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REVIEW PAPER

Discerning preschool teacher's experiences to enhance children's participation in mathematical play activities

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Abstract

This study analyses teachers' discernment of children's mathematical knowledge development during preschool play activities and defines how children's pre-knowledge supports or limits their learning possibilities. In total, two preschool teachers from different preschools participated in this research study. The data consists of two documentations of children's activities, written notes, and reflections from these two participated preschool teachers. The teachers chose the samples and defined them as important examples of how to merge mathematical knowledge development in free play. Using variation theory, the focus was to define what aspects of children's mathematical knowledge development were offered and how teachers discern children's pre-knowledge. The results show how the preschool teachers discern children's pre-mathematical knowledge in relation to development of abstract abilities, and the importance of identifying their participation in free play situations for promoting mathematical understanding.

Keywords Mathematical knowledge · Play · Preschool teachers · Variation theory

Introduction

In several countries, including Sweden, play-based learning is the current mandated pedagogy in the early years' curricula (Pyle et al., 2017), and has a long tradition in Swedish preschools (Helenius, 2018). Teachers' understanding of the relationship between play and knowledge development in children is a fundamental educational issue (Pyle et al., 2018). Fisher et al.'s (2013) research supports guided play as an optimal pedagogy for facilitating mathematics learning. Still, in Wickström et al.'s (2019) research the most observed context was to support the teaching and the learning of mathematics through direct instruction methods and teacher-directed play.

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However, "play learning argues" that children learn when they play and play when they learn, but no common definition of the concept of play exists (Helenius et al., 2016; Franzén, 2021). Some argue that play must be initiated by the children themselves, while others believe that play also involves adult-initiated activities, characterised by playfulness (Roskos and Christie, 2000). Child-initiated situations are supposed to give children the opportunity to choose their form of play, often endorsed as sociodramatic play, where groups of children practice imaginative role-playing by creating and following social rules pretending to be different family members (Elias and Berk, 2002). But this chosen play situation may not necessarily be free from teachers' involvement (van Oers, 2014). According to Pyle et al. (2018) research, teachers expressed confusion on finding a middle ground between free play and direct instruction. In play, children engage in role-play and construction, which develops their reasoning abilities to propose and test hypotheses while working collaboratively. This supports Sarama and Clements' (2009) argument of developing mathematical competencies at the core of mathematics and is in line with children's thinking. In addition, this social context helps develop children's motor skills, social interactions, and relationships. When children are engaged in a tolerant environment, where they are allowed to make mistakes, they can develop their interactions in a meaningful way (Perry and Dockett, 2007). However, there are few published studies of how free or spontaneous play-oriented situations contribute to mathematical learning, (Zippert et al., 2019). Flottorp and Vyas (2021) found, during collaborative conversation between preschool teachers in a university course, the importance of what the educator observed as children's mathematical learning in spontaneous play situations. Therefore, the researchers emphasise the need of more research to explore the relationship between mathematical knowledge and the educators' ability to follow up for example children's mathematical learning in spontaneous situations. Because of the limited research on spontaneous mathematical situations in preschool it is difficult for preschool teachers to identify and recognise in what way children's engagement with mathematical ideas promote mathematical development. This seems to be the case in Sweden (Helenius, 2018), where this research study takes place. Helenius argues that spontaneous situation can be as valuable as planned situations.

There is a tension between preparing children for school by focusing on the mathematics they need to learn and by using children's play and their own interests to support their mathematical curiosity (Fosse et al., 2018). There is also a tension between arguments for the necessary of subject matter knowledge in mathematics teacher education (Skott et al., 2018). However, Flottorp (2020) studies report how preservice teachers engaging with children in spontaneous situations, reflecting on the situations afterwards reveals that subject knowledge does not automatically lead to the ability to follow up the children's ideas. The informants argue that they have limited time to train this ability to identify children's mathematical thinking in spontaneous situations. A child may not be aware that they are using mathematical thinking to solve a real-world problem in this learning situation (Franzén, (2021). Franzén (2021) argues that if teachers recognise these behaviors, talk about what the child is doing, they encourage independent problem-solving, exploration, mathematical language acquisition and positive learning dispositions. Björklund et al. (2020)

see a strong direction towards research interest of number concepts and arithmetic skills in early childhood conferences. They argue for more research interest in what mathematics means for younger children and therefore the necessity of research in other topics of mathematics than only arithmetic skills. Sim and Xu (2017) identified a gap in the research field, regarding preschool teacher's difficulties to discern children developed mathematical knowledge in spontaneous play situations, which this study addresses.

Play as a mathematical activity

We need more research to explore the relationship between mathematical knowledge and the educators' ability to follow up children's mathematical development in spontaneous situations (Flottor and Vyas, 2021). For example, how the interplay between children being present, observing, and the preschool teacher's ability to respond to mathematical aspects in spontaneous situations. Farrugia (2021) recognised that children presented mathematical ideas even more when she as a researcher interacts with the children. Play in early childhood can be more effectively practiced and advocated for, when these connections are made (Wrigth, 2021). Common assumption is that children's play contributes to a future-oriented practice, central to all development and learning in childhood (Franzén, 2021). Play has been valued for its role in the education and upbringing of children based on the belief that through play the child moves forward. Several studies have shown that children learn mathematics through play (Björklund, 2008; Reikerås et al., 2012). The development of play skills in children has received increasing attention (Hännikäinen and Munter, 2018). However, knowledge about the relationships between play skills and skills in preschool children mathematics development is still lacking (Reikerås, 2020). Reikerås (2020) study's result showed how children's development level of play skills in some way correlate with the opportunity to develop mathematic skills. These findings underscore the importance of considering the children's' play when planning for and implementing mathematical activities. Mathematical skills might also be important for development of play skills, which needs to be explored in further research (Reikerås, 2020).

Bishop (1988) considers play to be a mathematical activity, with both children and adults as possible participants. Furthermore, Bishop (1988) noted that playing is related to *context* and is visualised in terms of *interactions between people*. Worthington and van Oers (2016) findings reveal that many play episodes included aspects of mathematics, and that these show how the children's home cultural knowledge underpinned their pretend play and informed their mathematics. Mathematics takes many forms in relation to its cultural context, but there are six general activities that tend to be common to all cultures when it comes to the production and use of mathematics: counting, measuring, locating, designing, playing, and explaining (Bishop, 1988). When preschool teachers were explicitly asked about the relationship between mathematics and their teaching based on Bishop's mathematical activities (1988), they found new ways to promote mathematical understanding in preschool (Helenius et al., 2016). Reikerås et al. (2020)

visualised in their study how children explored mathematics based on all Bishops six mathematical activities in a play-based outdoor activity. However, the most notable result was the teacher's role for promoting children to explore mathematics in this outdoor learning situation. The mathematical activity playing involves constructing and participating in games with varying degrees of formalised rules that all participants must follow (Bishop, 1988). Playing is considered to involve the mathematical activities of *modelling* (i.e., imagining via inner visualisations of reality), *abstracting* (i.e., identifying the relevant functions to focus on, each situation), guessing, estimating, predicting or assuming, and *hypothetical thinking* (i.e., imagining the potential actions necessary to continue the game/play and begin to engage in abstract thinking). The child has different concepts of reality, which it presents through different models used in play. For example, this could be through a business store or a physician–patient role-play, the themes and goals of the play can be modified by the players (Wood, 2009). When children propose hypotheses in play, they test new directions that may change the model. When children develop the ability to introduce different hypotheses, they also improve their spontaneous abstracting, and reorganise previously constructed mathematical knowledge into a new structure. The teacher's role is significant in enabling the child to develop an understanding of the various aspects of their surroundings (Weisberg et al., 2013; Reikerås et al., 2020).

The focus of this study is children's ability of abstraction (Bishop, 1988) in mathematics and preschool teachers' discernment of this ability. Mathematics is associated with being abstract and symbolic (Helenius, 2018), and understanding can arise from spontaneous abstracting (Williams, 2007). Spontaneous abstracting is the process of discovering complexities as generalisations that arise from abstract abilities (Sim and Xu, 2017; Williams, 2007). This study is based on the analysis of spontaneous free play activities. Free play activities are defined as play, which can be categorised as mathematical activities and contribute to mathematical understanding (Helenius et al., 2016). "This is not to say that mathematics is play, rather than some of the characteristics of playful situations allow those situations to be classified as mathematical. This is important because mathematics education in preschool, if mentioned at all, concentrates on content knowledge." (p. 14).

To analyse spontaneous free play activities, which promote mathematical knowledge development, the study's analysis is based on variation theory, and the assumptions of how to enhance learners' ability to discern aspects of the phenomenon that have not been previously identified (Holmqvist and Selin, 2019).

Problem statement and research focus

To contribute to the lack of research in the field of young children's spontaneous abstracting in mathematics, this study aims to analyse children's mathematical knowledge during preschool play activities and define how teachers' discernment of children's pre-knowledge supports or limits their learning possibilities, which lead to following research questions (RQ).

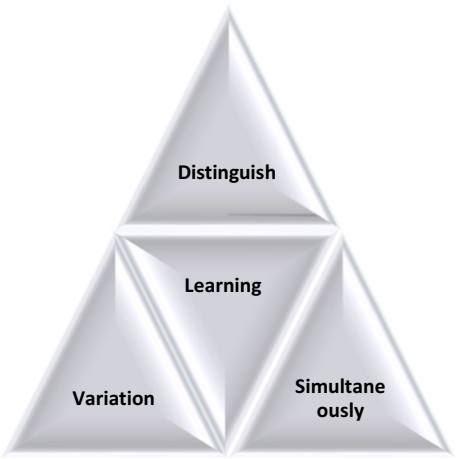

Fig. 1 Variation theory perspective for learning possibilities of the phenomenon

RQ 1 Which situations are defined as free play activities promoting children's spontaneous abstracting regarding mathematical knowledge development?
RQ 2 In what way do teachers discern children's pre-knowledge as support or limitation for mathematical spontaneous abstracting development?

Theoretical approach

In this study, variation theory (Marton, 2015) provides the analytical tools for exploring the necessary conditions for learning. As this study investigates preschool teachers' awareness of children's spontaneous abstracting in mathematics, in free play situations, the assumptions of variation can guide the design of education and enhance learners' abilities to discern aspects of the phenomenon is that have not previously been identified (Holmqvist and Selin, 2019; Svensson and Holmqvist, 2021). Ling et al. (2006) emphasized the importance of considering the wider context of the process by identifying aspects for students based on practical learning using real-life situations. From a variation theory perspective, the analysis identifies (see Fig. 1) how different aspects enable various *learning* abilities in different learning activities (Marton and Booth, 1997; Pang and Marton, 2003; Marton, 2015; Ekdahl, 2021). Several aspects of a phenomenon are discerned *simultaneously*, and as they *vary*, it becomes possible to *distinguish* certain aspects of the phenomenon crucial for learning (Lo et al., 2004; Marton, 2015) (Fig. 1)

Nevertheless, this could be comparable and valuable in understanding why an unexpected situation occurs in the learners' free play. This context enables teachers to develop a greater understanding of mathematics by experiencing playing itself as a mathematical activity and not merely as a means of developing mathematical knowledge through play (c.f. Helenius et al., 2016).

Methodology

This study is part of a one-and-a-half-year development project in a major city in Sweden that intends to strengthen preschool mathematics education. This study was conducted in a pre-school context, and the preschool teachers' intention was to develop children's mathematical knowledge, during a collaborative professional development project arranged by the city between 2014 and 2016.

Sample

Two preschool teachers (A and B) observed the children's play activities, based on Bishop's (1988) mathematical activity playing. The teachers have been working as preschool teachers for 15 respectively18 years. The data consists of written notes by the two preschool teachers with regards to the two documented observations, and notes from the collegial meeting from the two preschool teachers when discussing the two selected documentations. The two preschool teachers are from different preschools and have met during professional development.

Data collection

The data consists of two documented observations. The two observed documentations were based on mathematical aspects with regards to playing (Bishop, 1988) in free play situations. The first item of the documentation is a video-recorded (13 min 25 s) play sequence (preschool teacher A). The second item consists of three photos and notes from the preschool teacher (B) who documented the play situation. Furthermore, the data consists of notes (preschool teacher A and B) from reflections about the two selected documentations for this study during the collaborated meeting.

Empirical data was selected with regards to child-initiated situations, that is, situations in which the children chose the form of play (Wood, 2009). This study was undertaken to determine teachers' discernment of children's mathematical knowledge during preschool play activities, and to define how children's preknowledge supports or limits their learning possibilities. Therefore, it was important that the preschool teachers observed and selected the mathematical situations based on playing themselves, instead of I as a researcher. I typed transcripts of the two documented observations and the reflections from the collaborative meeting to search for connections to mathematical aspects regarding modelling, abstracting, and hypothetical thinking (Bishop, 1988). The selected results section (two documentations) primarily focuses on the critical differences of how children perceive the parts and the entirety (c.f. Holmquist and Selin, 2019) in the phenomenon playing in two different play situations. This is related to the first research question (RQ 1) regarding which situations are defined as free play activities promoting children's spontaneous abstracting regarding mathematical knowledge development.

In the next phase of the analysis, critical aspects are distinguished in the meeting between the teacher, the child, and the phenomenon (Marton, 2015). This relates to the second research question (RQ 2) regarding in what way do teachers discern children's pre-knowledge as support or limitation for mathematical spontaneous abstracting development. The two documented items are interpreted together with their reflections from their collegial conversations during the collegial meeting. This is related to the study's purpose of analysing teachers' discernment of children's mathematical knowledge development during preschool play activities: to define how children's pre-knowledge supports or limits their learning possibilities.

Result

Critical differences discernible within the phenomenon promoting mathematical knowledge development.

First documentation

In the following video-recorded documentation (13 m 25 s), the preschool teacher (A) observed three children (3–4 years of age) during a child-initiated play situation wherein they assume various roles in two scenarios, a picnic, and a camping modelling *simultaneity*. The children wear princess skirts over their trousers and are packing a suitcase when the video begins. Child two wears a princess's crown made of paper on his/her head. The following is an extract from the video (02:01-02:41) presented in Table 1 that provides examples of features of the components of the phenomenon of playing:

It is clear from this extract that wearing a crown is necessary for the play to continue. An aspect that child two distinguishes as a rule or a feature for the hypothetical thinking might be interpreted from putting a skirt (representing a crown) on child one's head. Child one confirms the rule through the expression "I am a princess." In the form of features characterising playing, the components can be interpreted

Table 1 Examples of features regarding Bishop's (1988) components in playing

Participen children	Features	Components
Child one	Oh, I am a princess	Abstracting
	Now, we shall go on a picnic	Modelling
Child two	But we cannot find a crown. Puts a skirt on the other child's head	Hypothetical thinking
Child one	Now, I am like grandmother	Hypothetical thinking
	but I am a princess	Abstracting
	I look like a man, you see	
Child two	But you are not	Hypothetical thinking
	Starts packing a suitcase	Modelling

Table 2 Examples of features understood as hierarchical regarding Bishop's (1988) components in playing

Participen children	Features	Components
Child two	Yes, we will be camping	Modelling
Child one	We are going on a picnic	Modelling
	Both children lie down between two couches	Hypothetical thinking
Child three	May I come with you?	
Child two	If you dress as a princess	Hypothetical thinking
	Child three accepts and puts on a skirt	Modelling
	Child three lies down	
Child one	No, we are going on a picnic	Modelling
Child two	Yes. Child one and child four sit on a bench	Hypothetical thinking
Child two	No! You shall not sit there; you shall sit on the floor. Child one and child three sit down, while child two starts putting quilts and pillows on the floor. […] You sleep on the ground while camping	

Table 3 Examples of abstracting features understood between Bishop's (1988) components in playing

Participen children	Features	Components
Child three	Where is my sleeping pad?	
Child two	This? Child two holds up the smaller sleeping pad	Abstracting
Child three	No, that is too small. That is no sleeping pad!	
Child two	We shall find a sleeping pad later!	
Child three	That is no sleeping pad! Child three abandons the play,	Hypothetical thinking
	and child one and two lie down on the small sleeping pad	Abstracting

as distinct in the above extract (Table 1). Each action performed by the children is interpreted separately. However, the features observed within and between the components may be understood as hierarchical to visualise breaking sequences in the play situation. Breaking involves changing the play pattern or model through various actions. In the following extract, simultaneity enables the identification of two models within the same play situation. Child one continues to prepare for the picnic while child two proceeds to another scenario, camping (05:44–05:58): (Table 2).

It is possible to abstract the events as features of modelling, connecting to a picnic model or/and a camping model until child one expresses an objection. Child one and child three sit on a bench since this abstracting feature is considered part of their modelling and is necessary to continue the play. For the play to continue, the children must identify the action that supports their hypothetical thinking: sitting up rather than lying down during the picnic model. The feature shared by both modelling is that all participants must dress as princesses. In the end, child two clarifies the rules of the camping modelling. All the children agree to sit on the floor. In the

next extract (Table 3), a small sleeping pad (a quilt) represents both a feature for abstracting and as a between abstracting and hypothetical reasoning (07:40–08:05).

The size of the sleeping pad is of crucial importance to child three. It enables us to identify critical differences, through the children's actions, over how the same phenomenon is experienced differently. In the previous extract (Table 3), critical differences emerged in relation to how the children experienced the three components. The sleeping pad's size is a feature for a sleeping pad size for laying on in a camping model that hinders the child to continue playing. Instead, this leads to child three abandoning the play utilizing hypothetical thinking about no longer being able to fit as the surface cannot be shared by three sleeping children, even if they could share it while sitting. This sleep size assessment is an example of mathematical thinking in this play activity. It could also be related to the fact that the two models varied too much towards the sleeping pad's size. In the picnic model, it is a quilt size for a feature abstracting to sit on than in the camping model is a feature for laying down. The rules as connected to features of selected attribute are also distinguished by the participating children, which prevents continued play. In a mathematical activity, the children could offer a larger area so that this play could continue.

In the next extract (Table 4), child one and child two are once again participating in the play. They are sitting on a blanket with some toy plates, toy cutlery and toy food, and pretending to eat from their plates (10:57–11:27):

The food that is discussed in this extract (Table 4) connects the two models' simultaneity in the play situation. Both children abstract toy food and imaginary food, making the food fit in both models. They also express verbally that it is imaginary. Through the ability to abstract, various ways of experiencing the same phenomenon can contribute to the play's continuance (hypothetical thinking). Despite the differences as features for the component abstracting, the play continues. Differences so far as to the children identifying different objects as substitutes to the potential events of the play. The interaction between the children continues, and the rules are expressed (what kind of object can be abstracted as a marshmallow) by child two,

Table 4 Examples of features distinguish different simultaneity regarding Bishop's (1988) component abstracting connected to the two models picnic and camping in the play situation

Participen children	Features	Components
Child two	You have cooked some nice food	
Child one	Next, we will have some candy, which I will pretend to mix with mustard. Makes a stirring motion with a plastic fork on the empty plate	Abstracting
Child two	This is soup. Raises an empty bowl. I think I will have some more	Abstracting
Child one	I have soup too. Stirring with the fork on the empty plate again	Abstracting
Child two	You have a marshmallow there; can you hand it over? Points at the toy food next to child one	Abstracting
Child one	Hands over an object depicting a slice of watermelon	Abstracting
Child two	No, not that one, that one. Points at a smaller, brown, cubical object	
Child two	No, that is a marshmallow	Abstracting

which seemed to help the other child to follow the play situation. Child two negotiated the first object but explained what he/she meant, and child one accepted the rules by picking up the other object.

It seems to be between the component's abstraction and hypothetical thinking that different abstracting features constitute critical differences in experiencing the same phenomenon for the three children and thereby develop their abilities to abstract and think hypothetically which is an important basis for developing mathematical knowledge regarding mathematical understanding. Difficulties or refusal to negotiate regarding the rules that apply to the acceptance of objects as abstraction leads to children abandoning this play. Depending on what categories of mathematical thinking are focused on, not all children are offered to develop mathematical knowledge in this play. At the time, the refusing to replace the quilt as an abstraction for a sleeping pad constituted a break for one of the children to continue the play. If another or larger object had been chosen as an abstraction for sleeping pads, everyone could have continued participating in the play. This indicates that the children negotiated the rules over what object they accepted for abstracting, contributing to abandoning the play and not developing mathematical knowledge.

Second documentation

In the second documentation, three children (five to six years of age) play using a cinema model. The preschool teacher states that "…the interaction is all right. All are familiar with the real setting of a cinema and the creativity is flourishing." The preschool teacher (B) describes how the children assume different roles in the play, such as a manager, a worker and a puppy guarding the cinema. "…there are toys for the guard puppy. The dog may talk over the telephone if needed.

In the play situation outlined above, the interaction is deemed to be acceptable when all children can identify the modelling. What is interpreted as a critical difference in experiencing the same phenomenon in this play situation is that a dog can talk on the telephone if needed. Toys for the dog, toys for the babies, and keyboards or walkie talkies for various work tasks are abstracted as possible hypotheses to develop the model for all children to participate in the play situation.

Discernible abilities by the teachers of children's mathematical development

The preschool teacher outlines the creativity, which is related to hypothetical thinking and contributes to developing the play in the second documentation. He/she documented these different situations in the play as following features in Table 5.

Modelling and abstraction seemed to be perceived as one part (and not separated from each other in the entirety) in the phenomenon playing. Changes regarding mathematical knowledge connected to the three components (modelling, abstracting and hypothetical thinking) seem to contribute to challenges for the preschool teachers.

The puppy that can talk over the telephone the preschool teacher (B) identifies as a critical aspect. This is a critical aspect to abstract in an obscure, changing model,

Table 5 Features distinguished by the preschool teacher based on Bishop's (1988) components for playing

Features	Components
Booking schedule adult movies	Modelling and abstraction
Booking schedule children movies	Modelling and abstraction
Play corner for babies	Modelling and abstraction
Keyboard for various work	Modelling and abstraction
Walkie talkies like phones	Modelling and abstraction
Toys for the little guarding puppy. It can talk on the phone when needed	Modelling and abstraction
All customers are made up. Suddenly someone calls when the film is about to start (it's the boss in the picture here). Then you can NOT book but have to hurry!	Hypothetical thinking

for the continuance of the play (hypothetical thinking). Because one of the children is still based on a concrete thinking where dogs cannot talk on the phone. The preschool teacher (B) identifies a critical aspect in the form of differences among the four children's experiences of the phenomenon. Specifically, it is the feature of abstracting that a dog can talk on the telephone, visualised based on a specific action, which leads to the collapse rather than the continuance of play. In contrast to the first documentation, this model is the same during the observed play situation. It is the role-player in the model, the puppy/dog, that changes in the model. The preschool teacher observed the children's interactions from a creativity perspective and then links to the problem-solving and the children's hypothetical thinking abilities to continue the play. The variation of the role-player puppy/dog is challenged by introducing non-familiar modelling in the cinema with the participating children and led to the collapse of play. It is this according to the preschool teacher that leads to the collapse of the play. While lacking in the experience of the context, the play focuses, the cinema, is not shared by all children, which limits participation. To abstract and think hypothetically in a group requires that the participants have a common experience of context (cinema) or content, which in this case does not exist for all children. In the first documentation, it appears that the child abandoned the play when critical aspects were identified. Critical aspects regarding the ability to abstract relevant functions to continue the play situation include the size of the sleeping pad in two different modelling (picnic and camping) simultaneously. This contributes to an awareness of the relationship between the parts, the entirety, and the phenomenon's structures to providing mathematical knowledge. By distinguishing the features for the ability to abstract specific functions for continued play, both verbal and gestor actions, the relationship's awareness became clear.

The play sequences seem to be broken when the children abandon them. This leads to awareness of the necessity to develop abstract objects or functions for a child to be a part of the play situation. This could be connected to developing abilities in play learning to develop mathematical knowledge. Participants must abide by both the implicit and explicit rules in a play situation. In these sequences, the implicit rules are connected to abstracting different objects and actions, leading

to the children abandoning the play. Suppose the rules change as in the first one through two different modellings (variation). Still, with the same materials and environment (invariance), the children need to agree on the changed rules of the sleeping pads by negotiating and participating in the play. Two children were doing this and presented their abstracting abilities of the features connecting to the sleeping pad in both modelling scenarios: picnic and camping. These abilities give expression to mathematical knowledge through the power to negotiate the rules regarding the same attribute in two play models simultaneously. In the second one, three of the children presented their abilities for the implicit rules by abstracting the dog's actions in two different role-players simultaneity and forming boundaries that linked to hypothetical thinking and contributed to the continuance of the play.

However, highlighting mathematical knowledge in relation to playing's also provides opportunities to identify the players' social interactions in a child-initiated play situation. The number of mathematical learning situations has increased related to mathematical content regarding the development of mathematical knowledge and thus opened opportunities for mathematically developed knowledge. The excerpt (in Table 6) includes reflections on children's various roles and rules in the play. The teaching of mathematical content in playing challenged the preschool teachers. This has contributed to the development of mathematical knowledge in child-initiated play situations. These perceptions may be interpreted as representing a deeper understanding of mathematical content regarding modelling and hypothetical thinking. Various roles and rules in the model contribute to different actions depending on the abilities to abstracting for promote hypothetical thinking.

The second documentation visualised how the phenomenon may vary too much during the play situation. This led to further reflections during the collegial conversations about the new guided teaching being implemented in playing. The feature between abstracting and hypothetical thinking—that a dog can talk over the telephone in the child-initiated play—is considered exclusive because it limits all participants' interactions. At the same time, the collective experience of the context

Table 6 Preschool teachers' (A and B) reflection during the collegial meeting

Preschool teacher	Before	After
A	Mathematics has become clearer and we find mathematics in activities where we did not find it before. We have observed in the children's play how they model and reason about their own reality	They assume various roles and lay down certain rules in the play, which is based on mathematical reasoning
B	[…] A bilingual child, from a different cultural background, wants to join in. This child has never been to the cinema and does not understand why a dog is talking on the telephone. The play collapses, and I intervene	Instead, a game as a play activity works splendidly. Here, the rules are clear and recurring. The terms are equal for all players. The joy is profound. We hope that this will be a way for a child to interact with others in the group in a positive manner, rather than being excluded

is crucial for the model (cinema) and decisive for the continuation of the play. Since this background knowledge lacked a child, the preschool teacher chooses to design and create an activity where all children can participate in the same context. Based on the three components of playing, the preschool teacher offered the child an opportunity to experience more features in relation to the components within the same phenomenon. With support of the three play components, the preschool teacher provided the same opportunities for mathematically developed knowledge but with more explicit rules as features (recognition for the child) through a play situation.

Through the phenomenon of playing (or play), which is considered to promote creativity and develop mathematical knowledge, the preschool teacher's role is challenged. Playing as a phenomenon may, Bishop notes, also consist of a more easily interpreted, regulated set of play rules to develop mathematical knowledge. This opportunity to discover more mathematical knowledge through playing may be seen to have contributed to the teachers' reflections (presented above) about their importance to guide the children when interruptions occur in free play situations initiated by the children. Instead of just observing in the free play situation, the teacher reflected about guided teaching situations with more explicit rules so that all children participating could contribute to the play situation. It requires observed and reflective observations based on in-depth subject theoretical knowledge.

Discussion

To contribute to the lack of research in the field of young children's spontaneous abstracting in mathematics, this study aims to analyses teachers' discernment of children's mathematical knowledge during spontaneous preschool play activities and define how children's pre-knowledge supports or limits their learning possibilities. Fosse et al. (2018) showed that mathematics in preschool is more linked to teaching and learning than it is in the play itself. Therefore, self-documented mathematics activities became important since it is the preschool teachers who have observed and selected the spontaneous mathematical learning opportunities for children's exploration of mathematics in preschool. The preschool teacher's role in introducing mathematical content in child-initiated situations, based on playing (Bishop, 1988), challenged the learning of new concepts in this study (see Table 6). In addition, these new concepts support and highlighted which situations the preschool teachers defined mathematical learning (Table 5) connected to Bishop's (1988) playing in the observed child-initiated situations (Helenius et al., 2016). The teacher must guide the children to develop an understanding of various aspects of objects, actions, rules, and language that can contribute to a mediating act for an increased understanding of what is happening (Franzén, (2021; Wickström et al., 2019). However, it is also vital that the teacher has specific knowledge and an understanding of how to present these aspects in the guided teaching situation, to offer children mathematical knowledge in free play situations (Wrigth, 2021). As a result, this happened when the preschool teacher (Table 6) reflected, in the second observation, about developing

mathematical knowledge by offering clearer rules so that all children gained an understanding (Farrugia, (2021; Flottorp and Vyas, 2021).

In the second observation, the preschool teacher indicated reflections about his/her supporting role (c.f. guiding role) within mathematical knowledge regarding play (Farrugia, (2021). Thus, the preschool teacher offered the same *understanding* concerning Bishop's (1988) three components in the mathematical activity, but with interpreted rules, which are more explicit in a game situation (Helenius et al., 2016; Wickström et al., 2019). This is possible to interpret when the preschool teacher offered new notions within the mathematical content that helped to visualise new notions within mathematical knowledge as in social interactions in the play itself (see Table 6). This was ascertainable because the preschool teacher offered new notions through guided teaching within the mathematical content and as a result, helped to visualise social interactions in the play (Pyle et al., 2017; Reikerås, 2020). According to Wickström et al. (2019) the educator in this situation took on a variety of roles and provided a range of play situations when he/she suggested mathematical knowledge development with regards to playing with more explicit rules as in a game situation (Table 6). Furthermore, this suggested a guided play situation regarding a familiar game situation in a broader cultural context for promoting spontaneous abstracting regarding mathematical knowledge development (Worthington and van Oers, 2016).

Notably, the result in this study showed (by using variation theory to analyses the two observed documentations by the preschool teachers) that abstracting arise as a critical aspect for children to develop mathematical knowledge based on Bishop's (1988) playing. and therefore, be a part of the play situation. Even in the reflections by the teachers this arise as a critical aspect. Instead of discussions connected to abstraction, they were discussing how the children modelled play based on their own reality and how they undertook different roles in the play situation. They talk about modelling from reality and role-playing during the reflections about the two documentations. However, it could link to discussions about how critical aspects regarding to spontaneous abstracting limit or support children's mathematical knowledge development in a free play situation (see Table 5). Sim and Xu (2017) identified a gap in previous research about preschool teachers' ability to discern how children develop their own knowledge in free play situations. In this study, children are limited or supported in developing their own mathematical knowledge depending on their abstract abilities (see Table 4 and 5). Reikerås et al. (2020) showed in their result the importance of the preschool teacher's interaction for promoting mathematical knowledge based on Bishops (1988) six mathematical activities of mathematical. Compared to the written reflections during the collegial conversations (Table 6) and the identified mathematical knowledge development (Table 5) connected to the components by playing (Bishop, 1988), this study defined how teachers' discernment of children's pre-knowledge supports or limits their learning possibilities in free play. In the two selected documentations, limitations, and support for developing knowledge are highlighted in relation to the collapse of the different free play situations (see Table 3). Through observed and collected information about children's potential learning from the teachers, we can presume that the abstract abilities are critical for mathematical developing knowledge regarding playing.

In the second documentation, the preschool teacher displays notions about variations in identifying children's experiences of the phenomenon in different guided teaching situations (see Table 6). Reikerås (2020) identified correlation between mathematical skills and play skills. By offering a changed situation yet retaining the same mathematical knowledge (Pang and Marton, 2003; Ekdahl 2021), the teacher provided the child with an opportunity to succeed even in the child-initiated play. Farrugia (2021) and Wrigth (2021) provide the importance of teacher's interaction with the children in spontaneous situations. The preschool teacher in this research study provided an opportunity to discuss what happened in the initial interaction between participants. As a result, it contributed to the children gaining additional mathematical knowledge and thus allowed them to be part of the free play (see Table 3 and 4). This encourages discussion about what we expect to happen in the changed situation. The teachers showed how they identified children's mathematical knowledge in new situations and how this contributed to more guided instructions through their collegial conversations (see Table 6). In this study mathematical knowledge is related to playing (Bishop, 1988). Worthington and van Oers (2016) showed how the home cultural context informed underlying mathematical ideas. By providing a wider responsibility to include more social activities through playing, such as in this study based on Bishop's (1988) six mathematical activities, may have contributed to increasing teachers' abilities to define and discern children's abstract abilities for promoting mathematical knowledge development in free play situations (see Table 3, 4, 5 and 6).

Conclusion

First, this study identified children's knowledge regarding playing and is presented through children's abilities to abstract both objects and actions as important for developing mathematical knowledge development. Everyday mathematics is assumed through a play-based teaching approach to develop children's mathematical knowledge. Therefore, emphasis in this study—regarding the findings about encouraging children's development of abstract abilities—is on the importance of identifying that all children must be inclusive in a free play situation to contribute to mathematical understanding.

Second, this study presented how both teachers and children showed difficulties in presented abstract abilities in child-initiated play situations.

Implication

This study's findings emphasise the necessity for future research on this topic, such as playing to provide and widen teachers' knowledge in identifying children's mathematical abstracting knowledge in spontaneous play situations. Furthermore, the findings contribute to an increased focus in the curricula which is of importance for both providing mathematical skills through play and play skills through mathematical knowledge development. The latter to increase the opportunities for all children

to participate in play and thereby offering opportunity to develop mathematical thinking.

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Data availability These data are available and secured on Malmö University's homepage.

Declarations

Competing interest The corresponding author states that there is no conflict of interest in this manuscript.

Ethical approval All preschool teachers in this study have agree to participate in this research study and are anonymizes.

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