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ARTICLE

Representation of the midnight sun in Greek and Indian astronomical texts

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Abstract
The midnight sun is a fascinating phenomenon observed near the Earth's poles. Whether it is known through observation or through theory, it is bound to provoke one's curiosity, since it involves the most familiar object in the sky, viz. the sun. This paper seeks to investigate the various ways in which this phenomenon was understood or considered in ancient literature, focusing on Indian astronomical literature. It is found that some texts primarily describe the phenomena at the poles themselves, while the remaining additionally describe the latitudes at which one would see days that are a few months long. A comparison is made across texts in order to facilitate an understanding of the methods and the accuracy of the texts discussed.

Keywords Bhāskara · Indian astronomy · Midnight sun · Polar night · Ptolemy · North pole · *Siddhānta* · Varāhamihira

1 Introduction

The midnight sun refers to a phenomenon observed in regions having high latitudes, where the sun remains above the horizon for more than 24 h, leading to the sun being visible at midnight. A related phenomenon is the polar night, where the sun remains under the horizon for more than 24 h. The phenomena of midnight sun and polar night occur due to Earth's axial tilt with respect to the ecliptic, that causes the poles and the region surrounding them to remain in perpetual darkness or light for a certain period of time. The longest daylight and night periods occur at the poles themselves: day time lasts from the March equinox to the September equinox in the north-pole and night time for the remaining 6 months of the year, with the reverse being the situation in the south-pole. The period of perpetual daylight or darkness decreases as the latitude decreases, until the polar circles of latitude (Arctic Circle in the north and Antarctic Circle in the south) after which these are no longer observed.

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Such a phenomenon is indeed spectacular to any observer since the sun, which is the most familiar object in the sky, does not set or rise for a period of time as it does elsewhere on the globe. Due to this, it was noted by observers who either visited places where these polar phenomena were active, or was deduced through calculation. The ideas and methods with which some ancient societies understood the phenomenon of the midnight sun are presented here, focusing on ancient India. This paper seeks to bring out the understanding, methods and accuracy with which the ancients discussed the midnight sun. By analyzing these aspects, it is possible to gain a deeper insight into their thought process.

2 Computation

As will be elaborated in the following sections, many astronomers provide values for the latitudes at which a midnight sun can be observed. In order to verify the accuracy of these figures, it is necessary to compute the latitude using the equivalent modern formulae.

Let the observations be made from some place with near-polar northerly latitude ϕ. The declination of the sun is represented by δ. The situation is represented in Fig. 1.

For the midnight sun to occur, the diurnal circle (*ahorātravṛtta*) must reach a point during summer such that it is completely above the horizon. Hence, there must be

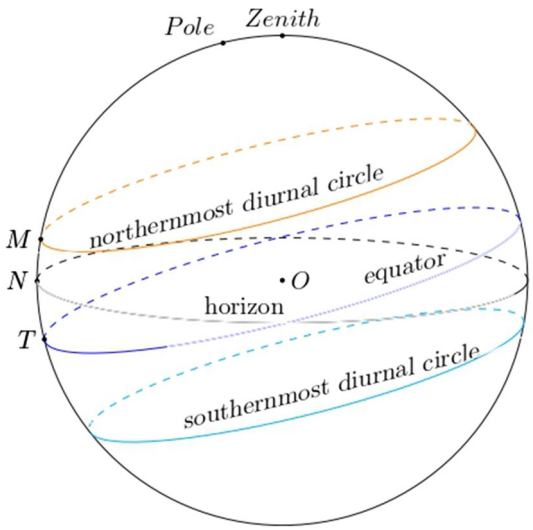

Fig. 1 An example of the celestial sphere at a near-polar latitude showing important circles and points in the context of the midnight sun

some value(s) of δ for which M remains above N, i.e., arc PM is less than arc PN, or equivalently, TN is less than TM (where P is the pole). This is given by the inequality:

$$\delta > 90 - \phi \tag{1}$$

The polar night will also occur given the same condition. Further, for solar longitude λ and maximum declination (*paramāpakrama*) ϵ, the declination of the sun is given by:

$$\sin\delta = \sin\lambda \sin\epsilon \tag{2}$$

Substituting δ from Eq. (2) into (1), we have:

$$\sin^{-1}(\sin\lambda \sin\epsilon) > 90 - \phi \tag{3}$$

This equation connects the latitude at which the midnight sun is visible to the longitude of the sun.

Several ancient texts mention the latitude of the places where the sun remains in the sky for a few months at a stretch. In Indian texts specifically, the latitudes are given for places where the sun is seen for 2 months, 4 months and 6 months. These are now calculated using formula (3).

The first case considered is that of the sun being continually visible for 2 months of the year. This happens when the solar longitude is between 60° and 120°.[1] Therefore, the first value of λ that satisfies (3) is 60°. Substituting this into (3), we get:

$$\sin^{-1}(\sin 60° \sin\epsilon) > 90 - \phi \tag{4}$$

The maximum declination is the same as the obliquity of the ecliptic. In Indian texts, this is usually taken as 24°. Incorporating this into the inequality, the following is the minimum latitude (rounded to the nearest arc second) that satisfies this inequality:

$$\phi = 69° \ 22' \ 31'' \tag{5}$$

Fig. 2 depicts this situation in case of the midnight sun. Similarly, if the sun is visible for 4 months in the year, corresponding to the solar longitudes 30° to 150°, the first longitude for which the inequality is satisfied is 30°. The latitude (rounded to the nearest arc second) is then equal to:

$$\phi = 78° \ 15' \ 58'' \tag{6}$$

This is represented in Fig. 3 for the midnight sun. The place where the sun is visible for 6 months of the year is the north pole, which has a latitude of 90°. The situation here is represented in Fig. 4 for the midnight sun. As seen, the horizon and the celestial equator coincide (the lower horizontal circle) and the highest diurnal circle of the sun in the sky is above this (the upper horizontal circle).

In summary, the values arrived at through computation are presented in Table 1.

3 Greek views

The references to the midnight sun and polar night in Greek literature are considered below in brief.

3.1 Homer

One of the earliest available sources in Ancient Greek literature, interpreted as referring to the polar night, is Homer's *Odyssey* (c. seventh century BCE). He states:

> ...And it reached the extreme boundaries of the deep-flowing ocean; where are the people and city of the Cimmerians, covered with shadow and vapor, nor does the shining sun behold them with his beams, neither when he goes towards the starry heaven, nor when he turns back again from heaven to earth but pernicious night is spread over hapless mortals ... (*Odyssey*, Book XI, 1891, pp. 146–147)

While the information given in the epic must not be taken literally, the above extract can be interpreted as showing

[1] The longitude here is reckoned from the March equinox (*sāyana meṣadi*) as is convention in the Indian texts. This implies that the sun will have maximum declination when its longitude is 90° and minimum declination when its longitude is 270°, which correspond to *sāyana karkādi* and *sāyana makarādi* in Indian reckoning.

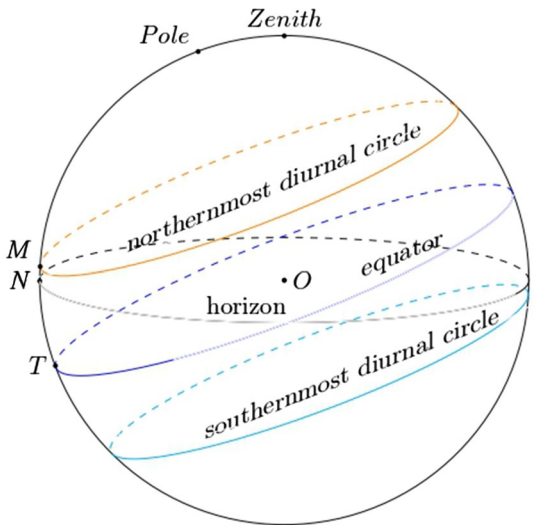

Fig. 2 The celestial sphere at latitude 69° 21′ 31″N where the midnight sun lasts for 2 months

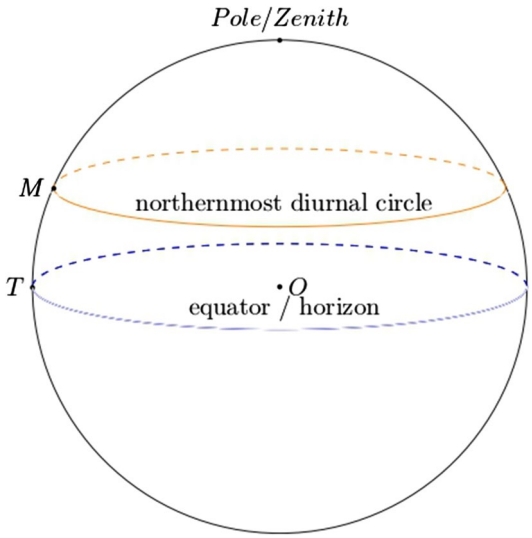

Fig. 4 The celestial sphere at latitude 90°N where the midnight sun lasts for 6 months

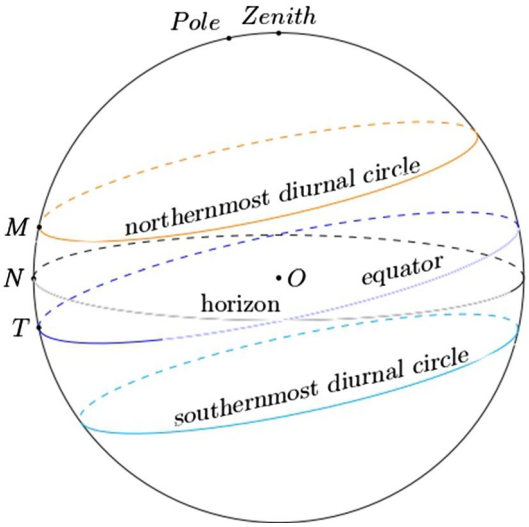

Fig. 3 The celestial sphere at latitude 78° 15′ 58″N where the midnight sun lasts for 4 months

at least some level of knowledge pertaining to the situation in the northerly latitudes, i.e., the fact that the sun does not shine there for extended periods of time. Given that the Greeks are located in the Mediterranean, they would not have seen this phenomenon directly. However, there were extensive trade routes moving northwards from the Mediterranean. Therefore, it is believed that this information was probably gathered from sailors, traders, and others who had visited the northern regions (Chevallier, 1984).

Table 1 Values for latitudes at which the midnight sun will be visible for 2, 4 and 6 months at a time (assuming 24° obliquity of the ecliptic, rounded to the nearest arc second)

Duration (months)	Latitude
2	69° 22′ 31″
4	78° 15′ 58″
6	90°

3.2 Pytheas

The Greek explorer Pytheas of Massalia is said to have sailed to the northern areas c. 325 BCE. He circumnavigated the British Isles and observed the polar night somewhere at a place that he designated as Thule, which was within the Arctic circle. This is believed to be Iceland, the Shetland Islands, or the coast of Norway. He then sailed back to the Mediterranean. The original record of his voyage is now lost, but he is referred to by several later sources (McPhail, 2014, p. 247).

3.3 Greek philosophers

Textual evidence for exploration of the Arctic after Pytheas is limited. Much of the development during and after this time is in the form of theory. Given that the midnight sun phenomenon is intimately related to the idea of Earth being a sphere, it is useful to consider how the concept of a spherical earth was conceived by the Greeks.

Greek philosophers from the fifth century BCE onwards considered the Earth to be a sphere (Dicks, 1970, p.72). To this effect, we can quote Plato (c. fourth century BCE):

> Well then, my friend," said he, "to begin with, the earth when seen from above is said to look like those balls that are covered with twelve pieces of leather ... (*Phaedo*, 2021, 110b)

Plato's *Phaedo* is presented as a dialogue where Socrates explains his ideas to others, including Plato. Plato does not provide any proof for the idea of a spherical earth. However, Plato's disciple Aristotle (c. fourth century BCE) expanded upon this idea by connecting it to astronomical observations:

> Either then the earth is spherical or it is at least naturally spherical. ... The evidence of the senses further corroborates this. How else would eclipses of the moon show segments shaped as we see them? ... There is much change, I mean, in the stars seen in Egypt and in the neighborhood of Cyprus which are not seen in the northerly regions; and stars, which in the north are never beyond the range of observation, in those regions rise and set. All of which goes to show not only that the earth is circular in shape, but also that it is a sphere of no great size ... (*De caelo*, 1922, 297b–298a)

Here, we see a development of thought, where the concept of a spherical earth, has been linked with physical observations. Various astronomers developed this concept of spherical earth after Plato and Aristotle. Given that the observations noted above are astronomical, it is not far-fetched to assume that the midnight sun was also considered in this context, especially during the time after Pytheas' voyage, however, several texts such as those of Eratosthenes are now lost (González, 2000), hence it is not possible to completely trace the development of ideas.

3.4 Ptolemy

A clear and detailed reference of the midnight sun comes from Ptolemy (c. second century CE). In his *Almagest* (II 6), he classifies locations of the world based on the length of the day at summer solstice, starting from 12 h at the equator to 24 h at the arctic circle. For places beyond the arctic circle, he provides calculated latitudes at which the sun would not set for several months successively (*Almagest*, 1984, II 6, pp. 34–39). The latitudes provided by him are reproduced in Table 2. Furthermore, in his opinion, the obliquity of the ecliptic amounts to 23° 51′ 20″ (*Almagest*, 1984, II 4). Hence, it becomes necessary to compute the latitude values separately. These have also been presented in the table, rounded to the nearest arc second.

Table 2 Ptolemy's midnight sun latitudes

Duration (months)	Latitude (Ptolemy)	Latitude (Computed)
1	67°	67° 0′ 18″
2	69° 30′	69° 29′ 51″
3	73° 20′	73° 22′ 58″
4	78° 20′	78° 20′ 0″
5	84°	83° 59′ 30″
6	90°	90°

Ptolemy's values are clearly quite accurate. This indicates that, by this time, the idea of midnight sun was fully incorporated into astronomy and geography.

4 Indian views

Indian authors of the *siddhāntas* (astronomical treatises) during the *siddhāntic* period (up to the twelfth century CE[2]) mention the phenomena of midnight sun and polar night, as occurring at the poles. It is not known if any Indians travelled that far north, or received information from others, including the Greeks, about the northerly latitudes directly. Instead, most mentions appear to have been derived through calculation from astronomical parameters.

Indians were aware of the spherical earth, just as the Greeks. The earliest text available in whole among the *siddhāntas*, the *Āryabhaṭīya*, mentions this:

> ... भूगोलः सर्वतो वृत्तः ।
> ... *bhūgolaḥ sarvato vṛttaḥ* |
> (*Āryabhaṭīya, Golapāda,* 1976, 6b)

The earth is spherical (lit. circular in all directions).

This idea is carried over in all *siddhāntas* including that of Bhāskara:

> भूमेः पिण्डः शशाङ्कज्ञकविरविकुजेज्यार्किनक्षत्रकक्षा-
> वृतैर्वृत्तो वृतः सन् ...।

[2] The earliest text that can be confidently dated and which is available in its complete form is *Āryabhaṭīya* of Āryabhaṭa, dating to the end of the fifth century CE. There are, however, several texts such as *Sūryasiddhānta*, which have not been dated conclusively. There is also an indication in the commentaries, as well as the *Pañcasiddhāntikā* of Varāhamihira, that there were other *siddhāntas* prior to this period, which are no longer available today. Hence, the beginning of the *Siddhāntic* period cannot be stated with adequate confidence. The period after the twelfth century did include various developments in Indian astronomy, however these are not considered in this paper.

bhūmeḥ piṇḍaḥ śaśāṅkajñakaviravikujejyārkinakṣatrakakṣā vṛtairvṛtto vṛtaḥ san ... |
(*Siddhāntaśiromaṇi, Golādhyāya, Bhuvanakośa*, 1981, 2)

The mass (*piṇḍa*?) of the earth, being surrounded by the orbits of the Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, and the stars, is a sphere (*vṛtta*, lit. circle).

Hence, the earth was understood to be a sphere in these texts. With respect to the midnight sun, texts commonly state the latitude at which the sun is visible for 2 months, 4 months and 6 months respectively or, in some cases, prescribe calculations for these latitudes. Specific references are now considered.

4.1 Āryabhaṭa

Āryabhaṭa (fl. 499 CE), in his *Āryabhaṭīya*, only mentions about the sun being visible for 6 months of the year at each of the poles. He explains this in the following half-verse:

रविवर्षार्धं देवाः पश्यन्त्युदितं रविं तथा प्रेताः ।

ravivarṣārdhaṃ devāḥ paśyantyuditaṃ raviṃ tathā pretāḥ |
(*Āryabhaṭīya, Golapāda,* 1976, 17a)

The gods (present at the north pole) see the sun risen for half a solar year, and similarly the demons (present at the south pole, see the sun for half a year).

As can be seen, the primary reference pertains to the north pole itself. Other descriptions of the midnight sun are not found.

4.2 Varāhamihira

Varāhamihira (sixth century CE) mentions the following in connection with the midnight sun in the *Pañcasiddhāntikā*, in a chapter that is not explicitly connected to any one of the five *siddhāntas* propounded in the text:

त्रिंशतीं त्रिसप्ततियुतां गत्वोदग्योजनत्रिभागं च ।
उज्जयिनीतो विरमति पर्यस्तोऽयं भगणगोलः ॥

षष्टिर्नाड्यस्तस्मिन् सकृदुदिते दृश्यते दिवसनाथः ।
परतः परतो बहुतरमा षण्मासादिति सुमेरौ ॥

योजनपञ्चनवांशांस्तृर्यधिकांश्च चतुःशतीमुदगवन्त्याः ।
गत्वा न धनुर्मकरौ कदाचिदपि दर्शनं व्रजतः ॥

तस्मादेव स्थानाद्व्यशीतियुक्तां चतुश्शतीं साग्राम् ।
नोदयमुपयान्त्यलिमृगघटचापधराः कदाचिदपि ॥

षडशीतिं पञ्चशतीं त्र्यंशोनं योजनं च तत एव ।
गत्वान्त्यं चक्रार्धं नोदेत्याद्यं न यात्यस्तम् ॥

लङ्कास्था भूलग्नां नभसो मध्यस्थितां च मेरुगताः ।
ध्रुवतारामीक्षन्ते तदन्तरालेऽन्तरोपगताः ॥

सकृदुदितः षण्मासान् दृश्योऽर्को मेरुपृष्ठसंस्थानाम् ।
मेषादिषु षट्सु चरन् परतो दृश्यः स दैत्यानाम् ॥

triśatīṃ trisaptatiyutāṃ gatvodagyojanatribhāgaṃ ca |
ujjayinīto viramati paryasto'yaṃ bhagaṇagolaḥ ||
ṣaṣṭirnāḍyastasmin sakṛdudite dṛśyate divasanāthaḥ |
parataḥ parato bahutaramā ṣaṇmāsāditi sumerau ||
yojanapañcanavāṃśāṃstryadhikāṃśca catuḥśatī mudagavantyāḥ |
gatvā na dhanurmakarau kadācidapi darśanaṃ vrajataḥ ||
tasmādeva sthānāddvyaśītiyuktāṃ catuśśatīṃ sāgrām |
nodayamupayāntyalimṛgaghaṭacāpadharāḥ kadācidapi ||
ṣaḍaśītiṃ pañcaśatīṃ tryaṃśonaṃ yojanaṃ ca tata eva |
gatvāntyaṃ cakrārdhaṃ nodetyādyaṃ na yātyastam ||
laṅkāsthā bhūlagnāṃ nabhaso madhyasthitāṃ ca merugatāḥ |
dhruvatārāmīkṣante tadantarāle'ntaropagatāḥ ||
sakṛduditaḥ ṣaṇmāsān dṛśyo'rko merupṛṣṭhasaṃsthānām |
meṣādiṣu ṣaṭsu caran parato dṛśyaḥ sa daityānām ||
(*Pañcasiddhāntikā*,1993, 13.21–27).

$373\frac{1}{3}$ *yojanas* north of Ujjain, the celestial sphere moves differently (*paryastaḥ*?). At that place, the sun is visible for 60 *nāḍīs* (1 whole day) having risen only once. It increases further and further [with increase in latitude] until it is for six months at the pole. $403\frac{5}{9}$ *yojanas* north of Ujjain, *Dhanu* and *Makara* are never seen. 482 and a fraction of *yojanas* north of that place (Ujjain) itself, *Vṛścika* (*ali*), *Dhanu* (*cāpadhara*), *Makara* (*mṛga*) and *Kumbha* (*ghaṭa*) never rise. $586\frac{2}{3}$ *yojanas* [north] from there (Ujjain), the latter half of the zodiac (*Tulādi*) never rises and the former half (*meṣādi*) never sets. Those present at the equator see the pole star on the ground (horizon), while those at the pole see it at the zenith, and those between these two places see it in between. Those present on *Meru* see the sun as rising once and remaining visible (risen), when it moves in the six *rāśis* beginning with *Meṣa*. The sun is visible [in the same way] for demons when in the other [six *rāśis*].

Varāhamihira has given the latitude of the places in terms of the *yojanas* from Ujjain. The latitude of Ujjain is taken to be 24° in *Pañcasiddhāntikā* (13.10). Based on the distance to the north pole from Ujjain, given above as $586\frac{2}{3}$ *yojanas*, it can be inferred that the circumference of the earth according

Table 3 Varāhamihira's midnight sun latitudes

Duration (months)	*Yojanas* north of Ujjain	Latitude
2	$403\frac{5}{9}$	69° 24′
4	~ 482	~ 78° 13′ 30″
6	$586\frac{2}{3}$	90°

to Varāhamihira, is 3200 *yojanas*. From this, we can calculate the latitudes of the other places mentioned above, which have been given in Table 3.

4.3 *Sūryasiddhānta*

The following verses in the *Sūryasiddhānta*[3] pertain to the midnight sun:

अयनान्ते विलोमेन देवासुरविभागयोः ।
नाडीषष्ट्या सकृदहर्निशाप्यस्मिन् सकृत्तथा ॥

तदन्तरेऽपि षष्ट्यन्ते क्षयवृद्धी अहर्निशोः ।
परतो विपरीतोऽयं भगोलः परिवर्तते ॥

ऊने भूवृत्तपादे तु द्विज्यापक्रमयोजनैः ।
धनुर्मृगस्थः सविता देवभागे न दृश्यते ॥

तथा चासुरभागे तु मिथुने कर्कटे स्थितः ।
नष्टच्छायामहीवृत्तपादे दर्शनमादिशेत् ॥

एकज्यापक्रमानीतैर्योजनैः परिवर्जिते ।
भूमिकक्षाचतुर्थांशे व्यक्षाच्छेषैस्तु योजनैः ॥

धनुर्मृगालिकुम्भेषु संस्थितोऽर्को न दृश्यते ।
देवभागेऽसुराणां तु वृषाद्ये भचतुष्टये ॥

मेरौ मेषादिचक्रार्धे देवाः पश्यन्ति भास्करम् ।
सकृदेवोदितं तद्वदसुराश्च तुलादिगम् ॥

ayanānte vilomena devāsuravibhāgayoḥ |
nāḍīṣaṣṭyā sakṛdaharniśāpyasmin sakṛttathā ||
tadantare'pi ṣaṣṭyante kṣayavṛddhī aharniśoḥ |
parato viparīto'yaṃ bhagolaḥ parivartate ||

ūne bhūvṛttapāde tu dvijyāpakramayojanaiḥ |
dhanurmṛgasthaḥ savitā devabhāge na dṛśyate ||
tathā cāsurabhāge tu mithune karkaṭe sthitaḥ |
naṣṭacchāyāmahīvṛttapāde darśanamādiśet ||
ekajyāpakramānītairyojanaiḥ parivarjite |
bhūmikakṣācaturthāṃśe vyakṣāccheṣaistu yojanaiḥ ||
dhanurmṛgālikumbheṣu saṃsthito'rko na dṛśyate |
devabhāge'surāṇāṃ tu vṛṣādye bhacatuṣṭaye ||
merau meṣādicakrārdhe devāḥ paśyanti bhāskaram |
sakṛdevoditaṃ tadvadasurāśca tulādigam ||
(*Sūryasiddhānta*, 2015, 61–67)

A day of 60 *nāḍīs* occurs once at the end of the solstices in the portions (hemispheres) of the gods and demons inversely (*vilomena*). A night of the same length also occurs once here. In the middle region, the increment and reduction of night and day are less than 60 [*nāḍīs*]. Beyond this, the celestial sphere revolves differently. The number of yojanas calculated from the declination, derived from the sine of two [*rāśis*], is subtracted from a quarter of the earth's circumference. At that distance from the equator, in the northern hemisphere, the sun is not visible when in *Dhanu* and *Makara* (*mṛga*), and in the southern hemisphere, when in *Mithuna* and *Karkaṭa*. In the quarter portion of the earth's circumference, where the shadow is lost (or not present), the sun is visible. The number of yojanas, calculated from declination, derived from the sine of one [*rāśi*], is subtracted from a quarter of the earth's circumference. At a place with that latitude, the sun is not visible, in the northern hemisphere, when in *Vṛścika* (*ali*), *Dhanu*, *Makara* (*mṛga*) and *Kumbha*, and in the southern hemisphere, when in the four *rāśis* beginning with *Vṛṣabha* (*vṛṣa*). At *Meru* (the North Pole), the gods see the sun, [which] rises [only] once, when it is in half the zodiac beginning with *Meṣa*, similarly the demons [for half the zodiac] beginning with *Tulā*.

The second half of verse 64 is obscure. In any case, the above verses state that method by which the latitudes are to be calculated. The first calculation produces the distance in *yojanas* from the equator to the point where the sun is visible for 2 months without setting. The second one does the same but for the point where the sun is visible for 4 months instead of two. The calculation is as follows:

First, the sine of two *rāśis* has to be calculated and from it the declination has to be derived. This is done using the formula stated above, which is reproduced here for convenience:

$$\sin\delta = \sin\lambda \sin\epsilon$$

With this declination, the equivalent *yojanas* have to be calculated. This is derived as follows: The value of the earth's circumference in *yojanas* is usually given in the text.[4] The circumference can be considered as a

[3] The currently available *Sūryasiddhānta* has not been confidently dated but is known to postdate Varāhamihira.

[4] *Sūryasiddhānta* (1.59) states that the diameter of the earth is 1600 *yojanas* and that the circumference is this value multiplied by the square root of 10, which amounts to about 5059.64 *yojanas*.

great circle, and hence equal to 360°. The declination δ must be thought of as an angle, and the corresponding *yojanas* must be derived proportionately. This gives the distance from the pole to the location in question. This result has to be subtracted from a quarter of the earth's circumference to get the *yojanas* to the location from the equator. This distance is of the place where the sun is visible for 2 months. Doing the same with the sine of one *rāśi* results in the place where the sun is visible for 4 months.

4.4 Brahmagupta

Brahmagupta (seventh century CE) provides the following regarding the midnight sun:

देवाः सव्यगमसुराः पश्यन्त्यपसव्यगं रविं क्षितिजे ।
विषुवति समपश्चिमगं निरक्षदेशस्थिताः पुरुषाः ॥

सौम्यमपमण्डलार्धं मेषाद्यं सव्यगं सदा देवाः ।
पश्यन्ति तुलाद्यर्धं दक्षिणमपसव्यगं दैत्याः ॥

पश्यन्ति देवदैत्या रविवर्षार्धमुदितं सकृत् सूर्यम् ।
शशिगाः शशिमासार्धं पितरो भूस्था नराः स्वदिनम् ॥

devāḥ savyagamasurāḥ paśyantyapasavyagaṃ raviṃ kṣitije |
viṣuvati samapaścimagaṃ nirakṣadeśasthitāḥ puruṣāḥ ||
saumyamapamaṇḍalārdhaṃ meṣādyaṃ savyagaṃ sadā devāḥ |
paśyanti tulādyardhaṃ dakṣiṇamapasavyagaṃ daityāḥ ||
paśyanti devadaityā ravivarṣārdhamuditaṃ sakṛt sūryam |
śaśigāḥ śaśimāsārdhaṃ pitaro bhūsthā narāḥ svadinam ||
(*Brāhmasphuṭasiddhānta, Golādhyāya,* 1966, 6–8)

On the day of equinox, the gods perceive the sun moving clockwise at the horizon, the demons perceive it as moving anticlockwise [at the horizon], and the humans living at the equator perceive it as moving westwards. The gods always see the northern half of the ecliptic that begins with *Meṣa* in a clockwise direction, while the demons always see the southern half of the ecliptic that begins with *Tulā* in an anticlockwise direction. [Because of this] the gods and demons see the sun risen once [in the sky] for half a solar year. The manes residing on [the far side of] the moon [see it] for half a lunar month. The humans, residing on earth, [see it] for their own day.

As is seen from these, the main references pertain to the situation at the poles themselves.

4.5 Lalla

Lalla (eighth century CE) mentions the following with respect to the midnight sun in *Śiṣyadhīvṛddhidatantra*:

पञ्चभिरधिकाः सप्ततिरंशा यस्मिन् पलस्य विषये स्युः ।
तत्र न वृश्चिककार्मुकमकरघटा दृश्यतां यान्ति ॥

pañcabhiradhikāḥ saptatiraṃśā yasmin palasya viṣaye syuḥ |
tatra na vṛścikakārmukamakaraghaṭā dṛśyatāṃ yānti ||
(*Śiṣyadhīvṛddhidatantra,* 16.20)

In that place where the latitude is 75°, *Vṛścika*, *Dhanu* (*kārmuka*), *Makara* and *Kumbha* (*ghaṭa*) are not visible.

Based on Bhāskara II's criticism of Lalla's value (quoted in the Discussion section below), it is known that Lalla's values for the latitude where the sun is visible for 2 months continually is 66° 30′.

4.6 Vaṭeśvara

The following reference is found pertaining to the midnight sun in Vaṭeśvara's *Gola* (c. ninth-tenth century CE):

यस्मिन् पलांशा रसकोशतुल्या न तत्र दृश्यो मकरो धनुश्च ।
पले शरागांशमिते ह्यदृश्याः कुम्भालिसप्ताङ्गमृगाः सदैव ॥

yasmin palāṃśā rasakośatulyā na tatra dṛśyo makaro dhanuśca |
pale śarāgāṃśamite hyadṛśyāḥ kumbhālisaptāṅgamṛgāḥ sadaiva ||
(*Vaṭeśvara Siddhānta and Gola*, 1986, 4.17)

Makara and *Dhanu* are not visible [at the place] where the latitude is 66°. *Vṛścika* (*ali*), *Dhanu* (*saptāṅga*[5]), *Makara* (*mṛga*) and *Kumbha* are never visible [at the place] where the latitude is 75°.

Since the specific *rāśis* are not visible, when sun is transiting through these *rāśis*, it will not be visible either. This is summarised in Table 4.

[5] The word *saptāṅga* literally means seven parts (or that which has seven parts). Based on the context and comparing the enumeration of *rāśis* with that of other authors, *saptāṅga* has been considered as a synonym of *Dhanu*. *Dhanu rāśi* is thought of as a half-horse half-man wielding a bow and arrow. Hence, *saptāṅga* could be a corrupt reading for *saptyaṅga—sapti* meaning horse and *aṅga* meaning part. This makes it a *bahuvrīhi* compound meaning, "he who has a horse as his part".

Table 4 Vaṭeśvara's midnight sun latitudes

Duration (months)	Latitude
2	66°
4	75°
6	–

4.7 Śrīpati

The following is mentioned by Śrīpati (eleventh century CE):

यत्राक्षांशाः सार्धषड्वृष्टिभागास्तस्मिन् देशे धन्विनक्रावदृश्यौ ।
पञ्चोपेता सप्ततिर्यत्र तत्र नो दृश्यन्ते नक्रचापालिकुम्भाः ॥

yatrākṣāṃśāḥ sārdhaṣaṭṣaṣṭibhāgāstasmin deśe dhanvinakrāvadṛśyau |
pañcopetā saptatiryatra tatra no dṛśyante nakracāpālikumbhāḥ ||
(*Siddhāntaśekhara*, 1947, 16.57)

At the place where the latitude is $66\frac{1}{2}^{\circ}$, there *Dhanu* (*dhanvin*) and *Makara* (*nakra*) are not visible. Where the latitude is 75°, *Vṛścika* (*ali*), *Dhanu* (*cāpa*), *Makara* (*nakra*) and *Kumbha* are not visible.

As in the previous cases, the visibility of *rāśis* translates to visibility of the sun. This is summarized in Table 5.

4.8 Bhāskara II

Bhāskara II (twelfth century CE), the author of *Siddhāntaśiromaṇi*, presents the following information with regard to the midnight sun:

त्र्यंशयुङ्नवरसाः पलांशका यत्र तत्र विषये कदाचन ।
दृश्यते न मकरो न कार्मुकं किं च कर्किमिथुनौ सदोदितौ ॥

यत्र साङ्घ्रिगजवाजिसम्मितास्तत्र वृश्चिकचतुष्टयं न च ।
दृश्यतेऽथ वृषभाच्चतुष्टयं सर्वदा समुदितं च लक्ष्यते ॥

यत्र तेऽथ नवतिः पलांशकास्तत्र काञ्चनगिरौ कदाचन ।
दृश्यते न भदलं तुलादिकं सर्वदा समुदितं क्रियादिकम् ॥

tryaṃśayuṅnavarasāḥ palāṃśakā yatra tatra viṣaye kadācana |
dṛśyate na makaro na kārmukaṃ kiṃ ca karkimithunau sadoditau ||

yatra sāṅghrigajavājisammitāstatra vṛścikacatuṣṭayaṃ na ca |
dṛśyate'tha vṛṣabhāccatuṣṭayaṃ sarvadā samuditaṃ ca lakṣyate ||
yatra te'tha navatiḥ palāṃśakāstatra kāñcanagirau kadācana |
dṛśyate na bhadalaṃ tulādikaṃ sarvadā samuditaṃ kriyādikam ||
(*Siddhāntaśiromaṇi, Golādhyāya, Tripraśnavāsanā*, 1981, 28–30)

Table 5 Śrīpati's midnight sun latitudes

Duration (months)	Latitude
2	66° 30′
4	75°
6	–

Table 6 Bhāskara's midnight sun latitudes

Duration (months)	Latitude
2	69° 20′
4	78° 15′
6	90°

In that place where the latitude is $69\frac{1}{3}^{\circ}$, *Makara* and *Dhanu* (*kārmuka*) are not visible, and *Karkaṭa* (*karki*) and *Mithuna* are always risen (above the horizon). [In that place where the latitude is] $78\frac{1}{4}^{\circ}$, the four [*rāśis*] beginning with *Vṛścika* are not seen, while the four beginning with *Vṛṣabha* are always risen. At *Meru* (*kāñcanagiri*, the North Pole) where the latitude is 90°, half of the zodiac beginning with *Tulā* is not visible at any time, while the half beginning with *Meṣa* (*kriyā*) is always visible.

Bhāskara states that at specific latitudes, certain *rāśis* are not visible. As with some of the previous authors, this is equivalent to the sun being invisible in those *rāśis*. The values are provided in Table 6.

5 Discussion

The references to the midnight sun in astronomical texts of Greek and Indian origin were explored in the previous sections. Among the astronomers mentioned above, it is observed that several have mentioned the latitudes for which the sun would be visible for 2 months and 4 months respectively. Such a practice is also observed in Ptolemy's *Almagest*, although it is for each month successively and not just for every 2 months. The result for 6 months is trivial, and hence it is not being considered. Table 7 summarizes the views of all of the above Indian authors and texts regarding the latitudes for the visibility of 2-month and 4-month-long days.

Āryabhaṭa and Brahmagupta have not been included since they only give description of the pole itself and not the latitudes leading up to it. *Sūryasiddhānta* presents the method for deriving these values. This is in the form of an

Table 7 Comparative of midnight sun latitudes in Indian texts

Duration (months)	Varāhamihira	Lalla	Vaṭeśvara	Śrīpati	Bhāskara II	Computed
2	69° 24′	66° 30′	66°	66° 30′	69° 20′	69° 22′ 31″
4	~ 78° 13′ 30″	75°	75°	75°	78° 15′	78° 15′ 58″

algorithm that relies on a sine table, the obliquity of the ecliptic and the circumference of the earth. The accuracy of the values derived using this algorithm thus depends on the accuracy of the sine table and the obliquity used. Hence, it is not appropriate to include it here.

The values given by Lalla, Vaṭeśvara and Śrīpati are far removed from the actual values. Lalla's value for the midnight sun latitude has been criticized by Bhāskara II in *Siddhāntaśiromaṇi*. He states:

षड्वृष्टिः सदला लवाः पलभवा यस्मिन् न तस्मिन् धनु-
र्नक्रश्चापि न वृश्चिको न च घटः पञ्चाद्रयो यत्र च ।

दृश्यः स्यादिति यत् सदा प्रलपितं लल्लेन गोले निजे
गोलज्ञ त्रिलवोनितास्त उदिताः केनोच्यतां हेतुना ॥

ṣaṭṣaṣṭiḥ sadalā lavāḥ palabhavā yasmin na tasmin dhanu-
rnakraścāpi na vṛściko na ca ghaṭaḥ pañcādrayo yatra ca |
dṛśyaḥ syāditi yat sadā pralapitaṃ lallena gole nije
golajña trilavonitāsta uditāḥ kenocyatāṃ hetunā ||
(*Siddhāntaśiromaṇi, Golādhyāya, Tripraśnavāsana,* 1981, 32)

[At that place] where the latitude is 66.5°, there *Dhanu* and *Makara* (*nakra*), and where [the latitude is] 75°, *Vṛścika* and *Kumbha* (*ghaṭa*) as well (apart from *Dhanu* and *Makara*), are not visible – this is said repeatedly (*sadā pralapitam*) by Lalla in his *Gola*. Tell me, O knower of spheres, why has he stated the values three degrees less [than what they are]?

Given that Varāhamihira had stated correct values much earlier, it is unclear why such a value is stated by them.

The values given by Varāhamihira and Bhāskara closely align with the computed values as is evident above. The values given by Ptolemy are also relatively accurate subject to his value of the obliquity of the ecliptic, as was seen earlier.

6 Possible antecedents

While these ideas are linked directly to astronomical theory, these certainly could not have arisen in a vacuum.

In case of the Greeks, it has been possible to trace out the development of the concept of the midnight sun over the course of various texts, even though there are gaps. Starting from stray references in Homer, through the injection of theory from philosophical schools, the astronomical discourse on the midnight sun attains a culmination in the treatise of Ptolemy. However, when considering the Indian texts, it is seen that Varāhamihira, who is one of the oldest dateable astronomers whose works have survived into the present, mentions the midnight sun in the same way that subsequent astronomers have. Hence, the recorded tradition does not give a direct indication of development with time. There are however, two pieces of information that shed some light on this matter.

First is the idea of enumerating latitudes where the midnight sun is apparent. In case of Ptolemy, this is done for places where the midnight sun will be visible for 1 month, 2 months and so on, starting at the Arctic Circle, until the North Pole. In case of the Indian astronomers, however, we see that they enumerate these latitudes only for a visibility of 2 months and 4 months, apart from the Pole itself. While several reasons can be conceived of for this, one possible idea is as follows. Most Indian authors enumerate the *rāśis* visible at each place. As the latitude increases beyond the Arctic Circle, the range of the zodiac that is always visible will increase on either side of 90°. The first instance where this range covers whole *rāśis* is when the midnight sun is 2 months long and the range spans two whole *rāśis*. The second instance is when the midnight sun is 4 months long and the range spans four whole *rāśis*. Hence, this preference may have been for the purposes of measuring in whole *rāśis* and not splitting a *rāśi* midway. The formulae for deriving these values in the *Sūryasiddhānta* also speak of whole *rāśis* being used in calculating the declination and distances, hence this supports this idea. This hints at the fact that the Indians may have arrived at the midnight sun latitudes through calculation and used it to demonstrate the theoretical principles of their treatises.

Another idea which is seen in Indian texts is a clear mention of the sun remaining in the sky for 6 months and remaining set for 6 months at the poles. This is also found in the works of Āryabhaṭa and Brahmagupta, apart from those of Varāhamihira. Unlike the previous idea which cannot be traced beyond Varāhamihira, we do find this concept in another form in literature that is older than these authors.

While non-astronomical texts do not describe the midnight sun directly, the idea that the gods (*devas*), who are said to live on Mount Meru, have a day that is equal to 1 year of humans is seen in several texts. In the *siddhāntas*, Meru is equated to the North Pole and these statements are interpreted in terms of astronomical theory. In non-astronomical works,

it is usually only a statement that is provided. Nevertheless, these statements have great antiquity since they can be traced back to the Vedas. Given their relevance to the present discussion, it seems appropriate to present some of these.

A clear comparison is found in the *Taittirīya Brāhmaṇa*:

एकं वा एतद्देवानामहः । यत्संवत्सरः ।

ekaṃ vā etaddevānāmahaḥ | yatsaṃvatsaraḥ |
(*Taittirīya Brāhmaṇa*, 2021, 3.9.22)

Or it is a day for the gods, what is a year [for humans].

This is a clear statement. The phrase "for humans" is added based on the context. From this, it is seen that there are Vedic statements linking the year to a day, specifically that of the gods. This idea is also observed in later literature.

For example, the *Manusmṛti* states:

दैवे रात्र्यहनी वर्षं प्रविभागस्तयोः पुनः ।
अहस्तत्रोदगयनं रात्रिः स्याद्दक्षिणायनम् ॥

daive rātryahanī varṣaṃ pravibhāgastayoḥ punaḥ |
ahastatrodagayanaṃ rātriḥ syāddakṣiṇāyanam ||
(*Manusmṛti*, 1886, 1.67)

The night and day of the gods is a year [for humans]. Of these, day represents sun's northern course (*uttarāyaṇa*, *udagayam*), while night represents sun's southern course (*dakṣiṇāyana*).

A couple of verses in the *Bhagavadgītā*, which is a section of the *Mahābhārata*, can be interpreted as expressing this in a cryptic manner:

अग्निर्ज्योतिरहः शुक्लः षण्मासा उत्तरायणम् ।
तत्र प्रयाता गच्छन्ति ब्रह्म ब्रह्मविदो जनाः ॥

धूमो रात्रिस्तथा कृष्णः षण्मासा दक्षिणायनम् ।
तत्र चान्द्रमसं ज्योतिर्योगी प्राप्य निवर्तते ॥

agnirjyotirahaḥ śuklaḥ ṣaṇmāsā uttarāyaṇam |
tatra prayātā gacchanti brahma brahmavido janāḥ ||
dhūmo rātristathā kṛṣṇaḥ ṣaṇmāsā dakṣiṇāyanam |
tatra cāndramasaṃ jyotiryogī prāpya nivartate ||
(*Bhagavadgītā*, 8.24–25 / *Mahābhārata*, 1999, 6.30.24–25)

Fire, light, day, white, and the six months of the northern course [of the sun] – those knowers of Brahman who leave [their bodies] at these times attain Brahman. Smoke, night, black and the six months of the southern course [of the sun] – the *yogī* [who leaves at these times] reaches the lunar light and returns.

In commentaries such as that of Śaṅkarācārya, this statement is interpreted in terms of the deities presiding over the times or elements, such as the deity of fire, deity of light, and so on. However, purely from a grammatical view, it is possible to connect these elements since they have been given in the same grammatical case. The connection relevant in this context is that of *ahaḥ* or day with *uttarāyaṇa* and *rātri* or night with *dakṣiṇāyana*. This association is interpreted based on the statement of *Manusmṛti* and other works.

Two *purāṇas* also mention the idea of a day and night of the gods being a human year clearly:

लौकिकेनैव मानेन अब्दो यो मानुषः स्मृतः ।
एतद्दिव्यमहोरात्रं शास्त्रेऽस्मिन्निश्चयो मतः ॥

दिव्ये रात्र्यहनी वर्षं प्रविभागस्तयोः पुनः ।
अहस्तत्रोदगयनं रात्रिः स्याद्दक्षिणायनम् ॥

laukikenaiva mānena abdo yo mānuṣaḥ smṛtaḥ |
etaddivyamahorātraṃ śāstre'sminniścayo mataḥ ||
divye rātryahanī varṣaṃ pravibhāgastayoḥ punaḥ |
ahastatrodagayanaṃ rātriḥ syāddakṣiṇāyanam ||
(*Vāyu Purāṇa*, 1987, 57.12)

The year of the humans, as is known in the world (i.e., the solar year), is considered to be equal to the year of the gods in this *śāstra* (subject, i.e., in *jyotiḥśāstra* or astronomy). The day and night of the gods constitutes one [human] year. They are divided into day (*ahaḥ*) which is the *uttarāyaṇa* (*udagayana*, northern course) and night which is the *dakṣiṇāyana* (southern course).

The description is the same as that of *Manusmṛti*. The second verse is almost entirely identical.

ऋतुत्रयं चायनं स्यात्तद्वयं च समा स्मृता ।
देवतानामहोरात्रं स च राम प्रकीर्तितः ॥

मेषादिषट्कगे सूर्ये तेषां दिवस उच्यते ।
तुलादिषट्कगे सूर्ये तेषां रात्रिः प्रकीर्तिता ॥

ṛtutrayaṃ cāyanaṃ syāttaddvayaṃ ca samā smṛtā |
devatānāmahorātraṃ sa ca rāma prakīrtitaḥ ||
meṣādiṣaṭkage sūrye teṣāṃ divasa ucyate |
tulādiṣaṭkage sūrye teṣāṃ rātriḥ prakīrtitā ||
(*Viṣṇudharmottara Purāṇa*, 1929, 1.73.12–13)

Three seasons (*ṛtu*, two months long) constitute one *ayana* (six months). Two of those make one year (*samā*). That, O Rāma, is the day and night (*ahorātra*) of the Gods. When the sun transits in the six [*rāśis*] beginning with *Meṣa*, it is their day. When the sun transits in the six [*rāśis*] beginning with *Tulā*, it is their night.

The northern and southern courses of the sun are given in terms of the *rāśis* through which the sun transits at that

time. In this context, it must be specified that, the midnight sun remains in the sky from the vernal equinox to the autumnal equinox, and remains set for the rest of the year. Sun's northern course which is called *uttarāyaṇa* is usually interpreted as starting from the winter solstice, when the sun actually starts moving towards the north. Similarly, *dakṣiṇāyana*, sun's southern course, begins at the summer solstice and proceeds for 6 months thence. However, "northern course" may be interpreted as the time when sun is in the northern part, i.e., from vernal to autumnal equinox, instead of winter to summer solstice, and similarly for "southern course," aligning this statement with the midnight sun observed at the poles. In other words, *uttarāyaṇa* can be interpreted as beginning when the sun transits into *Makara* or *Meṣa*, the latter being the same as the observed midnight sun.

It is not known which of these two interpretations was intended in Vedic and subsequent texts, since only the reference from *Viṣṇudharmottara Purāṇa*, among those considered above, makes a definite statement about the *rāśis* involved. The statement in this *purāṇa* does indeed match with the midnight sun, since it is stated that *uttarāyaṇa* begins when the sun transits into *Meṣa*.

In any case, even if the statements do align with the observed midnight sun, it is also not known whether these statements were made after understanding astronomical theory or by some other means. What complicates this effort to trace the development of the concept of the midnight sun is that Varāhamihira's *Pañcasiddhāntikā* provides all information relating to the midnight sun in a chapter that is not explicitly connected to any one *siddhānta* (as already noted above). Given this, it is not possible to trace this concept backwards from existing *siddhāntas* either. Hence the link between the Vedic association of a day of the gods with a year of humans, and the midnight sun of the *siddhāntas* remains elusive.

However, even if the exact intention of these statements and the development of these ideas into the later *siddhāntas* cannot be effectively traced, the fact that there are similar statements in earlier literature is evident.

7 Conclusion

The midnight sun, with its fascinating features, was referred to by several ancient astronomers who took interest in it. Among the Greeks, it is observed that stray references occur in the earliest texts and that with the development of theories in astronomy, it became possible to place these observations within a definite scheme and formulate a complete system, the culmination of which is represented in the *Almagest* of Ptolemy, that contains a clear description of the midnight sun. The voyage of Pytheas to the northern regions is a highlight in the record.

In the Indian tradition, it is observed that most authors mention at least the fact that the poles experience the midnight sun, and several others also mention other latitudes at which the midnight sun is observed for successive periods of time, viz., for 2 months and 4 months at a stretch. These mentions are distinct from those of the Greek tradition (as represented by Ptolemy), where the latitudes for each additional month of visibility are noted. The values noted by Lalla, Vaṭeśvara and Śrīpati were erroneous while those of Ptolemy, Varāhamihira and Bhāskara were found to align closely with the modern computed values.

Some facets of the Indian references are found to have a deeper history: Vedic literature does mention that the gods have a day that is equivalent to a human year, which corresponds to the situation at the poles. It cannot be said with certainty whether the requisite astronomical theories were available to them to make such a statement. Even then, a similarity is evident and this interpretation was also carried over into the *siddhāntas* as seen above. Given that most texts between the Vedas and the earliest known *siddhāntas* are not presently available, it is not possible to trace the development of thought that took place between these times. However, it is hoped that further analysis of extant texts and the discovery of new manuscripts will shed light on this matter.

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