



Optimal Formation of UAV Swarm for TDOA-Based Passive Target Tracking

Ui-Suk Suh¹ · Seul-Ki Han² · Won-Sang Ra¹

Received: 17 March 2020 / Revised: 24 June 2021 / Accepted: 5 August 2021 / Published online: 14 October 2021
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Abstract

An analytic solution to the optimal formation problem of an unmanned aerial vehicle (UAV) swarm is newly proposed for maximizing the performance of passive target tracking. Most previous techniques could not handle this problem efficiently because they have had difficulties in defining the performance index of a nonlinear target tracking filter in closed form. To overcome this limitation, the passive target tracking problem is investigated within the framework of the linear non-conservative robust Kalman filter (NCRKF) theory. Accommodating the merit of the suggested linear target tracking filter structure, its performance measure can be analytically expressed in terms of the UAV formation as well as the sensor accuracy. Therefore, it is easy to determine the optimal UAV formation by maximizing the target tracking performance in the worst case. In addition, our approach is very practical because it considers the estimation error characteristics of the actual passive target tracking filter and the communication range among UAVs in determining the optimal UAV formation. Through the simulations, the effectiveness of the proposed scheme is validated.

Keywords Optimal UAV formation · Passive target tracking · Fisher information matrix(FIM) · Non-conservative robust Kalman filter(NCRKF)

1 Introduction

Cooperative unmanned aerial vehicles (UAVs) are known to provide great versatility, resource efficiency, and survivability in various civilian and military applications compared to a single UAV [1–4]. An UAV swarm has been paid considerable attention by many researchers especially in passive target tracking. This is because an UAV swarm is able to provide enhanced performance even using low-grade sensors and to avoid the observability issue as well. Accordingly, the related studies have been mainly focused on the data fusion architecture and the formation of an

UAV swarm which directly affect the overall target tracking performance [5–7]. However, since there are difficulties in deriving the performance measure due to the nonlinear filter structure used for passive target tracking, a systematic way to determine the optimal UAV formation still remains as an unsolved problem.

The moving target tracking with a passive sensor has been recognized as one of the representative nonlinear estimation problems. This is because the available passive measurement such as TDOA(time difference of arrival) or AOA(angle of arrival) has a nonlinear relation with the target state variables that should be estimated. Since the random trajectory of a moving target is modeled as a nonstationary or cyclostationary time series, the passive target tracking can be addressed by the nonlinear Kalman filtering for Gauss-Markov random processes [8–11]. Theoretically speaking, the estimation performance of the Kalman filter is characterized by using a Fisher information matrix (FIM) equivalent to the inverse of error covariance matrix [12, 13]. However, different from the linear one, nonlinear filters often produce biased estimates which cause the performance degradation of target tracking. Therefore, the structure of a target

✉ Won-Sang Ra
wonsang@handong.edu

Ui-Suk Suh
uisuk.suh@handong.edu

Seul-Ki Han
sk_han@hyundai.com

¹ Department of Mechanical and Control Engineering,
Handong Global University, Pohang, South Korea

² Hyundai Motor Company, Seoul, South Korea

tracking filter is substantial in defining the practical target tracking performance measure.

It is well-known that the performance of a passive target tracking filter using multi-sensor measurements is influenced by the relative geometry between the target and the sensors. This implies that the optimal sensor placement is another significant factor to secure the satisfactory target tracking performance. However, since the error covariance matrix of a nonlinear Kalman filter varies with the target motion, it is hard to derive the necessary and sufficient condition for optimal sensor placement in closed-form [14]. To tackle this issue, most existing methods rely on gradient-based numerical algorithms which require computational burden [15–17]. Even worse, if moving platforms equip the sensors and control their formation continuously, the optimization problem will become increasingly complicated. In this circumstance, the numerical techniques would not be adequate for on-line determination of the optimal sensor placement.

Meanwhile, the parameter optimization of the prescribed sensor formation has been attempted [18–20]. This scheme assumes that the multiple sensors are placed on a line or circle with equal spacing. In this setting, the target tracking performance measure has just a single unknown parameter, hence the complexity of the sensor formation problem is drastically reduced. This method was usually applied for the limited case where the target is surrounded by multiple sensors. Although the far-field target tracking was sporadically studied, the result may be restrictive in real applications by ignoring the maximum communication range between sensors. Furthermore, the previous schemes could fail to find the global optimum because they omitted the bias estimation errors caused by nonlinear filters in formulating the sensor formation problem.

To overcome the aforementioned limitations, this paper presents an analytic solution to the optimal formation of an UAV swarm providing TDOA measurements for target tracking. In contrast to the existing methods, the estimation error properties of the passive target tracking filter are taken into account. To do this, the passive target tracking problem is settled by using a linear non-conservative robust Kalman filter (NCRKF) [21]. Exploiting the linear filter structure and the strong convergence of NCRKF, the FIM of the passive target tracking filter can be easily derived in closed form. It is also shown that, by taking determinant of the FIM, the object function of the UAV formation problem is simply expressed as a function of two factors; the relative position of the companion UAV with respect to the reference UAV as well as the standard deviations of the TDOA measurement noises. To cope with the time-varying relative geometry between target and UAV swarm, the optimal UAV formation is defined as the max-min solution of the derived objective function. Thus, the proposed analytic solution is able to guarantee the passive target tracking performance even in the worst case. Moreover, our solution is

practically meaningful because it is derived by considering the communication range among UAVs which is important in implementing the multi-UAV system. Through the computer simulations with typical passive target tracking scenarios, the validity of methodology is verified.

2 Linear Robust Passive Target Tracking Filter

This section outlines the TDOA-based passive target tracking problem in the setting of linear robust estimation theory. The detailed explanation is given in [21, 22].

Consider the typical engagement geometry illustrated in Fig. 1 involving two-dimensional tracking. In Fig. 1, (X_I, Y_I) are axes of the inertial frame, and the origin of the inertial frame is set as the initial position of the reference UAV. The X_I axis coincides with the initial line-of-sight (LOS) direction from the reference UAV to the target in the horizontal plane. The positions of the target and j -th UAV are (x_t, y_t) and (x_j, y_j) , respectively. Hereafter, the subscripts $j = 0$ and $j = 1 \sim N$ are used to denote the reference UAV and its companions. If the signal propagation speed c is known, the range difference (RD) r_j can be easily calculated from the acquired TDOA t_j . In such case, the RD measurement of the j -th UAV with respect to the reference UAV is simply defined as follows:

$$r_j \triangleq c \cdot t_j = d_{t,j} - d_{t,0}, \quad j = 0 \sim N \quad (1)$$

where

$$d_{t,j} = \sqrt{(x_t - x_0)^2 + (y_t - y_0)^2}.$$

In the above equation, $d_{t,j}$ indicates the distance from the j -th UAV to the target.

From the relationship (1), the linear uncertain measurement model can be derived after some tedious algebraic

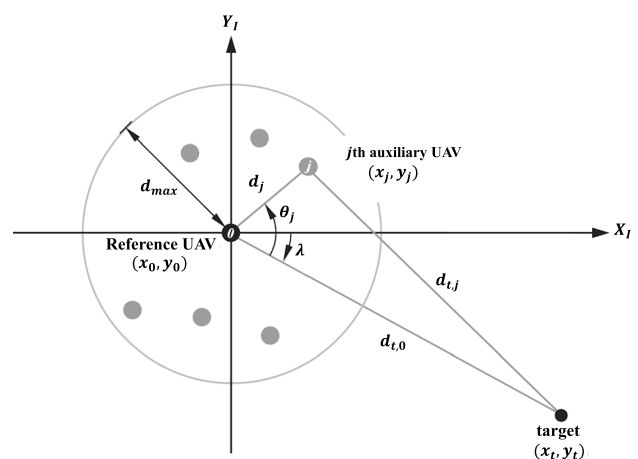


Fig. 1 Relative geometry of target and UAV swarm

manipulations [21]. This enables the TDOA-based target tracking problem to be dealt with in the setting of linear state estimation.

$$y_k = H_k x_k + v_k = (\tilde{H}_k - \Delta H_k) x_k + v_k \tag{2}$$

where

$$x \triangleq \begin{bmatrix} x_t - x_0 \\ y_t - y_0 \\ d_{t,0} \\ \dot{x}_t - \dot{x}_0 \\ \dot{y}_t - \dot{y}_0 \\ \ddot{d}_{t,0} \\ \ddot{x}_t - \ddot{x}_0 \\ \ddot{y}_t - \ddot{y}_0 \\ \ddot{\ddot{d}}_{t,0} \end{bmatrix}, \quad y \triangleq \begin{bmatrix} \vdots \\ y_j \\ \vdots \end{bmatrix}, \quad v \triangleq \begin{bmatrix} \vdots \\ v_j \\ \vdots \end{bmatrix},$$

$$H \triangleq \begin{bmatrix} \vdots \\ h_j & 0^{1 \times 6} \\ \vdots \end{bmatrix}, \quad \tilde{H} \triangleq \begin{bmatrix} \vdots \\ \tilde{h}_j & 0^{1 \times 6} \\ \vdots \end{bmatrix}, \quad \Delta H \triangleq \begin{bmatrix} \vdots \\ \Delta h_j & 0^{1 \times 6} \\ \vdots \end{bmatrix},$$

$$y_j \triangleq \tilde{r}_j^2 - (\tilde{x}_j - \tilde{x}_0)^2 - (\tilde{y}_j - \tilde{y}_0)^2 + \sigma_{x_j}^2 + \sigma_{y_0}^2 + \sigma_{y_j}^2 + \sigma_{x_0}^2 - \sigma_{r_j}^2,$$

$$h_j \triangleq -2 [(x_j - x_0) (y_j - y_0) r_j],$$

$$\tilde{h}_j \triangleq -2 [\tilde{x}_j - \tilde{x}_0 \tilde{y}_j - \tilde{y}_0 \tilde{r}_j],$$

$$\Delta h_j \triangleq -2 [\delta x_{j,0} \delta y_{j,0} \delta r_j],$$

$$v_j \triangleq \delta x_{j,0}^2 + \delta y_{j,0}^2 - \delta r_j^2 - 2(\tilde{x}_{j,0} \delta x_{j,0} + \tilde{y}_{j,0} \delta y_{j,0} - \tilde{r}_j \delta r_j) - \sigma_{r_j}^2 + (\sigma_{x_j}^2 + \sigma_{x_0}^2) + (\sigma_{y_j}^2 + \sigma_{y_0}^2).$$

In the above equation, $\tilde{\epsilon}$ and $\delta \epsilon$ are the noise corrupted measurement of the true variable ϵ and the measurement noise with the standard deviation σ_{ϵ} , respectively. The noise statistics of the j -th UAV position and the RD measurement are assumed as follows:

$$E \left\{ \begin{bmatrix} \delta x_j \\ \delta y_j \\ \delta r_j \end{bmatrix} \right\} = \mathbf{0}^{3 \times 1}, \quad var \left\{ \begin{bmatrix} \delta x_j \\ \delta y_j \\ \delta r_j \end{bmatrix} \right\} = diag(\sigma_{x_j}^2, \sigma_{y_j}^2, \sigma_{r_j}^2) \tag{3}$$

The measurement noise v_k and the stochastic parameter uncertainty ΔH_k satisfy the following statistics.

$$R_k \triangleq var\{v_k\} = diag(\dots R_j \dots) \tag{4}$$

$$W_k \triangleq E\{\Delta H_k^T R_k^{-1} \Delta H_k\} = \begin{bmatrix} \sum_{j=1}^N \frac{W_j}{R_j} & 0^{3 \times 6} \\ 0^{6 \times 3} & 0^{6 \times 6} \end{bmatrix} \tag{5}$$

$$V_k \triangleq E\{\Delta H_k^T R_k^{-1} v_k\} = \begin{bmatrix} \sum_{j=1}^N \frac{V_j}{R_j} \\ \mathbf{0}^{6 \times 1} \end{bmatrix} \tag{6}$$

where

$$R_j \triangleq var\{v_j\} = 2(2r_j^2 + \sigma_{r_j}^2)\sigma_{r_j}^2 + 2(\sigma_{x_j}^2 + \sigma_{x_0}^2)(2(x_j - x_0)^2 + \sigma_{x_j}^2 + \sigma_{x_0}^2) + 2(\sigma_{y_j}^2 + \sigma_{y_0}^2)(2(y_j - y_0)^2 + \sigma_{y_j}^2 + \sigma_{y_0}^2)$$

$$W_j \triangleq E\{\Delta h_j^T \Delta h_j\} = 4 \begin{bmatrix} \sigma_{x_j}^2 + \sigma_{x_0}^2 & 0 & 0 \\ 0 & \sigma_{y_j}^2 + \sigma_{y_0}^2 & 0 \\ 0 & 0 & \sigma_{r_j}^2 \end{bmatrix}$$

$$V_j \triangleq E\{\Delta h_j^T v_j\} = 4 \begin{bmatrix} (x_j - x_0)(\sigma_{x_j}^2 + \sigma_{x_0}^2) \\ (y_j - y_0)(\sigma_{y_j}^2 + \sigma_{y_0}^2) \\ -r_j \sigma_{r_j}^2 \end{bmatrix}$$

Without loss of generality, the target dynamics can be described using the standard constant acceleration motion model.

$$x_{k+1} = F_k x_k + u_k \tag{7}$$

where

$$F = \begin{bmatrix} I^{3 \times 3} & T \cdot I^{3 \times 3} & \frac{1}{2} T^2 \cdot I^{3 \times 3} \\ 0^{3 \times 3} & I^{3 \times 3} & T \cdot I^{3 \times 3} \\ 0^{3 \times 3} & 0^{3 \times 3} & I^{3 \times 3} \end{bmatrix}, \quad u \sim \mathcal{N}(0, Q).$$

In the above equation, T is the sampling period and u_k is the zero-mean white process noise with its variance Q_k .

Finally, according to [21], a linear passive target tracking filter is designed by applying the NCRKF theory to the uncertain linear state-space model (2) and (7).

(measurement update)

$$P_{k|k}^{-1} = P_{k|k-1}^{-1} + \tilde{H}_k^T R_k^{-1} \tilde{H}_k - W_k \tag{8}$$

$$\hat{x}_{k|k} = (I + P_{k|k} W_k) \hat{x}_{k|k-1} + P_{k|k} \tilde{H}_k^T R_k^{-1} (y_k - \tilde{H}_k \hat{x}_{k|k-1}) - P_{k|k} V_k \tag{9}$$

(time update)

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \tag{10}$$

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k} \tag{11}$$

where x and P are the estimate of the NCRKF and its error covariance matrix.

3 Optimal UAV Formation for TDOA-Based Target Tracking

3.1 Objective Function for Optimal UAV Formation

The ultimate goal of this paper is to determine the optimal UAV formation so that the best tracking performance can be achieved. It is a known fact that the performance of the target tracking filter is proportional to the amount of target information. Therefore, the optimal formation problem reduces the problem of determining the UAV formation, which maximizes the amount of target information. The amount of target information can be represented by FIM or the inverse of the estimation error variance when the given target tracking filter has a linear structure [23]. It implies that the target information is closely related to the filter structure for the passive target tracking problem. Therefore, we obtain the corresponding FIM of the NCRKF based passive target tracking filter and analyze its determinant to indicate tracking performance with respect to the UAV formation. Basic assumptions used for designing the optimal UAV formation are as follows:

- A1. The passive target tracking using the UAV swarm consists of reference UAV and N companion UAVs.
- A2. The companion UAVs are placed around the reference UAV with the consideration of the maximum possible communication range d_{max} as shown in Fig. 1. In the figure, the angular position θ_j of the j -th companion UAV is defined with respect to the reference LOS which is the direction from the reference UAV to the target. λ is the reference UAV-to-target LOS angle. d_j is the distance between the reference UAV and the j -th companion UAV. It is also assumed that d_{max} and d_j are not changed and maintained while passive target tracking is performed.
- A3. The measurement noises of UAV position have the same standard deviations denoted as σ_p . The standard deviations of the RD measurement noises are also the same.

$$\sigma_p \triangleq \sigma_{x_0} = \sigma_{y_0} = \sigma_{x_1} = \sigma_{y_1} \cdots \sigma_{x_N} = \sigma_{y_N}$$

$$\sigma_r \triangleq \sigma_{r_1} = \cdots = \sigma_{r_N}$$

- A4. The target is sufficiently far away from the UAV swarm maintaining the certain UAV formation. In addition, the target motion is negligible within the short time period.

Based on the above basic assumptions, the target information obtained by the proposed linear passive target tracking filter in Sect. 2 is easily computed. For more detailed derivation procedure, see Appendix A.

$$\mathcal{I}_{r,k}(\mathbf{x}) \approx k(\bar{\mathbf{H}}^T \bar{\mathbf{R}}^{-1} \bar{\mathbf{H}} - \bar{\mathbf{W}}), \tag{12}$$

where

$$\bar{\mathbf{H}} \triangleq \begin{bmatrix} \vdots \\ h_j \\ \vdots \end{bmatrix}, \quad \bar{\mathbf{H}} \triangleq \begin{bmatrix} \vdots \\ \tilde{h}_j \\ \vdots \end{bmatrix}, \quad \bar{\mathbf{R}} \triangleq \mathbf{R}.$$

It can be prove that the target information of NCRKF, $\mathcal{I}_{r,k}(\mathbf{x})$, is the same with that of optimal Kalman filter (OKF), $\mathcal{I}_{o,k}(\mathbf{x})$. Extracting the position information from (47) yields

$$\begin{aligned} \frac{1}{k} \mathcal{I}_{r,k}(\mathbf{x}) &\xrightarrow{a.s.} \frac{1}{k} \mathcal{I}_{o,k}(\mathbf{x}) \\ &= 4 \begin{bmatrix} \sum_{j=1}^N \frac{(x_j - x_0)^2}{\bar{R}_j} & \sum_{j=1}^N \frac{(x_j - x_0)(y_j - y_0)}{\bar{R}_j} \\ \sum_{j=1}^N \frac{(x_j - x_0)(y_j - y_0)}{\bar{R}_j} & \sum_{j=1}^N \frac{(y_j - y_0)^2}{\bar{R}_j} \end{bmatrix} \end{aligned} \tag{13}$$

where \bar{R}_j is obtained using (4) and assumption A3.

$$\bar{R}_j = 2\sigma_r^2(2r_j^2 + \sigma_r^2) + 8\sigma_p^2(d_j^2 + 2\sigma_p^2).$$

Using the assumption A2, the target and UAV positions in Fig. 1 are rewritten as follows:

$$(x_t, y_t) = d_{t,0}(c_\lambda, s_\lambda) \tag{14}$$

$$(x_j, y_j) = d_j(c_{\theta_j+\lambda}, s_{\theta_j+\lambda}) \tag{15}$$

where $c_\epsilon \triangleq \cos(\epsilon)$ and $s_\epsilon \triangleq \sin(\epsilon)$, respectively.

Now, let us define the amount of target position information by taking the dedeterminant of (13).

$$\begin{aligned} \mathcal{L}_p &\triangleq |\mathcal{I}_{o,k}(\mathbf{x})| \\ &= 4 \left[\sum_{j=1}^N \frac{(x_j - x_0)^2}{\bar{R}_j} \sum_{j=1}^N \frac{(y_j - y_0)^2}{\bar{R}_j} \right. \\ &\quad \left. - \left(\sum_{j=1}^N \frac{(x_j - x_0)(y_j - y_0)}{\bar{R}_j} \right)^2 \right] \end{aligned} \tag{16}$$

Provided that the target is sufficiently far away from the UAV swarm and the UAV swarm is guided toward the target, the following approximations (17)~(19) make sense.

$$d_{max} \ll d_{t,0} \tag{17}$$

$$(d_{t,0}s_\lambda - d_j s_{\theta_j+\lambda})^2 \ll (d_{t,0}c_\lambda - d_j c_{\theta_j+\lambda})^2 \tag{18}$$

$$c_\lambda \approx 1, \quad s_\lambda \approx 0 \tag{19}$$

Thus, the RD information is also approximated by

$$r_j \approx \sqrt{(d_{i,0} - d_j c_{\theta_j + \lambda})^2} - d_{i,0} \approx -d_j c_{\theta_j}. \tag{20}$$

Substituting (20) and $(x_j - x_0, y_j - y_0) \approx d_j(c_{\theta_j}, s_{\theta_j})$ into (16), the objective function for UAV formation is defined as

$$\mathcal{L}_p \approx 4 \left[\sum_{j=1}^N \frac{c_{\theta_j}^2}{\bar{R}_j} \sum_{j=1}^N \frac{s_{\theta_j}^2}{\bar{R}_j} - \left(\sum_{j=1}^N \frac{c_{\theta_j} s_{\theta_j}}{\bar{R}_j} \right)^2 \right], \tag{21}$$

where \bar{R}_j is replaced by

$$\bar{R}_j \approx 2\sigma_r^2 \left(2c_{\theta_j}^2 + \left(\frac{\sigma_r}{d_j} \right)^2 \right) + 8\sigma_p^2 \left(1 + 2 \left(\frac{\sigma_p}{d_j} \right)^2 \right).$$

3.2 Optimal Formation of UAV Swarm

From (21), we can confirm that the amount of target information \mathcal{L}_p depends on the relative position (θ_j, d_j) of companion UAVs with respect to the reference UAV, and the standard deviations (σ_r, σ_p) of measurement noises. \mathcal{L}_p is proportional to d_j , and inversely proportional to (σ_r, σ_p) . It means that the enhanced target tracking performance could be achieved as the distance between UAVs becomes larger and/or the sensor measurement is more accurate. However, the intuitive result on θ_j is difficult to be directly obtained from (21) because there is a somewhat complex nonlinear relationship between θ_j and \mathcal{L}_p . At this point, to make the analysis problem tractable, it is noteworthy that the position of the UAV measured by the navigation system is usually very accurate ($\sigma_p \ll 1$). Then, \bar{R}_j in (21) is again approximated as follows:

$$\bar{R}_j \approx 2\sigma_r^2 \left(2c_{\theta_j}^2 + \left(\frac{\sigma_r}{d_j} \right)^2 \right). \tag{22}$$

Meanwhile, if the target moves at high speed, the target LOS angle λ is rapidly changed. The variation of the LOS angle has a critical effect on the UAV formation because θ_j is determined with respect to the LOS direction as described in Fig. 1. Therefore, even while the UAVs are placed to maximize the amount of target information, they may not give a good tracking performance throughout the entire tracking period. To overcome the above limitation, it is desirable to find the UAV formation which maximize the amount of target information in the worst case. This leads to the following max-min problem.

$$\max_{\theta} \min_{\theta} \mathcal{L}_p \tag{23}$$

where \mathcal{L}_p is redefined by inserting (22) into (21).

$$\mathcal{L}_p \approx 4 \left[\sum_{j=1}^N \frac{c_{\theta_j}^2}{2\sigma_r^2 (2c_{\theta_j}^2 + (\frac{\sigma_r}{d_j})^2)} \sum_{j=1}^N \frac{s_{\theta_j}^2}{2\sigma_r^2 (2c_{\theta_j}^2 + (\frac{\sigma_r}{d_j})^2)} - \left(\sum_{j=1}^N \frac{c_{\theta_j} s_{\theta_j}}{2\sigma_r^2 (2c_{\theta_j}^2 + (\frac{\sigma_r}{d_j})^2)} \right)^2 \right] \tag{24}$$

A solution to this problem is given according to the following two cases. The cases are divided by the relationship between θ_j and (σ_r, d_j) .

Case 1. $|c_{\theta_j}| \gg \sigma_r/d_j$

The amount of target information \mathcal{L}_p is approximated and then rearranged as follows:

$$\begin{aligned} \mathcal{L}_p &\approx 4 \left[\sum_{j=1}^N \frac{c_{\theta_j}^2}{4\sigma_r^2 c_{\theta_j}^2} \sum_{j=1}^N \frac{s_{\theta_j}^2}{4\sigma_r^2 c_{\theta_j}^2} - \left(\sum_{j=1}^N \frac{c_{\theta_j} s_{\theta_j}}{4\sigma_r^2 c_{\theta_j}^2} \right)^2 \right] \\ &= \frac{N^2}{4\sigma_r^4} \left[\frac{1}{N} \sum_{j=1}^N t_{\theta_j}^2 - \left(\frac{1}{N} \sum_{j=1}^N t_{\theta_j} \right)^2 \right] \end{aligned} \tag{25}$$

where $t_{\epsilon} \triangleq \tan(\epsilon)$.

From (25), we can conclude that \mathcal{L}_p increases when the number of UAVs N is large, and the standard deviation of RD measurement noise σ_r is small. In addition, the term in parenthesis of (25) can be interpreted as the variance of t_{θ_j} , $j = 1, \dots, N$. Thus, this indicates that \mathcal{L}_p also increases as the variance t_{θ_j} becomes larger. In order to obtain the minimum value of \mathcal{L}_p , the stationary point of \mathcal{L}_p and its minimization condition are given by (26) and (27), respectively.

$$\begin{aligned} \frac{\partial \mathcal{L}_p}{\partial t_{\theta_N}} &= \frac{N^2}{4\sigma_r^4} \left[\frac{2}{N} t_{\theta_N} - \frac{2}{N^2} \sum_{j=1}^N t_{\theta_j} \right] = 0 \\ \rightarrow t_{\theta_N} &= \frac{1}{N-1} \sum_{j=1}^{N-1} t_{\theta_j} \end{aligned} \tag{26}$$

$$\frac{\partial^2 \mathcal{L}_p}{\partial t_{\theta_N}^2} = 2 - \frac{2}{N} > 0 \rightarrow N > 1 \tag{27}$$

As a well-known fact, since the UAV swarm-based passive target tracking problem needs at least 3 companion UAVs ($N \geq 3$), the minimization condition of (27) is always satisfied. Therefore, the minimizing solution of \mathcal{L}_p is to become $t_{\theta_N} = \frac{1}{N-1} \sum_{j=1}^{N-1} t_{\theta_j}$ defined in (26). By inserting (26) into (25), the minimum value of \mathcal{L}_p is obtained

$$\mathcal{L}_p^* = \frac{N(N-1)}{4\sigma_r^4} \left[\frac{1}{N-1} \sum_{j=1}^{N-1} t_{\theta_j}^2 - \left(\frac{1}{N-1} \sum_{j=1}^{N-1} t_{\theta_j} \right)^2 \right] \quad (28)$$

Now, the condition for maximizing the minimum value \mathcal{L}_p^* is obtained according to whether the number of UAVs N is an odd or even number.

1) N is an odd number

If the companion UAVs are placed symmetrically with respect to the target LOS direction, there exist the formation angle pairs (θ_i, θ_j) with the different sign and same value where $j = i + 1, \forall i, j$. In this case, because the number of UAVs N is an odd number, the second term on the right-hand side of (28) is always satisfies with $\frac{1}{N-1} \sum_{j=1}^{N-1} t_{\theta_j} = 0$ irrespective of the formation angle pairs. At this time, the formation angle of the N th companion UAV becomes $\theta_N = 0^\circ$ from (26). By the characteristics of tangent function, the first term on the right-hand side of (28) has the maximum value of the symmetry formation angle with $|\theta_j| = 90^\circ$. Therefore, the maximizing solution of \mathcal{L}_p^* is as follows:

$$\begin{aligned} \theta_N = 0^\circ \text{ and } \theta_i = 90^\circ \quad (i = 1, 3, \dots, N-2) \\ \text{and } \theta_j = -\theta_i \quad (i = 2, 4, \dots, N-1) \end{aligned} \quad (29)$$

2) N is an even number

In this case, \mathcal{L}_p^* is rearranged as follows:

$$\mathcal{L}_p^* \approx \frac{N(N-1)}{4\sigma_r^4} \left[\frac{1}{N-1} \sum_{j=1}^{N-2} t_{\theta_j}^2 + \frac{N}{(N-1)^2} t_{\theta_{N-1}}^2 \right] \quad (30)$$

Again, considering the situation of the formation angle pairs (θ_i, θ_j) with the different sign and same value where $j = i + 1, \forall i, j$, from the above equation, it can be deduced that \mathcal{L}_p^* is maximized when $|\theta_j| = 90^\circ (j = 1, \dots, N-1)$. This indicates $|\theta_N| \approx 90^\circ$ by (26). Therefore, the maximizing solution of \mathcal{L}_p^* can be defined by

$$\begin{aligned} \theta_i = 90^\circ \quad (i = 1, 3, \dots, N-1) \\ \text{and } \theta_j = -\theta_i \quad (j = 2, 4, \dots, N). \end{aligned} \quad (31)$$

Case 2. $|c_{\theta_j}| \ll \sigma_r/d_j$

In a similar way with Case 1, \mathcal{L}_p can be approximated by applying the assumption $|c_{\theta_j}| \ll \sigma_r/d_j$ to (24).

$$\mathcal{L}_p \approx \frac{1}{\sigma_r^8} \left[\sum_{j=1}^{N-1} d_j^2 c_{\theta_j}^2 \sum_{j=1}^{N-1} d_j^2 s_{\theta_j}^2 - \left(\sum_{j=1}^{N-1} d_j^2 s_{\theta_j} c_{\theta_j} \right)^2 \right] \quad (32)$$

The above equation is rearranged for convenience of the analysis.

$$\begin{aligned} \mathcal{L}_p &\approx \frac{1}{\sigma_r^8} \left[\sum_{i=1}^{N-1} \sum_{j=1}^{N-1} d_i^2 c_{\theta_i}^2 \cdot d_j^2 s_{\theta_j}^2 \right. \\ &\quad \left. - \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} (d_i c_{\theta_i} d_j s_{\theta_j})(d_j c_{\theta_j} d_i s_{\theta_i}) \right] \quad (33) \\ &= \frac{1}{\sigma_r^8} \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} d_i c_{\theta_i} d_j s_{\theta_j} (d_i c_{\theta_i} d_j s_{\theta_j} - d_i s_{\theta_i} d_j c_{\theta_j}) \end{aligned}$$

In (33), letting $p_{ij} \triangleq d_i c_{\theta_i} d_j s_{\theta_j} (d_i c_{\theta_i} d_j s_{\theta_j} - d_i s_{\theta_i} d_j c_{\theta_j})$, then \mathcal{L}_p can be regarded as the sum of all matrix element p_{ij} . Thus, using the fact that $p_{ij} = 0$ where $i = j$ and $p_{ij} + p_{ji} = (d_i c_{\theta_i} d_j s_{\theta_j} - d_i s_{\theta_i} d_j c_{\theta_j})^2$, the following equation is simply obtained.

$$\begin{aligned} \mathcal{L}_p &\approx \frac{1}{\sigma_r^8} \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} (d_i c_{\theta_i} d_j s_{\theta_j} - d_i s_{\theta_i} d_j c_{\theta_j})^2 \\ &= \frac{1}{\sigma_r^8} \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} d_i^2 d_j^2 s_{\theta_j - \theta_i}^2 \end{aligned} \quad (34)$$

At this point, the minimum amount of target information \mathcal{L}_p^* becomes 0 in the following condition.

$$\theta_i = \theta_j \text{ or } \theta_i = \theta_j \pm 180^\circ \quad (35)$$

From the result, we can understand that in the presence of considerable measurement noises, that is, $|c_{\theta_j}| \ll \sigma_r/d_j$, the UAV formation has a negligible effect on the target tracking performance. Thus it does not mean much to the problem.

Remark 1 The proposed scheme provides more general solution to the UAV formation problem than the existing methods because it considers practical views such as the measurement noise, the maximum communication range between UAVs, and the time-varying target motion. From (25), it is obvious that the amount of target information becomes larger as the number of UAVs increases or the measurement noise variance decreases. This conclusion accords with the previous approaches to the analysis of optimal UAV formation [15–17]. However, if the number of UAVs and the measurement noise properties are fixed, the amount of target information varies with the UAV formation. Unlike the previous results, (25) tells us that the optimal UAV formation maximizes the variance with respect to the formation angle θ_j . This analysis result implies that the good target tracking performance can be obtained by making the angular positions of the UAVs perpendicular to the target LOS direction and far away from it as much as possible.

Remark 2 In the UAV swarm-based target tracking problem, UAVs are generally assumed to be distributed evenly along a uniform circle so that the formation angle of j -th companion UAV is $2\pi j/N$ and with the reference UAV at the origin, according to a natural intuition in optimal UAV configuration. This is one of the most widely used formations in the existing researches [18, 24]. However, these methods are developed considering stationary target and UAVs; thus, it is not suitable for moving target scenario. As shown in Fig. 2, it is obvious that the conventional uniform formation expose their limitations in ensuring the enough amount of target information necessary to improve the target tracking performance.

4 Simulation Results

Computer simulations are carried out to validate the presented UAV formation algorithm. The performance of target tracking according to UAV formation is considered. The simulation parameters and conditions are summarized in Table 1. The target tracking is performed using TDOA information obtained from the UAV swarm, maintaining a particular UAV formation. At this time, it is assumed that the position and velocity of each UAV are individually given from its navigation sensor. It is also assumed that the target moves with constant velocity with a specific direction, where the reference and the three companion UAVs are guided towards the target. The trajectories of the target and the UAV swarm are shown in Fig. 3. For the given engagement scenario, the simulation results are obtained from 200 Monte Carlo trials.

The proposed UAV formation algorithm is compared with one of the most widely used formations, described in

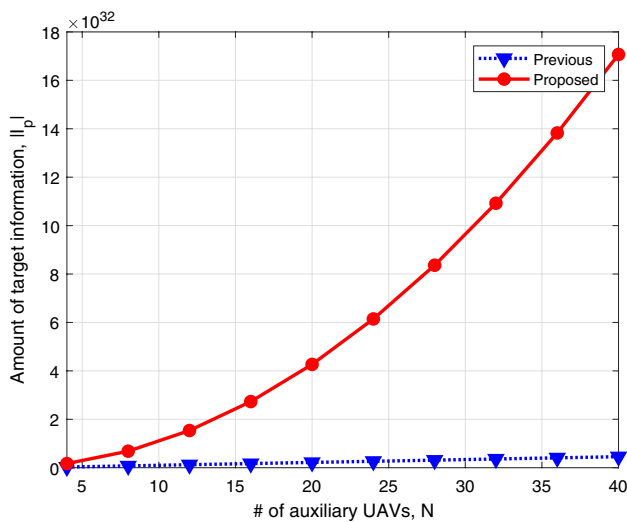
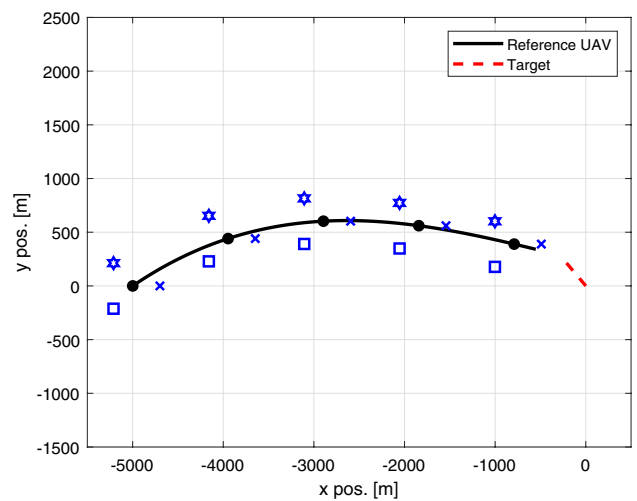
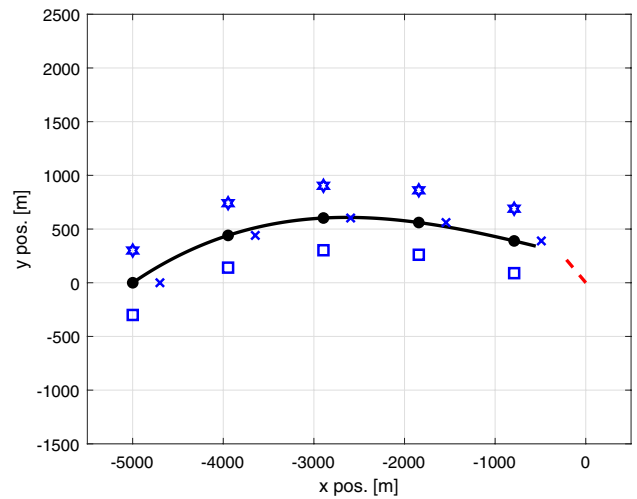


Fig. 2 Amount of target information



(a) uniform formation



(b) proposed formation

Fig. 3 Engagement scenario

Remark 2. We have applied three types of tracking filters to the addressed problem by adopting optimal KF(OKF), extended Kalman filter (EKF) and presented NCRKF based tracking filter. The OKF is used to predict the performance bound but is not implementable because it uses the unavailable measurement matrix H in (2). On the other hand, the EKF is known as one of the most commonly used techniques for the non-linear passive target tracking approach.

Figures 4, 5, 6 and 7 show the error means and the corresponding standard deviations of target position and velocity estimates. From the simulation result, the proposed NCRKF shows a fast convergence property, whereas the EKF does not due to its inherent non-linearity. In addition, it can be seen that the proposed formation in Fig. 3b tends to decrease the standard deviation of estimation error faster than the previous uniform formation shown in Fig. 3a. This implies

Table 1 Simulation condition

Item	Assumption
Target	Initial velocity $ v_t = 20[\text{m/s}]$, $\gamma_t = 135^\circ$ Initial position $(x_t, y_t) = (0, 0)[\text{m}]$
UAV	o # of companion UAV:
Swarm	$N = 3$ o Initial value: Velocity $ v_0 = 270[\text{m/s}]$, $\gamma_0 = 30^\circ$ Position $(x_0, y_0) = (-5000, 0)[\text{m}]$ $(x_j, y_j) = (x_0, y_0) + d_j(c_{\theta_j}, s_{\theta_j})[\text{m}]$ o UAV formation: Uniform formation $d_j = 300[\text{m}]$, $\theta_j = -\frac{2\pi}{3} + \frac{2\pi}{3}(j - 1)$, $j > 0$ Proposed formation $d_j = 300[\text{m}]$, $\theta_j = -\frac{\pi}{2} + \frac{\pi}{2}(j - 1)$, $j > 0$
Noise	RD $\sigma_{r_j}^2 = (0.5[\text{m}])^2$
Variance	UAV position $\sigma_{x_j}^2 = \sigma_{y_j}^2 = (0.5[\text{m}])^2$
Filter	o Proposed linear filter (NCRKF): $P_{0 0} = 1/9 \cdot \text{diag}([100^2, 100^2, 2 \cdot 100^2, \dots, 30^2, 30^2, 2 \cdot 30^2, \dots, 10^2, 10^2, 2 \cdot 10^2])$ $Q = \text{diag}([0.1^2, 0.1^2, 2 \cdot 0.1^2])$ o Non-linear filter (EKF): $P_{0 0} = 1/9 \cdot \text{diag}([100^2, 100^2, 30^2, 30^2, 10^2, 10^2])$ $Q = \text{diag}([0.1^2, 0.1^2])$, $R = \sigma_{r_j}^2 \cdot I^{3 \times 3}$ o SAMPLING: $T = 20[\text{ms}]$

that the proposed UAV swarm formation is a better selection from the viewpoint of target tracking performance, which is consistent with the results of the optimal formation analysis performed in Sect. 3.2.

In order to clearly show the effect of the UAV formation on the tracking performance, the normalized root mean squares errors (NRMSEs) of the position and velocity estimates are shown in Figs. 8 and 9. The NRMSE is calculated as follows:

$$\bar{E}_j = \frac{1}{D_j} \sqrt{\frac{1}{N} \sum_{k=1}^N \tilde{d}_j^2(k)} \tag{36}$$

where N is the number of Monte-Carlo trials, \tilde{d}_j is the position or velocity estimation error. Letting $\hat{\xi}$ be the estimate of the true vector ξ , then $\tilde{d}_j = (\hat{\xi} - \xi)^T (\hat{\xi} - \xi)$. D_j is the normalized constant which is defined as the standard deviation of the estimation error $\hat{\xi} - \xi$. It is chosen as the maximum value of the estimation error standard deviations obtained by using two different UAV formations is used for D_j .

From the NRMSEs of position and velocity as Figs. 8 and 9, the proposed formation provides more consistent estimation performance than the previous formation regardless of filter structures. This result corresponds to the fact mentioned in Remark 2. Consequently, the proposed formation algorithm is expected to be used as an attractive solution for guaranteeing the excellent performance of the passive target tracking based on the UAV swarm.

5 Conclusions

A novel UAV formation algorithm has been proposed for the passive target tracking using TDOA information measured by the UAV swarm. To make the UAV formation problem tractable, the non-conservative robust Kalman filter was used for passive target tracking. Exploiting its linear filter structure and estimation error properties, the UAV formation target information was derived in closed-form. It was shown that the proposed UAV formation maximizing the worst-case target information provides more reliable passive target tracking performance than the conventional uniform formation. Moreover, the proposed approach enables practical constraints overlooked in most existing methods, such as the maximum possible communication range between UAVs to determine the UAV formation. Through the computer simulation, it has been demonstrated that the proposed UAV formation technique provides exceptional tracking performance.

Appendix A

The FIM $\mathcal{J}_k(\mathbf{x})$ is generally defined by

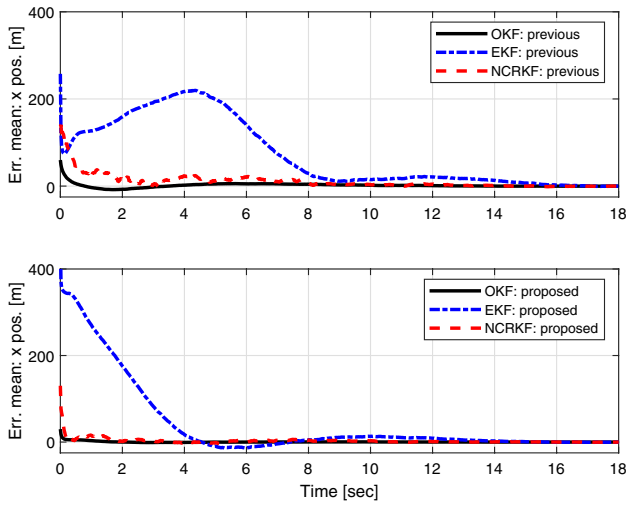
$$E\{(\hat{\mathbf{x}}_k - \mathbf{x})(\hat{\mathbf{x}}_k - \mathbf{x})^T\} \geq C_k(\mathbf{x}) \triangleq \mathcal{J}_k^{-1}(\mathbf{x}) \tag{37}$$

where $\hat{\mathbf{x}}_k$ is an unbiased estimate of parameter \mathbf{x} , which is to be estimated. $C_k(\mathbf{x})$ is the CRB (Cramer-Rao bound). The (i, j) th element of $\mathcal{J}_k(\mathbf{x})$ is given by

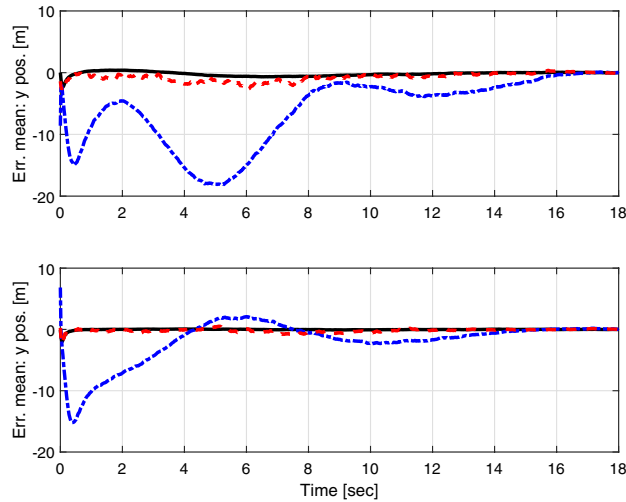
$$J_k(x)_{i,j} = E\left\{ \frac{\partial \log p(\mathbf{x}, \mathbf{y}^k)}{\partial x_i} \frac{\partial \log p(\mathbf{x}, \mathbf{y}^k)}{\partial x_j} \right\} \tag{38}$$

where $p(\mathbf{x}, \mathbf{y}^k)$ is the joint probability distribution of \mathbf{x} and stacked measurement vector $\mathbf{y}^k \triangleq \{y_l\}$, $l = 1, \dots, k$. The following target dynamics and measurement models are assumed.

$$\begin{aligned} \mathbf{x}_{k+1} &= F_k \mathbf{x}_k \\ \mathbf{y}_k &= H_k \mathbf{x}_k + \mathbf{v}_k \end{aligned} \tag{39}$$



(a) *x* position



(b) *y* position

Fig. 4 Error mean of target estimate: position

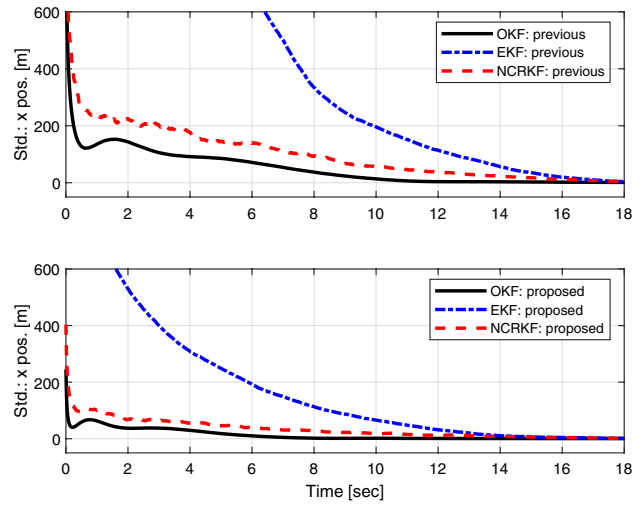
Under the assumption that the measurement noise v_k is a normally distributed random variable with mean zero and variance R_k , i.e., $y_k \sim \mathcal{N}(H_k x_k, R_k)$,

$$p(x, y^k) = \frac{1}{\prod_{l=1}^k (2\pi |R_l|)^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2} \sum_{l=1}^k (y_l - H_l x_l)^T R_l^{-1} (y_l - H_l x_l)\right) \quad (40)$$

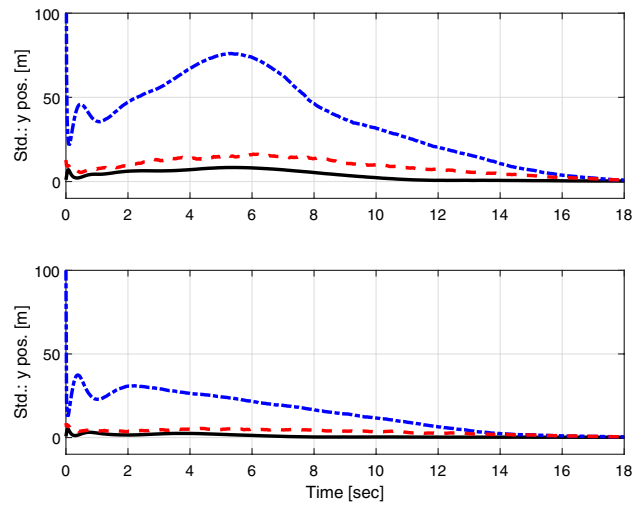
and the FIM is given by

$$\mathcal{J}_k(x) = (\mathcal{H}^k)^T (\mathcal{R}^k)^{-1} \mathcal{H}^k \quad (41)$$

where



(a) *x* position



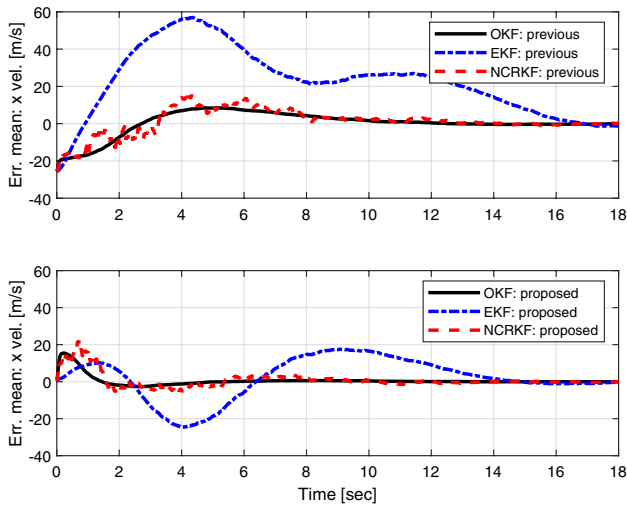
(b) *y* position

Fig. 5 Standard deviation of target estimate: position

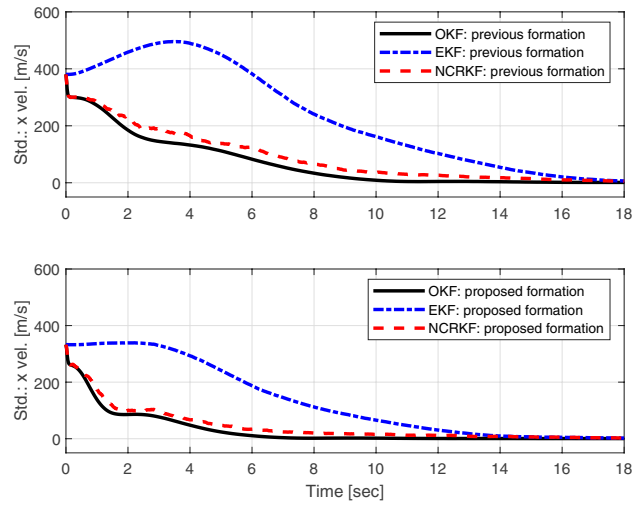
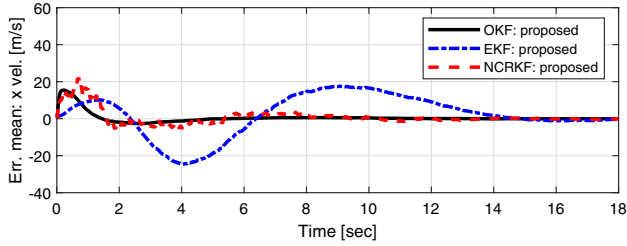
$$\mathcal{H}^k = H^k \Phi^k, \quad H^k = \begin{bmatrix} H^{k-1} & 0 \\ 0 & H_k \end{bmatrix}, \quad \mathcal{R}^k = \begin{bmatrix} R^{k-1} & 0 \\ 0 & R_k \end{bmatrix}$$

and the block matrix Φ^k is defined by

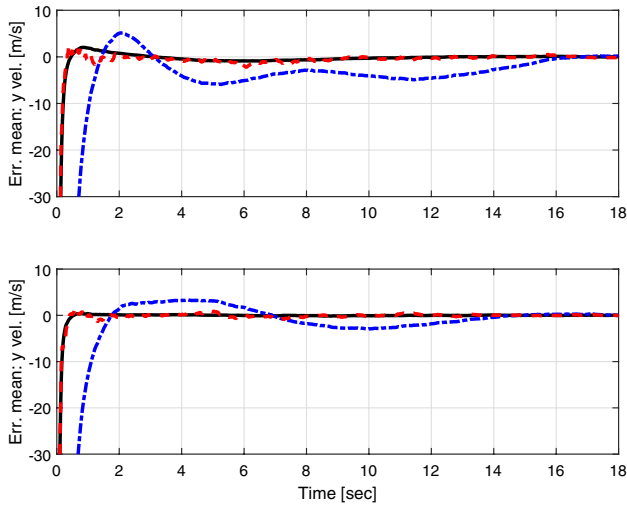
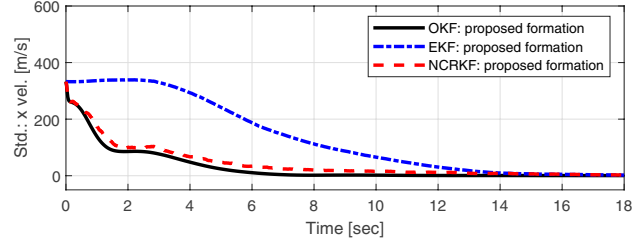
$$\Phi^k \triangleq \begin{bmatrix} \Phi(0, k) \\ \Phi(1, k) \\ \vdots \\ \Phi(k-1, k) \\ \Phi(k, k) \end{bmatrix} = \begin{bmatrix} \Phi(0, k-1)F_{k-1}^{-1} \\ \Phi(1, k-1)F_{k-1}^{-1} \\ \vdots \\ \Phi(k-1, k-1)F_{k-1}^{-1} \\ I^{n \times n} \end{bmatrix} = \begin{bmatrix} \Phi^{k-1}F_{k-1}^{-1} \\ I^{n \times n} \end{bmatrix}, \quad \Phi^0 = I^{n \times n}$$



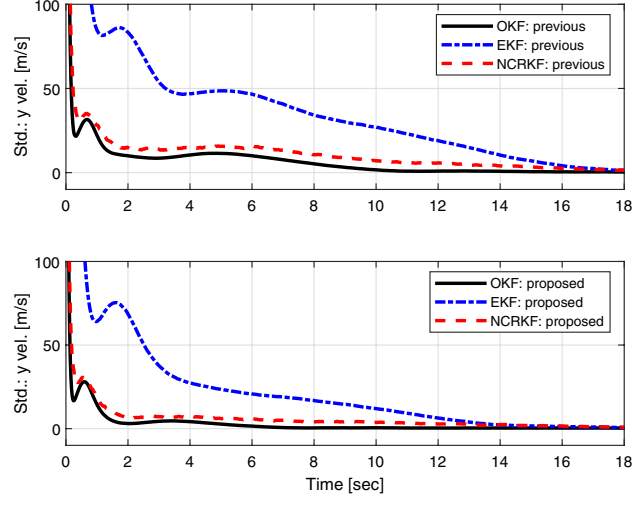
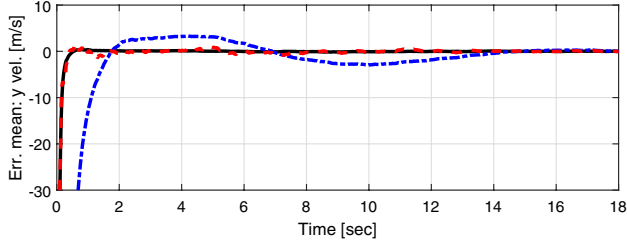
(a) *x* velocity



(a) *x* velocity



(b) *y* velocity



(b) *y* velocity

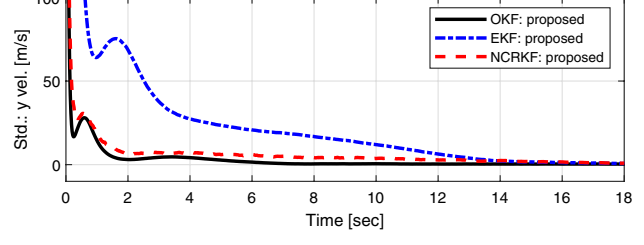


Fig. 6 Error mean of target estimate: velocity

Fig. 7 Standard deviation of target estimate: velocity

where

$$\Phi(j, k) \triangleq F_{j-1} \times \dots \times F_k \quad (j > k)$$

$$\Phi(j, k) \triangleq I^{n \times n} \quad (j = k)$$

$$\Phi(j, k) \triangleq (F_{k-1} \times \dots \times F_j)^{-1} \quad (j < k)$$

and

$$F^k \triangleq \begin{bmatrix} \Phi(0, 1) & \Phi(0, 2) & \dots & \Phi(0, k-1) & \Phi(0, k) \\ 0 & \Phi(1, 2) & \dots & \Phi(1, k-1) & \Phi(1, k) \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & \dots & 0 & \Phi(k-1, k) \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} F^{k-1} & \Phi^{k-1} F_{k-1}^{-1} \\ 0^{n \times (k-1)n} & 0^{n \times n} \end{bmatrix}, \quad F^1 = \begin{bmatrix} F_0^{-1} \\ 0^{n \times n} \end{bmatrix}.$$

Fig. 8 NRMSE of target estimate: position

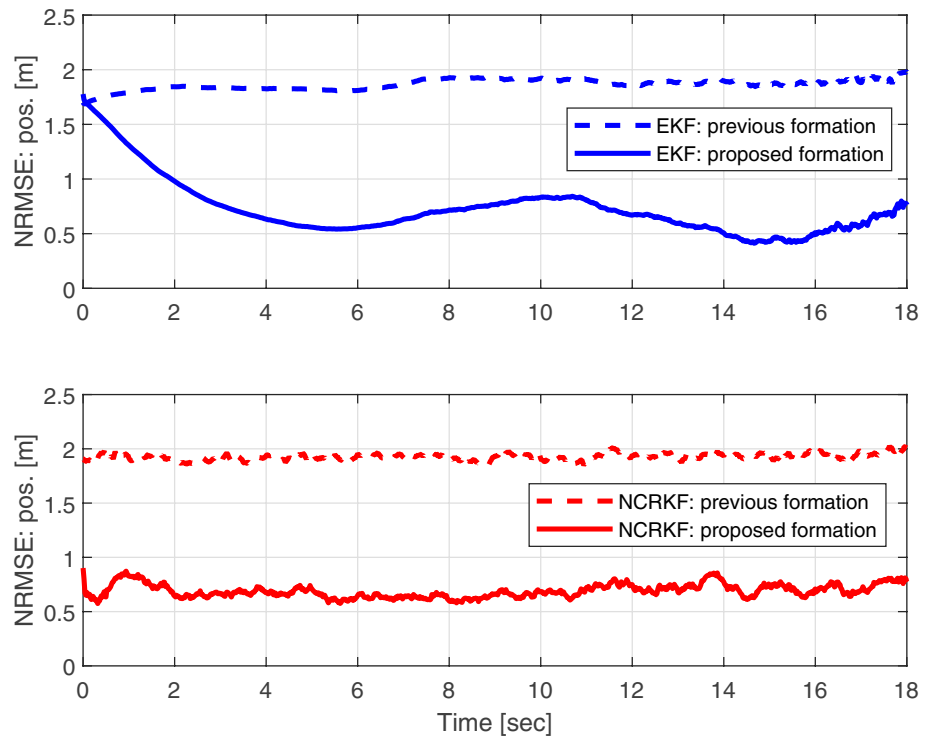
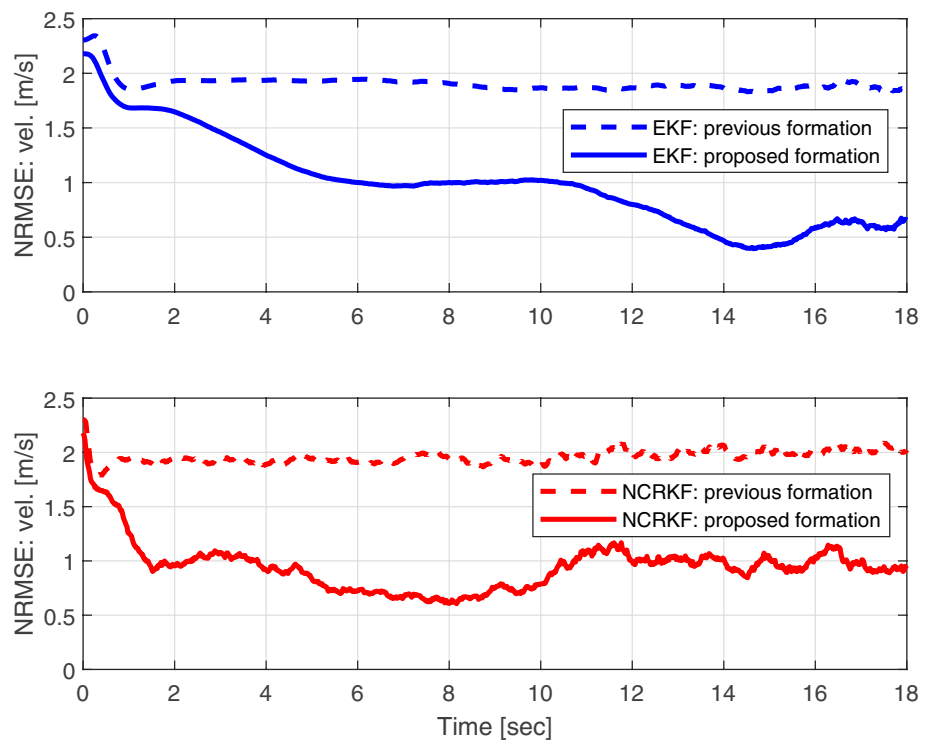


Fig. 9 NRMSE of target estimate: velocity



The detailed derivation procedure of the matrices can be found in [25].

The achievable amount of target information is defined by using weighting matrix \mathcal{W} .

$$\begin{aligned} \mathcal{I}_k &= \text{tr}\{\mathcal{W}\mathcal{J}_k\} = \text{tr}\{\mathcal{W}(\Phi^k)^T(H^k)^T(R^k)^{-1}H^k\Phi^k\} \\ &= \text{tr}\{(H^k)^T(R^k)^{-1}H^k\Phi^k\mathcal{W}(\Phi^k)^T\} \end{aligned} \tag{42}$$

where

$$\mathcal{W} = \begin{bmatrix} \mathcal{W}_p & 0 & 0 \\ 0 & \mathcal{W}_v & 0 \\ 0 & 0 & \mathcal{W}_a \end{bmatrix}$$

The weighting matrix consists of weighted values, \mathcal{W}_p , \mathcal{W}_v , and \mathcal{W}_a for target position, velocity, and acceleration respectively. In this paper, we focus on maximizing the amount of target position information, and thus the weighted values are designed as $\mathcal{W}_p = I$, $\mathcal{W}_v = 0$, $\mathcal{W}_a = 0$.

Consider the following system matrix.

$$F = \begin{bmatrix} I & T \cdot I & \frac{1}{2}T^2 \cdot I \\ 0 & I & T \cdot I \\ 0 & 0 & I \end{bmatrix} \tag{43}$$

By applying the above matrix F to (42), the achievable amount of target information becomes

$$\mathcal{I}_k = \sum_{j=0}^k H_j^T R_j^{-1} H_j \mathcal{W} = \sum_{j=0}^k \bar{H}_j^T \bar{R}_j^{-1} \bar{H}_j \tag{44}$$

where \bar{H}_j and \bar{R}_j denote the newly defined term corresponding to target position information of measurement matrix H_j and measurement error covariance R_j . The FIM depicts the characteristics that when the probability distribution is sharply peaked, the estimate may be more likely to be accurate than when the probability distribution is flatter. This means that the FIM includes the amount of target information carried by the corresponding measurement equation. Meanwhile, the probability distribution of the addressed filter framework is defined by [22]

$$\begin{aligned} p_r(\mathbf{x}, \mathbf{y}^k) &= \frac{1}{\prod_{l=1}^k (2\pi|R_l|)^{\frac{1}{2}}} \\ &\cdot \exp\left(-\frac{1}{2} \sum_{l=1}^k \{(\mathbf{y}_l - \tilde{H}_l \mathbf{x}_l)^T R_l^{-1} (\mathbf{y}_l - \tilde{H}_l \mathbf{x}_l) \right. \\ &\quad \left. - (\mathbf{x}_l^T W_l \mathbf{x}_l + \mathbf{x}_l^T V_l + V_l^T \mathbf{x}_l)\right\} \end{aligned} \tag{45}$$

In the same way as above, we can easily obtain the achievable amount of target information of the proposed filter.

$$\begin{aligned} \mathcal{I}_{r,k}(\mathbf{x}) &= \sum_{l=1}^k (\tilde{H}_l^T R_l^{-1} \tilde{H}_l - W_l) \mathcal{W} \\ &= \sum_{l=1}^k (\bar{H}_l^T \bar{R}_l^{-1} \bar{H}_l - \bar{W}_l). \end{aligned} \tag{46}$$

According to the problem formulation in this paper, especially described by basic assumptions of subsection 3.1, $\mathcal{I}_{r,k}(\mathbf{x})$ can be approximated as

$$\mathcal{I}_{r,k}(\mathbf{x}) \approx k(\bar{H}^T \bar{R}^{-1} \bar{H} - \bar{W}). \tag{47}$$

Appendix B

In this paper, the FIM of optimal KF and proposed NCRKF are defined respectively for the measurement equation of (2).

$$\mathcal{J}_{o,k}(\mathbf{x}) = -E \left\{ \frac{\partial^2 \ln p_o(\mathbf{x}, \mathbf{y}^k)}{\partial \mathbf{x}^2} \right\} \tag{48}$$

$$\mathcal{J}_{r,k}(\mathbf{x}) = -E \left\{ \frac{\partial^2 \ln p_r(\mathbf{x}, \mathbf{y}^k)}{\partial \mathbf{x}^2} \right\} \tag{49}$$

where

$$\begin{aligned} &\frac{\partial^2 \ln p_o(\mathbf{x}, \mathbf{y}^k)}{\partial \mathbf{x}^2} \\ &= -\frac{1}{2} \frac{\partial^2 \sum_{l=1}^k (\mathbf{y}_l - H_l \mathbf{x}_l)^T R^{-1} (\mathbf{y}_l - H_l \mathbf{x}_l)}{\partial \mathbf{x}^2} \\ &\quad - \frac{\partial^2 \ln p_r(\mathbf{x}, \mathbf{y}^k)}{\partial \mathbf{x}^2} \\ &= -\frac{1}{2} \left(\frac{\partial^2 \sum_{l=1}^k ((\mathbf{y}_l - \tilde{H}_l \mathbf{x}_l)^T R^{-1} (\mathbf{y}_l - \tilde{H}_l \mathbf{x}_l))}{\partial \mathbf{x}^2} \right. \\ &\quad \left. - \frac{\partial^2 \sum_{l=1}^k (\mathbf{x}_l^T W_l \mathbf{x}_l + \mathbf{x}_l^T V_l + V_l^T \mathbf{x}_l)}{\partial \mathbf{x}^2} \right) \end{aligned}$$

In the above equation, $p_o(\mathbf{x}, \mathbf{y}^k)$ and $p_r(\mathbf{x}, \mathbf{y}^k)$ are the joint probability density of optimal KF and NCRKF. The FIM of proposed filter can be rewritten by utilizing the optimal KF's FIM.

$$\begin{aligned} &E \left\{ \frac{\partial^2 \ln p_r(\mathbf{x}, \mathbf{y}^k)}{\partial \mathbf{x}^2} \right\} \\ &= E \left\{ \frac{\partial^2 \ln p_o(\mathbf{x}, \mathbf{y}^k)}{\partial \mathbf{x}^2} \right\} + E \left\{ \frac{\partial^2 (\Delta J_w - \Delta J_v - \Delta J_v^T)}{\partial \mathbf{x}^2} \right\} \end{aligned} \tag{50}$$

where

$$\begin{aligned}\Delta J_W &= S_W - E\{S_W\}, \\ S_W &\triangleq \sum_{l=1}^k \mathbf{x}_l^T \Delta H_l^T R^{-1} \Delta H_l \mathbf{x}_l \\ \Delta J_V &= S_V - E\{S_V\}, \\ S_V &\triangleq \sum_{l=1}^k \mathbf{x}_l^T \Delta H_l^T R^{-1} \mathbf{v}_l\end{aligned}$$

It can be easily known that the second term on the right-hand side of (50) becomes zero. That is, the FIM of the NCRKF converges to that of the optimal KF almost surely.

$$\mathcal{J}_{r,k}(\mathbf{x}) \xrightarrow{a.s.} \mathcal{J}_{o,k}(\mathbf{x}) \quad (51)$$

It seems a natural consequence as the NCRKF scheme is invented by compensating the difference between cost functions defined in least-squares estimation terms of optimal KF and standard KF without proper error compensating strategy.

Acknowledgements This work was supported in part by the Korea Electric Power Corporation(No. R19X001-51) and in part by the Korea Institute of Energy Technology Evaluation and Planning(KETEP) and the Ministry of Trade, Industry & Energy(MOTIE) of the Republic of Korea (No. 20214000000010).

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Ui-Suk Suh received the B.S. and M.S. degrees in mechanical and control engineering from Handong Global University, Pohang, South Korea, in 2015 and 2017, respectively. He is currently working toward the Ph.D. degree in mechanical and control engineering with Handong Global University, Pohang, South Korea. His research interest includes automatic target recognition, information fusion filter, autonomous vehicle control and related sensor technology.



Seul-Ki Han received her B.E. and Ph.D. degrees in Electrical and Electronic Engineering from Yonsei University, Seoul, Korea in 2009 and 2016, respectively. Since 2016, she has been with the Hyundai Motor Company, Korea, where she is currently a Senior Research Engineer. Her research interest includes filtering theory, multiple target tracking and automotive sensor fusion.



Won-Sang Ra received his B.S. degree in Electrical Engineering, M.S. degree in Electrical and Computer Engineering, and Ph.D. degree in Electrical and Electronic Engineering from Yonsei University, Seoul, Korea in 1998, 2000, and 2009, respectively. From March 2000 to February 2009, he was with the Guidance and Control Department, Agency for Defense Development, Daejeon, Korea, as a Senior Researcher. In 2015, he has collaborated as an Academic Visitor at the School of Aerospace, Transport, and Manufacturing, Cranfield University, United Kingdom. Since 2009, he has been with the School of Mechanical and Control Engineering, Handong Global University, Pohang, Korea, where he is currently a Professor. His main research topics are related to the robust filtering theory and its applications to autonomous vehicle guidance, control, and sensor fusion.