



Professor Dajun Guo: a true mathematician and educator

Jinxian Sun¹ · Xing-Bin Pan² · Yihong Du³ · Zhaoli Liu⁴ · Zhitao Zhang^{5,6}

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Dajun Guo was born in Sichuan province of China on January 23, 1934. He was an undergraduate student in Department of Mathematics of Sichuan University (1951–1955) and worked in 1956 with the position of assistant in Department of Mathematics of Shandong University. And then, he was promoted there to a lecturer (1960), an associate professor (1978) and a professor (1980). He was appointed as the Chairman of Department of Mathematics in Shandong University (1984–1989). As a visiting professor, D. Guo visited the University of Texas at Arlington (USA) (1985–1986), the University of Alberta (Canada) (1989–1990), and the University of Waterloo (Canada) (1994–1995). He has been the Editor-in-Chief of *Journal of Shandong University (Natural Science Edition)* (China), and has been a member of the editorial boards of *Advances in Mathematics* (China), *Northeastern Mathematical Journal* (China), *Abstract and Applied Analysis* (USA) and *Dynamics of Continuous, Discrete and Impulsive Systems* (Canada). He was the reviewer of *Mathematical Reviews* (USA). He was also the supervisor of twenty-six Ph. D. students of Department of Mathematics in Shandong University. His research area included nonlinear functional analysis, nonlinear integral equations and ordinary differential equations in Banach spaces.

Professor D. Guo was one of the three leaders of nonlinear functional analysis in China and they organized together the first, second and third congresses of nonlinear functional

This article is part of the topical collection dedicated to Prof. Dajun Guo for his 85th birthday, edited by Yihong Du, Zhaoli Liu, Xingbin Pan, and Zhitao Zhang.

✉ Jinxian Sun
jxsun7083@163.com

Xing-Bin Pan
xbpan@math.ecnu.edu.cn

Yihong Du
ydu@une.edu.au

Zhaoli Liu
zliu@cnu.edu.cn

Zhitao Zhang
zzt@math.ac.cn

Extended author information available on the last page of the article

analysis in China (Beijing 1980, Chengdu 1981 and Xiamen 1982), and he gave main reports in these congresses. He attended the “International Congress of Nonlinear Analysis and Applications” (Arlington, Texas, USA, 1986), and “International Congress of Differential Equations and Applications” (Columbus, Ohio, USA, 1988), and gave a twenty minutes talk in each congress. He also attended the “Third World Congress of Nonlinear Analysis” (Catania, Italy, 2000) and gave an one hour report in the congress.

His research project “The Solutions of Nonlinear Integral Equations and Applications” received the second prize of science and technology of Shandong Province of China (1981), and his monograph “Nonlinear Functional Analysis” (Shandong Science and Technology Press, Jinan, China, 1985, in Chinese) received the second prize of science and technology progress of the Ministry of Education of China (1998). The research project “Nonlinear Analysis and Applications” (D. Guo, J. Sun, D. Zhou, Z. Liu, P. Ji) received the first prize of science and technology progress of Shandong Province of China (1998). The research project “Nonlinear Integral Equations and Applications” (D. Guo, J. Sun, C. Huang) and the research project “Partial Ordering Method in Nonlinear Analysis” (D. Guo, J. Sun, Y. Du, D. Zhou) both received the second prize of science and technology progress of the National Education Committee of China (1988 and 1993). The research project “Nonlinear Functional Analysis and Applications” (D. Guo, J. Sun, C. Huang, G. Qi, Z. Liu) received the second prize of science and technology progress of Shandong Province of China (1993), and the research project “Nonlinear Functional Analysis and Its Applications” (D. Guo, Z. Liu, F. Liang, B. Lou) received the second prize of natural science of Shandong Province of China (2002). The teaching project “Writing Textbooks and Teaching Courses on Functional Analysis” (D. Guo, C. Huang, F. Liang) received the excellent teaching prize of the National Education Committee of China (1989).

In 1980–2000, Professor D. Guo was interested to establish new fixed point theorems for nonlinear operators. In 1981, by using the Leray-Schauder degree theory, he established in [31] (see also [51]) a new fixed point theorem: fixed point theorem of domain expansion and compression. This theorem was useful for discussing the existence of non-zero solutions and multiple solutions for nonlinear operator equations and global structure of eigenvalues and eigenvectors for nonlinear operators. In particular, he has used it to make an improvement of a theorem about the eigenvalues of nonlinear operators due to J. Cronin in 1972 (see [32, 51]). Several authors called it “Guo’s theorem” in their published papers (see, for example, J. Sun, *A generalization of Guo’s theorem and applications*, *J. Math. Anal. Appl.*, **126** (2) (1987), 566–573; D. Caponetti, A. Trombetta, G. Trombetta, *An extension of Guo’s theorem via k - ψ -contractive retractions*, *Nonlinear Anal.*, **64** (9) (2006), 1897–1907.) and it has been selected into three monographs (W. Chen, *Nonlinear Functional Analysis*, Gansu People’s Press, Lanzhou (China), 1982 (in Chinese), p.287, Theorem 18.9; [57] p.133, Theorem 2.5.1; [140] p.128, Theorem 3.6). Moreover, D. Guo collaborated with L.H. Erbe and W. Krawcewicz on extending this theorem to multivalued maps with an application to second order differential inclusions (see [73]). For decreasing and completely continuous operators, he obtained in [51] the existence and uniqueness of a positive fixed point and the convergence of the iterative sequence to the unique positive fixed point. As an application to a nonlinear integral equation interesting in nuclear physics, he improved a result due to C.A. Stuart in 1971 (Stuart’s result did not contain the conclusion of convergence of the iterative sequence). For decreasing operators without complete continuity, he obtained the same result (see [76, 110]) by strengthening an inequality condition and using a quite different method to drop the condition of complete continuity. In 1987, D. Guo and V. Lakshmikantham proposed the concept of a mixed monotone operator and its coupled fixed points and obtained the existence of a coupled

fixed point with minimax property and some iterative sequence converging to this coupled fixed point (see [54]). Many authors were interested in this concept and published many papers on this subject. Later, D. Guo obtained in papers [62] and [74] the existence and uniqueness of a positive fixed point and the convergence of the iterative sequence to the unique fixed point for a class of mixed monotone operators without the hypothesis on the continuity and compactness of the operators.

Professor D. Guo investigated nonlinear integral equations (especially, Hammerstein integral equations) for a long time (1960–2000). He used the topological degree theory, variational method and partial ordering method to discuss the existence of solutions, multiple solutions, existence of infinitely many solutions and applications to differential equations, and obtained a lot of new results (see [19, 20, 23–30, 36–39, 42–45, 49, 50, 52, 53, 56, 59–61, 66, 68, 70, 72, 75, 77, 79, 83, 86, 88, 90, 92, 138]). In paper [50], he used the critical point theory combining with the theory of linear integral equations to obtain the existence of infinitely many solutions of Hammerstein integral equations with applications to two-point boundary value problems for nonlinear ordinary differential equations. This result improved a result due to A. Ambrosetti and P.H. Rabinowitz in 1973 (that result could not apply to the two-point boundary value problems for nonlinear ordinary differential equations because the Green function did not satisfy the related conditions). By using the topological degree theory and partial ordering method, he investigated in papers [26, 29, 38] and [39] the sublinear, superlinear and mixed Hammerstein integral equations and obtained the existence of one nontrivial nonnegative solution and the existence of two nontrivial nonnegative solutions, respectively, and applied these results to the Dirichlet problems of elliptic partial differential equations. He also used the fixed point index theory to obtain the existence of two nontrivial nonnegative solutions for impulsive nonlinear Fredholm integral equations with applications to two-point boundary value problems for impulsive nonlinear ordinary differential equations (see [75]). In an early paper [23], he improved a result due to M.A. Krasnoselskii in 1956 on the existence of solutions to a class of Hammerstein integral equations by using the variational method and applied to a boundary value problem of elliptic partial differential equations, and in a recent paper [138], he obtained the existence of infinitely many solutions for a class of impulsive differential equations by using critical point theory to the equivalent Hammerstein integral equations. On the other hand, he collaborated with V. Lakshmikantham on investigation of a nonlinear integral equation arising in infectious diseases and obtained the existence of one nontrivial nonnegative periodic solution and the existence of two nontrivial nonnegative periodic solutions (see [53, 61]), which improved a result due to L.R. Williams and R.W. Leggett in 1982 on the existence of one nontrivial nonnegative periodic solution. In another paper [28], he discussed a Hammerstein integral equation in neutron transport theory, which was investigated by A. Pazy and P.H. Rabinowitz in 1969. This equation contained several physical parameters. A. Pazy and P.H. Rabinowitz proved that this equation has a unique nontrivial solution. D. Guo proved that this unique nontrivial solution continuously depended on these physical parameters, so it was a complementary of the result due to A. Pazy and P.H. Rabinowitz.

In recent years, Professor D. Guo was interested in impulsive nonlinear integro-differential equations in Banach spaces. This was a new area of investigation and there were only a few results. He used method of nonlinear functional analysis (topological degree theory, partial ordering method, etc.) combined with technique of differential equations (comparison results, upper and lower solutions, etc.) to investigate the solutions of first-order, second order, third-order and in general, n th-order impulsive nonlinear integro-differential equations in Banach spaces, and obtained a series of basic and new results (see

[78, 87, 89, 91, 94–106, 115–119, 123–125, 128, 129, 131–134, 136, 141]). These results made great contributions to the establishment of the general theory in this area. In paper [115], he established the general idea and some fundamental tools for discussing the existence of solutions of n th-order nonlinear impulsive integro-differential equations in Banach spaces. First, he selected the space of piecewise differentiable abstract functions as the basic space for discussion, and then, established the Taylor formula in this space ([115] Lemma 2), which enabled us to reduce the initial value problem of an n th-order impulsive integro-differential equation in a Banach space to an impulsive integral equation in the basic space, and so, reduce to find a fixed point of the corresponding operator in the basic space. Next, he established a formula about the Kuratowski measure of non-compactness of bounded sets in the basic space ([115], Lemma 7), which enabled us to prove the compactness of the corresponding operator in the basic space. Finally, the Schauder fixed point theorem implied that the corresponding operator has a fixed point in the basic space, and therefore, the original initial value problem had an solution. In addition, the construction of an example ([115], Example 1) as an application of the obtained result to an infinite system of scalar impulsive integro-differential equations is difficult and interesting. In three papers [125, 128] and [129], he discussed three boundary value problems for n th-order impulsive nonlinear integro-differential equations in Banach spaces. By using the basic space and the Taylor formula and the formula about the measure of non-compactness in the basic space, which were established in [115], and by applying the fixed point index theory, he obtained the existence of two positive solutions for each of the three boundary value problems. Of course, the three equivalent impulsive integral equations, and so, the three corresponding operators were different from each other, which were also different from that in [115]. In paper [119], he used a completely different method to deal with an initial value problem of an n th-order impulsive nonlinear integro-differential equation in a Banach space. The key was to establish a comparison result in the basic space, which enabled us to construct two monotone iterative convergent sequences of abstract functions such that the two limit abstract functions were the minimal and maximal solutions of the original initial value problems between a lower solution and an upper solution. In papers [133] and [134], he investigated two boundary value problems for n th-order impulsive nonlinear singular integro-differential equations in a Banach space. In order to overcome the difficulty arising from the singularities, he constructed a bounded closed convex set in the basic space for each boundary value problem apart from the singularities such that the corresponding operator mapped this bounded closed convex set into itself, and then, used the Schauder fixed point theorem. In another paper [141], he discussed the multiple solutions of a boundary value problem for a third-order impulsive nonlinear singular integro-differential equation in a Banach space. Instead of using the Schauder fixed point theorem, he used the fixed point theorem of cone expansion and compression with norm type, which was established by himself in [40] (see also [55]), and obtained the existence of two positive solutions for the boundary value problem. The key was to construct a completely new cone such that we could apply the fixed point theorem of cone expansion and compression with norm type twice apart from the singularities.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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Affiliations

Jinxian Sun¹ · Xing-Bin Pan² · Yihong Du³ · Zhaoli Liu⁴ · Zhitao Zhang^{5,6}

¹ Department of Mathematics, Jiangsu Normal University, Xuzhou, Jiangsu 221116, People's Republic of China

² Department of Mathematics, East China Normal University, Shanghai 200062, People's Republic of China

³ School of Science and Technology, University of New England, Armidale, NSW 2351, Australia

⁴ School of Mathematical Sciences, Capital Normal University, Beijing 100048, People's Republic of China

⁵ Academy of Mathematics and Systems Science, The Chinese Academy of Sciences, No. 55, Zhongguancun East Road, Beijing 100190, People's Republic of China

⁶ School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China