REVIEW

Thermo-mechanical coupled analysis based design of ventilated brake disc using genetic algorithm and particle swarm optimization

Ali Belhocine[1] · Dinesh Shinde[2] · Rahul Patil[3]

Abstract

The brake discs are subjected to thermal load due to sliding by the brake pad and fluctuating loads because of the braking load. This combined loading problem requires simulation using coupled thermo-mechanical analysis for design evaluation. This work presents a combined thermal and mechanical finite element analysis and evolutionary optimization-based novel approach for estimating the optimal design parameters of the ventilated brake disc. Five parameters controlling the design: Inboard plate thickness, outboard plate thickness, vane height, effective offset, and center hole radius were considered, and simulation runs were planned. 27 brake disc designs with design parameters as recommended by the Taguchi method (L27) were modeled using SOLIDWORKS, and the FEA simulation runs were carried out using ANSYS thermal & structural analysis tool. The fatigue life results were analyzed using a 3D surface plot for the effect of the design parameters on the response, contour plots for the determination of maximum response, and statistical regression analysis for model interpretation and predictive modeling. Finally, the two most accurate and widely used evolutionary optimization algorithms: genetic algorithm (GA) and particle swarm optimization (PSO) were applied to determine the optimal design parameters for the ventilated brake disc. The brake disc of design parameters predicted by GA and (PSO), gives 12.74% higher fatigue life compared to parametric analysis. These results have shown that the developed approach can be utilized effectively and reliably for solving, design ventilated brake disc problem in the industry.
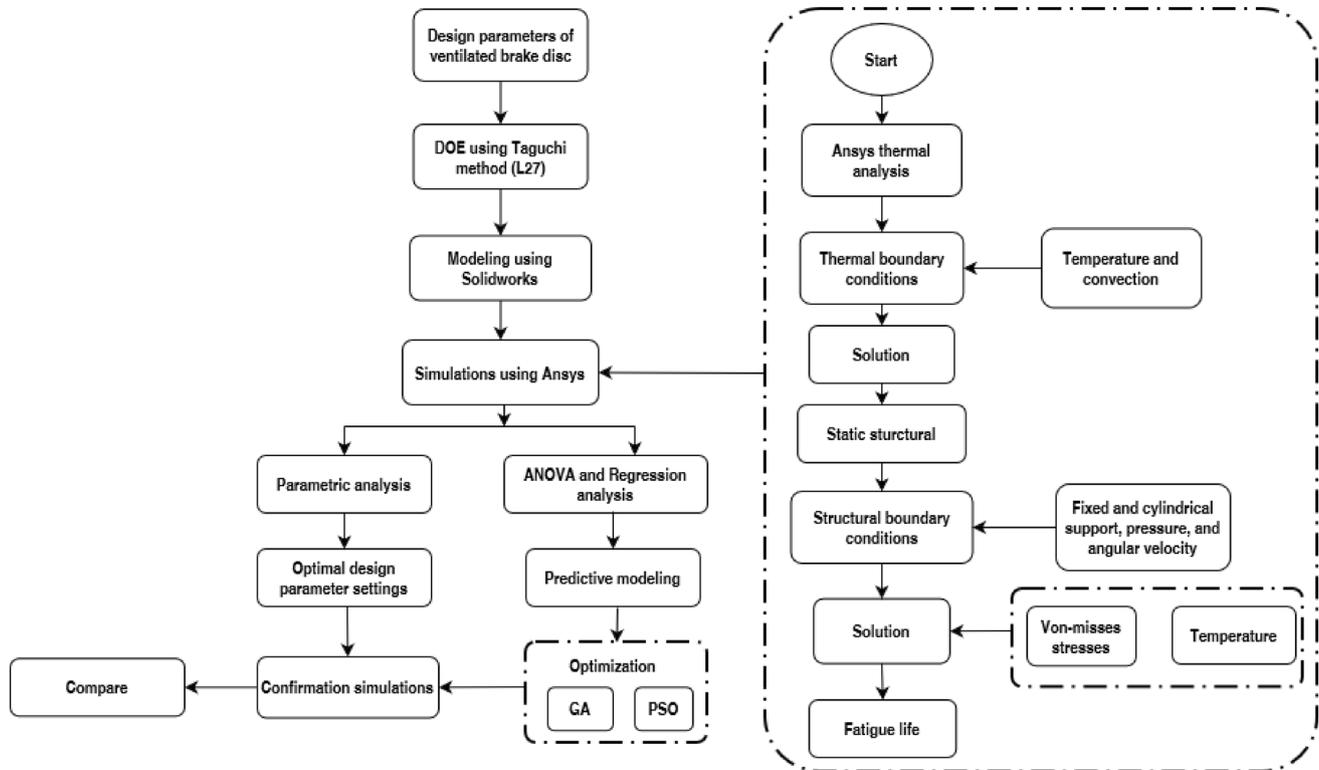
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Graphical abstract

Methodology of present study

Keywords Ventilated brake disc · Coupled thermo-mechanical analysis · Fatigue life · Parametric analysis · Genetic algorithm · Particle swarm optimization

1 Introduction

The automobile is a complex integration of electronic and mechanical components. Yim and Jo [1] carried out a simulation on the vehicle simulation package, CarSim to verify the efficiency of the proposed method. One of the major components is the braking system which is limited due to its shortcomings. Finite element methods are used extensively in many engineering applications to analyze new and existing designs [2, 3]. The brake friction pad slides over the brake disc to stop or control the automobile. This sliding generates heat, which the disc should dissipate to air. Thus, the brake disc is subjected to thermal as well as structural loads. It is ventilated to facilitate cooling and to relieve thermal stresses. They are ventilated at the surface where the brake friction pad comes in contact with the disc in terms of ventilation space and surface holes. The ventilation area, vanes dimensions, and angles are an important consideration during the design of these brake discs [4]. The

mechanical and thermal properties of the ventilated brake disc can be enhanced through design procedure based on the consideration of structural and thermal aspects. Composite materials are heterogeneous in nature and suffer from complex non-linear modes of failure, such as delamination, matrix crack, fiber-breakage, and voids, among others [5]. The material used for the brake disc is also a concern to designers. Normally, the brake discs are made of cast iron due to their superior tribological, mechanical, and economic properties. Due to the uneconomic and lengthy procedure for testing the brake disc to performance estimation of different designs, the simulation using finite element analysis (FEA) was preferred by the researchers. Belhocine and Bouchetara [6], Belhocine and Afzal [7] used the computational fluid dynamics tool of FEA for thermal and thermo-mechanical investigation of ventilated brake disc for design improvements. They estimated the heat transfer coefficient using the CFX tool of ANSYS. Pevec et al. [8] also utilized a similar tool to propose a numerical simulation model for

predicting the automotive brake disc cooling factor using a flow simulation study of air over the surface of the brake disc. Emphasis has been given to the wall surface temperature of the brake disc. Belhocine et al. [9] performed structural, contact analysis and prediction of the performance of a disc brake system with the help of FEA. They observed that the bolt holes suffer from stress concentration. They also predicted the contact pressure between the brake pad and disc. Some authors have examined the disc brake squeal using a complex Eigenvalue problem using finite element simulation. They combine vibrational equations and FEA using an enhanced contact mechanism. Others investigated thermo-elastic contact problems in brake discs made of functionally graded materials and compared them with ceramic and full-metal brake discs using FEM. Zhang et al. [10] applied FEM to analyze thermo-mechanical dynamics of the ventilated brake disc considering the temperature field, normal stress, and elasto-mechanical deformation with remarks that the structure of ventilation and thickness of inboard and outboard plates of ventilated brake discs plays a significant role in temperature distribution. Jian and Shui [11] used FEA and experimental configuration to understand the transient temperature field of the ventilated brake disc. Coupled thermal and structural (thermo-mechanical) analysis was utilized by various researchers to analyze the brake discs using finite element analysis; structural analysis includes the determination of stresses, strain, and deformations, whereas thermal analysis comprises temperature distribution, thermal stress, thermal strain, and heat transfer coefficient or heat flux. Duzgun [12] investigated and compared thermo-structural behavior of ventilated or non-ventilated different configurations of brake disc using FEA. They reported that ventilated brake disc reduced maximum thermal stress generation to 11–19% compared to its counterpart. Hwang and Wu [13] investigated ventilated brake discs for thermal stresses and temperature distribution. They build a 3D model with thermo-mechanical boundary conditions. Mahmoudi et al. [14] applied thermo-mechanical FEA to the brake disc of functionally graded material and compared results with aluminum and ductile cast iron disc. They elaborated the concept of thermal shock occurs at the surface of the brake disc. Jung et al. [15] studied the phenomenon of hot Judder experience by the ventilated brake disc using thermo-mechanical FEA. They investigated the disc surface temperature distribution as well as the generation hot area. Brake discs are normally subjected to fluctuating load due to intermittent braking application, which needs consideration of fatigue during analysis; hence the fatigue life estimation is crucial to accept the design methodology. Han et al. [16] analyzed the brake disc for thermal fatigue and thermo-mechanical coupled effect using FEA and experimental brake dynamometer test. They presented a counting method for fatigue life determination. In addition, they concluded that the disc material, i.e., gray cast iron has high or low fatigue depending on the increase of temperature. Zhang et al. [17] presented FEM fatigue model for high-speed trains using coupled thermo-mechanical analysis. They developed a model considering temperature and stress distribution on the brake discs.

In literature, it was observed that few of the works reported the fatigue life estimation of ventilated brake discs, but with limited trials of design. The current study presents a novel approach to determining the fatigue life of the ventilated brake disc considering the coupled thermo-mechanical analysis of multiple design trials. The design parameters of ventilated brake disc combination and their influence on fatigue life have been studied. Optimal parameters setting have been estimated for maximum fatigue life using genetic algorithm and particle swarm optimization.

2 Research methodology

2.1 Ventilated brake disc design

Ventilated brake disc design becomes complex due to the involvement of different geometries such as inboard & outboard plates, vanes, offset, etc. The vanes are placed in between the inboard and outboard plate to provide the cooling effect using airflow. They are oriented at different distances and angles. The design for ventilated brake disc should not only give better heat transfer but also attain adequate thermo-structural fatigue life since they are subjected to fluctuating load. Because of this, the ventilated brake disc should be designed based on coupled thermo-mechanical fatigue life instead of Von-Mises stresses and strains. In this work, a novel ventilated brake disc design shown in Fig. 1 is proposed and analyzed for combining thermal and mechanical fatigue life.

2.2 Taguchi (L27) orthogonal design

Five design parameters influencing the design of ventilated brake disc elaborated in Fig. 1 were considered in this study referring to the literature [18–20]. Structural integrity as well as the performance of the brake disc mainly depends on these parameters. Considering the parameters and levels, the experimental simulation was designed using Taguchi L27 orthogonal array (Table 1) to examine the effect of design parameters on the thermo-structural fatigue life, with the help of a software package MINITAB.

Table 2 shows the trial run sequence generated by the Taguchi design. With the help of randomization (27 simulation runs), there is a chance for each individual design parameter to contribute to the study.
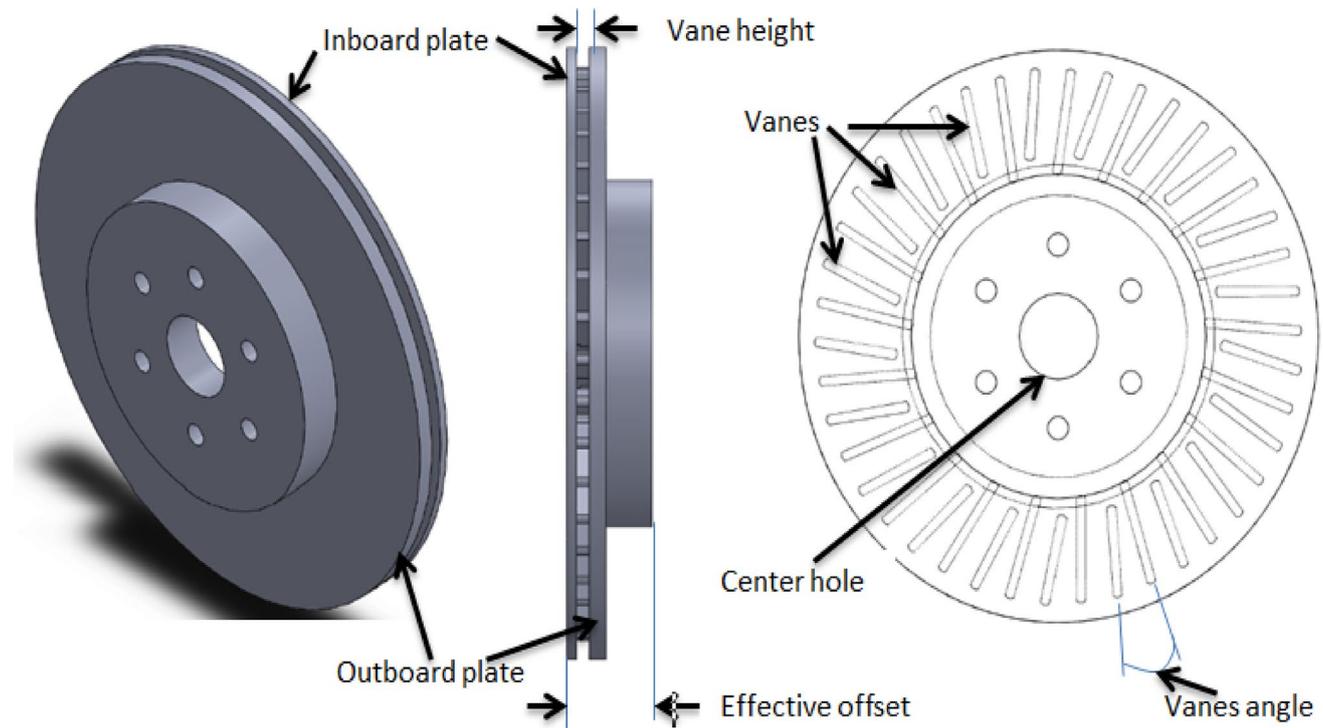
Fig. 1 Ventilated brake disc and design parameters

Table 1 Ventilated brake disc design parameters and levels

S. no.	Design parameters	Symbol	Levels		
			Level 1	Level 2	Level 3
1	Inboard plate thickness (mm)	T_i	3	7	11
2	Outboard plate thickness (mm)	T_o	3	7	11
3	Vane height (mm)	H	4	6	8
4	Effective offset (mm)	E	16	28	40
5	Center hole radius (mm)	R	20	25	30

2.3 Modeling and simulations

As recommended by Taguchi design (Table 2), brake discs were modeled using SolidWorks for each individual trial run with the set of parameters, and finite element simulation was carried out on steady-state thermal combined with a static structural module of ANSYS 20.0 separately. Figure 2 presents the methodology adopted in the present study. During FEA for fatigue life, S–N curve of the brake disc material (gray cast iron) was used considering low cycle and high cycle fatigue [16]. The material properties, loading, and boundary conditions were applied to replicate the operation of the disc brake (shown in Fig. 3). Following calculations were done for the structural loading referring to the following relations and assuming vehicle data used by Belhocine et al. [7].

Braking force on the brake pad,

$$F_{pad} = \frac{(30\%) \cdot \frac{1}{2}M \cdot V_0^2}{2 \cdot \frac{R_{rotor}}{R_{wheel}} \cdot \left(V_0 \cdot t_{stop} - \frac{1}{2}\left\{\frac{V_0}{t_{stop}}\right\}t_{stop}^2\right)} = 2591.6 \text{ N} \tag{1}$$

Braking pressure on the brake pad,

$$P = \frac{F_{pad}}{A_s \cdot \mu} = 1999537.073 \text{ Pa} = 1.999 \text{ MPa} \sim 2 \text{ MPa} \tag{2}$$

The rotational speed of the disc,

$$\omega = \frac{V_0}{R_{wheel}} = 105.26 \text{ rad/s} \tag{3}$$

The accuracy of the FEA model is sensitive to the type of mesh used. In view of this, the initial analysis was carried with coarse, medium, fine and edge refine meshing.

Table 2 Taguchi L27 design matrix

Trial no.	Design parameters (coded)									
	T_i	T_o	H	E	R	T_i	T_o	H	E	R
1	1	1	1	1	1	3	3	4	16	20
2	1	1	1	1	2	3	3	4	16	25
3	1	1	1	1	3	3	3	4	16	30
4	1	2	2	2	1	3	7	6	28	20
5	1	2	2	2	2	3	7	6	28	25
6	1	2	2	2	3	3	7	6	28	30
7	1	3	3	3	1	3	11	8	40	20
8	1	3	3	3	2	3	11	8	40	25
9	1	3	3	3	3	3	11	8	40	30
10	2	1	2	3	1	7	3	4	16	20
11	2	1	2	3	2	7	3	4	16	25
12	2	1	2	3	3	7	3	4	16	30
13	2	2	3	1	1	7	7	6	28	20
14	2	2	3	1	2	7	7	6	28	25
15	2	2	3	1	3	7	7	6	28	30
16	2	3	1	2	1	7	11	8	40	20
17	2	3	1	2	2	7	11	8	40	25
18	2	3	1	2	3	7	11	8	40	30
19	3	1	3	2	1	11	3	4	16	20
20	3	1	3	2	2	11	3	4	16	25
21	3	1	3	2	3	11	3	4	16	30
22	3	2	1	3	1	11	7	6	28	20
23	3	2	1	3	2	11	7	6	28	25
24	3	2	1	3	3	11	7	6	28	30
25	3	3	2	1	1	11	11	8	40	20
26	3	3	2	1	2	11	11	8	40	25
27	3	3	2	1	3	11	11	8	40	30

Finer hexahedron meshing was used here since it yields a more accurate flow-field than tetrahedron for the same edges resulting in improved results. Figure 4 reports the FEA mesh used in the analysis. The developed FEA model was evaluated for Von Mises stresses, considering the similar loading and boundary conditions to validate it. Figure 5 reveals that the model gives close results that of Han et al. [16].

The statistical model was analyzed using analysis of variance and the model for prediction of fatigue life for the given design parameters of the ventilated brake disc were presented.

2.4 Parametric analysis and optimization

3D response surface plots and contour plots were used to study the influence of the design parameters on the coupled thermo-mechanical fatigue life of ventilated brake disc and optimal design parameter settings were determined.

2.5 Optimization using genetic algorithm and particle swarm optimization

In this work, the selection of optimal design parameter settings for the ventilated brake disc was carried out using a genetic algorithm (GA) & particle swarm optimization (PSO).

Genetic Algorithm (GA) is one of the most chosen heuristic search optimization algorithms. It is a population-based algorithm which imitates the natural reproduction system based on Darvin's principle of survival of the fittest. It starts from the generation of the initial population of alternatives called chromosomes. The randomly selected chromosomes from the population are then subjected to mutation, i.e., fitness function evaluation, crossover, i.e., cross-replacements, and selection repeatedly until termination criteria are reached [21]. It works on the local search of the best solution considering fitness evaluation. Researchers used GA in diverse fields of application
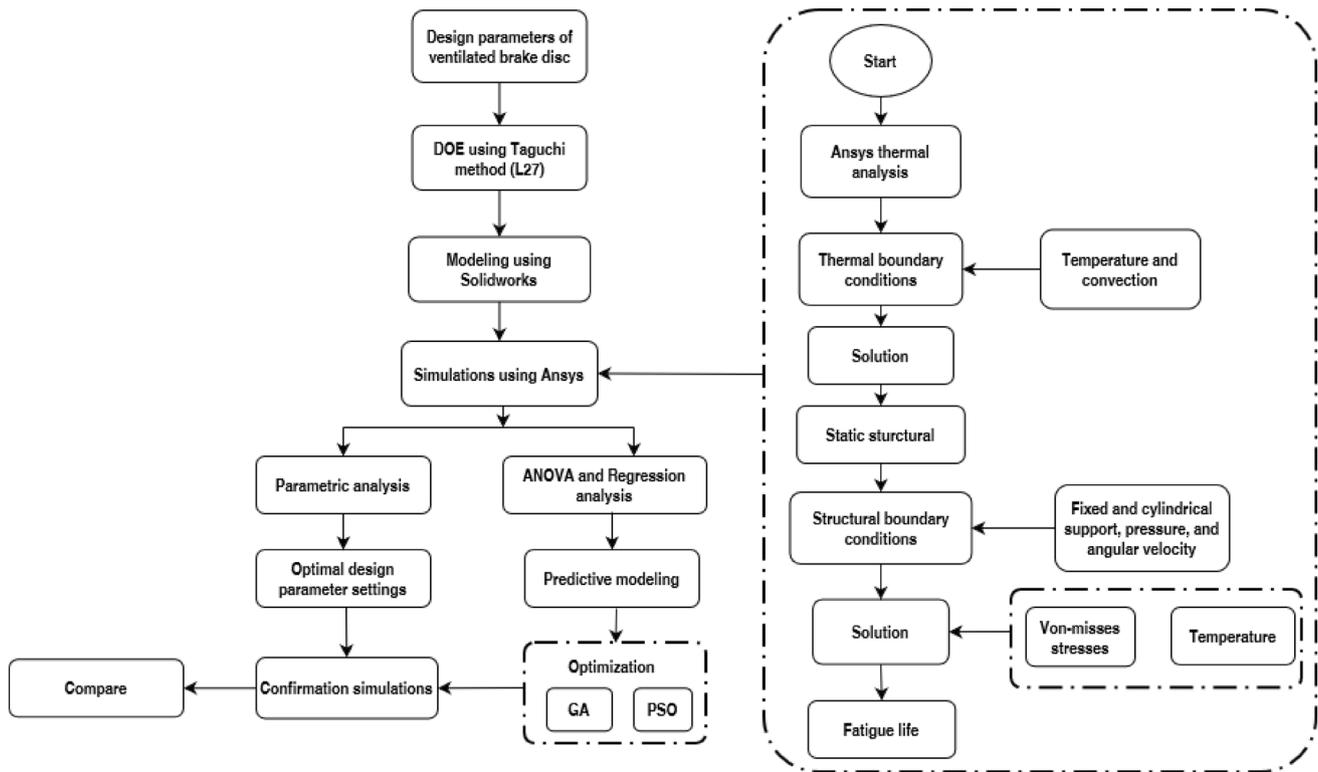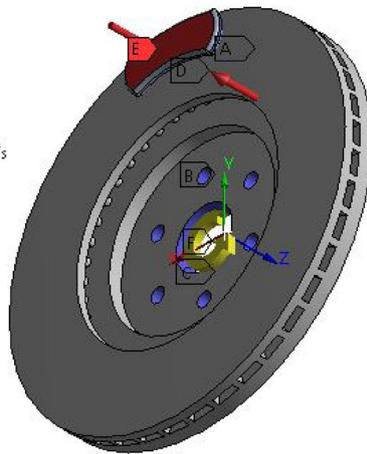
Fig. 2 Methodology of present study
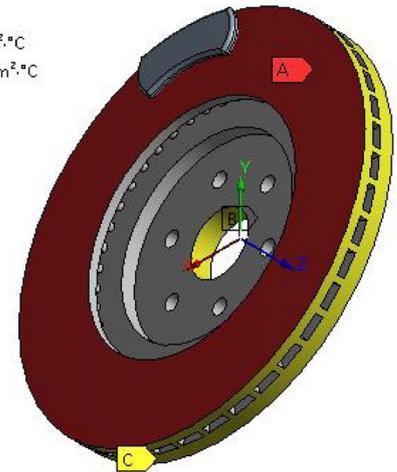
Fig. 3 FEA boundary conditions: **a** structural and **b** thermal

[22–24]. In the current study, GA implemented using the MATLAB programming platform.

PSO is one of the powerful optimization algorithms which mimic the swarm performance in a group of birds or fish. The swarming behavior depends on individuals and group positions, which makes the heuristic search easy to implement. It is based on the determination of a particle's new position using velocity factor considering global best and an individual's or local best position [25]. This methodology improves the search compared to techniques that
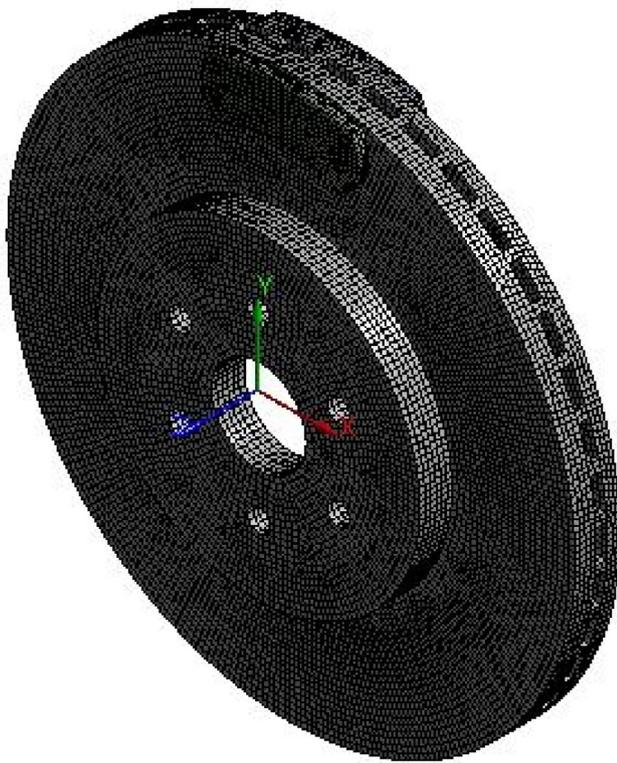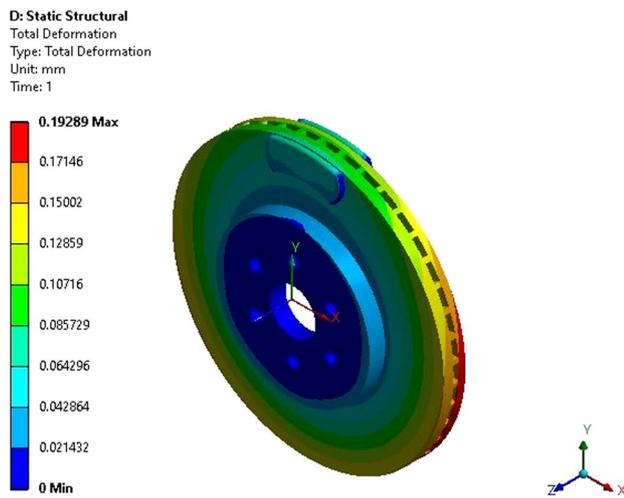
Fig. 4 Mesh model

Fig. 5 Total deformation of the brake disc for trial 1

work with local best such as GA. Scientists have used PSO in various engineering applications [26–29]. In the present work, PSO was utilized to obtain the optimal set of PIM process parameters for minimum weld lines, with the help of MATLAB programming.

3 Results and discussions

3.1 FEA simulations

Figure 6 shows the FEA simulation result of the ventilated brake disc design trial no. 1. The outer peripheral ring area of the inboard plate has the highest deformation. Simulations were carried out for total deformation and fatigue life; however, further study concentrated on fatigue life of the brake discs for braking performance. Total 27 simulation runs were carried out total deformation, Von-Mises stresses, and fatigue life result of 1st trial run is reported in Figs. 6–8. The deformation of the outer rim of the disc is maximum, especially of the outboard plate (Fig. 6). Stress concentration was seen at the bolting hole of the discs; also the outboard plate rim is subjected to maximum Von-Mises stresses (Fig. 7). Maximum and minimum fatigue life are the number of loading cycles the brake disc undergoes before failure. Different parts have shown different fatigue life due to structural integration (Fig. 8). Minimum life was seen near the bolting hole due to the stress concentration. The mean value of this fatigue life was taken for analysis. The present FEA model was validated for Von-Mises stresses and fatigue life. The gray cast iron has a low cycle and high cycle fatigue [16]; hence the current FEA model was evaluated for both and endorsed with literature as shown in Fig. 10. It was observed from Figs. 9, 10 that the current FEA model estimated close values of the responses. Table 3 shows the simulation results for 27 trial runs for design for ventilated brake disc using FEA.
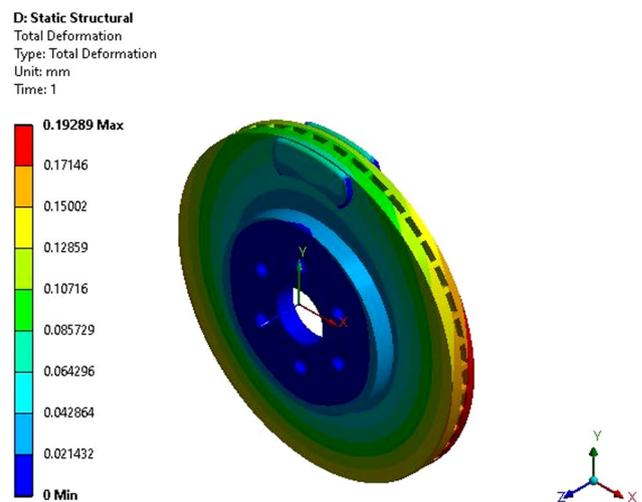

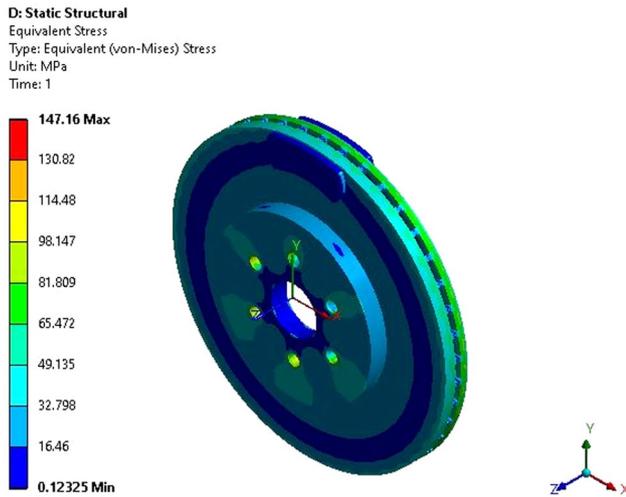**Fig. 6** Total deformation of the brake disc for trial 1
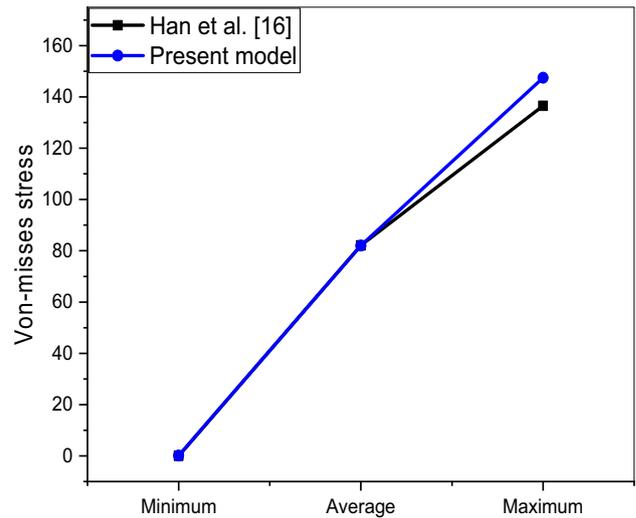
Fig. 7 Von-Mises stress of the brake disc for trial 1

Fig. 8 Fatigue life for trial no. 1

Fig. 9 Validation of the FEA model for Von-Mises stresses

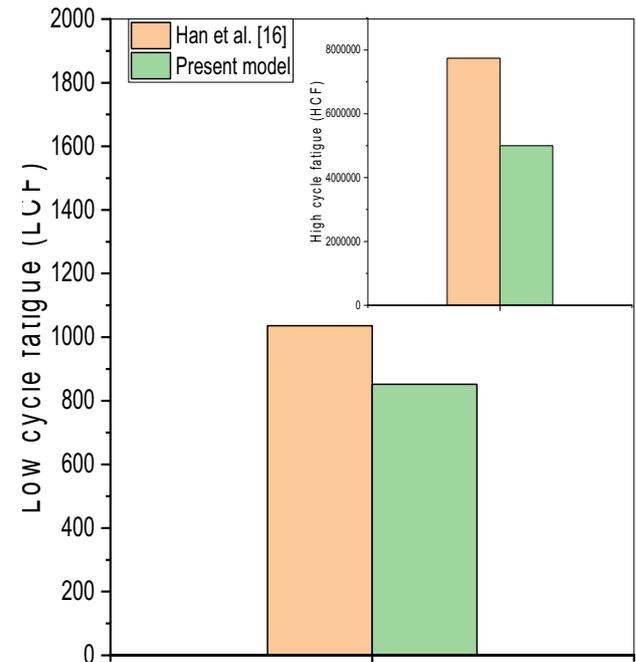**Fig. 10** Validation of the FEA model for LCF & HCF

3.2 Parametric analysis

Using the response surface model for fatigue life, the influence of design parameters on the fatigue life of ventilated brake disc were studied using response surface plots (Fig. 11) [30]. For 7 mm of inboard plate thickness, the fatigue life of the ventilated brake disc increases with outboard plate thickness up to 7 mm then decreases, also at all, the values of outboard plate thickness inboard plate thickness increases till mid-value (7 mm) then drops (Fig. 11a).

It was observed from Fig. 11b that vane height has a similar trend to the outboard plate thickness, but the fatigue life increases with inboard plate thickness for all the values of vane height. Figure 11c shows that fatigue life decreases with center hole radius for all the values of inboard plate thickness. The fatigue was appeared to be maximum for 6 mm vane height and 7 mm outboard plate thickness (Fig. 11d). Figure 11e reveals that fatigue life decreases with effective radius at lower values of outboard plate thickness,

Table 3 Fatigue life for different design trials

Trial no.	Fatigue life (cycles)	Trial no	Fatigue life (cycles)
1	5e6	15	3.1e6
2	4e6	16	5.2e6
3	3.9e6	17	4.7e6
4	5.1e6	18	2.8e6
5	4.5e6	19	5.7e6
6	5.4e6	20	3.4e6
7	3.9e6	21	4.1e6
8	4.6e6	22	3.7e6
9	2.5e6	23	5.12e6
10	4.9e6	24	4.67e6
11	3.8e6	25	5.1e6
12	3.2e6	26	4.87e6
13	5.1e6	27	2.106e6
14	2.8e6		

but the rate of decrement was higher at 40 mm. A similar pattern of variation was observed for the pair effective offset and center hole radius (Fig. 11f).

The stationary point found from the response surface can be used to identify that the point is a maximum output or minimum output or saddle point, this possibly done using contour plots (Fig. 12) [22]. The maximum value of fatigue life was observed at 7 mm inboard as well as outboard plate thickness, 6 mm vane height, 28 mm effective offset, and 20 mm center hole radius.

3.3 Statistical and regression analysis

The simulations conducted based on the Taguchi design were used for fitting a predictive mathematical model to closely describe the fatigue life of the ventilated brake disc. A standard statistical software package MINITAB was effectively used for conducting the regression analysis. The sum of squares (SS) is the difference between the square of the average and the individual value of the term. Mean square (MS) is the variance related to each parameter and its mathematical perception; it is obtained by dividing the degree of freedom (DF) to the sum of squares [23, 25]. F-value is the ratio between MS and SS. The significance of the model and terms is recognized by the high F-value displayed in Table 4. Here it is 40.45 for the model with a chance of 0.000%, no chance of occurrence of noise between the test and prediction of fatigue life, which reveals that the model is significant. The difference between R^2(adj) and predicted R^2 is 3.35% for the model confirms the same. The significant terms can be recognized by the value of "P-value." If it is below 0.05, it yields a significant term. P-value less than 0.005 for inboard plate thickness (T_i) and effective offset

(E) shows that they are significant parameters. The generated regression equation (Eq. 1) can be used to predict the fatigue life for the combination of design parameters. Figure 13 shows a comparison of actual & predicted fatigue life. It was observed that the present model predicts the fatigue life of ventilated brake discs with high accuracy. Overall variation of prediction as 5.25% is seen in fatigue life.

$$\text{Fatigue life (cycles)} = 589473 \times T_i + 473362 \times T_0$$
$$+ 380806 \times H + 619918 \times E - 65527 \times R$$
$$(4)$$

3.4 Optimal design parameters using GA & PSO

The maximization of the fatigue life of the ventilated brake disc was the objective for the present optimization problem which was defined considering the empirical relation (Eq. 1) generated by the predictive model to get the best design parameters.

Find $Z = (T_i, T_o, H, E, R)$;
To Maximize $f(Z) = \text{d}(Fatigue\ life)$;
Subjected to,

$$3 \le T_i \le 11$$
$$3 \le T_0 \le 11$$
$$4 \le H \le 8 \qquad\qquad (5)$$
$$16 \le E \le 40$$
$$20 \le R \le 30$$

GA & PSO was implemented to solve this optimization problem. Figure 14 shows the performance of GA with the help of average & best fitness observed after running MATLAB code for the objective function. Different parameters are needed to be considering in GA, such as chromosome length, population, generation, crossover rate, and mutation rate. Values of these parameters used in the MATLAB code are 12, 100, 140, 0.95, and 0.05 [31]. Fitness is the objective function of the optimization problem. Average fitness starts with a relatively higher value of fatigue life (1.8e9) and becomes minimum (8.32145e8) at 20th generation, whereas best fitness starts at 0.9e9 and yields minimum value (8.32145e8) at 20th generation. This variation was because of the random generation of a population of 100 individuals and calculation of fitness for different chromosomes randomly by GA. It was observed that irrelative to the variation of fitness, they converge to the same optimal value. After certain generations (10th), GA compares average fitness with the best fitness and both progress with better value (8.32145e8). The best optimal design parameters predicted by GA are given in Table 5.

A MATLAB code was established for the operation of PSO; the terms swarm size, acceleration factor, minimum
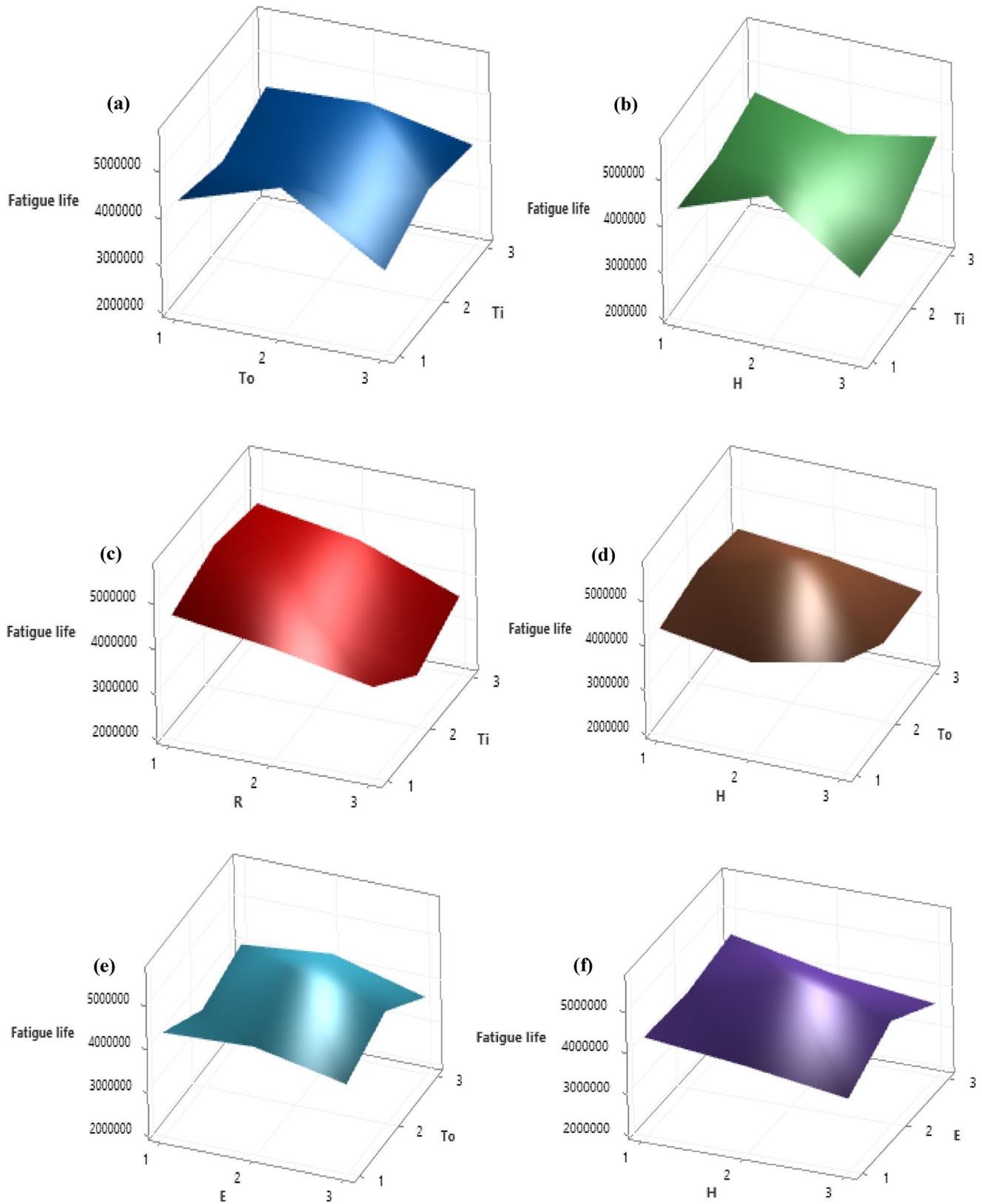
Fig. 11 Response surface plots for coded design parameter (1: level-1, 2: level-2, 3: level-3)
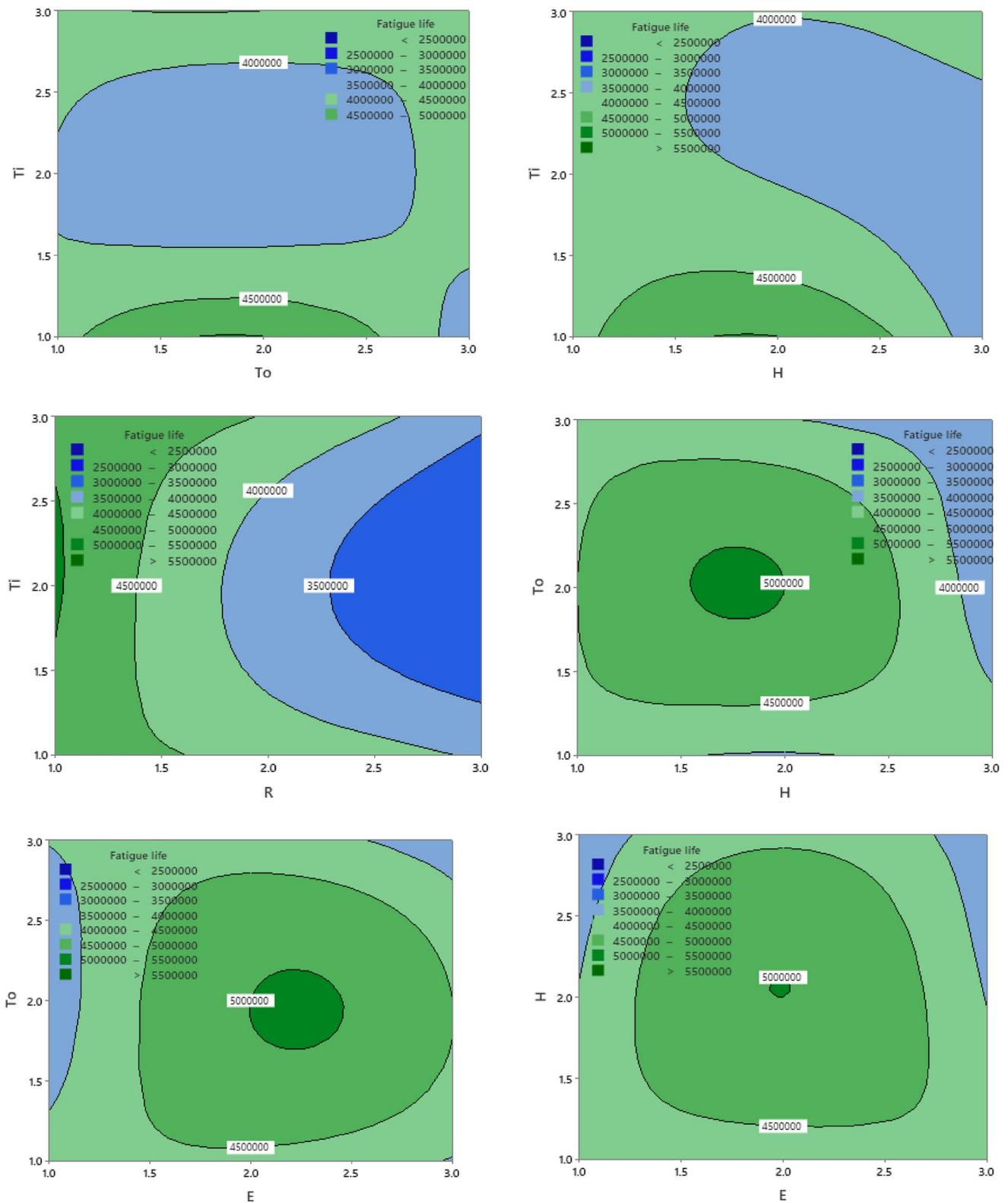
Fig. 12 Contour plots for coded design parameter (1: level-1, 2: level-2, 3: level-3)

Table 4 ANOVA analysis

| Source	DF	Adj SS	Adj MS	F-value	P-value	Remarks
Regression model	5	4.51043E+14	9.02086E+13	40.45	0.000	Significant
T_i	1	7.75572E+12	7.75572E+12	3.48	0.006	Significant
T_o	1	5.00128E+12	5.00128E+12	2.24	0.148	
H	1	3.23670E+12	3.23670E+12	1.45	0.241	
E	1	8.57753E+12	8.57753E+12	3.85	0.003	Significant
R	1	95,836,996,129	95,836,996,129	0.04	0.838	
Error	22	4.90623E+13	2.23010E+12			
Total	27	5.00105E+14		R^2	90.19%	
				R^2-adj	87.16%	
				R^2-pred	86%	
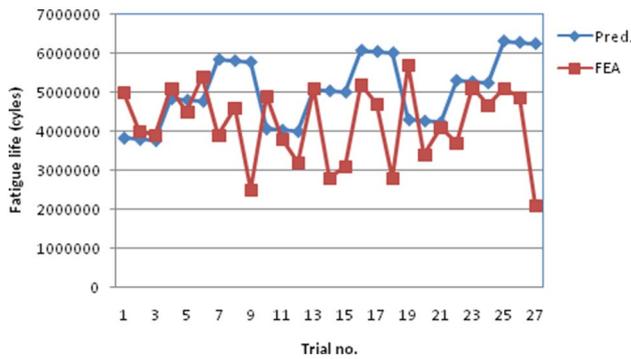
Fig. 13 Predicted vs. FEA results

and maximum inertia weights play a significant role in the accuracy of PSO in the evaluation of fitness function. Values for these terms were assumed as 100, 2, 0.4 & 0.9, respectively [32, 33]. PSO was run for 100 generations in the present study. Figure 15 shows the convergence of GA & PSO; we observed that PSO quickly (within 4 generations) convergences to the best solution to this problem. PSO predicts similar design parameters setting (Table 5) as predicted by GA, confirming the optimal solution to the problem. The confirmation tests were conducted with optimal design parameters of the ventilated brake disc, and results are reported in Table 5.
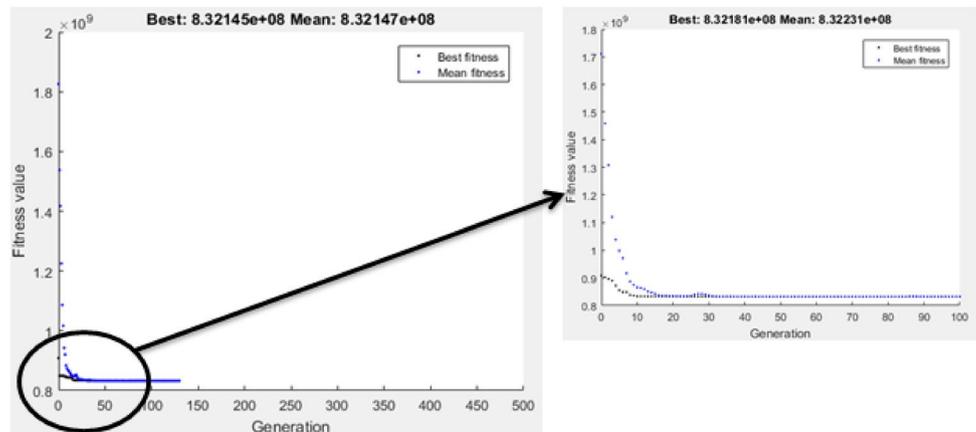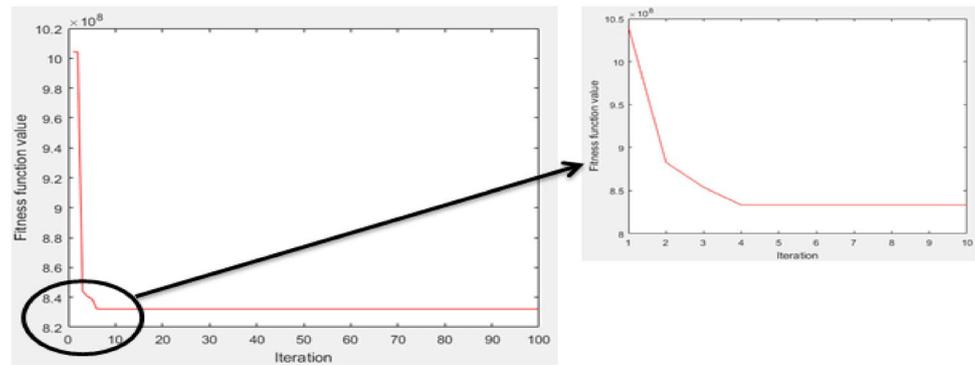
Fig. 14 Performance of GA

Table 5 Optimal design parameters settings and confirmation test result

Method	Inboard plate thickness, T_i (mm)	Outboard plate thickness, T_o (mm)	Vane height, H (mm)	Effective offset, E (mm)	Center hole radius, R (mm)	Fatigue life (cycle)
Parametric analysis	7	7	6	28	20	7.26e8
GA & PSO	4.9831	11	8	16	20	8.32e8

Fig. 15 Convergence of PSO

4 Conclusions

The present study concluded with the following remarks:

- Coupled thermal–mechanical finite element analysis effective tool to simulate the operating conditions of the brake disc, efficiently used in the present work. It examines the thermal as well as structural aspects for the design of ventilated brake disc.
- Present FEA model for ventilated brake disc can effectively use to simulate the exact braking conditions and the simulations significantly reduce the cost and time required for physical experimentation.
- High F-value and P-value less than 0.05 revealed that the present statistical and regression model is significant. Effective offset (E) is the most significant design parameter, followed by Inboard plate thickness (T_i), Outboard plate thickness (T_o), vane height (H), and hole radius (R). The regression model efficiently predicts the fatigue life for various combinations of design parameters.
- The 3D surface plot and contour plots are effective in estimating the optimal design parameters for maximum outcomes. Here the influences of different design parameters on the fatigue life of the disc have been studied. Also, the set of optimal design parameters were presented. Optimal design parameters of the ventilated brake disc obtained using parametric study are T_i of 7 mm, T_o of 7 mm, H of 6 mm, E of 28 mm, and R of 20 mm.
- GA & PSO were utilized to estimate the optimal design parameters of the ventilated brake disc, which are: T_i of 4.9831 mm, T_o of 11 mm, H of 8 mm, E of 16 mm, and R of 20 mm. They provided 12.74% enhancement in fatigue life compared with parametric analysis.
- The optimization results have been verified by the confirmation simulation test.

Declarations

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