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ORIGINAL PAPER

# Precise determination of the ascendant in the *Lagnaprakaraṇa*-IV

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**Abstract**
Mādhava's *Lagnaprakaraṇa* is an important astronomical text which discusses numerous innovative techniques of precisely determining the *udayalagna* or the ascendant. In previous papers, we have detailed the various methods of determining the ascendant described in the second and third chapters of this text. In this paper, we discuss a technique described in the fifth chapter, which makes use of two quantities known as the *viṣuvannara* and the *ayanāntaśaṅku*.

**Keywords** Ascendant · *Ayanāntaśaṅku* · *Dṛkkṣepakoṭikā* · *Dṛggati* · *Lagna* · *Lagnaprakaraṇa* · Mādhava · *Śaṅku* · *Viṣuvannara*

## 1 Introduction

In a series of earlier papers, we have discussed the various contributions of Mādhava to astronomy by way of developing ingenious computational techniques in the focused treatise *Lagnaprakaraṇa*. The first three papers presented techniques of determining astronomical quantities known as *prāṇakalāntara*, *cara* and *kālalagna*,[1] that are employed in determining the ascendant. These techniques are described in the first thirty verses of this text, corresponding to its first chapter. Three subsequent papers discussed the various techniques of precisely determining the *udayalagna* or the ascendant, described in verses 31–61 of the *Lagnaprakaraṇa*, corresponding to its second and third chapters.[2] In this paper, we discuss further methods of determining the ascendant described in verses 80–87, corresponding to the fifth chapter of this text.[3]

The following verses, belonging to the fifth chapter of the *Lagnaprakaraṇa*, first describe the techniques to determine the gnomons—*viṣuvannara* and *ayanāntaśaṅku*—corresponding to the equinoctial and solstitial ecliptic points, and then show how to determine the ascendant therefrom. As the equinoctial and solstitial ecliptic points are ninety degrees apart, the verses in this chapter can be considered as addressing a special case of the procedures laid out in our previous paper.[4]

In order to have a better appreciation and full comprehension of the contents of this paper, it should be read in conjunction with our earlier papers, as various physical and mathematical quantities described therein are employed here as well. In this regards, it may be reiterated that we employ the symbols $\lambda$, $\alpha$, $\delta$, and $z$ to respectively refer to the longitude, right ascension, declination, and zenith distance of a celestial object. The *kālalagna*, the latitude of the observer, and the obliquity of the ecliptic are denoted by the symbols $\alpha_e$, $\phi$, and $\epsilon$ respectively. It may also be mentioned that all the figures in this paper depict the celestial sphere for an observer having a northerly latitude $\phi$. In these figures, $N$, $S$, $E$, and $W$ denote the cardinal directions north, south, east, and west, while $P$ and $K$ denote the poles of the celestial equator and the ecliptic respectively.

## 2 Obtaining the *viṣuvannara* and the *ayanāntaśaṅku*

लम्बाहता कालविलग्नदोर्ज्या
व्यासार्धभक्ता विषुवन्नरं स्यात् ।
अन्त्यद्युजीवाहतकाललग्न-
कोटीगुणाद्यत् त्रिभमौर्विकाप्तम् ॥८०॥
तल्लम्बघातं पलमौर्विकान्त्य-
क्रान्त्योर्वधे कर्किमृगादिकत्वात् ।

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[1] See Kolachana et al. (2018b, 2018a; 2019a) respectively.
[2] See Kolachana et al. (2019b; 2020a, 2020b).
[3] We intend to bring out the contents of the fourth chapter of the *Lagnaprakaraṇa*, corresponding to verses 62–79, in a later paper.
[4] See Kolachana et al. (2020b).

स्वर्णं च कृत्वा त्रिगुणेन हृत्वा
लब्धं तु विद्यादयनान्तशङ्कुम् ॥८१॥
स्वल्पे विदिक्कोटिजलम्बघाते
कर्क्यादिकत्वं च भवेदमुष्य ।

*lambāhatā kālavilagnadorjyā*
*vyāsārdhabhaktā viṣuvannaraṃ syāt* |
*antyadyujīvāhatakālalagna-*
*koṭīguṇādyat tribhamaurvikāptam* ||80||
*tallambaghātaṃ palamaurvikāntya-*
*krāntyorvadhe karkimṛgādikatvāt* |
*svarṇaṃ ca kṛtvā triguṇena hṛtvā*
*labdhaṃ tu vidyādayanāntaśaṅkum* ||81||
*svalpe vidikkoṭijalambaghāte*
*karkyādikatvaṃ ca bhavedamuṣya* |

The Rsine of the *kālalagna* multiplied by the Rcosine of the latitude (*lamba*) and divided by the semi-diameter (*vyāsārdha*) would be the gnomon of the equinoctial ecliptic point (*viṣuvannara*). One should know that product of the Rcosine of the latitude (*lamba*) and that which is obtained by dividing the product of the last day-radius (*antyadyujīvā*) and the Rcosine of the *kālalagna* by the radius (*tribhamaurvikā*), applied positively or negatively to the product of the Rsine of the latitude (*palamaurvikā*) and [the Rsine of] the last declination (*antyakrānti*)—depending on [if the *kālalagna* is in the six signs] Cancer (*karki*) etc., or Capricorn (*mṛga*) etc.,—and [the result] divided by the radius (*triguṇa*), to be the gnomon of the solstitial ecliptic point (*ayanāntaśaṅku*).
[When the *kālalagna* is in Capricorn etc.,] if the product of the *vidikkoṭija* and the Rcosine of the latitude (*lamba*) is smaller [than the product of the Rsines of the latitude and the maximum declination], this [*ayanāntaśaṅku*] would be in Cancer (*karki*) etc.

These verses, in the *indravajrā* metre, give the following relation for the gnomon corresponding to the equinoctial point of the ecliptic:

$$\begin{aligned} viṣuvannara &= \frac{kālavilagnadorjyā \times lambajyā}{vyāsārdha} \\ &= \frac{R\sin\alpha_e \times R\cos\phi}{R}. \end{aligned} \tag{1}$$

They also give the following relation for the gnomon corresponding to the solstitial point of the ecliptic:

$$\begin{aligned} ayanāntaśaṅku &= [palamaurvikā \times antyakrāntijyā \pm \\ &\quad \frac{kālalagnakoṭīguṇa \times antyadyujīvā}{tribhamaurvikā} \times lambajyā] \\ &\quad \div triguṇa \\ &= \frac{|R\sin\phi \times R\sin\epsilon \pm \frac{R\cos\alpha_e \times R\cos\epsilon}{R} \times R\cos\phi|}{R}. \end{aligned} \tag{2}$$

The above relations can be simply derived by employing (1) of Kolachana et al. (2020b) to determine the *śaṅku* of the Sun when it is at these positions.

## 2.1 Obtaining the *viṣuvannara*

When the Sun is at either equinoctial point, its declination ($\delta$) and ascensional difference ($\Delta\alpha_s$) are zero, while its right ascension ($\alpha$) can be 0 or 180 degrees. Substituting these values in (1) of Kolachana et al. (2020b) yields (1) of this paper.

This relation can also be derived geometrically as shown in Fig. 1. In this figure, $\Gamma$ is the vernal equinoctial point, and $Y$ is its projection on the horizon. The planar right-angled triangle $\Gamma XY$ lies in a plane perpendicular to the horizon and parallel to the prime meridian. Its side $\Gamma Y$ represents the *viṣuvannara* or the gnomon dropped from the vernal equinoctial point. Its hypotenuse $\Gamma X$ is equal to the Rsine of the arc $\Gamma E$, which is nothing but the *kālalagna* ($\alpha_e$). Finally, the angle $\Gamma\hat{X}Y$ represents the angle between the planes of the equator and the horizon. Thus,

$$\Gamma Y = viṣuvannara, \;\; \Gamma X = R\sin\alpha_e, \;\; \Gamma\hat{X}Y = \phi' = 90 - \phi.$$

The planar right-angled triangle $TOT'$ lies in the plane of the prime meridian, which is perpendicular to the horizon. Here, $T$ represents the intersection of the equator and the prime meridian, and $T'$ is its projection on the horizon. Thus, the side $TT'$ is perpendicular to the horizon, the hypotenuse $OT$ is equal to the radius of the celestial sphere, and the angle $TOT'$ represents the angle between the planes of the equator and the horizon. Therefore, we have

$$TT' = R\cos\phi, \quad OT = R, \quad \text{and} \quad T\hat{O}T' = \phi' = 90 - \phi.$$

It can be clearly seen that the two right-angled triangles $\Gamma XY$ and $TOT'$ are similar. Applying the rule of proportionality to the sides of these two triangles, we have

$$viṣuvannara = \frac{R\cos\phi}{R} \times R\sin\alpha_e,$$

which is the required relation.[5] It may be noted that the same relation can be obtained in a similar manner when the autumnal equinoctial point is above the horizon.

[5] This result could have been obtained directly from the right-angled triangle $\Gamma XY$ by employing the relation $\Gamma Y = \Gamma X \times \sin(90-\phi)$. The given method only serves to illustrate the procedure of *trairāśika*, or the rule of three, which was generally the preferred method for deriving relations in the Indian mathematical and astronomical tradition.

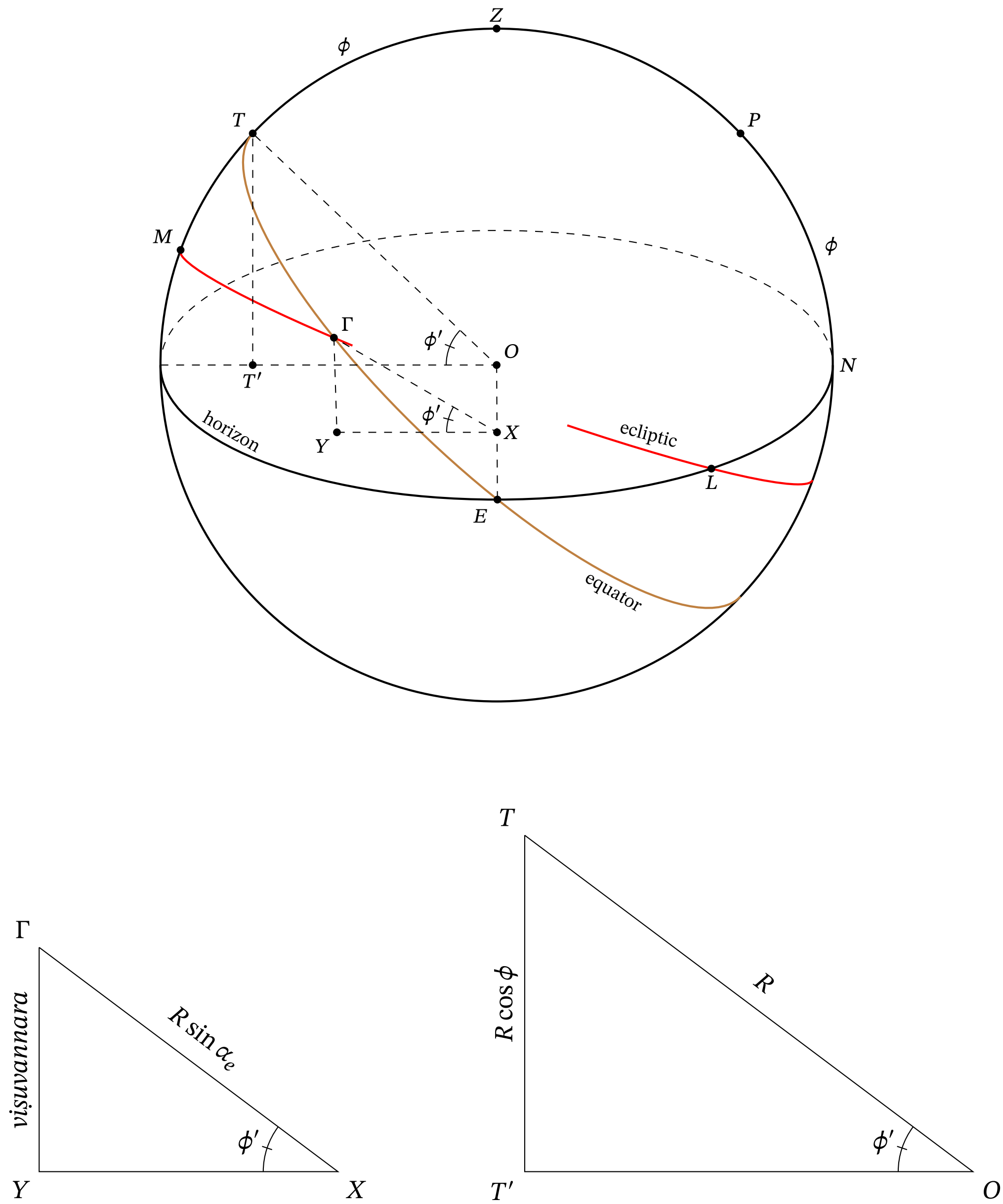



**Fig. 1** Determining the *viṣuvannara*

## 2.2 Obtaining the *ayanāntaśaṅku*

When the Sun is at the solstitial points, its declination is $\pm\epsilon$, and its right ascension is 90 or 270 degrees. Its 'instantaneous' ascensional difference, calculated using (16) of Kolachana (2018a), would be

$$R\sin\Delta\alpha_s = \frac{R\times R\sin\phi\times R\sin\epsilon}{R\cos\phi\times R\cos\epsilon}.$$

It can be seen that substituting these values in (1) of Kolachana et al. (2020b) yields (2) of this paper.

The geometric derivation for the *ayanāntaśaṅku* is somewhat involved, but very interesting. This derivation can be carried out for three different scenarios where (i) the *kālalagna* is in the range of 90 to 270 degrees (i.e. Cancer etc.) and the summer solstitial point is above the horizon, (ii) the *kālalagna* is in the range of 270 to 270 + $\Delta\alpha_m$ degrees,[6] or 90 – $\Delta\alpha_m$ to 90 degrees, and the summer solstitial point is above the horizon, and (iii) the *kālalagna* is in the range of 270 + $\Delta\alpha_m$ to 90 – $\Delta\alpha_m$ degrees and the winter solstitial point is above the horizon. The latter two cases together constitute the *mṛgādi* (Capricorn etc.) period of the *kālalagna*.

6 Where $\Delta\alpha_m$ is the maximum ascensional difference at a given latitude.

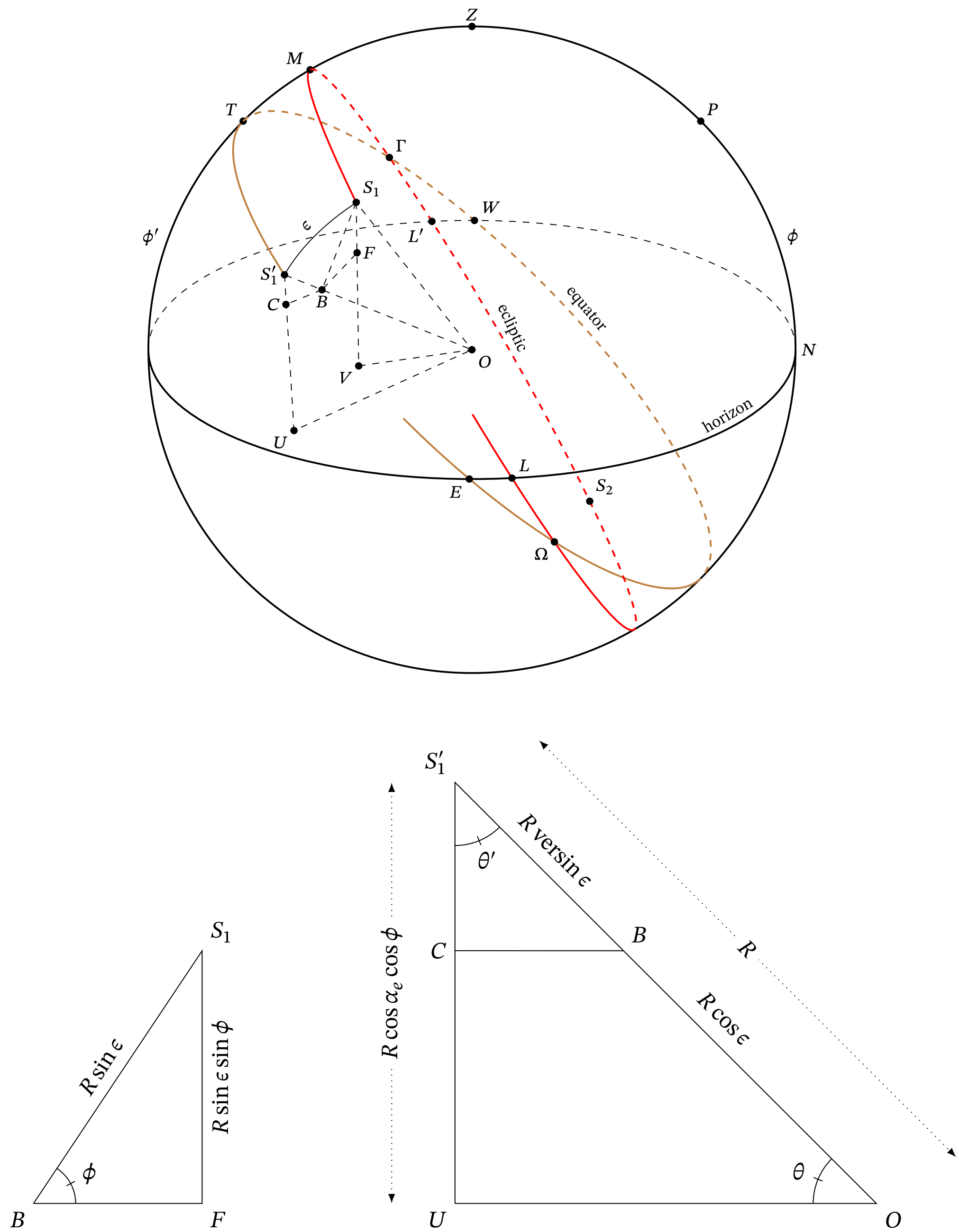



**Fig. 2** Determining the *ayanāntaśaṅku* when the *kālalagna* is *karkyādi*

The construction required for determining the *ayanāntaśaṅku* when the *kālalagna* is in the range of 90 to 270 degrees is shown in Fig. 2. In this figure, $S_1$ is the summer solstitial point, and $V$ is its projection on the horizon. Thus, $S_1V$ corresponds to the *ayanāntaśaṅku*. The equatorial point $S_1'$ corresponds to the right ascension of $S_1$, and $U$ is its projection on the horizon. Here, $OS_1$ and $OS_1'$ are the radii of the celestial sphere, and $S_1B$ is the perpendicular from $S_1$ dropped on $OS_1'$. This perpendicular corresponds to the Rsine of the arc $S_1S_1'$, which is nothing but the declination ($\epsilon$) of $S_1$. Thus, we have

$$S_1B = R\sin\epsilon, \quad OB = R\cos\epsilon, \quad \text{and} \quad BS_1' = R\,\text{versin}\,\epsilon.$$

Now, let $BF$ and $BC$ be the perpendiculars dropped from $B$ onto $S_1V$ and $S_1'U$ respectively, which makes them parallel to the horizon. Then, it can be seen from the figure that the measure of the gnomon $S_1V$ is given by

$$S_1V = S_1F + S_1'U - S_1'C. \tag{3}$$

The derivation of each of the quantities in the right hand side of the above expression is discussed below.

### 2.3 Obtaining $S_1F$

$S_1F$ can be obtained by considering Fig. 3a. This figure depicts an instant when the *kālalagna* is exactly 90 degrees. Thus, the vernal equinoctial point ($\Gamma$) is on the prime meridian, and $S_1$ and $S_1'$ are located on the six o' clock circle, the latter coinciding with the east cardinal point ($E$). In this case, the radius $OS_1'$ is located on the horizon, and the perpendicular $S_1B$ dropped from $S_1$ onto $OS_1'$ (in the plane of the six o' clock circle) is equal to the Rsine of the arc $S_1S_1'$, whose measure is $\epsilon$. That is, $S_1B = R\sin\epsilon$. As $B$ lies on the horizon, the perpendicular $BF$ from $B$ meets the gnomon of $S_1$ at its foot. Thus, $F$ is the projection of $S_1$ on the horizon.

Now, it can be seen that the planar right-angled triangle $S_1BF$ is perpendicular to the horizon, and parallel to the plane of the prime meridian. In this triangle, the angle $S_1\hat{B}F$ is equal to the angle between the planes of the six o' clock circle and the horizon, which is equal to the latitude ($\phi$) of the observer. Therefore, we have

$$S_1F = S_1B \times \sin\phi = R\sin\epsilon\sin\phi. \tag{4}$$

As the relative positions of $S_1$ and $S_1'$ are fixed, the measure of $S_1F$ is also fixed, irrespective of the location of $S_1$ on the celestial sphere. Thus, $S_1F$ will have the same measure in Fig. 2 as well.

### 2.4 Obtaining $S_1'U$

$S_1'U$ can be determined from Fig. 3b. The celestial sphere depicted here is the same as that depicted in Fig. 2. In this figure, the planar right-angled triangle $S_1'UH$ is perpendicular to the horizon, and parallel to the plane of the prime meridian. Its hypotenuse $S_1'H$ is equal to the Rsine of the arc $S_1'E$. As $\Gamma S_1' = 90$, and $\Gamma E = \alpha_e$, we have $S_1'E = \alpha_e - 90$. Thus, $S_1'H = R\cos\alpha_e$. The angle $S_1'\hat{H}U = \phi' = 90 - \phi$ gives the measure of the angle between the planes of the equator and the horizon. Therefore, we have

$$S_1'U = S_1'H \times \sin(90-\phi) = R\cos\alpha_e\cos\phi. \tag{5}$$

### 2.5 Obtaining $S_1'C$

$S_1'C$ can be obtained by considering the similar triangles $S_1'CB$ and $S_1'UO$ in Fig. 2. In the right-angled triangle $S_1'CB$, we have

$$BS_1' = R\,\mathrm{versin}\,\epsilon, \qquad B\hat{S}_1'C = \theta' = 90-\theta,$$

and thus,

$$S_1'C = R\,\mathrm{versin}\,\epsilon\,\sin\,\theta.$$

However, from the right-angled triangle $S_1'UO$, we have $\sin\theta = \cos\alpha_e\cos\phi$. Therefore,

$$S_1'C = R\cos\alpha_e\cos\phi\,\mathrm{versin}\,\epsilon. \tag{6}$$

Substituting (4), (5), and (6) in (3), we obtain the relation for the gnomon corresponding to the summer solstitial point when the *kālalagna* is in the range of 90 to 270 degrees:

$$S_1V = R\sin\epsilon\sin\phi + R\cos\alpha_e\cos\epsilon\cos\phi. \tag{7}$$

For an observer in the northern hemisphere, the summer solstitial point does not set immediately when the *kālalagna* goes beyond 270 degrees, but continues to be above the horizon for a time period corresponding to the maximum ascensional difference ($\Delta\alpha_m$) at that latitude. Similarly, the summer solstitial point rises earlier by a time interval of $\Delta\alpha_m$ before the *kālalagna* reaches 90 degrees. The former case, when the *kālalagna* is in the range 270 to $270+\Delta\alpha_m$ degrees, is shown in Fig. 4a. The latter case, when the *kālalagna* is in the range of $90-\Delta\alpha_m$ to 90 degrees, is shown in Fig. 4b. In both these cases, $S_1$ is above the horizon, but $S_1'$ is below it. In either scenario, employing a similar construction as shown in Fig. 2, the gnomon of the summer solstitial point can be shown to be equal to

$$\begin{aligned} S_1V &= S_1F - S_1'U + S_1'C \\ &= R\sin\epsilon\sin\phi - R\cos\alpha_e\cos\epsilon\cos\phi. \end{aligned} \tag{8}$$

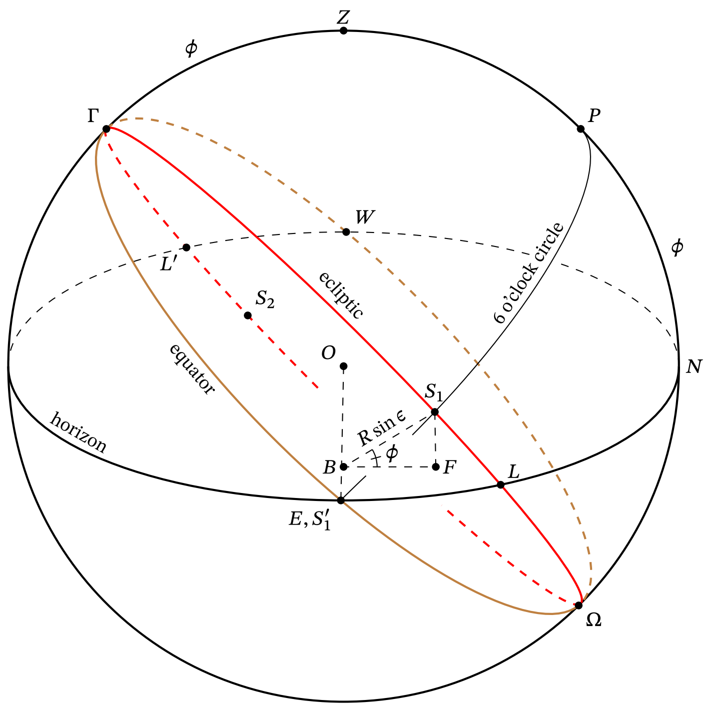
Finally, when the *kālalagna* is in the range of $270+\Delta\alpha_m$ to $90-\Delta\alpha_m$ degrees, the winter solstitial point ($S_2$) is above the horizon, as shown in Fig. 5. Employing a similar construction as shown in Fig. 2, the gnomon corresponding to $S_2$ can be shown to be equal to

$$S_2V' = R\cos\alpha_e\cos\epsilon\cos\phi - R\sin\epsilon\sin\phi. \tag{9}$$

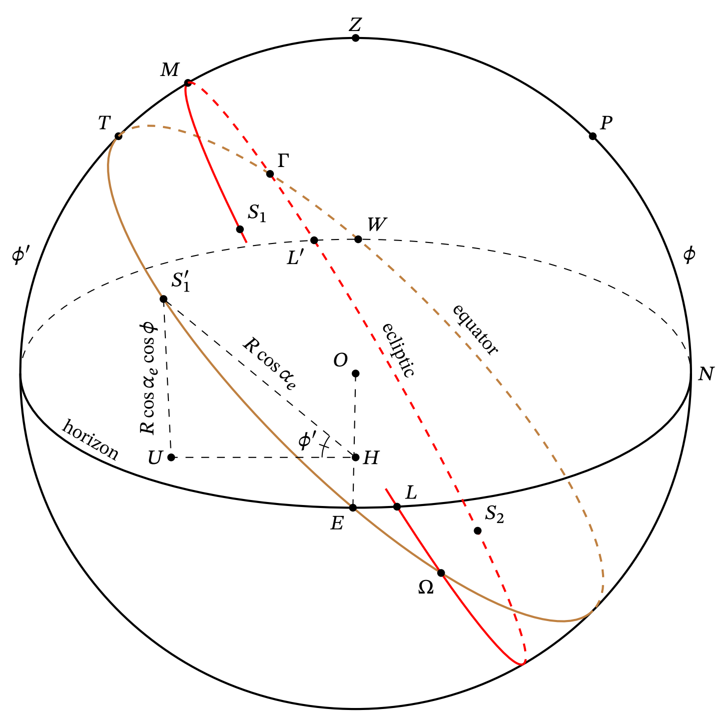
Taken together, (7), (8), and (9) yield (2), and also satisfy the conditions for addition and subtraction of the constituent terms as stated in the verse.

### 2.6 Distinguishing between the summer and winter solstitial points when the *kālalagna is mṛgādi*

We have seen that either the summer or winter solstitial points can be above the horizon when the *kālalagna* is in the range of 270 to 90 degrees. Their corresponding gnomons are given by the relations (8) and (9) respectively. To obtain positive values for these gnomons, it can be seen that the expression $R\cos\alpha_e\cos\epsilon\cos\phi$ has to be smaller than $R\sin\epsilon\sin\phi$ in the former instance, and greater in the latter instance. Therefore, depending upon the relative magnitudes of these two expressions, one can determine which solstitial point is above the horizon. The first half of verse 82 thus

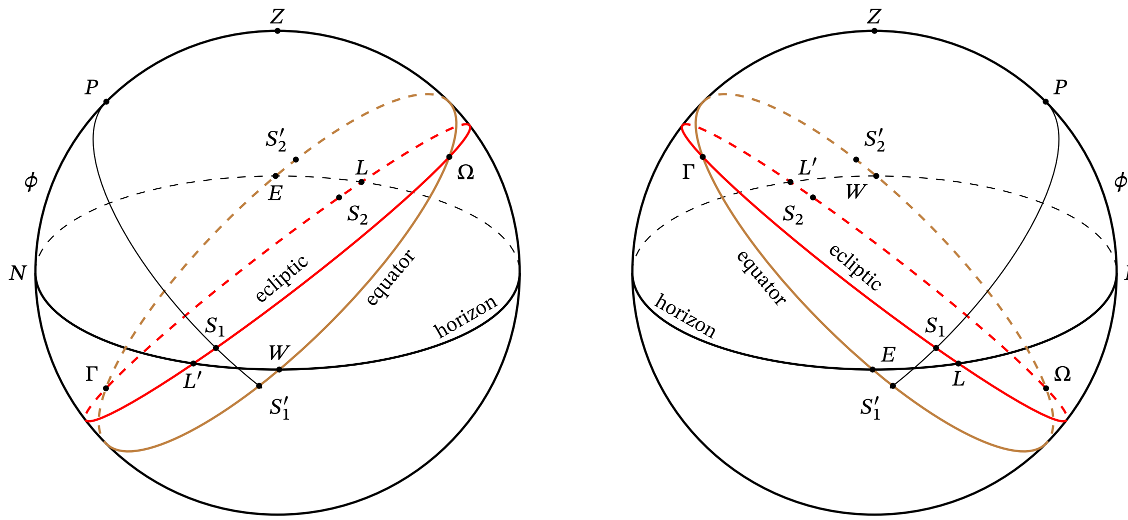



**(a)** Obtaining $S_1F$.




**(b)** Obtaining $S_1'U$.

**Fig. 3** Obtaining $S_1F$ and $S_1'U$ for determining the *ayanāntaśaṅku*




**(a)** $270 < \alpha_e < 270 + \Delta\alpha_m$ **(b)** $90 - \Delta\alpha_m < \alpha_e < 90$

**Fig. 4** Instances of the summer solstitial point above the horizon when the *kālalagna* is *mṛgādi*

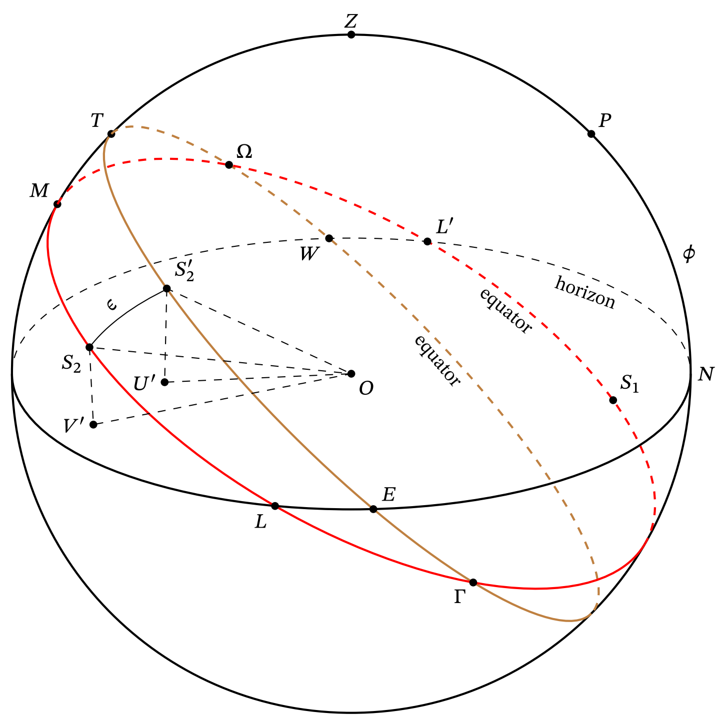



**Fig. 5** Instance of the winter solstitial point above the horizon when the *kālalagna* is *mṛgādi*

states that, when the *kālalagna* is *mṛgādi*, the gnomon calculated using (2) would correspond to the summer solstitial point (*karki* or the solstitial point at a longitude of 90 degrees) when the quantity[7]

$$\frac{R\cos\alpha_e \times R\cos\epsilon}{R} \times R\cos\phi$$

in the numerator is smaller than the product $R\sin\phi \times R\sin\epsilon$.

## 3 Relationship between *viṣuvannara*, *ayanāntaśaṅku*, and *dṛkkṣepakoṭikā*

शङ्क्वोस्तयोरत्र तु वर्गयोगात्
मूलं च दृक्क्षेपगुणस्य कोटिः ॥८२॥

[7] The author uses the phrase *vidikkoṭijalambdaghāta* to refer to this quantity in the verse. As *lamba* refers to $R\cos\phi$, the term *vidikkoṭija* is to be understood as the quantity $\frac{R\cos\alpha_e \times R\cos\epsilon}{R}$. The etymology of this term is unclear.

*śaṅkvostayoratra tu vargayogāt*
*mūlaṃ ca dṛkkṣepaguṇasya koṭiḥ* ||82||

The square root of the sum of the squares of those two gnomons [described in the previous verse] is the Rcosine of the *dṛkkṣepa*.

This half-verse, in the *indravajrā* metre, gives the following relation:

$$\textit{dṛkkṣepakoṭikā} = \sqrt{(\textit{viṣuvannara})^2 + (\textit{ayanāntaśaṅku})^2}. \tag{10}$$

The above relation is a special case of (7) in Kolachana et al. (2020b), where the *viṣuvannara* and the *ayanāntaśaṅku* (which are gnomons dropped from points ninety degrees apart on the ecliptic) replace the *śaṅku* and *dṛggati* (which too are gnomons dropped from points ninety degrees apart on the ecliptic). That is, if the *śaṅku* is measured at either of the equinoctial points, then the *dṛggati* will be measured at the corresponding solstitial points and vice-versa. Therefore, the above relation is just a special case of (7) in Kolachana et al. (2020b).

## 4 Determining the *udayalagna* from the *viṣuvannara*

व्यासार्धनिघ्नात् विषुवन्नराद्यत्
दृक्क्षेपकोट्याप्तफलस्य चापम् ।
तदेव तत्रोदयलग्नमाहुः
आद्ये पदे कालविलग्नकस्य ॥८३॥
तस्य द्वितीये तु पदे धनुस्तत्
चक्रार्धतः शुद्धमुशन्ति लग्नम् ।
चक्रार्धयुक्तं च पदे तृतीये
संशोधितं मण्डलतश्चतुर्थे ॥८४॥

*vyāsārdhanighnāt viṣuvannarādyat*
*dṛkkṣepakoṭyāptaphalasya cāpam* |
*tadeva tatrodayalagnamāhuḥ*
*ādye pade kālavilagnakasya* ||83||
*tasya dvitīye tu pade dhanustat*
*cakrārdhataḥ śuddhamuśanti lagnam* |
*cakrārdhayuktaṃ ca pade tṛtīye*
*saṃśodhitaṃ maṇḍalataścaturthe* ||84||

That arc, which is of the quotient obtained from the division of the semi-diameter (*vyāsārdha*) multiplied *viṣuvannara* by the Rcosine of the *dṛkkṣepa* (*dṛkkṣepakoṭi*), itself is stated to be the rising ecliptic point (*udayalagna*) there, in the first quadrant of the *kālalagna*. Indeed in the second, third, and fourth quadrants of that (*kālalagna*), that arc [respectively] subtracted from a semi-circle (*cakrārdha*), added by a semi-circle, and subtracted from the full circle (*maṇḍala*), is stated to be the [rising] ecliptic point (*lagna*).

These two verses, in the *upajāti* and *indravajrā* metres respectively, give the following procedure to determine the *udayalagna* using the *viṣuvannara* and the *dṛkkṣepakoṭi*, which have been defined earlier in this chapter:

$$\textit{udayalagna} = \textit{cāpa}\left(\frac{\textit{viṣuvannara} \times \textit{vyāsārdha}}{\textit{dṛkkṣepakoṭi}}\right) \quad [\textit{kālalagnasya ādye pade}]$$

$$\textit{udayalagna} = \textit{cakrārdha} - \textit{cāpa}\left(\frac{\textit{viṣuvannara} \times \textit{vyāsārdha}}{\textit{dṛkkṣepakoṭi}}\right) \quad [\textit{kālalagnasya dvitīye pade}]$$

$$\textit{udayalagna} = \textit{cakrārdha} + \textit{cāpa}\left(\frac{\textit{viṣuvannara} \times \textit{vyāsārdha}}{\textit{dṛkkṣepakoṭi}}\right) \quad [\textit{kālalagnasya tṛtīye pade}]$$

$$\textit{udayalagna} = \textit{maṇḍala} - \textit{cāpa}\left(\frac{\textit{viṣuvannara} \times \textit{vyāsārdha}}{\textit{dṛkkṣepakoṭi}}\right). \quad [\textit{kālalagnasya caturthe pade}]$$

Taking $\lambda_l$ as the longitude of the *udayalagna*, and denoting the *dṛkkṣepakoṭi* as $R\cos z_d$, the above relations can be expressed in mathematical notation as follows:

$$\lambda_l = R\sin^{-1}\left(\frac{\textit{viṣuvannara} \times R}{R\cos z_d}\right) \tag{11}$$
[*kālalagna* in the first quadrant]

$$\lambda_l = 180 - R\sin^{-1}\left(\frac{\textit{viṣuvannara} \times R}{R\cos z_d}\right) \tag{12}$$
[*kālalagna* in the second quadrant]

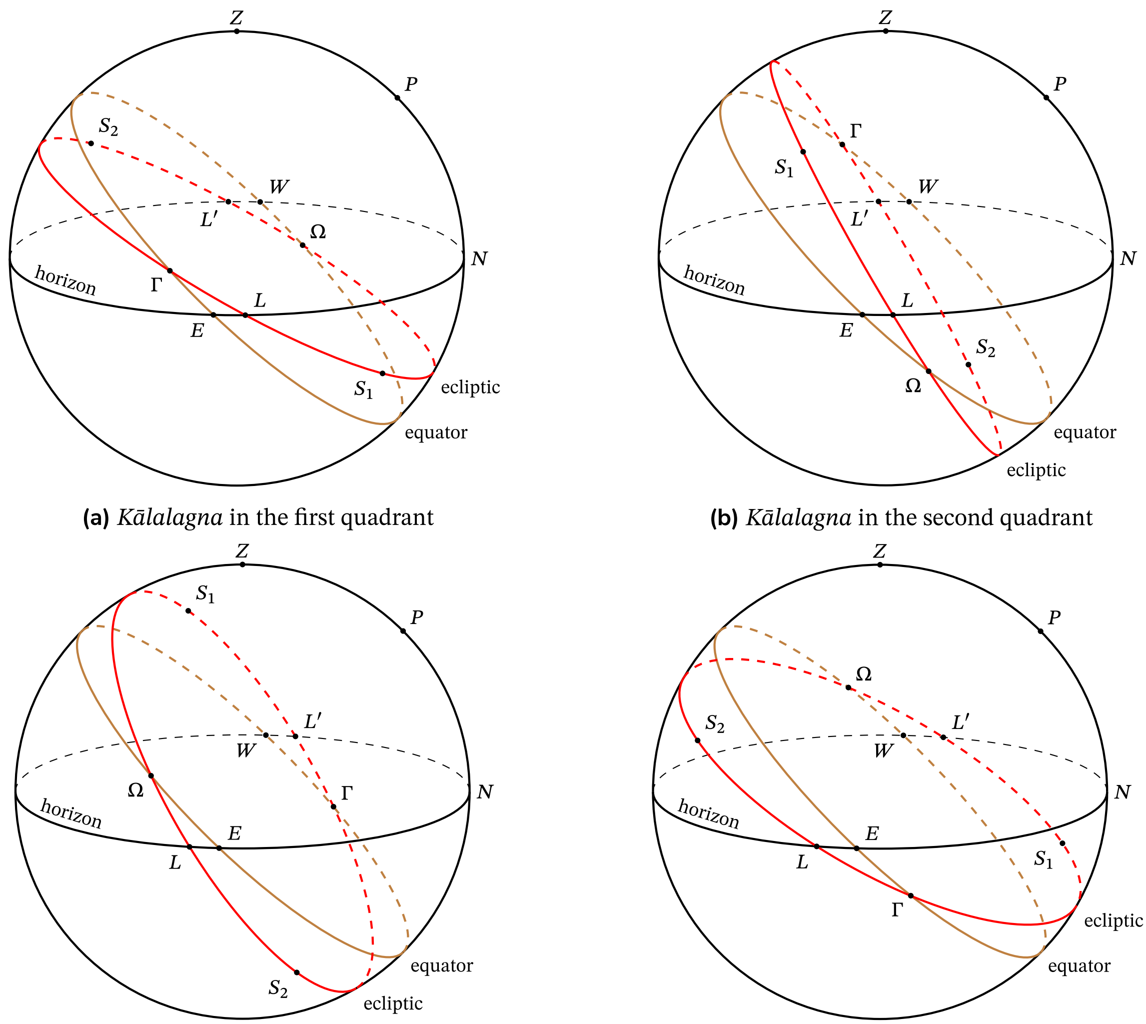
$$\lambda_l = 180 + R\sin^{-1}\left(\frac{\textit{viṣuvannara} \times R}{R\cos z_d}\right) \tag{13}$$
[*kālalagna* in the third quadrant]

$$\lambda_l = 360 - R\sin^{-1}\left(\frac{\textit{viṣuvannara} \times R}{R\cos z_d}\right). \tag{14}$$
[*kālalagna* in the fourth quadrant]

Here, the *viṣuvannara* is obtained using (1).

The *viṣuvannara* is the gnomon corresponding to the equinoctial points. Verses 58–60[8] detail the general procedure for obtaining the *udayalagna* given the position of the

[8] See Kolachana et al. (2020b).




**(a)** *Kālalagna* in the first quadrant

**(b)** *Kālalagna* in the second quadrant

**(c)** *Kālalagna* in the third quadrant

**(d)** *Kālalagna* in the fourth quadrant

**Fig. 6** Determining the *udayalagna* from the *viṣuvannara* and the *ayanāntaśaṅku*

Sun, and the measure of its gnomon (*śaṅku*). Therefore, the relations for *udayalagna* given in these two verses can be obtained by considering the Sun to be at either equinoctial point, taking the *śaṅku* to be equal to the *viṣuvannara*, and applying (12)–(15) of Kolachana et al. (2020b). However, to demonstrate the physical significance of the relations given in these two verses, here we derive (11)–(14) of this paper with the help of Figs. 6a–6d, which show the *kālalagna* in different quadrants.

## 4.1 Obtaining the ecliptic arc from the equinoctial point to the horizon

As the longitudes of the equinoctial points, as well as the separation of the rising and setting ecliptic points are known, the *udayalagna* can be determined by obtaining the arcs $\Gamma L$, $L'\Gamma$, $\Omega L$, and $L'\Omega$ in each of the Figs. 6a–6d respectively. We have shown that (16) of Kolachana et al. (2020b) gives the measure of the arc from the Sun to the rising ecliptic point ($SL$) when the Sun is in the eastern hemisphere, and the measure of the arc from the setting ecliptic point to the Sun ($L'S$) when it is in the western hemisphere. Thus, the arcs $\Gamma L$ or $L'\Gamma$ can be obtained by simply considering the Sun to be present at the vernal equinoctial point ($\Gamma$), and employing (16) of Kolachana et al. (2020b). If the Sun were at the vernal equinoctial point, its *śaṅku* would be equal to the *viṣuvannara*, given by (1), and thus (16) of Kolachana et al. (2020b) would reduce to

$$\Gamma L \text{ or } L'\Gamma = R\sin^{-1}\left(\frac{vi\d{s}uvannara \times R}{R\cos z_d}\right). \qquad (15)$$

Similarly, the arcs $\Omega L$ and $L'\Omega$ can be obtained by considering the Sun to be present at the autumnal equinoctial point. In this case too, the *śaṅku* of the Sun would be equal to the *viṣuvannara*, given by (1),[9] and (16) of Kolachana et al. (2020b) would reduce to

$$\Omega L \text{ or } L'\Omega = R\sin^{-1}\left(\frac{viṣuvannara \times R}{R\cos z_d}\right). \qquad (16)$$

### 4.2 Obtaining the *udayalagna*

When the *kālalagna* is in the first quadrant, as shown in Fig. 6a, the vernal equinoctial point is above the horizon and in the eastern hemisphere. Here, the longitude of the *udayalagna* is given by the arc $\Gamma L$ which is directly obtained from (15). This is the result stated in (11).

When the *kālalagna* is in the second quadrant, the vernal equinoctial point is above the horizon and in the western hemisphere, as shown in Fig. 6b. Here, the longitude of the *udayalagna* is given by the arc $\Gamma L = L'L - L'\Gamma$, where $L'L = 180$, and $L'\Gamma$ is obtained using (15). Thus, we get (12).

When the *kālalagna* is in the third quadrant, the autumnal equinoctial point is above the horizon and in the eastern hemisphere, as shown in Fig. 6c. Here, the longitude of the *udayalagna* is given by the arc $\Gamma L = \Gamma\Omega + \Omega L$, where $\Gamma\Omega = 180$, and $\Omega L$ is obtained using (16). Thus, we obtain (13).

Finally, when the *kālalagna* is in the fourth quadrant, as shown in Fig. 6d, the autumnal equinoctial point is above the horizon and in the western hemisphere. Here, the longitude of the *udayalagna* is given by the arc $\Gamma L = \Gamma\Omega - L'\Omega + L'L$, where $\Gamma\Omega = L'L = 180$, and $L'\Omega$ is obtained using (16). Thus, we obtain the result stated in (14).

## 5 Determining the *udayalagna* from the *ayanāntaśaṅku*

व्यासार्धनिघ्नादयनान्तशङ्कोः
दृक्क्षेपकोट्याप्तधनुर्धनर्णम् ।
निजायनान्ते युगयुक्पदत्वात्
कृत्वा भवेदौदयिकं विलग्नम् ॥८५॥
मृगादियाते सति काललग्ने
तत्कोटिजीवाफललम्बघाते[10]।
स्वल्पे परक्रान्तिगुणाक्षघातात्
पदान्यता तस्य च कल्पनीया ॥८६॥

*vyāsārdhanighnādayanāntaśaṅkoḥ*
*dṛkkṣepakoṭyāptadhanurdhanarṇam* |
*nijāyanānte yugayukpadatvāt*
*kṛtvā bhavedaudayikaṃ vilagnam* ||85||
*mṛgādiyāte sati kālalagne*
*tatkoṭijīvāphalalambaghāte* |
*svalpe parakrāntiguṇākṣaghātāt*
*padānyatā tasya ca kalpanīyā* ||86||

The result obtained by applying the arc of the quotient obtained from the division of the product of the *ayanāntaśaṅku* and the semi-diameter (*vyāsārdha*) by the Rcosine of the *dṛkkṣepa* (*dṛkkṣepakoṭi*) to [the longitude of] the own solstitial point (*nijāyanānta*) positively or negatively, depending on even or odd quadrants [of the *kālalagna*], would be the rising ecliptic point (*audayikaṃ vilagnam*).
When the *kālalagna* is in Capricorn etc. (*mṛgādi*), if the product of the result obtained from its Rcosine (*koṭijīvā*) and the Rcosine of the latitude (*lamba*) is smaller than the product of the Rsine of the maximum declination (*parakrāntiguṇa*) and [the Rsine of] the latitude (*akṣa*), the quadrant of that [*kālalagna*] should be considered to be otherwise (*padānyatā*) [i.e., odd as even, and even as odd, for the purpose of deciding whether to add or subtract the obtained arc to the *nijāyanānta*, in determining the *udayalagna*].

Verses 85, in the *upajāti* metre, describes the following procedure to determine the *udayalagna* using the *ayanāntaśaṅku* and the *dṛkkṣepakoṭi*, which have been defined earlier in this chapter:

$$udayalagna = nijāyanānta + dhanuṣ\left(\frac{ayanāntaśaṅku \times vyāsārdha}{dṛkkṣepakoṭi}\right) \quad [yukpada]$$

$$udayalagna = nijāyanānta - dhanuṣ\left(\frac{ayanāntaśaṅku \times vyāsārdha}{dṛkkṣepakoṭi}\right). \quad [ayukpada]$$

Taking $\lambda_l$ as the longitude of the *udayalagna*, denoting the *dṛkkṣepakoṭi* as $R\cos z_d$, and noting that the longitudes of the summer ($S_1$) and winter ($S_2$) solstitial points are 90 and 270 degrees respectively, the above relations can be expressed in mathematical notation as follows:

$$\lambda_l = 90 - R\sin^{-1}\left(\frac{ayanāntaśaṅku \times R}{R\cos z_d}\right) \qquad (17)$$

[*kālalagna* in the first quadrant]

[9] It may be noted that the gnomons corresponding to the vernal and autumnal equinoctial points are equal.

[10] तत्कोटिजीवापललम्बघाते in the manuscripts. A likely transcribing error. This phrase has to refer to the quantity $R\cos\alpha_e \cos\epsilon \times R\cos\phi$. Such an interpretation is only possible when *pala* is replaced with *phala*.

$$\lambda_l = 90 + R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right) \tag{18}$$

[*kālalagna* in the second quadrant]

$$\lambda_l = 270 - R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right) \tag{19}$$

[*kālalagna* in the third quadrant]

$$\lambda_l = 270 + R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right). \tag{20}$$

[*kālalagna* in the fourth quadrant]

Here, the *ayanāntaśaṅku* is obtained using (2), taking care of the quadrant of the *kālalagna* in which it is sought to be applied. It may be noted that the 'own solstitial point' (*nijāyanānta*) of the *kālalagna* refers to $S_1$ when the *kālalagna* is in the first two quadrants, and $S_2$ when the *kālalagna* is in the third and fourth quadrants. This is only used to represent the quantities 90 and 270 degrees in the above relations, and does not indicate which solstitial point is above the horizon in each of these cases.

The above relations can be directly derived by considering the Sun to be present at either solstitial point, taking its *śaṅku* to be the appropriate *ayanāntaśaṅku*, and employing (12)–(15) of Kolachana et al. (2020b). However, to demonstrate the physical significance of the relations given here, we derive (17)–(20) of this paper with the help of Figs. 6a–6d, which depict the *kālalagna* in different quadrants.

As the longitudes of the solstitial points, as well as the separation of the rising and setting ecliptic points are known, the *udayalagna* can be determined by obtaining the arcs $L'S_2$, $S_1L$, $L'S_1$, and $S_2L$ in each of the Figs. 6a–6d respectively. These arcs can be obtained by considering the Sun to be present at $S_1$ or $S_2$ and applying (16) of Kolachana et al. (2020b). Here, however, we would have to replace the *śaṅku* of the Sun with the respective *ayanāntaśaṅkus* of $S_1$ and $S_2$. In our discussion of verses 80–81, we have shown that the relation for the *ayanāntaśaṅku* varies depending upon the quadrant of the *kālalagna*. Therefore, these different cases are dealt separately below.

### 5.1 Obtaining the *udayalagna* when the *kālalagna* is *karkyādi*

When the *kālalagna* is in the second and third quadrants, the summer solstitial point is above the horizon and the *ayanāntaśaṅku* is given by (7). Substituting this quantity in (16) of Kolachana et al. (2020b), we have

$$S_1L \text{ or } L'S_1 = R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right). \tag{21}$$

[*ayanāntaśaṅku* using (7)]

When the *kālalagna* is in the second quadrant, as shown in Fig. 6b, the summer solstitial point lies above the horizon in the eastern hemisphere. Here, the longitude of the *udayalagna* is given by the arc $\Gamma L = \Gamma S_1 + S_1L$, where $\Gamma S_1 = 90$, and $S_1L$ is given by (21). Thus, we obtain (18).

When the *kālalagna* is in the third quadrant, as shown in Fig. 6c, the summer solstitial point lies above the horizon in the western hemisphere. Here, the longitude of the *udayalagna* is given by the arc $\Gamma L = \Gamma S_1 - L'S_1 + L'L$, where $\Gamma S_1 = 90$, $L'L = 180$, and $L'S_1$ is given by (21). Thus, we have $\Gamma L = 270 - L'S_1$, which is the same as (19).

### 5.2 Obtaining the *udayalagna* when the *kālalagna* is *mṛgādi* and the winter solstitial point is above the horizon

We have shown in our discussion of verses 80–81 that when the *kālalagna* is in the range of $270 + \Delta\alpha_m$ to $90 - \Delta\alpha_m$ degrees,[11] the winter solstitial point is above the horizon, and the *ayanāntaśaṅku* is given by (9). Substituting this quantity in (16) of Kolachana et al. (2020b), we have

$$S_2L \text{ or } L'S_2 = R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right). \tag{22}$$

[*ayanāntaśaṅku* using (9)]

Figure 6a depicts a situation where the *kālalagna* is in the first quadrant, and the winter solstitial point ($S_2$) lies above the horizon in the western hemisphere. In this case, we have $\Gamma L + 360 = \Gamma S_2 - L'S_2 + L'L$, where $\Gamma S_2 = 270$, $L'L = 180$, and $L'S_2$ is given by (22). Thus, the longitude of the *udayalagna* is given by the arc $\Gamma L = 90 - L'S_2$, which is the same as (17).

Figure 6d depicts a situation where the *kālalagna* is in the fourth quadrant, and the winter solstitial point lies above the horizon in the eastern hemisphere. In this case, the longitude of the *udayalagna* is given by the arc $\Gamma L = \Gamma S_2 + S_2L$, where $\Gamma S_2 = 270$, and $S_2L$ is obtained using (22). Thus, we obtain (20).

### 5.3 Obtaining the *udayalagna* when the *kālalagna* is *mṛgādi* and the summer solstitial point is above the horizon

We have shown in our discussion of verses 80–81 that when the *kālalagna* is in the range of $90 - \Delta\alpha_m$ to 90 or 270 to $270 + \Delta\alpha_m$ degrees, the summer solstitial point is above the horizon, and the *ayanāntaśaṅku* is given by (8). Substituting this quantity in (16) of Kolachana et al. (2020b), we have

[11] Here, $\Delta\alpha_m$ is the maximum ascensional difference at a given latitude.

$$S_1L \text{ or } L'S_1 = R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right). \quad (23)$$

[*ayanāntaśaṅku* using (8)]

Figure 4b depicts a scenario where the *kālalagna* is in the first quadrant, and the summer solstitial point is above the horizon in the eastern hemisphere. Here, the longitude of the *udayalagna* is given by the arc $\Gamma L = \Gamma S_1 + S_1 L$, where $\Gamma S_1 = 90$, and $S_1 L$ is given by (23). Therefore, we have

$$\lambda_l = 90 + R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right). \quad (24)$$

$[90 - \Delta\alpha_m < \alpha_e < 90]$

Figure 4a depicts a scenario where the *kālalagna* is in the fourth quadrant, and the summer solstitial point is above the horizon in the western hemisphere. In this case, the longitude of the *udayalagna* is given by the arc $\Gamma L = \Gamma S_1 - L'S_1 + L'L$, where $\Gamma S_1 = 90$, $L'L = 180$, and $L'S_1$ is given by (23). Thus, we obtain

$$\lambda_l = 270 - R\sin^{-1}\left(\frac{ayan\bar{a}nta\acute{s}a\dot{n}ku \times R}{R\cos z_d}\right). \quad (25)$$

$[270 < \alpha_e < 270 + \Delta\alpha_m]$

Comparing (24) with (17), it is seen that the arc has to be added in the former case, and subtracted in the latter, though both relations pertain to the first quadrant of the *kālalagna*. Similarly, comparing (25) and (20), it is seen that the arc has to be subtracted in the former case, and added in the latter, though both relations pertain to the fourth quadrant of the *kālalagna*. Thus, we find that (24) and (25) are an exception to the rule given in verse 85, which states that the appropriate arc is to be added to or subtracted from the *nijāyanānta* in the even and odd quadrants respectively. This exception is addressed in verse 86 (in the *upajāti* metre), which states that the even quadrant of the *kālalagna* has to be considered as odd, and the odd as even, for the purpose of determining between the addition or subtraction of the arc to the *nijāyanānta* in these two cases.[12] This is equivalent to stating that the appropriate arc has to be added to the *nijāyanānta* in the odd quadrant, and subtracted from it in the even quadrant, when the *kālalagna* is *mṛgādi* and the summer solstitial point is above the horizon. As can be seen, this exception to the rule satisfies both (24) and (25).

[12] It may be noted that verse 86 cleverly alludes to these two cases by giving the relative magnitude of the expressions

$$\frac{R\cos\alpha_e \times R\cos\epsilon}{R} \times R\cos\phi$$

and $R\sin\phi \times R\sin\epsilon$ in the numerator of (2). As we have already shown in our discussion of verses 80–81, the former is smaller than the latter when the *kālalagna* is *mṛgādi* and the summer solstitial point is above the horizon. It may also be noted that the compound expression *tatkoṭi-jīvāphala* in this verse refers to the 'result' obtained from the Rcosine of the *kālalagna*, or the quantity $\frac{R\cos\alpha_e \times R\cos\epsilon}{R}$ in the numerator of (2). The same quantity has also been referred to as the *vidikkoṭija* in the first quadrant of verse 82.

## 6 Determining the *lagnakarṇa*

लम्बाप्तो लग्नकर्णः स्यात् त्रिज्यापरनराहतेः ।
तेन वा लग्नमानेयं स्फुटीकरणवर्त्मना ॥८७॥

*lambāpto lagnakarṇaḥ syāt*
*trijyāparanarāhateḥ* |
*tena vā lagnamāneyaṃ*
*sphuṭīkaraṇavartmanā* ||87||

The *lagnakarṇa* would be the quotient obtained from the division of the product of the radius (*trijyā*) and the *paranara* (i.e. *paraśaṅku*) by the Rcosine of the latitude (*lamba*). The rising ecliptic point (*lagna*) can also be computed from it by means of the *sphuṭīkaraṇa* process.

This verse, in the *anuṣṭubh* metre, defines a quantity known as the *lagnakarṇa* in terms of the *paranara* or the *paraśaṅku* ($R\cos z_d$), and the *lambajyā* ($R\cos\phi$) as follows:

$$\begin{aligned} lagnakar\d{n}a &= \frac{paranara \times trijy\bar{a}}{lambajy\bar{a}} \\ &= \frac{R\cos z_d \times R}{R\cos\phi}. \end{aligned} \quad (26)$$

The verse further states that the *udayalagna* can be computed from this quantity by means of a process known as *sphuṭīkaraṇa*. This process is described later in the *Lagnaprakaraṇa*. We only present this verse here for the sake of completeness, and intend to discuss this procedure in detail in a forthcoming paper.

## 7 Discussion

In our previous paper,[13] we disc ussed the procedure of determining the ascendant described in verses 53–61, constituting the third chapter of the *Lagnaprakaraṇa*. There, the author presents two quantities, known as *śaṅku* and *dṛggati*, which are the gnomons corresponding to the Sun and an ecliptic point ninety degrees behind the Sun. In that paper, we have shown how these two quantities, along with a third quantity known as *dṛkkṣepakoṭi*, have been manipulated to precisely calculate the ascendant.

Similarly, in the verses discussed in this paper, the author defines two gnomons corresponding respectively to the

[13] See Kolachana et al. (2020b).

equinoctial and solstitial ecliptic points, and together with the *dṛkkṣepakoṭi*, again precisely determines the ascendant. The relations for the gnomons in terms of the *kālalagna*, just as in the case of the *śaṅku* and *dṛggati*, are quite innovative, and capture the variation in these two quantities accurately. While the variation in the *viṣuvannara* is fairly straightforward, the variation of the *ayanāntaśaṅku* is more complex, having different relations in different quadrants of the *kālalagna*. The text captures the variation in the measure of the *ayanāntaśaṅku* in different scenarios particularly well, revealing a deep study and strong comprehension of this topic by the author. Later verses discuss the means to precisely determine the ascendant using the *viṣuvannara* and the *ayanāntaśaṅku*, based on various possible values of the *kālalagna*. Exceptions to the stated rules are made abundantly clear, showing that the author has carefully considered all possible scenarios. Thus, Mādhava once again lives up to the epithet of "*golavid*", bestowed upon him by later scholars.

## 8 Conclusion

In this paper, we discussed the fifth chapter of *Lagnaprakaraṇa*, which describes yet another sophisticated technique to precisely determine the ascendant. The chapter ends with a tantalising verse, describing a quantity known as *lagnakarṇa*, and hinting at a process known as *sphuṭīkaraṇa* for determining the ascendant. We intend to discuss this fascinating procedure and other contributions of the *Lagnaprakaraṇa* in forthcoming papers.

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