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ORIGINAL ARTICLE

Health Status Evaluation of Catenary Based on Normal Fuzzy Matter-Element and Game Theory

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Abstract

At present, there is no unified standard for the health status evaluation of electrified railway catenary in China. The current catenary evaluation model only considers quantitative detection indicators, without qualitative indicators such as weather, which is one-sided to some extent. Thus, an improved catenary status evaluation model is constructed with both quantitative indicators and qualitative indicators. In this evaluation model, the normal fuzzy matter-element method is used to determine the correlation value of each grade, and the weighted average principle is used to re-determine the status grade of catenary when the maximum correlation principle fails. Meanwhile, entropy weight method and particle swarm optimization algorithm to optimize analytic hierarchy process method are combined to improve the shortcomings of single weight method, and game theory is used to determine the subjective and objective weight coefficients, so as to reduce the influence of subjective experience. Select a Chinese railway catenary in 2018 as an example for verification analysis, the results show that the model constructed in this paper can effectively help professionals to make correct judgments on the health status of catenary, and provide a new idea and method for the comprehensive evaluation of the catenary operation status, which has certain practicability.

Keywords Catenary · Fuzzy matter-element evaluation · Game theory · Combined weights · Weighted average principle

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1 Introduction

With the rapid development of electric energy and engineering technology, railway transportation power has gradually changed from the traditional internal combustion and steam locomotive traction to electric locomotive traction. More and more railway lines have been electrified. The increase of electrified mileage is inseparable from the safe operation of catenary. Catenary is a special power supply line of electrified railway. Its structure and working mode are very complex. It is usually set up over the railway track and arranged in the open air. It is vulnerable to the high-speed impact of locomotive pantograph, which has become a weak link in the traction power supply system, accounting for 80% of all faults in the traction power supply system. Health status evaluation of catenary is an important link to ensure the safe operation of electrified railway. It is of great significance to know the actual situation of catenary in time and improve the operation reliability of catenary [1–3].

In recent years, fault Petri net [4], reliability analysis method, image method [5], fault tree analysis [6], grey clustering [7], fuzzy mathematics and other methods have been widely used in the reliability evaluation of catenary, power system and other industries. Y. Yang et al. [8] calculated the reliability of catenary based on the failure rate and repair rate of parts, and realized the status evaluation of catenary by fault tree method and reliability theory. However, this method needed to count the status of a large number of components, with a large workload, and the evaluation results couldn't be quantified. H. B. Cheng et al. [9] proposed trigonometric membership function to determine the membership degree of each indicator belonging to the evaluation grade, and used fuzzy mathematics and entropy weight method to evaluate the health status of catenary. The entropy weight method ignored the influence of equilibrium and didn't consider the experience of catenary professionals, which might lead to the deviation of evaluation results. S. B. Liu et al. [10] proposed entropy weight method and analytic hierarchy process (AHP) to combine weight, and grey clustering was used to evaluate the health status of catenary. Both subjective and objective effects are considered by the combined weighting method, but the weighting coefficients were selected by experience, and the consistency check method of AHP was too single and lacked theoretical basis. Ref. [11] introduced the influence of weather conditions on the catenary, and established the failure rate and repair rate models of various components by using the credibility theory to evaluate the reliability of the catenary, but the catenary evaluation model was not established, which lacked integrity. In [12], the reliability of pantograph system was analyzed by combining Bayesian network with fuzzy mathematics, and the weak links affecting the safety of pantograph were found according to the reliability results. In the above references, some of them did not establish the status evaluation model of catenary, so it was impossible to carry out quantitative evaluation on catenary; some of the evaluation models only included the indicators detected in 6C system, and the weather conditions such as rain, snow, strong wind, thunder, etc., as well as the number of failures of catenary, failure ratio and other indicators were not considered.

As the main source of power transmission for electric locomotives, catenary operates in the open air for a long time and is easily affected by various weather conditions. It has been proved that most faults of catenary are related to the weather. Therefore, it is necessary to add weather and historical operation into the evaluation system. According to the safety work rules of catenary, the operation and maintenance rules of catenary and other railway rules and regulations, this paper integrates the weather condition and the failure of catenary into the status evaluation model of catenary. On this basis, the following works are completed: First, in accordance with the relevant standards and specifications, establish a more scientific and complete evaluation model that includes catenary safety indicators, catenary smoothness indicators and current collection performance indicators between pantograph and catenary, weather indicators and historical operation condition indicators; Secondly, in view of the shortage of weight determination methods in [9, 10], this paper combines entropy weight method with AHP method, and then proposes a weight determination method based on entropy weight method and Particle Swarm Optimization (PSO) to optimize AHP, and uses game theory to assign weight coefficients, effectively reducing the impact of human factors; Thirdly, the fuzzy theory and matter-element analysis method are combined organically, and the weighted average correlation principle is used to evaluate the final status of catenary when the maximum correlation principle fails.

2 Establishment of Catenary Health Evaluation Model

2.1 Selection of Indicators and Establishment of Evaluation Model

Like the power grid transmission line, catenary is also a very complicated electrical and mechanical system, which undertakes the current transmission function in the process of railway transportation. According to relevant standards and technical specifications, the establishment of catenary evaluation model should follow the principles of objective and reasonable, clear hierarchy, qualitative and quantitative combination. Quantitative indicators can be divided into four categories: catenary safety indicators, catenary smoothness indicators, current collection performance indicators between pantograph and catenary and catenary historical operation condition indicators; the qualitative indicators include weather condition during the operation of catenary, etc.

Safety indicators, that is, indicators to ensure the operation safety of catenary and train, are mainly geometric parameters, including contact wire height, stagger value, span, superelevation, side clearance and so on. If the contact wire is too high, it will lead to poor contact of pantograph and catenary system, which will cause offline; if it is too low, it will increase contact wear and shorten the service life of components. The stagger value is an important parameter to ensure the uniform friction between pantograph and contact wire. Reasonable stagger value will prevent the pantograph from falling off. Span, superelevation, side clearance and other parameters are also important parameters affecting the safety of catenary, which need to meet the safety standards.

Catenary smoothness indicators, which are used to evaluate the local unevenness of catenary, mainly include two indicators: the height difference within one span and the hard spot. Hard spot is an indicator that affects catenary operation. It will cause damage and arc burn of pantograph and catenary system, and even lead to pantograph and catenary failure. The smoothness of contact wire is the premise of good current collection.

Current collection performance indicators between pantograph and catenary, which affect the current transmission quality between pantograph and contact wire. Good current collection performance will ensure the safe operation of catenary and train. Current collection performance indicators include pantograph-catenary contact pressure, locator gradient, maximum arcing time, etc. Pantograph-catenary contact pressure is the most direct factor affecting pantograph-catenary current collection performance, and appropriate contact pressure can guarantee current collection performance; the sudden change of contact pressure will directly affect current collection, and even cause power supply failure of catenary and train. In the ideal catenary, the arcing time should be 0 ms. However, in the actual catenary operation, the arcing often occurs, which is harmful to the pantograph-catenary system, the conductor is worn and the current collection quality is reduced, so it is necessary to eliminate the arcing in time.

Weather indicators, which have great influence on the operation of catenary. The catenary is exposed outdoors for a long time, which is susceptible to various types of weather. Weather such as rain, snow, thunder, and fog may cause catenary breakdown, train power failure and other faults. According to technical standards and weather standards [13, 14], it is feasible and reasonable to incorporate weather indicators into health status evaluation of catenary.

Catenary historical operation indicators, including normal operation time of catenary and components, failure times, the proportion of primary defects and the proportion of secondary defects. At present, China's catenary standards divide the defects into two levels, and the first-level defects are more serious.

According to the selection of the above five categories indicators and catenary actual situation, a multi-level health status evaluation model of catenary is constructed, which includes 5 first-level indicators and 17 s-level indicators, as shown in Fig. 1.

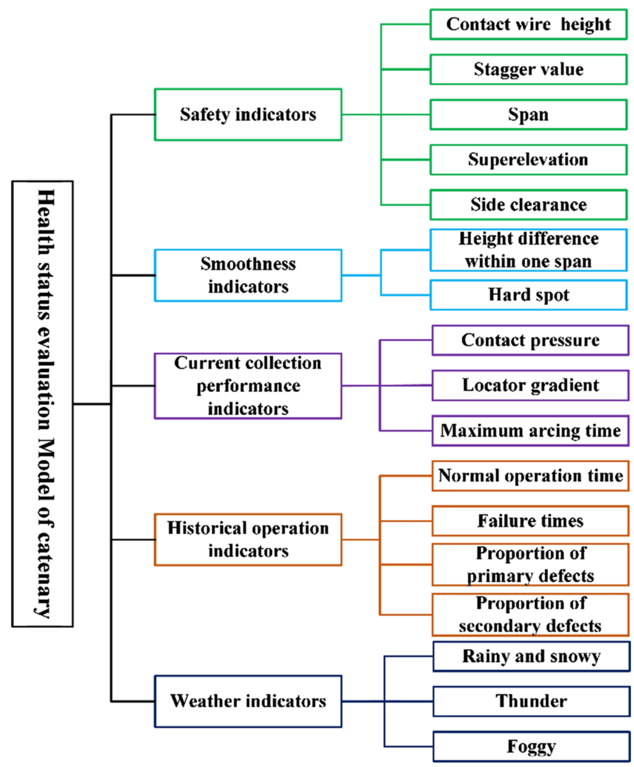

Fig. 1 Catenary health status evaluation model

2.2 Classification of Catenary Evaluation Grade

After catenary status evaluation model is established, it is necessary to determine the grade of catenary operation status. In order to judge the actual catenary situation more reasonably, combined with expert experience, this paper divides the catenary status into five grades: excellent, good, general, warning and failure. The specific description is as follows:

(1) "Excellent" status: Catenary is in the best status;
(2) "Good" status: Catenary status is slightly worse than the previous status, but almost no failure occurs;
(3) "General" status: Catenary status can meet the operation requirements of the railway, and generally no major accidents will occur;
(4) "Warning" status: There are hidden dangers in catenary. Although there is no failure temporarily, it is likely to occur in the future. In this status, a warning is required and staff are reminded to check in time;
(5) "Failure" status: Catenary has a fault and needs to be repaired and corrected immediately.

3 Combined Weights

The weights are mainly divided into subjective weights, objective weights and combined weights. Subjective weights include analytic hierarchy process [15] and so on. Objective weights include entropy weight method [16], variation coefficient method [17], principal component analysis method [18], etc. Combined weights combine subjective and objective weights with certain rules to seek more reasonable weights distribution.

3.1 Data Preprocessing

Suppose there are n evaluation objects, and m evaluation indicators. Standardize the original data, so that the standardized data are in the range of 0–1.

For the larger the better indicator, the standardization equation is

$$x^*(i,j) = \frac{x(i,j) - \min_j(x(i,j))}{\max_j(x(i,j)) - \min_j(x(i,j))} \tag{1}$$

For the smaller the better indicator, the standardization equation is

$$x^*(i,j) = \frac{\max_j(x(i,j)) - x(i,j}{\max_j(x(i,j)) - \min_j(x(i,j))} \tag{2}$$

where $x(i,j)$ is the original value of the indicator, $\max_j(x(i,j))$ and $\min_j(x(i,j))$ are the maximum original data and the minimum original data of the j-th indicator respectively, and $x^*(i,j)$ is the data after standardized processing.

In the catenary evaluation model established in this paper, side clearance and normal operation time are processed according to the larger the better indicators; the remaining indicators are standardized according to the smaller the better indicators.

3.2 PSO-AHP Weight Method

Analytic hierarchy process uses 1–9 scale method to compare the two evaluation indicators to build a judgment matrix P, and uses Eqs. (3) and (4) to judge whether the matrix meets the consistency requirements:

$$CI = \frac{\lambda_{\max} - r}{r - 1} \tag{3}$$

$$CR = \frac{CI}{RI} \tag{4}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the judgment matrix, r is the order of the judgment matrix ($r \leq m$), CI is the consistency index, CR is the consistency ratio of the judgment matrix, RI is the standard value of the average random consistency index. The RI values of order 1–9 judgment matrices are (0, 0, 0.52, 0.89, 1.12, 1.26, 1.36, 1.41, 1.46). When $CR < 0.1$, the judgment matrix meets the consistency requirements, and then the subjective weight of each indicator is calculated. When $CR > 0.1$, the judgment matrix needs to be adjusted until the consistency check requirements are met.

When the judgment matrix P meets the consistency check requirements, the weight of each indicator can be solved, as shown in Eq. (5).

$$P\omega = \lambda_{\max}\omega \tag{5}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the judgment matrix, and ω is the obtained weight value.

However, the consistency check method of AHP method is too single, lacks theoretical basis, and is highly vulnerable to subjective influence of decision-makers, and then the weights obtained will be inaccurate. Based on this, this paper uses PSO algorithm to optimize AHP, and then calculates new weights. The consistency index between AHP and PSO-AHP is compared, and the weights corresponding to the smaller value are taken as the final subjective weights values.

PSO-AHP weights method transforms the above problems into a constraint optimization problem, as shown in Eq. (6), and the specific optimization steps are shown in Fig. 2.

$$\begin{cases} F = CR = \min \frac{1}{r} \sum_{j=1}^{r} \left| \sum_{k=1}^{r} (p_{jk}\omega_k) - r\omega_j \right| \\ s.t. \begin{cases} \omega_k > 0 \\ \sum_{k=1}^{r} \omega_k = 1 \end{cases} \end{cases} \tag{6}$$

where F is the consistency index, ω_k is the weight value, as the optimization variable, r is the judgment matrix order, and p_{jk} is the importance of the j-th indicator relative to the k-th indicator in the judgment matrix.

3.3 Entropy Weight Method

The entropy value of the j-th indicator is

$$K_j = -\frac{\sum_{i=1}^{n} v(i,j) \cdot \ln v(i,j)}{\ln n} \tag{7}$$

where $v(i,j)$ is the characteristic proportion of the i-th object under the j-th indicator, and $v(i,j) = \frac{x^*(i,j)}{\sum_{i=1}^{n} x^*(i,j)}$.

The entropy weight of the j-th indicator is

$$\omega_j = \frac{1 - K_j}{\sum_{j=1}^{m} (1 - K_j)} \tag{8}$$

The entropy weight obtained by the above equation is the weight of each indicator, and satisfies $\omega_j > 0$, $\sum_{j=1}^{m} \omega_j = 1$.

3.4 Game Theory Combines Weights

In order to make up for the deficiency of balance and randomness of entropy weight method, and fully consider the advantages and disadvantages of subjective weights and objective

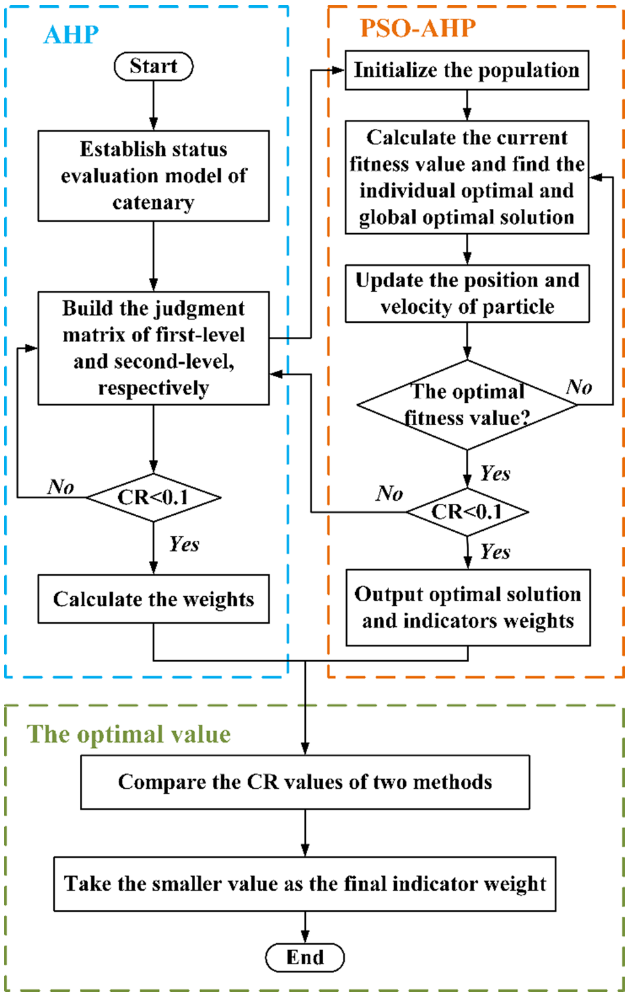

Fig. 2 Flow chart of PSO-AHP weights

weights, PSO-AHP method is proposed on the basis of entropy weight method to obtain more accurate and objective weight. The combined weight method of game theory [19] is used to determine the weight coefficient, as shown in Eq. (9).

$$\omega_j = a_1\omega_j^1 + a_2\omega_j^2 \tag{9}$$

where ω_j^1 is objective weight, ω_j^2 is subjective weight, ω_j is the combined weight, and $a_1 + a_2 = 1$.

Game theory transforms the above process of solving a_1 and a_2 into solving min $\left\| a_1\omega_j^1 + a_2\omega_j^2 \right\|_2$. The first derivative condition of the optimal solution is obtained, and the system of linear equations is

$$\begin{vmatrix} \omega_j^{1^T}\omega_j^1 & \omega_j^{1^T}\omega_j^2 \\ \omega_j^{2^T}\omega_j^1 & \omega_j^{2^T}\omega_j^2 \end{vmatrix} \begin{vmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \end{vmatrix} = \begin{bmatrix} \omega_j^{1^T}\omega_j^1 \\ \omega_j^{2^T}\omega_j^2 \end{bmatrix} \tag{10}$$

By solving the above linear equations, the optimal values of a_1 and a_2 can be obtained, and the combined weight of each indicator can be determined finally, thus forming the fuzzy matter-element matrix R_ω of catenary indicators weights.

$$R_\omega = \begin{bmatrix} C_1 & C_2 & \cdots & C_m \\ \omega_1 & \omega_2 & \cdots & \omega_m \end{bmatrix} \tag{11}$$

4 Improved Fuzzy Matter-Element Evaluation Method

4.1 Fuzzy Matter-Element Theory

The matter-element method is to describe things with three elements {things, characteristics and quantities} in order to make a comprehensive analysis. If the quantity value in the matter-element has ambiguity, it is called fuzzy matter-element [20, 21], which is recorded as $\{N, C, \mu(x)\}$. If each indicator has K membership grades, the k-th fuzzy matter-element matrix of n objects is

$$R_{\mu k} = \begin{bmatrix} & N_1 & N_2 & \cdots & N_n \\ C_1 & \mu_k(x^*(1,1)) & \mu_k(x^*(2,1)) & \cdots & \mu_k(x^*(n,1)) \\ C_2 & \mu_k(x^*(1,2)) & \mu_k(x^*(2,2)) & \cdots & \mu_k(x^*(n,2)) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_m & \mu_k(x^*(1,m)) & \mu_k(x^*(2,m)) & \cdots & \mu_k(x^*(n,m)) \end{bmatrix} \tag{12}$$

where $R_{\mu k}$ is the m-dimensional compound fuzzy matter-element $(k = 1, 2, \cdots, K)$ of n research objects at the k-th grade, C_j is the j-th indicator $(j = 1, 2, \cdots, m)$, N_i is the i-th research object $(i = 1, 2, \ldots, n)$,$\mu_k(x^*(i,j))$ is the membership of j-th indicator at the k-th grade.

4.2 Fuzzy Matter-Element of Normal Membership

(1) Quantitative indicators membership

Commonly used membership functions include triangle, trapezoid, rectangle, normal and ridge [22, 23]. Compared with other functions, the normal distribution function can better characterize the characteristics of catenary and more objectively reflect the operation status of catenary. Therefore, this paper selects the fuzzy normal distribution membership function to solve the membership values of catenary indicators, as shown in Fig. 3. According to the principle of 3σ, 6σ is used as the definition domain of each membership function in this paper, and the membership calculation of each grade is shown in Eqs. (13) - (17). Among them, 0, a, b, c, d are the standard values of the status grade thresholds corresponding to each indicator, respectively. $\mu_1 \sim \mu_5$ correspond to failure, warning, general, good and excellent status, respectively.

$$\mu_1 = \begin{cases} 0 & x^*(i,j) < 0 \\ e^{-\frac{x^*(i,j)^2}{2\sigma^2}} & 0 \le x^*(i,j) \le a \end{cases}, \sigma = \frac{a}{3} \tag{13}$$

$$\mu_2 = \begin{cases} e^{-\frac{(a-x^*(i,j))^2}{2\sigma^2}} & 0 < x^*(i,j) \le a \ , \sigma = \frac{a}{3} \\ e^{-\frac{(x^*(i,j)-a)^2}{2\sigma^2}} & a < x^*(i,j) \le b \ , \sigma = \frac{b-a}{3} \end{cases} \tag{14}$$

$$\mu_3 = \begin{cases} e^{-\frac{(b-x^*(i,j))^2}{2\sigma^2}} & a < x^*(i,j) \le b \ , \sigma = \frac{b-a}{3} \\ e^{-\frac{(x^*(i,j)-b)^2}{2\sigma^2}} & b < x^*(i,j) \le c \ , \sigma = \frac{c-b}{3} \end{cases} \tag{15}$$

$$\mu_4 = \begin{cases} e^{-\frac{(c-x^*(i,j))^2}{2\sigma^2}} & b < x^*(i,j) \le c \ , \sigma = \frac{c-b}{3} \\ e^{-\frac{(x^*(i,j)-c)^2}{2\sigma^2}} & c < x^*(i,j) \le d \ , \sigma = \frac{d-c}{3} \end{cases} \tag{16}$$

$$\mu_5 = \begin{cases} e^{-\frac{(d-x^*(i,j))^2}{2\sigma^2}} & c < x^*(i,j) \le d \\ 1 & x^*(i,j) > d \end{cases}, \sigma = \frac{d-c}{3} \tag{17}$$

(2) Qualitative indicators membership

Qualitative indicators cannot be directly reflected by specific data, and the above method cannot be applied to determine the membership of qualitative indicators. This paper uses Delphi method [24, 25] to determine. The specific steps of Delphi method to determine membership are as follows:

Step 1: Select and determine specific evaluation indicators, and form an expert consultation group;

Step 2: Collect and compare the first judgment opinions of each expert;

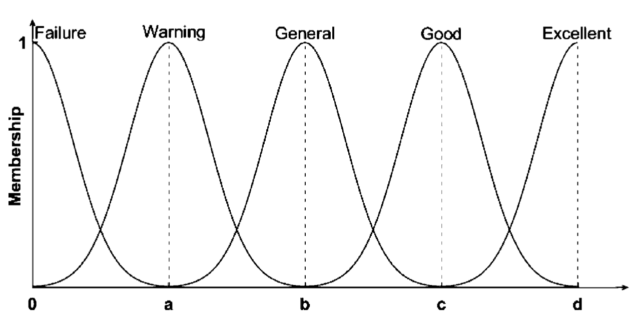

Fig. 3 Membership function of fuzzy normal distribution

Step 3: Feedback the results to the experts, and ask them to decide whether they need to modify their judgment;

Step 4: Collect opinions and feedback results for 3–4 times repeatedly until the experts no longer modify their opinions;

Step 5: Determine the membership of each qualitative indicator belonging to the corresponding grade. If there are g experts in total, and h ($h \le g$) experts evaluate the indicator "rain and snow weather" as "general", then the membership of the indicator at the "general" grade is $\frac{h}{g}$. Other indicators and grades are also treated.

4.3 Fuzzy Matter-Element Matrix of Correlation Coefficient

Correlation degree is a measure of the degree of association between two things. Under certain conditions, the correlation degree function and membership degree function can be exchanged equally. When a specific value in the correlation function is determined, the corresponding function value can be obtained, which is the correlation coefficient. The correlation coefficient is determined by Eq. (18).

$$\delta_{kij} = \mu_k(x^*(i,j)) \tag{18}$$

where δ_{kij} is the correlation coefficient of the j-th indicator of the i-th evaluation object, which belongs to the k-th grade.

According to Eqs. (12) and (18), the fuzzy matter-element matrix of correlation coefficient as shown in Eq. (19) is established.

$$R_{\delta k} = \begin{bmatrix} & N_1 & N_2 & \cdots & N_n \\ C_1 & \delta_{k11} & \delta_{k21} & \cdots & \delta_{kn1} \\ C_2 & \delta_{k12} & \delta_{k22} & \cdots & \delta_{kn2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_m & \delta_{k1m} & \delta_{k2m} & \cdots & \delta_{knm} \end{bmatrix} \tag{19}$$

The correlation degree of each grade is calculated according to Eq. (20)

$$R_k = R_\omega \oplus R_{\delta k} \tag{20}$$

where R_k is the correlation degree of the k-th grade, "$\oplus$" is the fuzzy matter-element operator, and the algorithm is multiplication before addition.

4.4 Principle of Grade Determination

In general, the maximum relevance principle can effectively evaluate, but there are also inaccurate results. For example, for a certain research object, the evaluation grade determined according to the principle of maximum relevance is "excellent", while the relevance degree of "good" grade and "warning" grade are very close to the maximum relevance degree.

In this case, the maximum relevance principle is likely to lose part of the information, and eventually leads to unreasonable evaluation results. This paper is improved as follows:

(1) Calculate the validity index of the maximum correlation degree.

$$\alpha=\frac{K\beta-1}{2\gamma(K-1)} \tag{21}$$

where β is the maximum correlation degree of a certain research object, γ is the second largest correlation degree, and K is the number of grades. When $\alpha < 0.5$, the maximum correlation principle fails; when $0.5 \le \alpha < 1$, the maximum correlation principle is effective; when $\alpha \ge 1$, the maximum correlation principle is very effective.

(2) When the maximum correlation principle fails, the catenary status evaluation grade shall be re-determined according to the principle of weighted average correlation [26]. The calculation equation of weighted average principle is as follows:

$$R = \frac{\sum_{j=1}^{K} r_{\delta j}\lambda_j}{\sum_{j=1}^{K} r_{\delta j}} \tag{22}$$

where $r_{\delta j}$ is the correlation value of the j-th grade, $\lambda_j = \{1,2,3,4,5\}$, R is the correlation value of weighted average principle. This paper determines that the R value in the range of $[j-0.5, j+0.5]$ belongs to the j-th grade.

Based on the above, this paper applies the fuzzy matter-element method and the combination optimization weighting method to the catenary status evaluation. The specific steps are shown in Fig. 4.

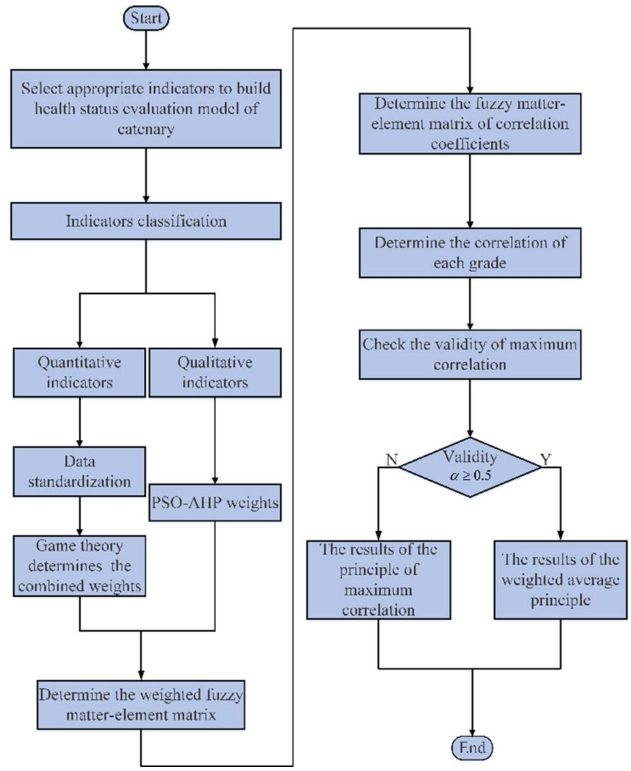

Fig. 4 Steps of catenary health status evaluation

5 Experimental Verification

Select the 2018 catenary detection data of a railway power supply section in China as the sample. According to the catenary health status evaluation model established previously, starting from the kilometer mark K0+356, 10 groups of detection data at different kilometer marks are selected to form the analysis sample in this paper. The standardized data of quantitative indicators are shown in Table 1.

5.1 Fuzzy Matter-Element Matrix of Correlation Coefficient

With reference to the relevant standards of catenary and the experience of experts, the thresholds of each quantitative indicator at each grade are determined, as shown in Table 2.

Qualitative indicators can't be directly reflected by data. In this paper, Delphi method is used to determine the correlation value of each grade, and the fuzzy matter-element matrix of correlation coefficient is composed of quantitative indicators and qualitative indicators.

Taking the "general" status as an example, the fuzzy matter-element matrix with correlation coefficient is calculated according to Eq. (19), as shown below.

$$R_{\delta 3}=\begin{bmatrix} & N_1 & N_2 & \cdots & N_{10} \\ C_1 & 0.0046 & 1.52\times10^{-8} & \cdots & 5.27\times10^{-7} \\ C_2 & 1.52\times10^{-8} & 0 & \cdots & 0.0302 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{17} & 0.2 & 0.1 & \cdots & 0.25 \end{bmatrix}$$

5.2 Weights Fuzzy Matter-Element

In the catenary health status evaluation model established in this paper, $C_1 \sim C_{14}$ are quantitative indicators, and the combined weights are determined by the combination of entropy weight method and subjective weight method; $C_{15} \sim C_{17}$ are the qualitative indicators, and the weights

Table 1 Standardization results of catenary indicators

Indicators	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{10}
C_1	0.6185	0.0000	0.7301	0.5645	0.5974	0.5974	0.2415	0.7447	1.0000	0.7585
C_2	1.0000	0.0000	0.9938	0.4814	0.3882	0.4720	0.3851	0.2205	0.5994	0.2795
C_3	0.5229	0.4575	0.1601	0.6471	0.6471	0.6830	0.0000	0.4771	1.0000	0.3954
C_4	0.9057	1.0000	0.7264	0.0000	0.7170	0.5660	0.6321	0.0189	0.3113	0.4906
C_5	0.0331	0.5108	0.0000	0.4561	0.2432	0.6029	0.0978	0.3727	1.0000	0.4187
C_6	0.4082	0.6122	0.0000	1.0000	0.9184	0.1633	0.8980	0.4694	0.6939	0.7143
C_7	0.2105	0.0789	1.0000	0.5789	0.2105	0.1053	0.0000	1.0000	0.1579	0.7895
C_8	0.7000	0.7361	0.9895	0.7585	0.0000	0.6682	0.8266	0.9068	0.7948	1.0000
C_9	0.0000	0.5714	0.3571	0.8163	0.7143	1.0000	1.0000	0.9524	0.3571	0.9524
C_{10}	0.6364	0.7909	0.4545	0.5182	0.9091	1.0000	0.5909	0.0000	0.2091	1.0000
C_{11}	0.2857	1.0000	0.4286	0.7143	1.0000	1.0000	0.0000	0.7143	1.0000	1.0000
C_{12}	0.2857	1.0000	0.2857	0.7143	1.0000	1.0000	0.0000	0.4286	1.0000	1.0000
C_{13}	0.6000	1.0000	0.2000	0.0000	1.0000	1.0000	0.7140	0.5000	1.0000	1.0000
C_{14}	0.2000	1.0000	0.4000	0.5000	0.0000	0.0000	0.1430	0.2500	1.0000	1.0000

Table 2 Standardization results of catenary detection indicators thresholds

	Failure	Warning	General	Good	Excellent
C_1	0	0.2	0.4	0.6	1
C_2	0	0.25	0.5	0.75	1
C_3	0	0.25	0.5	0.75	1
C_4	0	0.222	0.519	0.741	1
C_5	0	0.05	0.125	0.5	1
C_6	0	0.25	0.5	0.75	1
C_7	0	0.25	0.5	0.75	1
C_8	0	0.2	0.4	0.7	1
C_9	0	0.438	0.7	0.875	1
C_{10}	0	0.2	0.4	0.6	1
C_{11}	0	0.5	0.7	0.9	1
C_{12}	0	0.25	0.5	0.75	1
C_{13}	0	0.25	0.5	0.75	1
C_{14}	0	0.25	0.5	0.75	1

are determined by the subjective weight method. For the five first-level indicators, the subjective weight method is also used to determine the weights.

(1) Subjective weights

The judgment matrices of 5 first-level indicators and 17 s-level indicators are constructed by category, respectively. The *CR* values and weights values obtained by AHP method and PSO-AHP method are solved and compared respectively. The results are shown in Tables 3, 4, 5, 6, 7, 8. Table 3 can be regarded as the contribution of 5 first-level indicators in the catenary evaluation system. The larger the weight value, the greater the role of such indicator in the catenary.

Table 3 Subjective weights comparison of first-level indicators

Indicators	AHP	PSO-AHP
B_1	0.3392	0.2909
B_2	0.1260	0.2827
B_3	0.3392	0.1872
B_4	0.0501	0.1198
B_5	0.1455	0.1194
CR	0.0308	0.0095
Consistency check	The *CR* value of PSO-AHP is smaller	

Table 4 Subjective weights comparison of safety indicators

Indicators	AHP	PSO-AHP
C_1	0.2580	0.2557
C_2	0.5139	0.3525
C_3	0.0529	0.1815
C_4	0.1223	0.1775
C_5	0.0529	0.0328
CR	0.0304	0.0263
Consistency check	The *CR* value of PSO-AHP is smaller	

The parameters of PSO algorithm are set as follows: the number of iterations is 100, the population size is 30, the learning factors are $c_1 = c_2 = 1.49445$, the speed interval is [-0.5, 0.5], and the position interval is [1/9, 9].

From Table 3, 4, 5, 6, 7, 8, it can be seen that the *CR* values obtained by PSO-AHP method is smaller and more consistent for the judgment matrices of order 3 and above (including order 3); for the 2nd order matrix, the *RI* value in AHP method is 0, so the *CR* value cannot be calculated, while the *CR* value obtained by PSO-AHP can be satisfied.

Table 5 Subjective weights comparison of catenary smoothness indicators

Indicators	AHP	PSO-AHP
C_6	0.25	0.3396
C_7	0.75	0.6604
CR	–	0.0698
Consistency check	The *CR* value of PSO-AHP is smaller	

Table 6 Subjective weights comparison of current collection performance indicators

Indicators	AHP	PSO-AHP
C_8	0.4054	0.3117
C_9	0.1140	0.2561
C_{10}	0.4806	0.4322
CR	0.0279	0.0215
Consistency check	The *CR* value of PSO-AHP is smaller	

Table 7 Subjective weights comparison of historical operation indicators

Indicators	AHP	PSO-AHP
C_{11}	0.5596	0.3547
C_{12}	0.2494	0.1704
C_{13}	0.0955	0.2151
C_{14}	0.0955	0.2598
CR	0.0163	0.0073
Consistency check	The *CR* value of PSO-AHP is smaller	

Table 8 Subjective weights comparison of weather indicators

Indicators	AHP	PSO-AHP
C_{15}	0.2583	0.2687
C_{16}	0.6370	0.6066
C_{17}	0.1047	0.1247
CR	0.0370	0.0164
Consistency check	The *CR* value of PSO-AHP is smaller	

Therefore, PSO-AHP method is used to determine the subjective weights of the detection indicators in this paper.

(2) Entropy weight method

Entropy weight method can only be used for quantitative indicators. It mainly determines the weight value of each indicator according to the degree of data difference. The entropy weights of 14 s-level indicators in this paper are shown in Fig. 5.

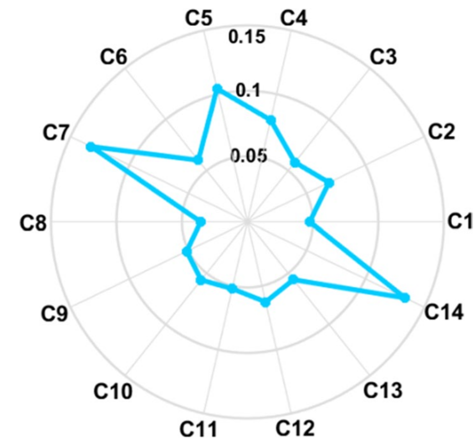

Fig. 5 Entropy weights of catenary quantitative indicators

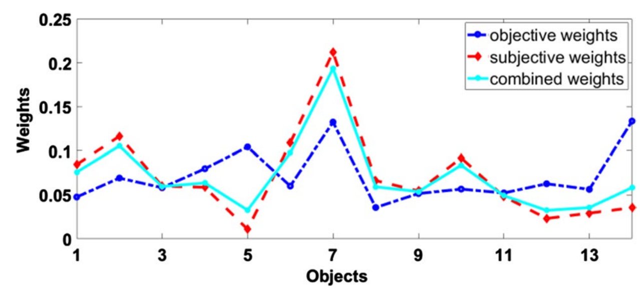

Fig. 6 Combined weights of quantitative indicators

From Fig. 5, we can see that the entropy weights of the 14 s-level indicators have certain differences, indicating that the contribution of each indicator to the operation of the catenary system is different. Among these indicators, C_7 and C_{14} have larger weight values, which can explain that hard spot and proportion of secondary defects have a greater impact on catenary, which is also consistent with the actual operation.

(3) Combined weights

The weight coefficients of entropy weight method and PSO-AHP method are determined by game theory, which are 0.2329 and 0.7671 respectively. The combined weights of quantitative indicators are shown in Fig. 6. Combined weights both consider the influence of subjective weights and objective weights. Compared with single weighting method, the combined weighting method is more objective and reasonable. Using game theory to give weight coefficients can eliminate the randomness of subjective factors, which is more persuasive.

Compared with the weight coefficients determined by expert experience, the weight coefficients determined by this method is more accurate and objective, avoiding the influence of subjective factors. At the same time, it is consistent with the expert experience method, that is, the proportion of subjective weights is greater than that of objective weights.

By using the weights of the first-level indicators and the second-level indicators, the combined weights of all indicators are finally obtained as shown in Table 9.

5.3 Fuzzy Matter-Element Evaluation Results of Catenary

According to Eq. (20), the fuzzy matter-element results of catenary status in this section are obtained, and the evaluation grade is determined according to the principle of maximum correlation, and the results are shown in Table 10.

In general, the maximum correlation principle can effectively evaluate the results, but there are cases where the results are inaccurate. For example, for the first object in Table 10, the evaluation result determined according to the principle of maximum correlation is the "warning" grade, but it can be found from the table that the correlation of the "good" grade of is very close to the maximum correlation. Similarly, for the 10th evaluation object, the evaluation result determined according to the principle of maximum correlation is "excellent", but the correlation of "good" grade is very close to the maximum correlation. In this case, the maximum correlation principle is likely to lose part of the information, and eventually lead to unreasonable evaluation results.

Based on the above, this paper uses Eq. (21) to examine the effectiveness of the principle of maximum correlation, and the results are shown in Fig. 7.

It can be seen from the figure that only the 3rd and 7th research objects have a validity of more than 0.5, and the principle of maximum correlation is effective; the remaining eight research objects are invalid and need to be determined again with the principle of weighted average. The catenary grade finally determined in this paper is shown in Table 11.

From Table 11, it can be concluded that among the 10 sets of data, the evaluation grade of the 7th set of data is "failure", which indicates that the catenary at this point needs to be repaired to eliminate the fault in time. The remaining 9 objects are "general" or above. The catenary status can meet the operation requirements of the railway. Generally, no major accidents will occur, but close monitoring

Table 9 Combined weights of catenary indicators

Indicators	Weights	Indicators	Weights
C_1	0.0678	C_{10}	0.0748
C_2	0.0941	C_{11}	0.0439
C_3	0.0532	C_{12}	0.0289
C_4	0.0568	C_{13}	0.0318
C_5	0.0291	C_{14}	0.0520
C_6	0.0872	C_{15}	0.0287
C_7	0.1728	C_{16}	0.0647
C_8	0.0528	C_{17}	0.0133
C_9	0.0481		

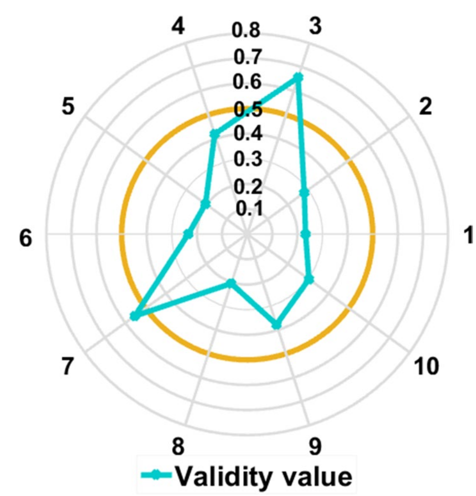

Fig. 7 Validity values of the maximum correlation

Table 10 Catenary evaluation results of the maximum correlation principle

Objects	Failure	Warning	General	Good	Excellent	Results
1	0.0817	0.2724	0.1417	0.2500	0.1512	Warning
2	0.2896	0.0588	0.1238	0.1449	0.2560	Failure
3	0.1470	0.1680	0.1005	0.1634	0.3653	Excellent
4	0.0978	0.0314	0.3841	0.2977	0.1104	General
5	0.1229	0.1904	0.1562	0.1938	0.2453	Excellent
6	0.1491	0.1046	0.1651	0.2219	0.2643	Excellent
7	0.3456	0.1221	0.1086	0.1891	0.1260	Failure
8	0.1626	0.1806	0.2413	0.0943	0.2689	Excellent
9	0.0471	0.2642	0.0797	0.1441	0.3333	Excellent
10	0.0032	0.1053	0.1257	0.3424	0.3523	Excellent

Table 11 Final grades of catenary

Objects	R	Results
1	3.1301	General
2	3.0218	General
3	–	Excellent
4	3.3163	General
5	3.2731	General
6	3.3842	General
7	–	Failure
8	3.1333	General
9	3.5208	Good
10	4.0068	Good

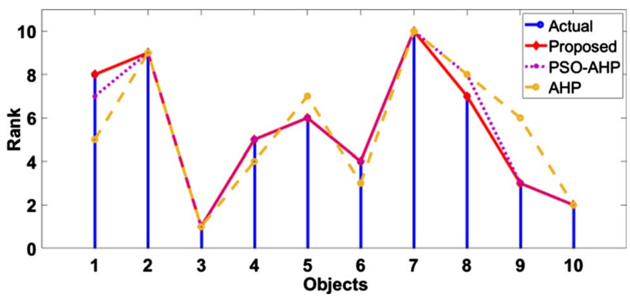

Fig. 8 Comparison of evaluation ranking results of different weight methods

and regular detection are required to prevent the status from deteriorating.

In order to further verify the accuracy of the combined weight method used in this paper to determine the evaluation grade, the evaluation results of this method, PSO-AHP method and AHP method are sorted respectively, and compared with the actual situation. The results are shown in Fig. 8. It can be seen from the figure that the ranking of evaluation results obtained by the weight method used in this paper is consistent with the actual situation of catenary, and the ranking results of PSO-AHP method and AHP method are different from the actual situation.

In 10 groups of samples selected, the ranking accuracy of the proposed method is 100% and the evaluation accuracy is 90%. The results are both higher than those of PSO-AHP and AHP. The reason why the evaluation accuracy is lower than the sorting accuracy is that there is a group of sample with misjudgment, but it does not affect its sorting importance in the whole 10 groups of samples, so these two kinds of accuracy rates are not in conflict.

Proper and reasonable ranking results can effectively help the catenary practitioners to understand the status of catenary in time and make reasonable judgments. As the catenary is easily affected by weather and other special factors, together with the increase of pantograph catenary wear, the status of catenary will also change. Therefore, it is necessary for catenary practitioners to further regularly check the parameters of catenary and make timely judgments.

6 Conclusions

In this paper, the fuzzy evaluation and matter-element analysis are combined organically. Based on the traditional evaluation system, the weather indicators and historical operation condition indicators are added. The operation status evaluation model of catenary including quantitative and qualitative indicators is rebuilt. The evaluation results are compared with the actual situation of catenary in this section, which realized the objective health status evaluation of catenary. Through research and analysis, the following conclusions are drawn:

(1) The entropy weight method is used to determine the objective weights of quantitative indicators, the PSO-AHP method is used to determine the subjective weights, and the game theory is used to determine the subjective and objective weight coefficients, which improves the traditional single weight method and the lack of artificial coefficients, and the combined weights obtained are more scientific and reasonable.
(2) On the basis of the traditional fuzzy matter-element method, the normal distribution function and Delphi method are used to improve the correlation, which fully considers the ambiguity and randomness between the indicators and the evaluation grade, and the obtained correlation can better represent the catenary operation status.
(3) For the case where the principle of maximum correlation is invalid, the principle of weighted average is proposed to replace the principle of maximum correlation, so as to make the status grade of catenary more in line with the actual situation and facilitate the correct judgment of catenary practitioners.
(4) The evaluation results are ranked from excellent to failure, and compared with the results of single weighting method. The results show that the evaluation results of this paper are consistent with the actual situation. The evaluation method of this paper can provide strong support for the status evaluation of catenary.

In this paper, PSO algorithm is used to optimize AHP, and other meta heuristic algorithms (such as difference algorithm) can be used to replace PSO algorithm. The method proposed in this paper can provide a new idea for the health evaluation of catenary, and can also be applied to other engineering fields.

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Compliance with Ethical Standards

Conflict of Interest This is the first submission of this manuscript and no parts of this manuscript are being considered for publication elsewhere. All authors have approved this manuscript. No author has financial or other contractual agreements that might cause conflicts of interest.

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