; Tversky and Kahneman, 1992). This study focuses on the cross evaluation method based on the existing research results of psychological parameters. The psychological parameters will directly use the research results of Tversky and Kahneman; that is, $\alpha=\beta=0.88$, and $\lambda=2.25$. In this study, management objectives are divided into organizational objectives, personal objectives and composite objectives. The formation and calculation process of cross efficiency based on these three types of management objectives is shown below.

3.4.1 The Steps of Cross Evaluation Based on Organizational Objectives

Step1: The organizational objective $\left(\theta^{OO}\right)$ is formulated according to the self-evaluation efficiency of evaluated DMUs and macro control policies, see Fig. 2 for the organizational objective setting process linked by the blue arrow.

Step2: When the DMU_k evaluates DMUj (j = 1, 2,…, n; j ≠ k), DMU_k compares the self-evaluation efficiency $\left(\theta_{jj}^{*}\right)$ of DMUj with the value of the organizational goal $\left(\theta^{OO}\right)$. If the θ^{OO} is larger, the prospect value ${S_{j1}}^{k}$ is the loss of DMU_j. On the contrary, if the prospect value ${S_{j2}}^{k}$ is a gain, the evaluated DMU_k, for its personal benefit, will then consider selecting a group of weights from the self-evaluation weights to maximize ${S_{j1}}^{k}$ and minimize ${S_{j2}}^{k}$, and that group of weights will be used to evaluate DMU_j, see Fig. 2 for the performance evaluation process linked by the red arrow.

Step3: Here, DMU_k not only evaluates its peer DMU_j (j = 1, 2,…, n; j ≠ k), but also accepts cross evaluation from these peers. The process of cross evaluation is shown as step 2. The DMU's self-evaluation efficiency θ_{kk}^{*} is solved by Model (2), and the cross evaluation scores $\left(\theta_{jk}\right)$ from peers are solved by Models (6)–(7). There are n-1 peers for DMU_k. Therefore, the cross efficiency $\theta_{OO-k}^{\text{cross - efficiecy}}$ of DMU_k is the combination of θ_{kk}^{*} and θ_{jk}(j = 1, 2,…, n; j ≠ k).

3.4.2 The Steps of Cross Evaluation Based on Personal Objectives

Step 1: The personal objective $\left(\theta^{PO}\right)$ of DMU_k (k = 1, 2,…, n) is generally related to the DMU's current level and expectations for the future, see Fig. 3 for the personal objective setting process linked by the blue arrow.

Step 2: The DMU_k compares its own self-evaluation efficiency $\left(\theta_{kk}^{*}\right)$ with the personal objective $\left(\theta^{PO}\right)$. If θ^{PO} is larger, the prospect value ${S_{k1}}^{k}$ is a loss for DMU_k. Conversely, if, the prospect value ${S_{k2}}^{k}$ is a gain, DMU_k, will then consider selecting a group of weights from the self-evaluation weights to minimize ${S_{k1}}^{k}$ and maximize ${S_{k2}}^{k}$, and this group of weights is used to evaluate DMU_j, see Fig. 3 for the performance evaluation process linked by the red arrow.

Step3: Here, DMU_k's self-evaluation efficiency θ_{kk}^{*} is solved by Model (2), and the cross evaluation score θ_{kj} from peers is solved by Models (8)–(9). Similarly, the cross

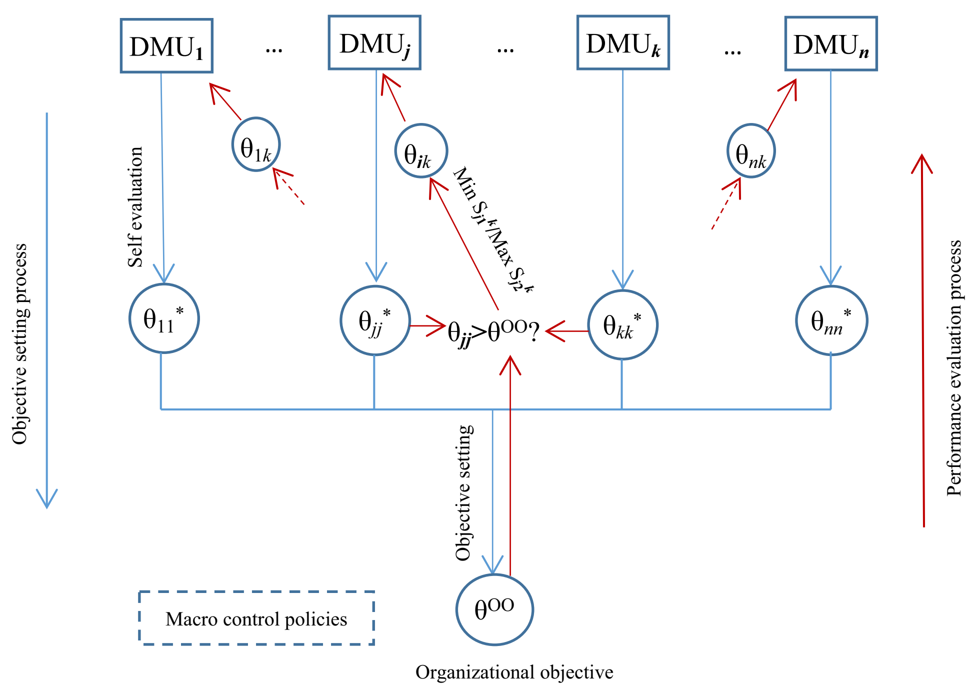
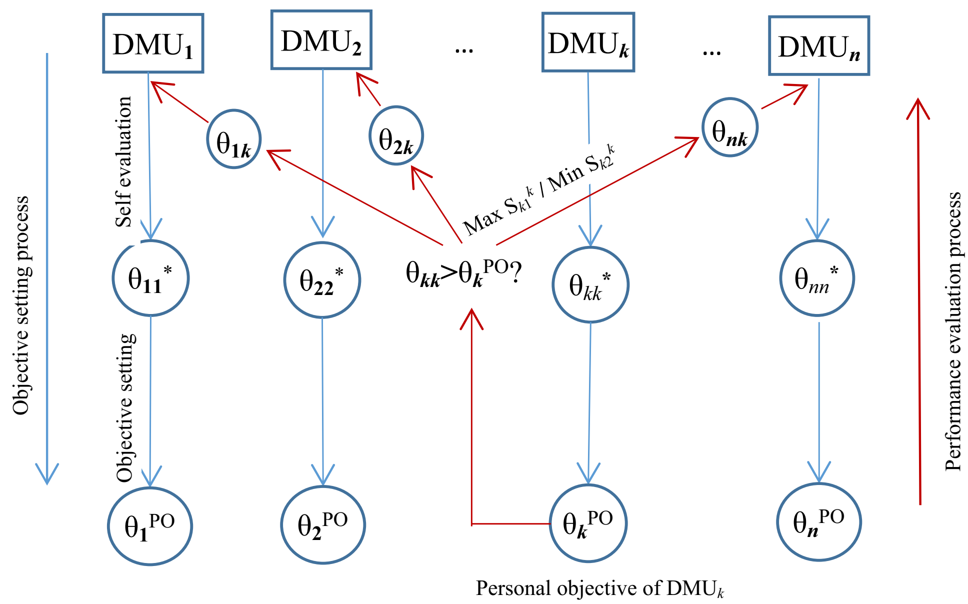


Fig. 2 Formation and calculation process of cross efficiency based on organizational objectives

Fig. 3 Formation and calculation process of cross efficiency based on personal objectives



efficiency $\theta_{PO-k}^{\text{cross - efficiecy}}$ of DMU$_k$ is the combination of θ_{kk}^{*} and θ_{jk}(j = 1, 2,…, n; j ≠ k).

3.4.3 The Steps of Cross Evaluation Based on Composite Objectives

Next, DMU$_k$ may get cross evaluations from peers based on organizational objectives or personal goals. The type of management objective directly affects the ranking of DMUs. Therefore, DMUs should first simultaneously determine the importance of the organizational objectives and personal objectives. Step 1: Determine the organizational objective's importance (μ) according to the actual application, as well as the importance of personal goals (1-μ). The composite objectives are equal to $\theta^{OO} \times \mu + \theta_k^{PO} \times (1\text{-}\mu)$. Step 2: Calculate the cross efficiency based on composite objectives. Cross efficiency is the average value of self-evaluation and peer cross evaluation, which is based on composite objectives.

4 Illustration Example

In this section, an example in the energy industry is used to illustrate the effectiveness and rationality of the cross efficiency evaluation method proposed in this study. The data used in this example comes from the 2018 Annual Performance Statistics Report of energy enterprises, provided by the Fujian Science and Technology Bureau. The source data involves 23 enterprises. To avoid disclosing enterprise information, the data has been anonymized (with enterprise names hidden) and represented as DMUs. Description of source data is shown in Table 1.

The source data present the following issues: (1) there are invalid data points, such as a "product sales revenue" value of 0, indicating that the enterprise has no product sales, and

Table 1 Description of source data

	Input			Output		
	R&D Personnel	R&D Expenses (thousands)	Product Sales Revenue (thousands)	New product sales revenue (thousands)	Total profit (thousands)	Net profit (thousands)
DMU1	4045.2	108.8	4	225.4	121.1	110.23
…	…	…	…	…	…	…
	3248	78	0	3340	−4249	−4249
…	…	…	…	…	…	…
DMU23	132,712.49	81,870	27	7326.01	6675	6675
Max.	273,103	199,285	101	16,392	58,347.56	52,377.74
Min.	2314.73	0	4	225.4	−63,113	−63,113

consequently, no new product sales or profits; (2) there is strong endogeneity between "total profit" and "net profit", as well as between "product sales revenue" and "new product sales revenue"; and (3) there are many negative values in "total profit" and "net profit" whereas the method proposed in this paper (DEA method) is only applicable to positive data.

Before processing the data using the model proposed in this paper, the following steps were taken: (1) preprocessing the source data by deleting invalid data points, resulting in the removal of 2 invalid entries and leaving 21 valid entries; (2) due to the strong endogeneity between "net profit" and "total profit", and their small data difference making them interchangeable, only one was selected, with "net profit" chosen as one of the output indicators; (3) using the ratio of "new product sales revenue" to "product sales revenue" as a new output indicator, "new product sales revenue rate" to eliminate the end-ogeneity between these two indicators; (4) converting the units of "net profit" and "R&D expenses" from "thousand" to "million"; and (5) standardizing the "net profit" data to address the issue of negative values. The preprocessed indicators and data are shown in Table 2.

This example is divided into four parts, taking organizational objectives, personal objectives and composite objectives as reference points, respectively, to illustrate the effectiveness of the cross efficiency evaluation method. Finally, the method proposed in this study is compared with the traditional method. All psychological parameters related to cross efficiency are $\alpha=\beta=0.71$, $\lambda=2.25$。

4.1 Cross Evaluation Based on Organizational Objectives

To reflect the impact of organizational objectives on cross efficiency, seven different organizational objectives are selected for the example, namely $\theta^{OO-1}=0.4$, $\theta^{OO-2}=0.5$, $\theta^{OO-3}=0.6$, $\theta^{OO-4}=0.7$, $\theta^{OO-5}=0.8$, $\theta^{OO-6}=0.9$ and $\theta^{OO-7}=1$. Solve the self-evaluation efficiency of each DMU according to Model (2) and obtain the DMU's peer evaluation efficiency according to Models (6) or (7). Then, calculate the cross efficiency of each DMU according to the self-evaluation score and peer-evaluation scores. The cross efficiencies of these 21 enterprises under the seven organizational objectives are shown in Table 3.

To intuitively show the change of the cross efficiency of the 21 DMUs under the seven organizational objectives, the 21 DMUs are ranked according to the cross efficiency value in Table 3. The ranking of DMUs s is also graphically illustrated, as shown in Fig. 4. Combined with Table 2, it is not difficult to find that the relationship between the

Table 2 Description of preprocessed data

	Inputs		Outputs		CCR efficiency
	x_1: R&D Personnel	x_2: R&D Expenses (million)	y_1: New Product Sales Revenue Rate	y_2: Net Profit (million)	
DMU1	4	2.25	0.03	0.55	1
DMU2	7	13.37	0.18	0.56	0.6177
DMU3	15	44.88	0.95	0.56	0.7932
DMU4	60	90.93	0.99	0	0.2241
DMU5	49	30.96	0.56	0.73	0.2436
DMU6	90	16.92	0.12	0.80	0.1971
DMU7	26	37.05	0.77	0.55	0.4053
DMU8	46	125.70	0.85	0.67	0.2354
DMU9	6	7.51	0.43	0.56	1
DMU10	5	18.69	0.25	0.55	0.9475
DMU11	5	8.56	0.08	0.55	0.8
DMU12	20	21.90	1	0.71	0.6976
DMU13	13	4.88	0.001	0.54	0.4527
DMU14	4	2.34	0.14	0.55	1
DMU15	14	13.91	1	0.55	1
DMU16	15	4.90	0.47	0.55	1
DMU17	14	23.47	0.71	0.56	0.6849
DMU18	12	45.63	1	0.55	1
DMU19	25	81.18	0.27	1	0.2951
DMU20	5	8.85	0.02	0.55	0.8
DMU21	27	73.26	0.62	0.60	0.3005

Table 3 Cross efficiency and CCR-efficiency of the 21 DMUs under the seven organizational objectives

	θ^{OO-1}	θ^{OO-2}	θ^{OO-3}	θ^{OO-4}	θ^{OO-5}	θ^{OO-6}	θ^{OO-7}
DMU1	0.4815	0.6728	0.7195	0.7586	0.7958	0.8446	0.4815
DMU2	0.3614	0.4804	0.5226	0.3614	0.3614	0.3614	0.3614
DMU3	0.4887	0.5901	0.6062	0.6332	0.4887	0.4887	0.4887
DMU4	0.1236	0.1207	0.1235	0.1236	0.1236	0.1236	0.1236
DMU5	0.1501	0.1501	0.1501	0.1501	0.1501	0.1501	0.1501
DMU6	0.0586	0.0572	0.0572	0.0572	0.0586	0.0586	0.0586
DMU7	0.2784	0.277	0.277	0.277	0.2784	0.2784	0.2784
DMU8	0.153	0.153	0.153	0.153	0.153	0.153	0.153
DMU9	0.8017	0.8361	0.8503	0.8695	0.8924	0.9182	0.8017
DMU10	0.5251	0.616	0.6405	0.6819	0.7236	0.7595	0.5251
DMU11	0.3637	0.5197	0.5689	0.6134	0.3586	0.3586	0.3637
DMU12	0.4946	0.568	0.5974	0.4946	0.4946	0.4946	0.4946
DMU13	0.1521	0.1488	0.1488	0.1521	0.1521	0.1521	0.1521
DMU14	0.7558	0.7775	0.8043	0.8405	0.8872	0.9267	0.7558
DMU15	0.6898	0.7617	0.7838	0.8097	0.8341	0.8701	0.6898
DMU16	0.4637	0.5406	0.5901	0.6436	0.6911	0.7239	0.4637
DMU17	0.4675	0.5503	0.579	0.4675	0.4675	0.4675	0.4675
DMU18	0.6054	0.7033	0.7204	0.7465	0.7719	0.797	0.6054
DMU19	0.1486	0.1486	0.1486	0.1486	0.1486	0.1486	0.1486
DMU20	0.2702	0.4904	0.5378	0.5866	0.2629	0.2629	0.2702
DMU21	0.1989	0.1989	0.1989	0.1989	0.1989	0.1989	0.1989

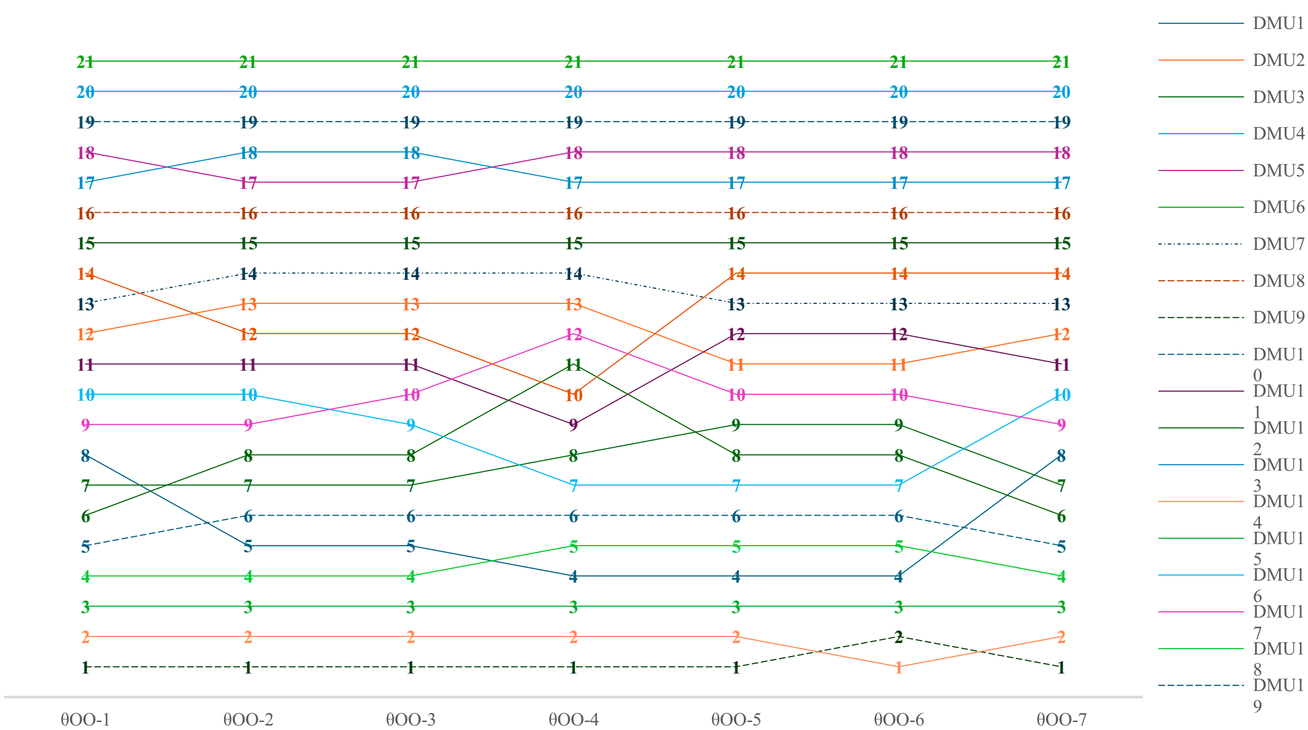


Fig. 4 Ranking trend of the 21 DMUs under the seven organizational objectives

self-evaluation scores of DMUs and the values of organizational objectives affects the ranking of DMUs. On the one hand, except for DMU19, the ranking of DMUs with self-evaluation scores of less than 1 (such as DMU2, DMU3, DMU7, DMU11, DMU12, DMU16 and DMU20) is greatly affected by the value of organizational objectives. On the other hand, cross efficiency and the ranking of a DMU would be lower than that under other organizational objectives when the DMU's self-evaluation efficiency is close to the organizational objectives. For example, DMU12, whose self-evaluation efficiency is 0.6976, ranks the lowest and the sixth under θ^{OO-6} ($\theta^{OO-6}=0.9$). Similar situations can also be seen in DMU17, DMU3 and other DMUs.

The above results show that the sampled enterprises will receive benevolent evaluations from their peers when the resource allocation level is higher than the organizational objective. If the contrary is true, an enterprise will face aggressive evaluation from its peers. The resource allocation levels of many enterprises in the example have always been higher than the organizational objectives. Therefore, their cross evaluation efficiencies have not changed much, and their rankings are relatively stable. On the contrary, the enterprises whose resources need to be optimized face organizational objectives; the cross efficiency changes greatly, and the ranking is relatively unstable. Therefore, these enterprises sometimes face aggressive evaluations from their peers, while sometimes they receive benevolent evaluations from their peers. This leads to greater changes in cross efficiency under different organizational objectives, as well as great fluctuations in rankings.

4.2 Cross Evaluation Results Based on Personal Objectives

To reflect the role of personal objectives in cross evaluation, this part considers various personal objectives for each enterprise. For example, seven groups of personal objectives are selected for each enterprise, as shown in Table 4. The cross efficiency of these 21 DMUs is calculated according to "the steps of cross evaluation based on personal objectives", which is shown in Table 5.

As can be seen from Table 5, the cross efficiencies of each DMU are different under different personal objectives. This finding shows that personal objectives, as reference points, have an impact on the cross efficiency of the 21 DMUs. To more intuitively present the trend of the cross efficiency of the 21 sampled enterprises under the seven personal objectives (Θ^{PO-t}, $t=1, 2,\ldots, 7$), the cross efficiency ranking of the 21 DMUs is shown in Fig. 5. As can be seen from Fig. 5, only a few of the 21 DMUs showed changes in rankings under personal objectives, such as DMU1, DMU5, DMU7, DMU10 and DMU20. Obviously, the cross efficiency of each DMU varies in line with the change of personal objectives, but has little impact on the ranking trend of DMUs. This finding indicates that the change trend of cross efficiency among DMUs is consistent under personal objectives. Comparing Figs. 5 and 4, both personal objectives and organizational objectives have impacts on enterprise ranking. However, organizational objectives have a greater impact on ranking than personal objectives, which is more consistent with the impact of macro-control than micro adjustment in management applications.

4.3 Evaluation Results Under Composite Objectives

Enterprises often not only face either organizational objectives or personal objectives; they can also face organizational objectives and personal objectives at the same time; that is, composite objectives. There is an important problem for composite objectives, which is determining the importance of organizational objectives relative to personal objectives. The importance is determined by practical application. For the sake of brevity, let's assume that organizational objectives are as important as personal objectives; that is, composite objectives. This is calculated by the following formula:

$$\Theta^{CO} = \Theta^{OO} \times 0.5 + \Theta^{PO} \times 0.5$$

The cross efficiency of the 21 DMUs based on seven composite objectives is shown in Table 6.

As can be seen from Table 6, the cross efficiency of the 21 DMUs is obviously affected by the composite objectives. To visually show the change trend of cross efficiency under the composite objective, the ranking of the 21 DMUs in Table 6 is graphically shown in Fig. 6. According to Fig. 6

Table 4 Seven personal objectives of 21 DMUs

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}
Θ^{PO-1}	1	0.7	0.75	0.8	0.85	0.9	0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Θ^{PO-2}	1	1	0.7	0.75	0.8	0.85	0.9	0.95	1	1	1	1	1	1	1	1	1	1	1	1	1
Θ^{PO-3}	1	1	1	0.7	0.75	0.8	0.85	0.9	0.95	1	1	1	1	1	1	1	1	1	1	1	1
Θ^{PO-4}	1	1	1	1	0.7	0.75	0.8	0.85	0.9	0.95	1	1	1	1	1	1	1	1	1	1	1
Θ^{PO-5}	1	1	1	1	1	0.7	0.75	0.8	0.85	0.9	0.95	1	1	1	1	1	1	1	1	1	1
Θ^{PO-6}	1	1	1	1	1	1	0.7	0.75	0.8	0.85	0.9	0.95	1	1	1	1	1	1	1	1	1
Θ^{PO-7}	1	1	1	1	1	1	1	0.7	0.75	0.8	0.85	0.9	0.95	1	1	1	1	1	1	1	1

For simplicity in formatting, "D" represents "DMU"

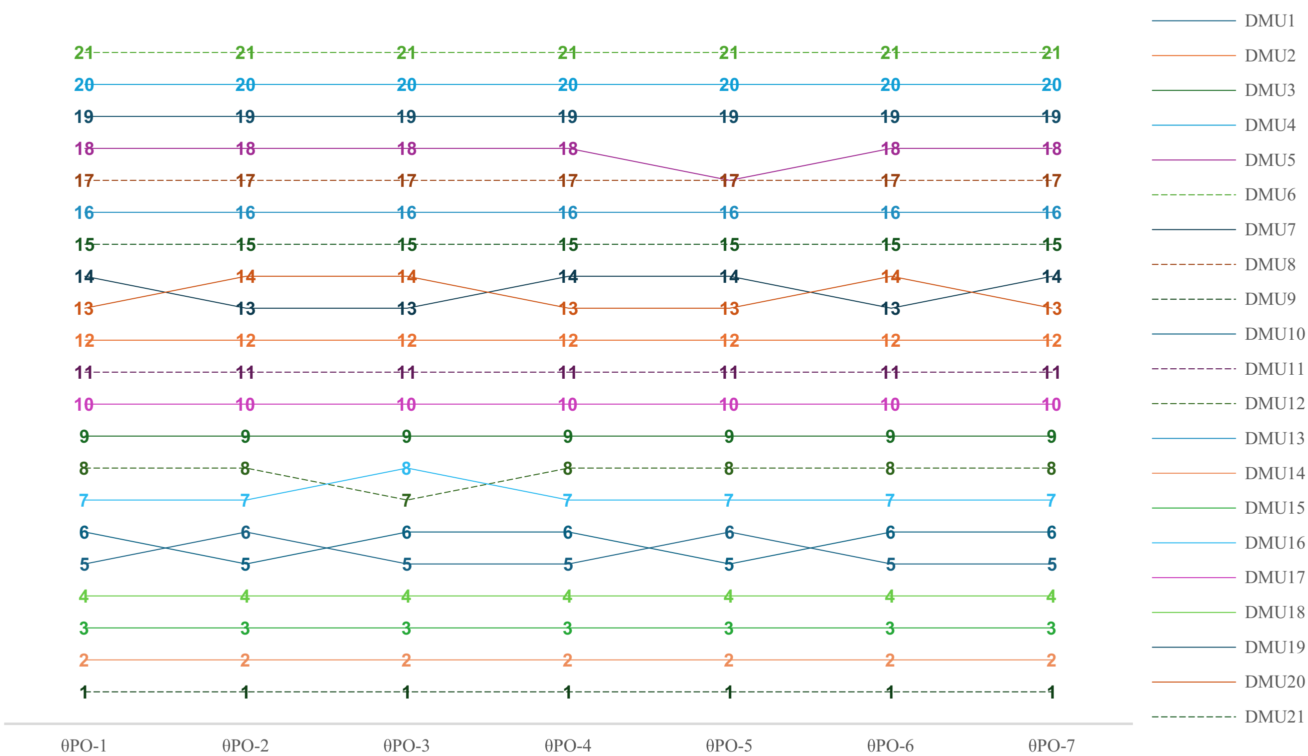


Fig. 5 Rankings trend of 21 DMUs under seven personal objectives

Table 5 Cross efficiency of 21 DMUs under seven personal objectives

	θ^{PO-1}	θ^{PO-2}	θ^{PO-3}	θ^{PO-4}	θ^{PO-5}	θ^{PO-6}	θ^{PO-7}
DMU1	0.5427	0.5367	0.5425	0.545	0.5347	0.5408	0.5444
DMU2	0.3707	0.3737	0.3702	0.3706	0.3723	0.3726	0.3702
DMU3	0.4998	0.5006	0.4989	0.4988	0.499	0.4994	0.4997
DMU4	0.1257	0.1269	0.1276	0.127	0.1268	0.128	0.1269
DMU5	0.156	0.156	0.1561	0.1555	0.1563	0.1564	0.1562
DMU6	0.0709	0.0715	0.0714	0.0707	0.0705	0.0709	0.0715
DMU7	0.2828	0.287	0.2871	0.2834	0.2834	0.2837	0.2824
DMU8	0.1564	0.1568	0.1563	0.1565	0.1563	0.1569	0.1564
DMU9	0.8196	0.8166	0.8142	0.8146	0.8164	0.8146	0.8139
DMU10	0.5398	0.5466	0.5381	0.5376	0.5402	0.5377	0.5381
DMU11	0.3755	0.3741	0.3782	0.3764	0.3769	0.3794	0.3824
DMU12	0.5113	0.5064	0.5123	0.5097	0.5072	0.5074	0.5068
DMU13	0.1748	0.1804	0.1793	0.1781	0.1802	0.1789	0.1789
DMU14	0.7997	0.7972	0.8002	0.7994	0.8026	0.798	0.7975
DMU15	0.7272	0.7049	0.7091	0.7144	0.7112	0.7142	0.7254
DMU16	0.5117	0.5142	0.5109	0.5185	0.5146	0.5128	0.5135
DMU17	0.4791	0.4754	0.4756	0.4776	0.4769	0.4768	0.4834
DMU18	0.6181	0.6189	0.619	0.619	0.6198	0.6194	0.6184
DMU19	0.155	0.1528	0.1528	0.153	0.1527	0.1535	0.1528
DMU20	0.2899	0.2839	0.2855	0.286	0.2857	0.283	0.2865
DMU21	0.2033	0.2037	0.2043	0.2032	0.2035	0.2038	0.2042

Table 6 Cross efficiency of 21 DMUs under seven composite objectives

	Θ^{CO-1}	Θ^{CO-2}	Θ^{CO-3}	Θ^{CO-4}	Θ^{CO-5}	Θ^{CO-6}	Θ^{CO-7}
DMU1	0.506	0.6047	0.631	0.6518	0.6652	0.6927	0.5129
DMU2	0.3651	0.4271	0.4464	0.366	0.3668	0.367	0.3658
DMU3	0.4932	0.5454	0.5525	0.566	0.4939	0.4941	0.4942
DMU4	0.1244	0.1238	0.1256	0.1253	0.1252	0.1258	0.1252
DMU5	0.1524	0.153	0.1531	0.1528	0.1532	0.1532	0.1531
DMU6	0.0635	0.0644	0.0643	0.0639	0.0646	0.0648	0.065
DMU7	0.2802	0.282	0.282	0.2802	0.2809	0.2811	0.2804
DMU8	0.1543	0.1549	0.1546	0.1547	0.1546	0.1549	0.1547
DMU9	0.8089	0.8263	0.8322	0.842	0.8544	0.8664	0.8078
DMU10	0.531	0.5813	0.5893	0.6098	0.6319	0.6486	0.5316
DMU11	0.3685	0.4469	0.4735	0.4949	0.3678	0.369	0.3731
DMU12	0.5013	0.5372	0.5548	0.5021	0.5009	0.501	0.5007
DMU13	0.1612	0.1646	0.1641	0.1651	0.1662	0.1655	0.1655
DMU14	0.7734	0.7873	0.8022	0.8199	0.8449	0.8623	0.7767
DMU15	0.7047	0.7333	0.7464	0.7621	0.7727	0.7922	0.7076
DMU16	0.4829	0.5274	0.5505	0.581	0.6029	0.6183	0.4886
DMU17	0.4722	0.5128	0.5273	0.4726	0.4722	0.4722	0.4755
DMU18	0.6105	0.6611	0.6697	0.6827	0.6958	0.7082	0.6119
DMU19	0.1512	0.1507	0.1507	0.1508	0.1507	0.1511	0.1507
DMU20	0.2781	0.3871	0.4117	0.4363	0.2743	0.273	0.2784
DMU21	0.2007	0.2013	0.2016	0.2011	0.2012	0.2014	0.2016

and Table 6, it is not difficult to find that the rankings of DMUs with CCR efficiency equal to 1 (such as DMU1, DMU9, DMU10, DMU14, DMU15, etc.) are less affected by composite objectives. In addition, the change of ranking is within three places, which indicates that DMUs with higher performance levels maintain ranking advantages under an objectives incentive. As can be seen from Fig. 6, the influence of the seven composite objectives on the ranking of the 21 DMUs is significantly different. The ranking of DMUs has changed greatly under the Θ^{CO-4}, this clearly indicates that the value of the composite objective will also affect the ranking of DMUs. In addition, the ranking under composite objectives is similar to that under organizational objectives, which again shows that organizational objectives have a greater impact on enterprise ranking.

4.4 Method Comparison

This part shows the advantages and significance of the proposed method through a comparison with classical methods, including benevolent and neutral cross efficiency evaluation methods.

The cross efficiencies of the 21 DMUs under the organizational objective, personal objective and composite objective are compared, see Fig. 7. It is not difficult to find that the performance level of each DMU under the organizational objective and composite objective is similar; The performance level of each DMU under the personal objective is relatively decentralized, and the cross efficiency of the 21 DMUs with lower CCR efficiency is lower. Conversely, the cross efficiency of DMUs with higher CCR efficiency is higher.

The methods proposed in this paper with classical methods are compared. As can be seen from Fig. 7, the cross efficiency of the DMUs based on management objectives is more consistent with the aggressive cross efficiency evaluation, which in turn is lower than the benevolent cross efficiency and more centralized than the neutral cross efficiency.

According to management theory, the organizational objectives are generally higher than the current performance levels for all DMUs. Faced with a higher level of organizational objectives, each DMU accepts aggressive evaluation from the DMU's peers, resulting in lower cross efficiency based on the organizational objective. The effect is also closer to the aggressive cross evaluation method. The DMU in the neutral cross efficiency evaluation method only considers its own interests and is indifferent to peer performance, so the evaluation results are relatively scattered. Personal objectives are generally higher than a DMU's current level, and may be higher or lower than that of the DMU's peers, so peers in turn may get a higher or lower level evaluation from the DMU. Therefore, the cross efficiency method based on personal objectives is between benevolent and aggressive efficiency. As is known from this study, the cross evaluation method based on organizational objectives is more suitable for performance evaluation under market macro-control,

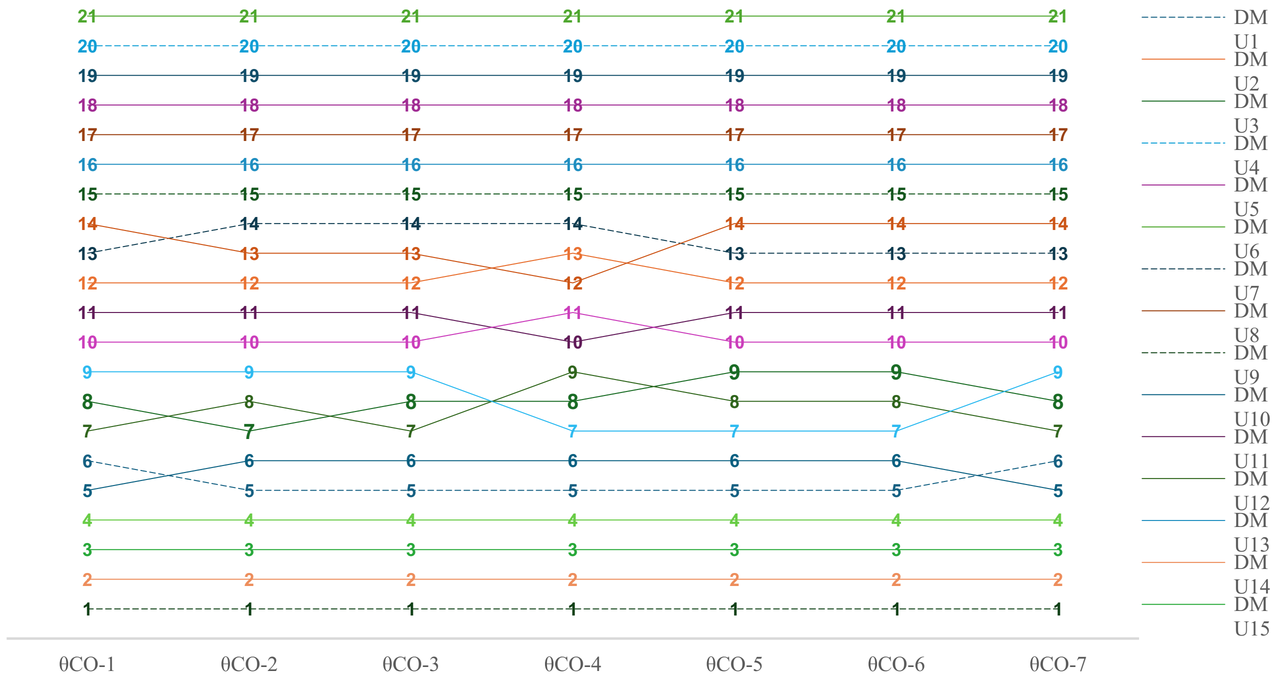


Fig. 6 Ranking trend of the 21 DMUs under the seven composite objectives

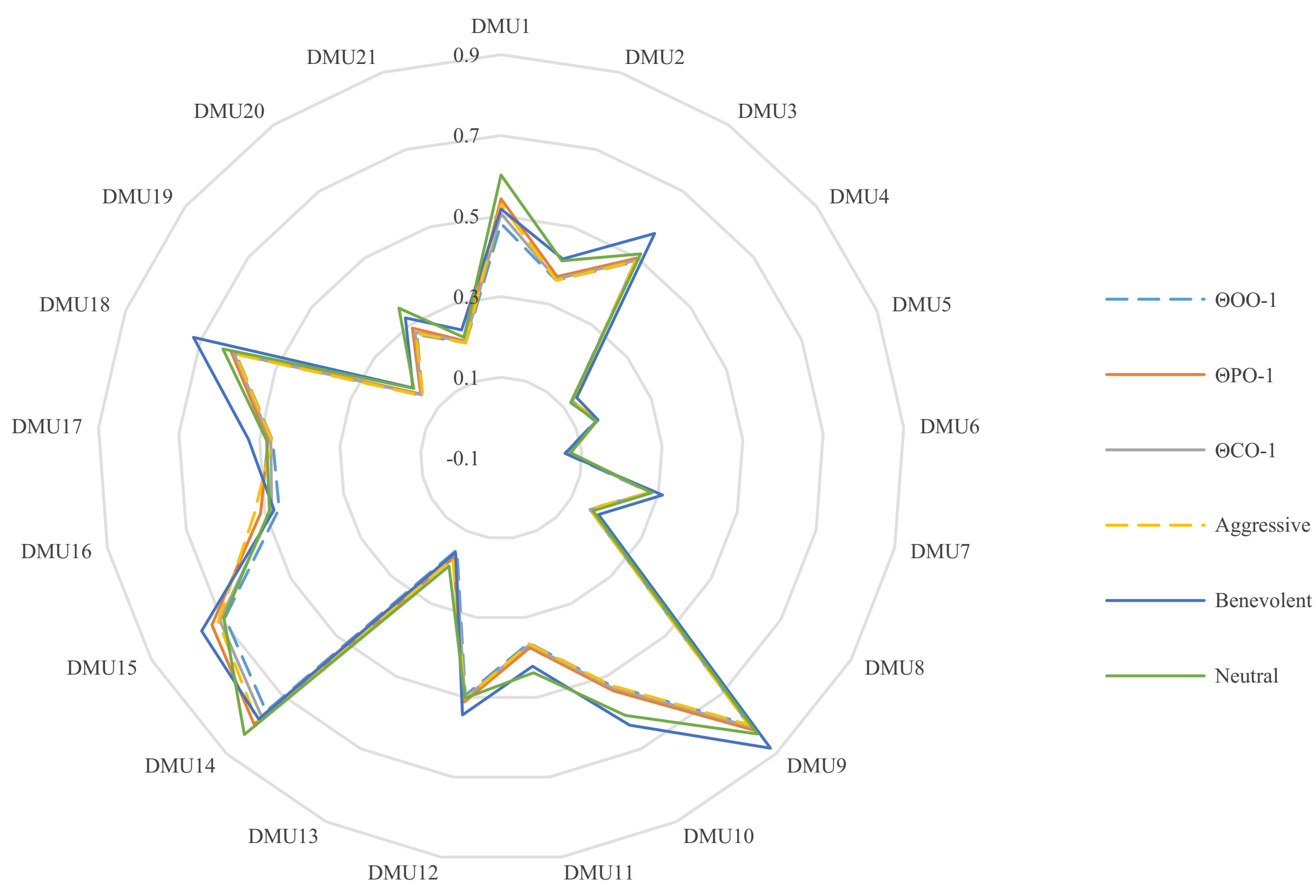


Fig. 7 Comparison of rankings between the proposed method and classical methods

where organizational objectives can be adjusted to adapt to different performance evaluation needs. Compared with the aggressive cross evaluation method, the method based on organizational objectives breaks the practice of blindly suppressing peers, but the method can still control the strength and extent of regulation. The cross evaluation method based on personal objectives can also increase the differentiation between DMUs, and is applicable to relevant evaluations, such as qualification or clearance evaluations.

5 Conclusions

Cross-efficiency evaluation is an important performance evaluation method for ranking DMUs. Most existing studies have often taken DMUs, which are virtual or specific, as the evaluation criteria. However, as the management objective, the evaluation criteria are representative and one-sided. Performance level is selected as the management objective in this paper to reflect the bounded psychology of the DMUs when facing the management objective. Organizational objectives, personal objectives and composite objectives are the reference points for performance evaluation. The DMUs whose performance levels are higher than the management objectives will receive expanded excellent scores from peers. Meanwhile, DMUs whose performance levels are lower than the management objectives will obtain expanded negative scores from peers. The corresponding management methods proposed by this paper should be set according to different application backgrounds, to increase the flexibility of cross efficiency evaluation methods. To confront the risk avoidance psychology of DMUs, cross efficiency evaluation models based on prospect theory are constructed in this study to improve the applicability of the method. Impact and significance of this method at the policy level:

The cross-efficiency evaluation method, which accounts for bounded rationality, incorporates actual performance levels as management objectives, better aligning with real-world conditions. This approach enables policymakers to more accurately assess the actual performance of various decision-making units, thereby avoiding biases from overly idealized frontier settings and enhancing the scientific and rational foundation of policy formulation.

By accurately assessing performance levels DMUs, policymakers can allocate resources more effectively. For instance, in distributing public resources, the government can prioritize allocation based on each unit's performance, optimizing efficiency and enhancing social welfare.

By using cross-efficiency evaluation with performance levels as management objectives, policymakers can assess how decision-making units perform under various performance targets. This helps identify which units excel and which fall short, enabling policymakers to develop targeted measures to improve overall management performance.

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Data Availability No datasets were generated or analysed during the current study.

Declarations

Conflict of Interest The authors declare no conflict of interest.

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