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ORIGINAL ARTICLE

Analysis of MRT Precoding Massive MIMO for Railway & Maritime Terminal in Multi-cell Environment

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Abstract

This paper extends downlink multi-user massive multiple-input-multiple-output (MIMO) with maximum ratio transmission (MRT) precoding for rapidly movable railway and maritime terminal in multi-cell environment and analyzes its outage probability. In a railway & maritime adjacent multi-cell environment, especially for users (regarded as railway train or ship terminal) at the cell edge, inter-cell interference (ICI) causes the performance degradation of communication quality. Therefore, it is necessary to establish an analytical model to consider the above-mentioned problem. In this paper, we analyze the signal-to-interference and noise ratio (SINR) distribution of downlink multi-user massive MIMO with MRT precoding in a movable railway & maritime adjacent multi-cell environment and derive a new mathematical formula for the outage probability. We also compare the derived results with simulation results to confirm their consistency.

Keywords Railway and Maritime adjacent multi-cell · Massive MIMO · Precoding · Probability

1 Introduction

Massive MIMO has emerged as a critical technology for next-generation wireless communication systems. It enables users to share the same frequency resources simultaneously using large-scale array antennas at the base station [1] [2]. Precoding is crucial in massive MIMO as it suppresses inter-user interference and improves antenna gain. MRT precoding has received much attention among various precoding techniques due to its simplicity in signal processing [3] [4]. For example, it was shown that MRT outperforms ZF in low SNR regions and when the ratio of the number of BS antennas to the number of users is small.

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The performance of a massive MIMO system is often evaluated using metrics such as achievable data rate and signal-to-interference-plus-noise ratio (SINR) [3, 4, 7]. Additionally, the outage probability, representing the probability of SINR falling below a statistical threshold, is a critical indicator for assessing communication stability. Although simulation approaches are practical, performance evaluation of massive MIMO systems using them requires extensive time due to the system's increasing complexity with the number of antennas. To perform accurate and efficient performance analysis of Massive MIMO, it is necessary to formulate the analysis considering inter-cell interference (ICI), which significantly affects communication quality at cell boundaries. This enables more precise and comprehensive analysis, contributing to developing next-generation wireless communication systems. Recently, LTE-railway and LTE-maritime are also considered the cell-edge performance for achieving the broadband data transmission [8]-[13]. However, in this system, down link signals are directly transmitted from overlapping multi-cells to users, trains & ships, thus although ICI is very severe, statistical modeling or analysis is still insufficient situation. While existing analytical models of traditional massive MIMO-OFDM with MRT precoding have been proposed [14] [15], they are also mainly limited to a single-cell environment and need to consider

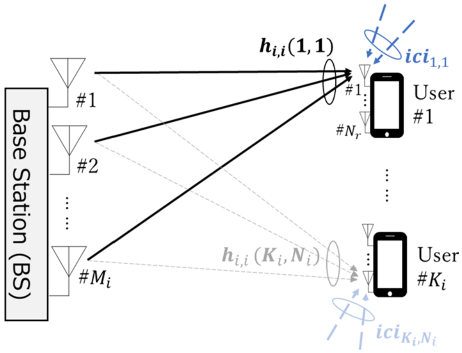

Fig.1 System model

ICI adequately. To address this issue, this paper presents a new mathematical model for the outage probability of MRT precoding in a movable railway & maritime terminal adjacent multi-cell environment that considers ICI. Additionally, we analyze the SINR distribution of MRT precoding in the same environment to comprehensively understand system performance. By utilizing this analytical approach, we can simplify the performance analysis and improve the accuracy of our evaluations, providing valuable insights for developing next-generation railway & maritime based wireless communication systems.

2 System Model

Precoding is a technique that exploits transmit diversity by weighting the transmitted data. It enables IUI suppression and user (regarded as railway train or ship terminal) multiplexing for simultaneous connections. In a multi-user MIMO, linear precoding techniques include MRT, ZF, and transmit Wiener precoding. However, MRT precoding is famous for its ability to maximize the signal-to-noise ratio (SNR) with simple signal processing. In this paper, we analyze a massive MIMO system with MRT precoding. Also, we assume a downlink multi-user massive MIMO system in a movable Railway & maritime terminal adjacent multi-cell environment, where the number of base station (BS) antennas is $M_i (M_i \geq 100)$, the number of users is K_i, and the number of antennas per user is N_i in the i-th cell as shown in Fig. 1.

At railway and marine terminals, there are numerous multipaths due to sea level, cargo, and other factors. Therefore, Rayleigh flat-fading channel is considered. $\boldsymbol{h}_{i,j}(u,r) \in \mathbb{C}^{1\times M_i}$ denotes the channel vector from the BS of the j-th cell to the r-th antenna of the u-th user in the i-th cell. Each entry of the channel vector is independent and identically distributed (i.i.d) following $CN(0,1)$, which means the symmetric complex Gaussian distribution with zero-mean and unit-variance. We also assume the estimated channel state information (CSI) is perfect. The channel matrix $\boldsymbol{H}_{i,j}$ is expressed as

$$\boldsymbol{H}_{i,j} = \left[\boldsymbol{h}_{i,j}^T(1,1), \cdots, \boldsymbol{h}_{i,j}^T\left(1,N_i\right), \cdots, \boldsymbol{h}_{i,j}^T\left(K_i,N_i\right)\right]^T \tag{1}$$

In this model, different signals are transmitted to each user's antenna so that the data symbol vector $\boldsymbol{X}_i$ is represented as

$$\boldsymbol{X}_i = \left[x_i(1,1), \cdots, x_i\left(1,N_i\right), \cdots, x_i\left(K_i,N_i\right)\right]^T \tag{2}$$

Signal power is normalized to $\mathbb{E}\left(\left|x_i(u,r)\right|^2\right) = 1$, where $\mathbb{E}$ means the expectation operation. Since MRT precoding is a method of multiplying the Hermitian transpose of the channel matrix $\boldsymbol{H}$ by the data symbol vector $\boldsymbol{X}$, the received signal matrix $\boldsymbol{Y}_i$ becomes

$$\boldsymbol{Y}_i = \sqrt{\frac{P_i}{M_iK_iN_i}}\boldsymbol{H}_{i,i}\boldsymbol{H}_{i,i}^H\boldsymbol{X}_i + \sum_{\substack{l=1 \\ l\neq i}}^{L} \sqrt{\frac{P_l}{M_lK_lN_l}}\boldsymbol{H}_{i,l}\boldsymbol{H}_{l,l}^H\boldsymbol{X}_l + \boldsymbol{Z}_i, \tag{3}$$

where P_i is the average total transmit power of the BS in the i-th cell, $(\bullet)^H$ denotes the matrix Hermitian transpose, the second term is the inter-cell interference (ICI) received from the other $L-1$ cells and the third term $\boldsymbol{Z}_i = \left[z_i(1,1), \cdots, z_i(K_i,N_i)\right]^T \in \mathbb{C}^{K_iN_i\times 1}$ is the additive white Gaussian noise (AWGN) vector, each element follows $CN(0,1)$. In this model, the u-th user is assumed to be at the cell edge and each cell has the same number of users. In this case, since ICI distribution has the same distribution of IUI, the total interference can be expressed as the form of ICI and IUI multiplication. Here, we define the ξ as the power ratio of ICI and IUI. Therefore, the received signal at the r-th antenna of the u-th user is expressed as

$$y_i(u,r) = \sqrt{\frac{P_i}{M_iK_iN_i}}\boldsymbol{h}_{i,i}(u,r)\boldsymbol{h}_{i,i}^H(u,r)x_i(u,r) + \sqrt{(1+\xi)}\sqrt{\frac{P_i}{M_iK_iN_i}}\sum_{\substack{k=1 \\ k\neq u}}^{K_i}\sum_{\substack{n=1 \\ n\neq r}}^{N_i}\boldsymbol{h}_{i,i}(u,r)\boldsymbol{h}_{i,i}^H(k,n)x_i(k,n) + z_i(u,r), \tag{4}$$

where the first term is the desired signal, the second term is the interference signal and the third term is noise. From (4), the SINR for the r-th antenna of the u-th user is

$$\mathrm{SINR}_i(u,r) = \frac{\frac{P_i}{M_iK_iN_i}\left|\boldsymbol{h}_{i,i}(u,r)\boldsymbol{h}_{i,i}^H(u,r)\right|^2}{1+\frac{(1+\xi)P_i}{M_iK_iN_i}\sum_{\substack{k=1\\k\neq u}}^{K_i}\sum_{\substack{n=1\\n\neq r}}^{N_i}\left|\boldsymbol{h}_{i,i}(u,r)\boldsymbol{h}_{i,i}^H(k,n)\right|^2}. \tag{5}$$

3 Outage Probability Analysis

3.1 Interference Signal Distribution

The interference signal power of the r-th antenna of the u-th user is as follows,

$$U_i(u,r) = \frac{1}{M_i}\sum_{\substack{k=1\\k\neq u}}^{K_i}\sum_{\substack{n=1\\n\neq r}}^{N_i}\left|\boldsymbol{h}_{i,i}(u,r)\boldsymbol{h}_{i,i}^H(k,n)\right|^2. \tag{6}$$

Equation (6) is the sum of $K_iN_i - 1$ streams, each stream follows a gamma distribution $\phi(x;1,1)$. The gamma distribution is the following distribution as

$$\phi(x;k,\theta) = \frac{1}{(k-1)!\theta^k}x^{k-1}e^{-\frac{x}{\theta}}, \tag{7}$$

where k is called the shape parameter and θ is called the scale parameter. Taking into account the distribution and correlation of each stream from [16], the probability density function (PDF) in (6) can be expressed as

$$U_i(u,r) : f(y) = (1-\kappa)\sum_{j=0}^{\infty}\kappa^j\phi\left(y;K_iN_i+j-1,\frac{1}{\tau}\right), \tag{8}$$

where

$$\kappa = \frac{K_iN_i-1}{\sqrt{M_i}+K_iN_i-2}, \tau = \frac{\sqrt{M_i}}{\sqrt{M_i}-1}.$$

From (8), the expectation and variance of $U_i(u,r)$ are calculated as

$$E\left(U_i(u,r)\right) = K_iN_i - 1, \tag{9}$$

$$Var\left(U_i(u,r)\right) = K_iN_i - 1 + \frac{(K_iN_i-1)(K_iN_i-2)}{M_i}. \tag{10}$$

Equation (8) is an addition of infinite terms, however in practice, it can only be computed with finite terms. Therefore, using the Taylor expansion, the PDF of $U_i(u,r)$ can be rewritten as follows,

$$f(y) = \lambda\kappa^{-(K_iN_i-2)}\left[e^{-\lambda y} - e^{-\tau y}\sum_{n=0}^{K_iN_i-3}\frac{(\kappa\tau y)^n}{n!}\right], \tag{11}$$

where $\lambda = \frac{\sqrt{M_i}}{\sqrt{M_i}+K_iN_i-2}$.

3.2 Desired Signal Distribution

Next, we consider the desired signal. The amplitude of the desired signal $\sqrt{D_i(u,r)}$ for the r-th antenna of the u-th user is normalized as

$$\sqrt{D_i(u,r)} = \frac{1}{M_i}\left|\boldsymbol{h}_{i,i}(u,r)\boldsymbol{h}_{i,i}^H(u,r)\right|. \tag{12}$$

Observing (12), this equation follows a gamma distribution $\phi\left(x;M_i,\frac{1}{M_i}\right)$. The power is obtained by squaring the amplitude. Therefore, the expectation and variance of the desired signal power $D_i(u,r)$ are as follows,

$$E\left(D_i(u,r)\right) = 1 + \frac{1}{M_i}, \tag{13}$$

$$Var\left(D_i(u,r)\right) = \frac{4}{M_i} + \frac{10}{M_i^2} + \frac{6}{M_i^3}. \tag{14}$$

3.3 SINR Distribution and Outage Probability

Substituting (11) and (12) into (13), SINR is expressed as

$$\mathrm{SINR}_i(u,r) = \frac{P_iM_i}{K_iN_i} \bullet \frac{D_i(u,r)}{1+\frac{(1+\xi)P_i}{K_iN_i}U_i(u,r)}. \tag{15}$$

Consider the distribution of Equation (15). $D_i(u,r)$ has zero-variance when $M_i \to \infty$ from (14), meaning that the desired signal power becomes deterministic in a massive MIMO system with a very large number of transmit antennas. On the other hand, for $1 + \frac{(1+\xi)P_i}{K_iN_i}U_i(u,r) = I_i(u,r)$, the variance is calculated from (10), as follows

$$\begin{aligned}&\mathrm{Var}\left(I_i(u,r)\right)\\&= \frac{(1+\xi)^2P_i^2\left(K_iN_i-1\right)}{K_i^2N_i^2}\left[1+\frac{K_iN_i-2}{M_i}\right]\\&> \frac{(1+\xi)^2P_i^2\left(K_iN_i-1\right)}{K_i^2N_i^2}.\end{aligned} \tag{16}$$

Observing (16), $\mathrm{Var}\left(I_i(u,r)\right)$ is non negligible value when P_i, K_i, N_i take reasonable values, and is significantly larger than the variance of the desired signal power when $(1+\xi)^2P_i^2M_i >> K_iN_i$.

Therefore, it can be seen that the distribution of (15) is highly dependent on the distribution of the interference signal power. Moreover, $D_i(u,r)$ is assigned the expected value, and $U_i(u,r)$ is treated as a random variable following

Table 1 Simulation parameters

Parameters	Values
Number of BS antennas M	256
Number of users K	15
Number of antennas per users N	2
Average total transmit power P	$10dB$
Threshold γ_{th}	$10dB$

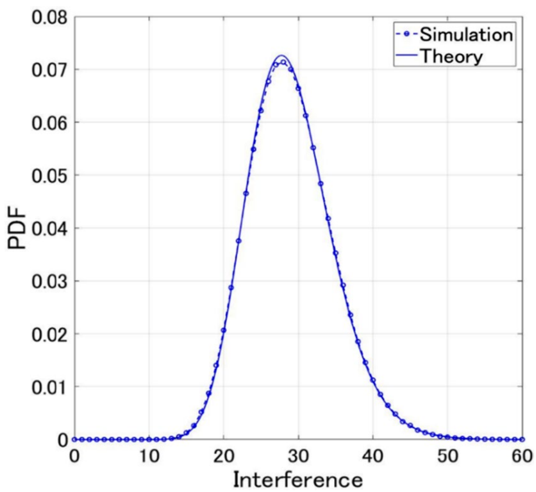

Fig. 2 Interference power distribution

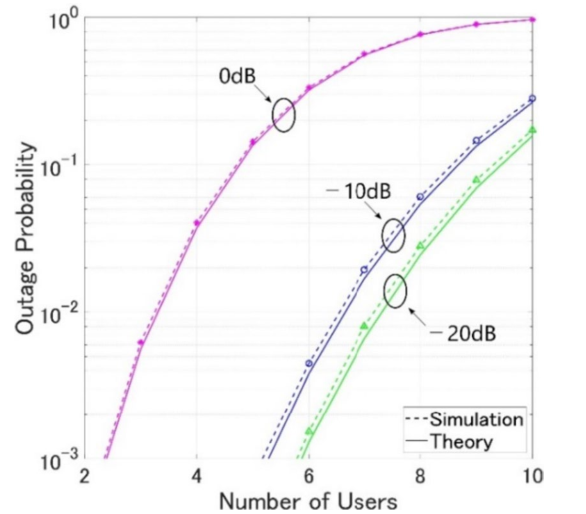

Fig. 3 Outage probability per number of users for various ICI power of 0 dB, −10 dB, −20 dB

the PDF in (11). Outage probability is defined as the probability that the SINR is less than the threshold SINR γ_{th}. Let γ_{th} is the threshold SINR, and the outage probability P_{out} is expressed as

$$P_{out} = \mathbb{P}\left(\frac{P_iM_i}{K_iN_i}\bullet\frac{1+\frac{1}{M_i}}{1+\frac{(1+\xi)P_i}{K_iN_i}U_i(u,r)} < \gamma_{th}\right), = P\left(U_i(u,r) > \frac{M_i+1}{\gamma_{th}(1+\xi)} - \frac{K_iN_i}{P_i(1+\xi)}\right), \tag{17}$$

where $\mathbb{P}$ means the probability and $\rho = \frac{M_i+1}{\gamma_{th}(1+\xi)} - \frac{K_iN_i}{P_i(1+\xi)}$. Using (11), the outage probability can be formulated as

$$P_{out} = \int_{\rho}^{\infty} \lambda\kappa^{-(K_iN_i-2)}\left[e^{-\lambda y} - e^{-\tau y}\sum_{n=0}^{K_iN_i-3}\frac{(\kappa\tau y)^n}{n!}\right]dy = \kappa^{-(K_iN_i-2)}e^{-\lambda\rho}\left[1-(1-\kappa)e^{-\kappa\tau\rho}\sum_{n=0}^{K_iN_i-3}\sum_{m=0}^{n}\kappa^n\frac{(\tau\rho)^m}{m!}\right]. \tag{18}$$

4 Performance Comparison

The simulation parameters are listed in Table 1. Figure 2 shows the PDF of the interference signal power for simulation and theoretical results. Observing Fig. 2, the PDF of the simulation and theoretical results is tightly matched.

Figure 3 shows the outage probability per users. Outage probability is getting worse as the number of users increases. On the other hand, the system sum rate increases monotonically with the number of users. This shows the importance of outage probability analysis.

Figure 4 shows the outage probability for various number of BS antennas. The outage probability rapidly improves with increasing the number of BS antenna. It can be also seen that the outage probability differs significantly depending on the amount of ICI power. For achieving the outage probability of 10^{-2} the required number of BS antennas for ICI power of 0 dB, −10 dB, −20 dB is 890, 510 and 460, respectively. This indicates the need for a mathematical model that takes ICI into account.

Figure 5 shows the outage probability per SNR when $M_i = 500$. We can see that outage probability improves with increasing transmit power. However, it does not decrease to zero. This is because the interference power increases as the transmit power increases. Therefore, it is found that increasing the number of transmit antennas is the most effective way to improve the outage probability.

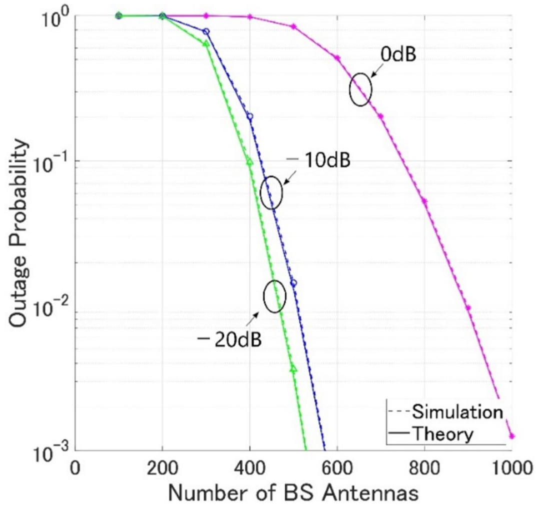

Fig. 4 Outage probability per number of BS antennas for various ICI power of 0 dB, −10 dB, −20 dB

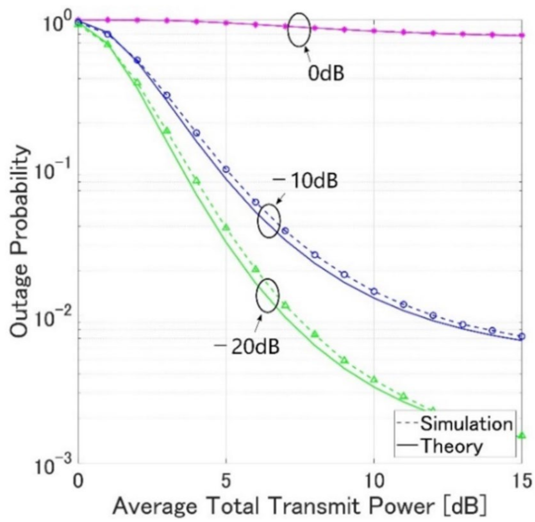

Fig. 5 Outage probability per average total transmit power for various ICI power of 0 dB, −10 dB, −20 dB

5 Conclusion

In this paper, we have analyzed a downlink multi-user massive MIMO system with MRT precoding for railway and maritime terminal in multi-cell environment and derived the mathematical formula for the outage probability. From the simulation result, we confirm that the derived mathematical formula is well matched. We also confirmed that increasing the number of BS antennas is the most effective way to improve the outage probability for MRT precoding. We were able to demonstrate the need for an analytical model that takes ICI into account, which is a novelty of this paper. By using the derived formula, the number of transmit antennas, the number of receive antennas, and the transmitting power can be determined when designing a system or foreseeing the movable Railway & maritime communication environment.

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