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Recent advances in system reliability optimization driven by importance measures

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Abstract System reliability optimization problems have been widely discussed to maximize system reliability with resource constraints. Birnbaum importance is a well-known method for evaluating the effect of component reliability on system reliability. Many importance measures (IMs) are extended for binary, multistate, and continuous systems from different aspects based on the Birnbaum importance. Recently, these IMs have been applied in allocating limited resources to the component to maximize system performance. Therefore, the significance of Birnbaum importance is illustrated from the perspective of probability principle and gradient geometrical sense. Furthermore, the equations of various extended IMs are provided subsequently. The rules for simple optimization problems are summarized to enhance system reliability by using ranking or heuristic methods based on IMs. The importance-based optimization algorithms for complex or large-scale systems are generalized to obtain remarkable solutions by using IM-based local search or simplification methods. Furthermore, a general framework driven by IM is developed to solve optimization problems. Finally, some challenges in system reliability optimization that need to be solved in the future are presented.

Keywords importance measure, system performance,

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reliability optimization, optimization rules, optimization algorithms

1 Introduction

The concept of reliability emerged after World War I to compare the operation safety of key weapons; this concept has been applied to technical systems for more than a century (Rausand and Høyland, 2003). A detailed history of reliability theory from the 1950s to 1990s was presented by summarizing some critical reliability progresses (Knight, 1991). Reliability is an important discipline applied in the entire product lifecycle of design, manufacturing, operation, maintenance, life extension, and end-of-life. System reliability optimization problems (ROPs) are popular topics in reliability engineering and play an essential part in engineering projects, such as in nuclear power plants (Čepin, 2019), energy storage systems (Mohamad and Teh, 2018), automotive systems (Yu et al., 2018), and traction drive systems of rail transit (Lin et al., 2018).

Generally, system reliability could be improved by increasing component reliability, adding parallel redundancy of components, reassigning the positions of interchangeable components, or combining these methods (Kuo and Prasad, 2000). System ROPs can be classified into five types, namely, ROP, redundancy allocation problem (RAP), reliability-RAP (RRAP), component assignment problem (CAP), and complex problem (CP). ROP (Gopal et al., 1980; Kuo et al., 1987; Coit and Smith, 1996; Mettas, 2000; Prasad and Kuo, 2000) is a general and straightforward system ROP that aims to achieve optimal reliability allocation to maximize system reliability considering the cost constraints. RAP (Liang and Smith, 2004; Ramirez-Marquez and Coit, 2004; Onishi et al., 2007; Kulturel-Konak et al., 2003) aims to achieve the maximum system reliability by allocating the redundant components suitably with the cost constraints. RRAP (Chern, 1992; Tian et al., 2008; Yeh and Hsieh, 2011; Garg

and Sharma, 2013; Abouei Ardakan and Zeinal Hamadani, 2014) is a type of system ROP for obtaining the optimal solution with the highest system reliability by adjusting the component reliability and redundancy with consideration of the limitations (reliability, cost, weight, and volume). CAP (Derman et al., 1974; Zhu et al., 2011; Levitin et al., 2017b) aims to generate optimal assignment with maximum system reliability by assigning n available components into n positions. CP (Pant et al., 2015; Peng et al., 2016; Yu et al., 2017; Zhao et al., 2019a) is a type of difficult system ROP for complex systems with large-scale or complicated tasks to maximize system performance with limited resources.

Importance measures (IMs) are used to evaluate the effect of component reliability on system reliability (Birnbaum, 1969). IMs are useful tools in reliability engineering (Compare et al., 2017), risk analysis (Fang et al., 2017), and system reliability optimization (Fu et al., 2019a). These measures can help reliability engineers to find a better solution rapidly because they can identify the weakest links of the system, which are the premise and foundation of system design, maintenance, and resource configurations. During the system design period, component importance can help designers determine cost-effective design ideas with relatively high system reliability and low cost rapidly. The IMs can be applied in the design of communication systems (Liu et al., 2018), modern digital systems (Borgonovo et al., 2016), power industry (Espiritu et al., 2007), embedded systems (Aliee et al., 2016), and head-up display systems (Dui et al., 2017b). For the maintenance process, component importance can find the weakest components to improve its reliability, such as the integrated IM, considering the repair and failure rates (Si et al., 2012a; 2012b; 2013; Zhao et al., 2013; Dui et al., 2019). The IMs can also be found in wind turbine systems (Wu and Coolen, 2013; Dui et al., 2017c), electrical networks (Hilber and Bertling, 2004; Vu et al., 2016), control systems of computerized numerical control lathes (Xiahou et al., 2018), propeller plane systems (Dui et al., 2017a), and automobile tire systems (Fu et al., 2019b). For limited resources, IMs can determine the components that can generate the greatest system reliability improvement by arranging several resources reasonably to reduce system risk. The IMs are used for the bus test systems (Fang et al., 2016), critical infrastructure (Xu et al., 2020), navigation systems (Baroud and Barker, 2018), water distribution pipelines (He and Yuan, 2019), and resilient power systems (Wang et al., 2019).

IMs have been applied successfully in the complex engineering field to analyze reliability, safety, and risk in the past 60 years. The development of IMs in system reliability optimization can be classified into three stages.

(1) The first stage (before the 1990s) is an initial development of the system ROPs by applying the Birnbaum importance to solve these problems. The studies

focused on the system reliability optimization for small-scale systems with series or parallel structures, which established the mathematical model of optimization problems and proposed some optimization rules based on the Birnbaum importance to solve these problems (Tillman et al., 1977). The extensions of the Birnbaum importance are used to solve optimization problems because assigning resources to the component with the highest importance can improve the system's performance significantly. In this stage, some optimization rules are proposed to attain better solutions for optimization problems considering some IMs, such as the criticality importance (Barlow and Proschan, 1975), redundant importance (Boland et al., 1988), and Δ -importance (Xie and Shen, 1989).

(2) The second stage (from 1990 to 2011) is a further exploration of the system reliability optimization based on the IMs. The heuristic methods are used to generate optimal results based on efficient heuristic rules. The IM-based heuristic rules are proposed to solve system ROPs (Lin and Kuo, 2002; Yao et al., 2011; Zuo and Kuo, 1990). These heuristic rules clarify the mechanism of IMs in solving system ROPs. Moreover, this stage is the solid foundation of system reliability optimization.

(3) The third stage (after 2011) is an in-depth exploration of IMs for solving system ROPs. The book *Importance Measures in Reliability, Risk, and Optimization* provided a comprehensive contribution of IMs in reliability engineering (Kuo and Zhu, 2012). The IMs are also used to deal with some system ROPs related to the design, maintenance, and resource allocation. This book made IM a popular topic in reliability research. The mathematical models of system reliability optimization have remarkably changed, including the following tendency: (a) the diversity and multilevel of the objective functions are considered (Abouei Ardakan and Rezvan, 2018; Bretas et al., 2018), such as system availability and the remaining useful time of the system; (b) the states of components obtain multiple levels (Yeh and Chu, 2018; Jiang et al., 2019; Zaretalab et al., 2020); (c) the system structure has become complicated (Su et al., 2018; Xiang and Yang, 2018); and (d) the relationship among the components becomes complicated (Li et al., 2018; Mi et al., 2018). Many intelligent algorithms, such as artificial bee colony algorithm (Ghambari and Rahati, 2018), directional bat algorithm (Chakri et al., 2018), boundary swarm optimization method (Yeh, 2019), and butterfly optimization algorithm (Arora et al., 2018), are introduced to solve the system ROPs. For the system ROPs of CP, some IMs are developed to deal with mathematical models, such as IMs for reconfigurable systems (Si et al., 2014) and IMs for the consecutive k -out-of- n systems with sparse d (Shen et al., 2015; Shen and Cui, 2015).

The methods for solving all problems can be divided into two categories. The first category refers to IM-based optimization rules, which can obtain the results rapidly

based on IM analysis using ranking and heuristic methods. The former can obtain the optimization results directly by IM ranking, and the latter can obtain the results by using IM-based rules iteratively. The differences between these methods are presented as follows: The heuristic methods require many iterations, but the ranking methods only need one iteration. In the second category, the IM-based optimization algorithms are used to solve some CPs or large-scale problems based on the optimization rules. According to the purpose of the optimization rules, the IM-based optimization algorithms are divided into IM-based local search and simplification methods. IM-based local search methods are used in the sub-process of intelligent algorithms to improve the performance of the corresponding algorithms, while simplification methods can abridge the objectives or screen the critical factors of the optimization problems. Therefore, IM-based optimization rules are used to solve simple problems, whereas IM-based optimization algorithms are introduced to solve CPs.

The remainder of this work is organized as follows. Section 2 briefly illustrates the details of IMs, including their significance and extensions, based on Birnbaum importance. Section 3 summarizes the IM-based optimization rules for system ROPs. Section 4 investigates the IM-based optimization algorithms for system ROPs. Section 5 provides a general optimization framework driven by IMs for the system ROPs. Finally, Section 6 proposes the challenges in system reliability optimization in future research.

2 IMs for system ROPs

The Birnbaum importance of component i for binary systems was first proposed in 1969 to measure the impact of component reliability on the system reliability of binary state systems (Birnbaum, 1969). The Birnbaum importance of component i , which is denoted as $I^{\text{BM}}(i)$, can be evaluated by:

$$I^{\text{BM}}(i) = \frac{\partial R_\phi(\mathbf{P})}{\partial P_{i1}} = \Pr\{\Phi(\mathbf{X}) = 1 | X_i = 1\} - \Pr\{\Phi(\mathbf{X}) = 1 | X_i = 0\},$$

$$(i = 1, \dots, n), \quad (1)$$

where n is the number of components in the system; P_{i1} is the probability of component i at state 1, which is the reliability of component i in the binary state systems; $R_\phi(\mathbf{P})$ is the function of system reliability, in which $\mathbf{P} = (P_{11}, \dots, P_{i1}, \dots, P_{n1})$; X_i is the state of component i , where $X_i = 1$ indicates that component i is working, and $X_i = 0$ indicates that this component is failed in this binary state

system; and $\Phi(\mathbf{X})$ is the system structure-function, in which $\mathbf{X} = \{X_1, \dots, X_i, \dots, X_n\}$, where $\Phi(\mathbf{X}) = 1$ illustrates that the system is functioning, and $\Phi(\mathbf{X}) = 0$ indicates that the system is failed.

2.1 Significance of IMs

Birnbaum importance can be demonstrated from two points of view (e.g., probability principle and the gradient geometrical meaning) to understand its significance better.

2.1.1 Probability principle of Birnbaum importance

We use random experiments to introduce the significance of the component importance for the IMs. A random experiment E has n basic events $\omega = (\omega_1, \dots, \omega_i, \dots, \omega_n)$, and each basic event may or may not occur. The probability that event j will occur is represented by $\Pr(E_j) = f_j(\Pr(\omega_1), \Pr(\omega_2), \dots, \Pr(\omega_n))$ ($j = 1, \dots, 2^n$). The random variable $x(E_j)$ represents the price of the event E_j , in which $x(E_j) = 1$ when E_j occurs and $x(E_j) = 0$ if E_j does not occur. According to the set value $x(E_j)$, x represents the average price of the random experiment E that consists of a set event $\{E_1, E_2, \dots, E_{2^n}\}$. Then, x can be calculated by:

$$x = \sum_{j=1}^{2^n} x(E_j) \cdot \Pr(E_j) = \sum_{j=1}^{2^n} x(E_j) \cdot f_j(\Pr(\omega_1), \Pr(\omega_2), \dots, \Pr(\omega_n)). \quad (2)$$

The effect of the basic event ω_i on the random experiment E can be represented by $\frac{\partial x}{\partial \Pr(\omega_i)}$, which represents the price change of the random experiment E with the probability change in the basic event ω_i . Thus, $\frac{\partial x}{\partial \Pr(\omega_i)}$ can also be used to represent the probability principle of Birnbaum importance.

2.1.2 Gradient geometrical meaning of Birnbaum importance

The gradient is a vector that represents the fastest directions of function change at a known point. In a rectangular coordinate system, the gradient of the function $f(y_1, \dots, y_i, \dots, y_n)$ refers to the partial derivative of f with respect to y_i (Dui et al., 2013):

$$\nabla f = \mathbf{grad} f = \frac{\partial f}{\partial y_1} y_1 + \dots + \frac{\partial f}{\partial y_i} y_i + \dots + \frac{\partial f}{\partial y_n} y_n, \quad (3)$$

where y_i ($i = 1, 2, \dots, n$) is the orthogonal unit vector that points in the coordinate direction as y_i changes.

$\Pr\{\Phi(\mathbf{X}) = 1\} = f(P_{11}, P_{21}, \dots, P_{n1})$ can be considered a function of parameters $P_{11}, P_{21}, \dots, P_{n1}$ in Eq. (1). The

gradient of $f(P_{11}, P_{21}, \dots, P_{n1})$ is presented as follows (Dui et al., 2013):

$$\begin{aligned} \nabla f &= \frac{\partial \Pr\{\Phi(\mathbf{X}) = 1\}}{\partial P_{11}} P_{11} + \frac{\partial \Pr\{\Phi(\mathbf{X}) = 1\}}{\partial P_{21}} P_{21} + \dots \\ &+ \frac{\partial \Pr\{\Phi(\mathbf{X}) = 1\}}{\partial P_{n1}} P_{n1} \\ &= I^{\text{BM}}(1)P_{11} + I^{\text{BM}}(2)P_{21} + \dots + I^{\text{BM}}(n)P_{n1}, \end{aligned} \quad (4)$$

where gradient is the sum of the product of the Birnbaum importance and the corresponding component reliability. The gradient at one point $(P_{11}, P_{21}, \dots, P_{n1})$ shows the direction of improving the system reliability most rapidly, and the magnitude refers to the improvement of the system reliability in this direction.

$$I^{\text{S}^{\text{BM}}}(i) = \frac{\sum_{\mathbf{X}} (\Pr\{\Phi(\mathbf{X}) = 1 | X_i = 1\} - \Pr\{\Phi(\mathbf{X}) = 1 | X_i = 0\})}{2^n} \quad (5)$$

The criticality importance (Barlow and Proschan, 1975), considering system failure, refers to the ratio of the decrease in system reliability when component i fails to the probability of system failure. The criticality importance of component i , $I^{\text{C}}(i)$, can be calculated as follows:

$$\begin{aligned} I^{\text{C}}(i) &= \Pr\{\Phi(1_i, \mathbf{X}) - \Phi(0_i, \mathbf{X}) = 1, X_i = 0 | \Phi(\mathbf{X}) = 0\} \\ &= \frac{1 - P_{i1}}{1 - R(\mathbf{P})} \cdot I^{\text{BM}}(i), \end{aligned} \quad (6)$$

where 1_i and 0_i are the reliability vector of all components when the reliability of component i is one and zero, respectively.

The Birnbaum importance of component i for CAP $I^{\text{BM}}_{\text{CAP}}(i)$ is provided to determine the impact degree of the component on system reliability (Papastavridis, 1987). $I^{\text{BM}}_{\text{CAP}}(i)$ is the extension of Birnbaum importance in CAP, which has been derived from Eq. (1):

$$\begin{aligned} I^{\text{BM}}_{\text{CAP}}(i) &= \frac{\partial R(n)}{\partial P_{i1}} = R(n | X_i = 1) - R(n | X_i = 0) \\ &= \frac{R(i-1)R'(n-i) - R(n)}{1 - P_{i1}}, \end{aligned} \quad (7)$$

where $R(j)$ is the reliability of the consecutive- k -out-of- j : F subsystem in consecutive- k -out-of- n : F system, which has components from position 1 to position j ; and $R'(j)$ is the reliability of the consecutive- k -out-of- j : F subsystem, in which the components are from position $(n-j+1)$ to position n .

Δ -importance is proposed to represent the improvement of the system reliability when the reliability of component i

2.2 Extensions of IMs for system ROPs

The Birnbaum importance was extended into many different forms to solve various engineering problems. All the extensions can be classified into three categories according to the system states: IMs for (1) binary state systems, (2) multistate systems, and (3) continuous systems.

2.2.1 IMs for binary state systems

Birnbaum structure importance is a structure importance that can be calculated by Birnbaum importance when all the reliability of components is equal to 0.5. Birnbaum structure importance is an extension of Birnbaum importance when all the component reliability is the same. The expression of Birnbaum structure importance $I^{\text{S}^{\text{BM}}}$ can be obtained by:

changes from P_{i1} to P'_{i1} because the Birnbaum importance of component i is independent with the reliability of component i (Xie and Shen, 1989). The Δ -importance of component i , $I^{\Delta}(i)$, can be evaluated by:

$$\begin{aligned} I^{\Delta}(i) &= \Pr\{\Phi(\mathbf{X}) = 1 | P'_{i1}\} - \Pr\{\Phi(\mathbf{X}) = 1\} \\ &= (P'_{i1} - P_{i1}) \cdot I^{\text{BM}}(i). \end{aligned} \quad (8)$$

The parallel redundant importance of component i , $I^{\text{PR}}(i)$, refers to the increase in system reliability by adding the parallel redundancy of component i (Shen and Xie, 1990), which can be calculated by:

$$\begin{aligned} I^{\text{PR}}(i) &= R(P_{11}, \dots, P_{i1} + (1 - P_{i1})P_{i1}^*, \dots, P_{n1}) \\ &- R(P_{11}, \dots, P_{i1}, \dots, P_{n1}) \\ &= P_{i1}^* (1 - P_{i1}) I^{\text{BM}}(i), \end{aligned} \quad (9)$$

where P_{i1}^* is the reliability of the redundant component for component i .

For binary systems, the potential improvement importance of component i , $I^{\text{IP}}(i)$, is the improvement of system reliability when component i is working, which can be calculated by (Aven and Jensen, 2000):

$$\begin{aligned} I^{\text{IP}}(i) &= \Pr\{\Phi(\mathbf{X}) = 1 | X_i = 1\} - \Pr\{\Phi(\mathbf{X}) = 1\} \\ &= (1 - P_{i1}) \cdot I^{\text{BM}}(i), \end{aligned} \quad (10)$$

where the maximum potential improvement is the product of the maximum reliability improvement and the Birnbaum importance of this component.

The risk achievement worth of component i , $I^{\text{RAW}}(i, t)$, is the ratio of the risk when component i is at the failed state to the nominal value of the risk (Zio and Podofillini, 2003a), indicating the relative increase in the risk because of the failure of component i , which can be calculated by:

$$I^{\text{RAW}}(i, t) = \frac{F_i^+(t)}{F(t)}, \tag{11}$$

where $F(t)$ is the nominal value of the risk at time t , and $F_i^+(t)$ is the risk when component i fails.

The risk reduction worth of component i , $I^{\text{RRW}}(i, t)$, is the ratio of the nominal value of the risk to the risk when component i is at the working state (Zio and Podofillini, 2003a) to evaluate the potential of component i in reducing the risk, which can be calculated by:

$$I^{\text{RRW}}(i, t) = \frac{F(t)}{F_i^-(t)}, \tag{12}$$

where $F_i^-(t)$ is the risk when component i is working.

Considering the lifetime distribution of components, the Birnbaum importance can be represented by the conditional probability that difference exists between the conditional probability that system lifetime is larger than t when the component lifetime is larger than t and when component lifetime is not larger than t (da Costa Bueno, 2005). The Birnbaum importance of component i that considers lifetime at time t , $I_T^{\text{BM}}(i, t)$, is calculated by:

$$I_T^{\text{BM}}(i, t) = \Pr(\tau > t | S_i > t) - \Pr(\tau > t | S_i \leq t), \tag{13}$$

where τ is the lifetime of the system, and S_i is the lifetime of component i .

The Bayesian importance of component i , $I^{\text{Bay}}(i)$, is the conditional probability that component i fails with the system (Singpurwalla, 2006):

$$I^{\text{Bay}}(i) = \Pr\{X_i = 0 | \Phi(\mathbf{X}) = 0\}. \tag{14}$$

When a system fails, identifying which components have caused the failure, as well as the influence degree of components on the system failure, is important.

The availability importance of component i is the effect of the availability of component i on the availability of the entire system (Barabady and Kumar, 2007). Availability is a function related to the failure or repair rate. Therefore, the failure rate-based availability importance of component i , $I_\lambda^{\text{A}}(i)$, is given by:

$$I_\lambda^{\text{A}}(i) = \frac{\partial A(n)}{\partial A_i} \frac{\partial A_i}{\partial \lambda_i}, \tag{15}$$

where $A(n)$ is the system availability, A_i is the availability of component i , and λ_i is the failure rate of component i .

Similarly, the repair rate-based availability importance of component i , $I_\mu^{\text{A}}(i)$, is given by:

$$I_\mu^{\text{A}}(i) = \frac{\partial A(n)}{\partial A_i} \frac{\partial A_i}{\partial \mu_i}, \tag{16}$$

where μ_i is the repair rate of component i .

The cost-effective IM $I^{\text{CEIM}}(i)$ is proposed by combining the Birnbaum importance with the total cost of failure (Gupta et al., 2013):

$$I^{\text{CEIM}}(i) = \frac{\Delta g_i(P'_{i1})}{g(\mathbf{P})} / \frac{\sum_{i=1}^n E(C_i)}{E(C_i)}, \tag{17}$$

where $\Delta g_i(P'_{i1})$ is the change in the probability of system failure when the component reliability of component i changes from P_{i1} to P'_{i1} , $g(\mathbf{P})$ is the probability of system failure without the improvement of component reliability, and $E(C_i)$ is the expected cost when component i fails.

For reconfigurable systems, such as consecutive k -out-of- n systems, the approximation of the reconfigurable importance for component i , $I^{\text{opt}}(i)$, can be calculated by (Si et al., 2014; Dui et al., 2018):

$$I^{\text{opt}}(i) = \lim_{\Delta \rightarrow 0} \frac{R^{\text{opt}}(P_{i1} + \Delta, \mathbf{P}) - R^{\text{opt}}(P_{i1}, \mathbf{P})}{\Delta}, \tag{18}$$

where Δ is the change in component reliability; and $R^{\text{opt}}(P_{i1}, \mathbf{P})$ and $R^{\text{opt}}(P_{i1} + \Delta, \mathbf{P})$ are system reliabilities of the optimal configurations before and after improving the reliability of component i by Δ , respectively, $\mathbf{P} = (P_{11}, \dots, P_{i1}, \dots, P_{n1})$.

The Birnbaum importance of component i in mission k , $I_k^{\text{PM}}(i, t)$, is an extension of Birnbaum importance for the phased-mission systems (Li et al., 2015):

$$I_k^{\text{PM}}(i, t) = \frac{\partial R_k(t)}{\partial R_{k,i}(t)}, \tag{19}$$

where $R_{k,i}(t)$ is the component reliability of component i in mission k at time t ; and $R_k(t)$ is the mission reliability for mission k at time t , represented by the reliabilities of components when $R_k(t) = f(R_{k,1}(t), \dots, R_{k,i}(t), \dots, R_{k,m_k}(t))$, in which m_k represents the number of components in the mission k .

The component maintenance priority $I_{ji}^{\text{M}}(t)$ is proposed to evaluate the importance of component j once component i fails (Wu et al., 2016):

$$I_{ji}^{\text{M}}(t) = H_{ji} \frac{\partial R_\phi(\lambda_i, P_{i1}(t))}{\partial P_{j1}(t)}, \tag{20}$$

where

$$H_{ji} = \begin{cases} 1 & \text{if } \Phi(1_1, \dots, 1_{i-1}, 0_i, 1_{i+1}, \dots, 1_n) = 0 \\ \Phi(0_i, 0_j, 1_{ij}) & \text{if } \Phi(1_1, \dots, 1_{i-1}, 0_i, 1_{i+1}, \dots, 1_n) = 1 \end{cases},$$

$(0_i, 0_j, 1_{ij})$ represents the states of the system when components i and j fail but other components are working;

$\lambda_i = \chi(\Phi(1_1, \dots, 1_{i-1}, 0_i, 1_{i+1}, \dots, 1_n) = 0)$, where $\chi(\cdot)$ is an indicator function; and $P_{il}(t)$ represents the reliability of component i at time t in the binary system.

The cost-based parallel redundancy importance $I_C^{PR}(i)$ is introduced by combining the cost function with Birnbaum importance (Wu and Wu, 2017):

$$I_C^{PR}(i) = I^{BM}(i) \cdot \max(C(r_i, n_i)) / C(r_i, n_i), \quad (21)$$

where $C(r_i, n_i)$ is the cost of adding redundancy for component i , r_i is the reliability of component i and its redundant components, and n_i is the redundant number of component i .

The global component importance of component i , $I^{GC}(i)$, depends not only on the tasks of each phase but also on the components' nominal mean time between failure (MTBF) and their uncertain levels (Wu et al., 2018):

$$I^{GC}(i) = \frac{E(D(R(\mathbf{M}|x_{\sim i})))}{D(R(\mathbf{M}))} = \frac{D(R(\mathbf{M})) - D(E(R(\mathbf{M}|x_{\sim i})))}{D(R(\mathbf{M}))}, \quad (22)$$

where \mathbf{M} is the nominal MTBF vector of all components; $\sim i$ represents nearly all components, except for component i ; $E(D(R(\mathbf{M}|x_{\sim i})))$ is the total effects caused by component i and its interaction effect with other components; $D(E(R(\mathbf{M}|x_{\sim i})))$ is the average decrement in the variance of the mission success probability without considering component i , depending on all the components themselves and the interaction among them except for component i ; and $D(R(\mathbf{M}))$ is the variance of mission success probability.

The interval-valued Birnbaum importance of component i , $I_{[\cdot]}^{BM}(i)$ is similar to that of Birnbaum importance, which uses the interval-valued reliability to calculate the Birnbaum importance (Qiu et al., 2018):

$$I_{[\cdot]}^{BM}(i) = [R(n|X_i = 1)] - [R(n|X_i = 0)] = [\underline{R}(n|X_i = 1) - \bar{R}(n|X_i = 0), \bar{R}(n|X_i = 1) - \underline{R}(n|X_i = 0)], \quad (23)$$

where $[R(n|X_i = 1)]$ is the interval-valued system reliability when component i is working; $[R(n|X_i = 0)]$ is the interval-valued system reliability when component i fails; $\underline{R}(\cdot)$ is the lower boundary of system reliability; and $\bar{R}(\cdot)$ is the upper boundary of system reliability.

The importance $I^{c-IM}(i, t)$ based on the cut set, including edge i , is developed for the network with n edges when the edge failures follow a counting process $\{N(t), t \geq 0\}$ (Du et al., 2019). $I^{c-IM}(i, t)$ is the conditional probability that a cut, including edge i , fails when the network fails, which is shown by:

$$I^{c-IM}(i, t) = \frac{\Pr\{\text{a cut } C \in C_i \text{ is down at } t\}}{\Pr\{\text{the network is down at } t\}} = \frac{\sum_{k=0}^n \Pr(N(t) = k)F(k, 0_i)}{\sum_{k=0}^n \Pr(N(t) = k)F(k)}, \quad (24)$$

where $F(k)$ is the destruction spectrum (D-spectrum) of the network, which is the probability that the network fails when exactly k randomly selected edges fail; $F(k, 0_i)$ is the D-spectrum of edge i , which is equal to the probability that the network fails when randomly selected k edges fail, including edge i ; and $\Pr(N(t) = k)$ is the probability that k edges fail at time t .

The importance based on the path set of edge i , $I^{p-IM}(i, t)$, is the conditional probability that a path including edge i works when the network works at time t , which is shown by:

$$I^{p-IM}(i, t) = \frac{\Pr\{\text{a path } P \in P_i \text{ is up at } t\}}{\Pr\{\text{the network is up at } t\}} = \frac{\sum_{k=0}^n \Pr(N(t) = k)F'(n-k, 1_i)}{\sum_{k=0}^n \Pr(N(t) = k)F'(n-k)}, \quad (25)$$

where $F'(k)$ is the probability that the network works if randomly selected k edges work; $F'(n-k, 1_i)$ (or $F'(n-k, 0_i)$) is the probability that the network works (fails) if randomly selected $n-k$ edges, including edge i , work (fail); and $F'(n-k)$ is equal to $F'(n-k, 0_i) + F'(n-k, 1_i)$.

The component reliability boundary, manufacturing difficulty, and feasibility are considered for improving the component reliability. A generalized Birnbaum IM (GBIM) $I^{GB}(i)$ is proposed by considering the cost-reliability relation function to evaluate the contribution of individual components to improve the system reliability (Si et al., 2019):

$$I^{GB}(i) = \frac{\partial R_s}{\partial c_i} = \frac{\partial R_s}{\partial P_{il}} \frac{\partial P_{il}}{\partial c_i}, \quad (26)$$

where R_s is the system reliability; and c_i is the cost of the component related to component reliability range, manufacturing complexity, and technology feasibility.

2.2.2 IMs for multistate systems

For the multistate systems, the concept of system performance is defined based on the expected utility U (Griffith, 1980), which can be calculated by:

$$U = \sum_{j=1}^M a_j \Pr(\Phi(\mathbf{X}) = j), \quad (27)$$

where $0 = a_0 \leq a_1 \leq \dots \leq a_M$ represents the performance levels of the system at different states, which corresponds

to the system state vector $\{0, 1, \dots, M\}$; $a_0 = 0$ is the state of the system with the lowest performance level.

The Griffith importance $I_m^G(i)$ is the generalization of Birnbaum importance for evaluating the impact of component i on the system performance (Griffith, 1980):

$$I_m^G(i) = \sum_{j=1}^M (a_j - a_{j-1}) \times [\Pr(\Phi(m_i, \mathbf{X}) \geq j) - \Pr(\Phi((m-1)_i, \mathbf{X}) \geq j)], \quad (28)$$

where $\Phi(m_i, \mathbf{X})$ is the system state when the component i is at state m . $I_m^G(i)$ evaluates the importance of component i at state m in multistate systems; it indicates the change in the system performance when the component i degrades from state m to state $m-1$.

The Birnbaum sensitivity measure for the generalized phased-mission systems $I^{BS}(i)$ is proposed to consider the effect of the failure of component i on the unreliability of the system (Xing and Dugan, 2002):

$$I^{BS}(i) = \frac{\partial Q(\mathbf{q})}{\partial q_i} = \frac{\partial(Q \cdot P_u + \bar{P}_u)}{\partial q_i} = -\bar{Q} \cdot \frac{\partial(P_u)}{\partial q_i} + P_u \frac{\partial(Q)}{\partial q_i}, \quad (29)$$

where q_i is the probability when component i fails; $Q(\mathbf{q})$ is the failure function of the corresponding perfect coverage model system, in which $\mathbf{q} = \{q_1, \dots, q_i, \dots, q_n\}$; Q is the probability that the system fails, and $\bar{Q} = 1 - Q$; and P_u is the probability that no single-point failure occurs, and $\bar{P}_u = 1 - P_u$.

The binary Birnbaum importance is generalized to the multistate, where the system or component works (or fails) when $\Phi(\mathbf{X}) \geq k_0$ (or $\Phi(\mathbf{X}) < k_0$) (Zio and Podofillini, 2003b), where k_0 is the required performance level. The multistate Birnbaum importance of component i in this system $I_{k_0}^{BM}(i)$ is shown by:

$$I_{k_0}^{BM}(i) = \Pr\{\Phi(\mathbf{X}) \geq k_0 | X_i \geq k_0\} - \Pr\{\Phi(\mathbf{X}) \geq k_0 | X_i < k_0\}. \quad (30)$$

Griffith importance only measures the effect of a particular component on the system performance. However, it does not determine the ranks of the components or states based on the important levels. Therefore, Wu importance of component i at state m , $I_m^{Wu}(i)$, is proposed to overcome the deficiency of Griffith importance by considering the performance utility (Wu and Chan, 2003):

$$I_m^{Wu}(i) = P_{im} \sum_{j=0}^M a_j \Pr(\Phi(m_i, \mathbf{X}) = j), \quad (31)$$

where $I_m^{Wu}(i)$ illustrates the contribution of component i at state m to the system performance U , which is the sum of $I_m^{Wu}(i)$ ($m = 0, 1, \dots, M_i$), given that $U = \sum_{m=0}^{M_i} I_m^{Wu}(i)$.

The multistate redundancy importance of component i , $I^{MRI}(i)$, is an IM based on the improvement of the system reliability (Ramirez-Marquez et al., 2006):

$$I^{MRI}(i) = \Pr\{\Phi(x_i^+, \mathbf{X}) \geq d\} - \Pr\{\Phi(\mathbf{X}) \geq d\}, \quad (32)$$

where d represents the constant demand of system; and x_i^+ represents the addition of the same components for component i during the system design. $I^{MRI}(i)$ is an estimation of the profit earned by adding redundancy or leveling up the state.

The risk reduction worth of component i in the multistate system is extended by considering the required component condition α (Zio et al., 2007), called the generalized performance achievement worth measure, $I_\alpha^{PAW}(i)$, which is shown by:

$$I_\alpha^{PAW}(i) = \frac{L}{L_i^{>\alpha}}, \quad (33)$$

where L is the nominal loss of the system; α is the determined and known required component condition, and the system operates better when the component condition is more than α ; and $L_i^{>\alpha}$ is the expected loss of the entire system when the condition of component i remains above level α .

The mean absolute deviation (MAD) importance $I^{MAD}(i)$ is proposed to measure the expected absolute deviation of reliability for the multistate systems with multistate components caused by the various performance levels and corresponding probabilities of a specified component (Ramirez-Marquez and Coit, 2007), which emphasizes the probabilities of the operation state of the system and the failure state of a component:

$$I^{MAD}(i) = \sum_l \Pr(l_i) |\Pr(\Phi(l_i, \mathbf{X}) \geq d) - \Pr(\Phi(\mathbf{X}) \geq d)|, \quad (34)$$

where l_i represents the state of component i .

Griffith importance studies the situation that a component degrades from the state m to the state $m-1$. However, it does not consider the case that a component degrades from the state m to any other lower states $\{m-1, m-2, \dots, 0\}$. For that reason, the integrated IM of component states $I_{m,l}^{IIM}(i)$ is introduced to extend the Griffith importance by considering the transition rates among different component states (Si et al., 2012c):

$$\begin{aligned} I_{m,l}^{IIM}(i) &= P_{im} \cdot \lambda_{m,l}^i \sum_{j=1}^M (a_j - a_{j-1}) \\ &\quad \times [\Pr(\Phi(m_i, \mathbf{X}) \geq j) - \Pr(\Phi(l_i, \mathbf{X}) \geq j)] \\ &= P_{im} \cdot \lambda_{m,l}^i \sum_{j=1}^M a_j \\ &\quad \times [\Pr(\Phi(m_i, \mathbf{X}) = j) - \Pr(\Phi(l_i, \mathbf{X}) = j)], \quad (m > l), \quad (35) \end{aligned}$$

where $\lambda_{m,l}^i$ is the failure rate of component i transferring from state m to state l . $I_{m,l}^{iIM}(i)$ is the expected change in the system performance when component i degrades from state m to state l .

The multistate component performance importance for component i at state j , $I_j^{MCP}(i)$, can be calculated by (Roychowdhury and Bhattacharya, 2019):

$$I_j^{MCP}(i) = \frac{\partial h(\mathbf{P})}{\partial P_{ij}} = \frac{\partial h(P_1, \dots, P_i, \dots, P_n)}{\partial P_{ij}}, \quad (36)$$

where $h(\mathbf{P})$ is the system performance function; P_i is the probability set of all states for component i ; and P_{ij} represents the probability of component i at state j .

2.2.3 IMs for continuous systems

The continuum structure system is introduced by considering that the system or component states are any value in a segment $[0, 1]$ (Baxter, 1984; 1986). The structure function $\Phi : [0, 1]^n \rightarrow [0, 1]$ of a continuous system is non-decreasing in each argument, satisfying $\Phi(0_1, 0_2, \dots, 0_n) = 0$ and $\Phi(1_1, 1_2, \dots, 1_n) = 1$.

For continuous systems, $[0, \beta]$ represents the failure states of the system and $[\beta, 1]$ represents the working states. The Birnbaum importance of component i at a level $\beta \in (0, 1]$, $I_\beta^{CS}(i, t)$, is evaluated by (Kim and Baxter, 1987):

$$I_\beta^{CS}(i, t) = \Pr(\Phi(\mathbf{X}(t)) \geq \beta | X_i(t) \geq \delta_i^\beta) - \Pr(\Phi(\mathbf{X}(t)) \geq \beta | X_i(t) < \delta_i^\beta), \quad (37)$$

where δ_i^β denotes the key element of component i , and $0 < \delta_i^\beta < 1$ for all $\beta \in (0, 1]$.

The lower variance of the importance for all components can obtain a balance system, which can eliminate bottlenecks or overly reliable components (Zio and Podofillini, 2007). The balanced IM I^B can be calculated by:

$$I^B = \frac{1}{\sqrt{\bar{I}^2 - \bar{I}^2}},$$

$$\bar{I}^2 = \frac{1}{n} \sum_{i=1}^n I_i^2, \quad \bar{I} = \frac{1}{n} \sum_{i=1}^n I_i, \quad (38)$$

where I_i is the importance of component i . I^B can be used in binary, multistate, or continuous systems.

The maintenance-based IM for the continuous system is the system performance improvement by maintaining the component i for the unit maintenance time (Cai et al., 2018). Therefore, the maintenance-based importance of component i , $I^U(i, t)$, can be expressed as follows:

$$I^U(i, t) = \frac{d\Delta U_i(t)}{dt} = \int_{m_i=0}^{\infty} p(m_i) \int_{l_i > m_i}^{\infty} \mu_{m_i, l_i}(t) \int_{u=0}^{\infty} a(u) [p_{l_i}(u) - p_{m_i}(u)] du dl_i dm_i, \quad (39)$$

where $\Delta U_i(t)$ is the performance improvement of system when considering time t to maintain the component i ; $p(m_i)$ represents the probability when the component i is at state m ; $\mu_{m_i, l_i}(t)$ is the density function when the state of component i transits from m to l at maintenance time t ; $a(u)$ is the utility function of the system when its state is at u ; and $p_{l_i}(u)$ and $p_{m_i}(u)$ represent the probability of the system at state u when component i is at state l_i and m_i , respectively.

The extended importance indexes have a relatively close relationship with the Birnbaum importance. Most IMs for binary systems can be derived by the Birnbaum importance directly, such as the $I_S^{BM}(i)$, $I^C(i)$, $I_{CAB}^{BM}(i)$, $I^\Delta(i)$, $I^{PR}(i)$, $I^{IP}(i)$, $I^{opt}(i)$, $I_k^{PM}(i, t)$, $I_{ji}^M(t)$, and $I_{[.]}^{BM}(i)$. Some cost-related IMs can also be derived by Birnbaum importance by introducing the change in system performance for unit cost, such as $I_\lambda^A(i)$, $I_\mu^A(i)$, $I^{CEIM}(i)$, $I_C^{PR}(i)$, $I^{GC}(i)$, and $I^{GB}(i)$. Birnbaum importance considers the relationships between the system performance when component i is perfect, the system performance when component i fails, and the current system performance. However, some importance indexes consider these factors to analyze the importance level of component i , such as $I^{RAW}(i, t)$, $I^{RRW}(i, t)$, $I_T^{BM}(i, t)$, $I^{Bay}(i)$, $I^{c-IM}(i, t)$, and $I^{p-IM}(i, t)$. Griffith importance has extended the Birnbaum importance into multistate systems considering the system performance. Almost all IMs for multistate systems and continuous systems are derived by the Griffith importance.

The development of IMs extended by the Birnbaum importance can be summarized in Fig. 1. The selected IMs are used to optimize system ROPs and can be divided into three categories based on the system states as binary systems, multistate systems, and continuous systems.

3 IM-based optimization rules for system reliability optimization

Some scholars have proposed the IM-based optimization rules to solve the system ROPs, in which its validity is proven mathematically or through numerical experiment results. Optimization rules can be classified into two categories, namely, optimization rules by IM-based ranking and heuristic methods. The references for system reliability optimization can be analyzed in Table 1 by considering the problems, system states, IMs, and optimization rules.

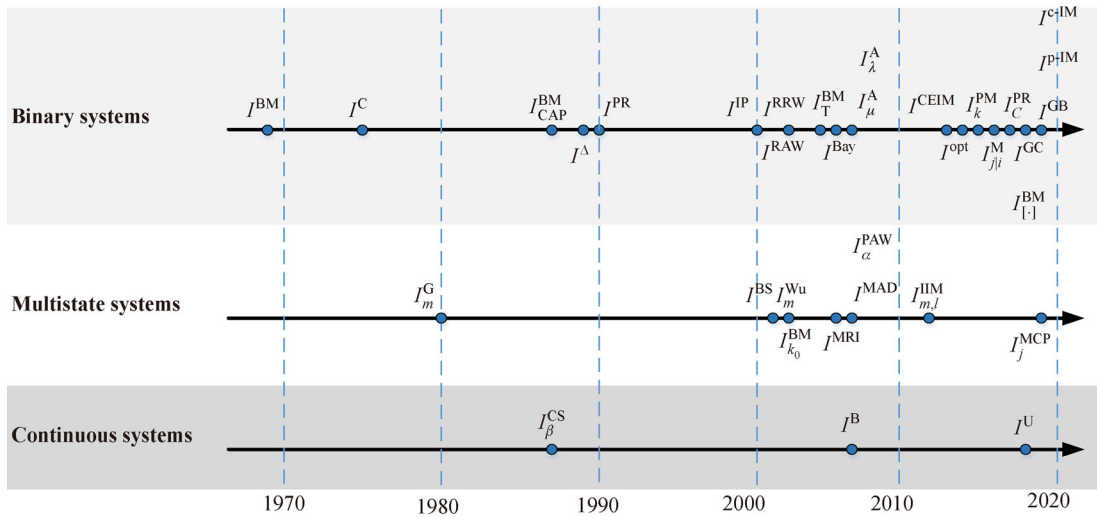


Fig. 1 Extensions and classifications of Birbaum importance for system reliability optimization.

Table 1 Reference analysis of optimization rules based on IMs

| References | Problems | Systems | IMs | Rules |
|--------------------------------------|----------|------------|----------------------------|-----------|
| Barabady and Kumar (2007) | ROP | Binary | I_λ^A or I_μ^A | Ranking |
| Zio et al. (2007) | ROP | Multistate | I_α^{PAW} | Ranking |
| Gupta et al. (2013) | ROP | Binary | I^{CEIM} | Ranking |
| Wu et al. (2016) | ROP | Binary | I_{ji}^M | Ranking |
| Roychowdhury and Bhattacharya (2019) | ROP | Multistate | I_j^{MCP} | Ranking |
| Boland et al. (1988) | RAP | Binary | I^{PR} | Ranking |
| Shen and Xie (1990) | RAP | Binary | I^{PR} | Ranking |
| da Costa Bueno (2005) | RAP | Binary | I_T^{BM} | Ranking |
| Ramirez-Marquez and Coit (2007) | RAP | Multistate | I^{MAD} | Ranking |
| Bhattacharya and Roychowdhury (2014) | RAP | Binary | I_S^{BM} or I^{BM} | Ranking |
| Bhattacharya and Roychowdhury (2016) | RAP | Binary | I^{Bay} | Ranking |
| Zuo and Kuo (1990) | CAP | Binary | I_{CAP}^{BM} | Heuristic |
| Lin and Kuo (2002) | CAP | Binary | I_{CAP}^{BM} | Heuristic |
| Yao et al. (2011) | CAP | Binary | I_{CAP}^{BM} | Heuristic |
| Zhu et al. (2017) | CAP | Binary | I_{CAP}^{BM} | Heuristic |
| Qiu et al. (2018) | CAP | Binary | $I_{[-]}^{BM}$ | Heuristic |

3.1 Optimization rules for ROP

The availability allocation problems are solved to determine the optimal solution related to reliability and maintainability; the constraints of these problems are the cost for maximizing system availability. For repairable systems, the availability IMs (I_λ^A and I_μ^A) of component i based on failure and repair rates are the partial derivative of system availability with respect to the component avail-

ability and failure and repair rates (Barabady and Kumar, 2007). The strategy indicates that the component with the largest available IM can have the greatest effect on system availability. The research indicates that the proposed strategy can be applied in the availability allocation problems based on the ranking of IM, wherein performance is demonstrated through numerical examples.

The multistate railway network is modeled, in which each rail section can remain at different states as a component, and the speed depends on the degradation

tracks and traffic conditions (Zio et al., 2007). The generalized performance achievement worth can be represented by the delay decrement when the traveling speed in the rail section is higher than the determined level. The results of different cases indicate that relaxing the speed requirements on the sections with high importance can result in a large decrement of the overall delay. Therefore, the order of relaxing sections should be determined by the ranking of I_{α}^{PAW} .

Maintenance cost plays a critical role in improving system performance, and the IM that combines reliability with cost is useful in determining which components can generate the highest improvement in the system performance using the same cost constraints. I^{CEIM} can be used to rank the primary events based on the cost-effective approach for inspection, replacement of components, and maintenance in the engineering systems (Gupta et al., 2013). If the system performance needs to be improved with limited budget, then the component with high I^{CEIM} must be prioritized.

Performing preventive maintenance (PM) frequently may require a great deal of time and can reduce system availability because of the increase in downtime. Therefore, engineers perform PM on several components simultaneously. A component maintenance priority I_{ji}^{M} is a typical IM for selecting components in PM (Wu et al., 2016). The maintenance policy is introduced by selecting some components with higher I_{ji}^{M} , and the proposed policy

can determine the optimal number of components to minimize the expected cost with a given time based on the ranking of I_{ji}^{M} .

A multistate component performance importance is presented to evaluate the effect of the probability of a component at one state on the system performance (Roychowdhury and Bhattacharya, 2019). I_j^{MCP} is proven useful for improving system performance. The multistate component performance importance can maximize the system performance at the design stage by improving the component performance with the largest importance repeatedly. The proposed method is based on the ranking of I_j^{MCP} , and its effectiveness is demonstrated by numerical experiments.

By analyzing the references of the optimization rules for ROP, most of the optimization rules are based on importance ranking, and heuristic-based optimization rules are seldom used to solve the ROP. As such, the procedures of ranking-based optimization rules for ROP are summarized in Fig. 2. For ROP, the proper IM should be selected first by analyzing the optimization model. Second, the IM is calculated, and the components are ranked in decreasing order of IM values. Third, the reliability improvement of the component with the largest importance is determined. Then, the remaining resource is checked. If no resource remains, the final solution is outputted; otherwise, the second step is repeated until all resources are exhausted.

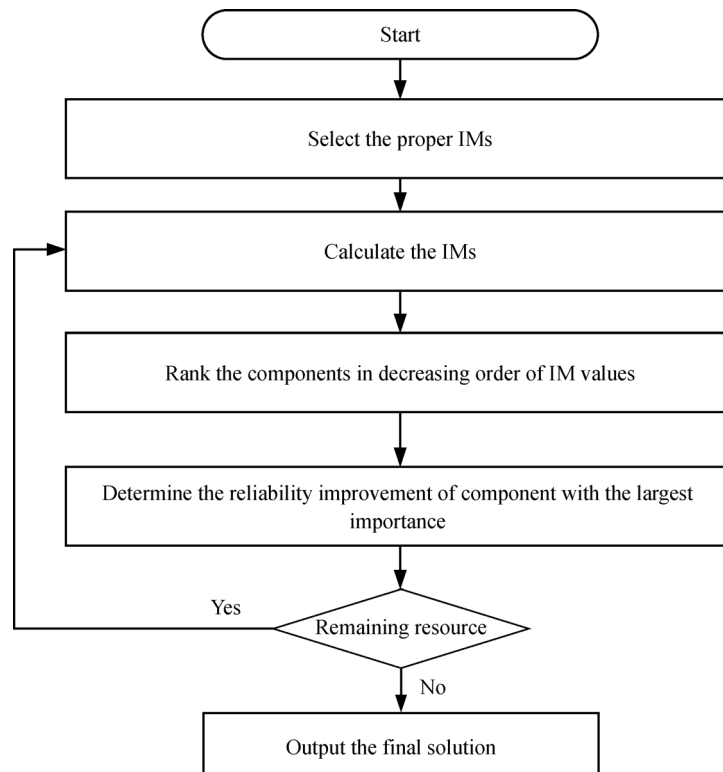


Fig. 2 Procedures of ranking-based optimization rules for ROP.

3.2 Optimization rules for RAP

The redundancy importance that considers active redundancy is introduced into coherent systems to evaluate the changes in system reliability by adding one redundant component (Boland et al., 1988). The parallel redundancy importance I^{PR} is used to identify the priority of components in improving system reliability. The redundant component should be assigned to the component with the largest I^{PR} . The proposed rule is based on the rank of component reliability. Its effectiveness is verified by mathematical derivation and numerical experiments for the series-parallel systems and parallel-series systems.

To obtain the largest improvement of system reliability, parallel redundancy should be added to the components that generate the largest improvement in terms of system reliability, especially when the cost, space, and weight are limited. When the cost of each redundant component is equal, the component, which can generate the largest system reliability increment, results in parallel redundancy (Shen and Xie, 1990). Ranking the components based on I^{PR} to identify the most profitable component is a reasonable approach.

The system lifetime optimization problem allocates a redundant component for maximizing the system lifetime. The problem in allocating a spare part into a k -out-of- n :F system with dependent components is investigated because adding the redundancy to different components can result in additional lifetimes (da Costa Bueno, 2005). The main contribution of this work is that, for a series (parallel) system, performing a minimal standby (active) redundancy operation and allocating it to the weakest component with the largest I_T^{BM} is stochastically recommended.

For coherent systems, the optimal solution with maximum reliability improvement is generated by allocating a redundant component to the appropriate components based on the IMs (Bhattacharya and Roychowdhury, 2014). The proposed optimal method of iteratively adding one redundant component is extended to solve the problem with unknown information about the components' reliabilities. For situation 1, in which we do not know the component reliabilities, the redundant component with the same reliability should be assigned to the component with the highest Birnbaum structure importance I_S^{BM} . For situation 2, in which the reliabilities of all components are the same, the redundant component should be allocated to the component with the highest Birnbaum importance I^{BM} . For situation 3, supposing that all component reliabilities are different, the redundant component should be added to component with the largest $(1 - P_{i1})I^{BM}(i)$. For situation 4, in which the reliabilities of redundant components may be not the same but the reliabilities of components are known, a redundant component should be added to the component with the highest $P'_{i1}(1 - P_{i1})I^{BM}(i)$.

Bayesian importance is introduced to assign redundant components to maximize system reliability (Bhattacharya and Roychowdhury, 2016). Some rules are applied to optimize RAP, as follows: (1) For series systems, system reliability can be maximized by allocating redundancy to the component with the highest I^{Bay} ; (2) For any coherent systems with non-overlapping subsystems, in which components' reliabilities in the same subsystem are the same, adding redundancy to the subsystem with high I^{Bay} can improve system reliability.

By analyzing the optimization rules that are based on the ranking of importance, the procedures of ranking-based optimization rules for RAP can be summarized in Fig. 3. For RAP, the proper IM should be selected first by analyzing the optimization model. Second, we calculate and rank the IM in decreasing order according to the importance of components. Third, we determine the redundancy allocation of the component with the largest importance. Then, we check the remaining resource. If no resources remain, the final solution is outputted; otherwise, we return to the second step until all resources are exhausted.

Some heuristic-based optimization rules are used to maximize system performance. A heuristic approach based on MAD is developed to maximize the reliability of multistate systems with multistate components (Ramirez-Marquez and Coit, 2007). The RAP for multistate systems with multistate components is to the consideration of the allocation of redundant components with a limited budget to maximize system reliability with the given demand. The procedures of MAD-based heuristics are summarized in Table 2. The results of numerical experiments with different system complexities illustrate that the MAD-based heuristic can solve this type of optimization problem effectively by using the optimization rules based on heuristic methods.

3.3 Optimization rules for CAP

The optimal invariant design of consecutive k -out-of- n systems aims to determine the optimal assignment once the ranking of components' reliabilities is known. On the one hand, the optimal invariant design can identify the optimal invariant assignment of some typical consecutive- k -out-of- n systems; on the other hand, the non-existence of optimal invariant assignment in other consecutive k -out-of- n systems should also be proven (Zuo and Kuo, 1990). Systems without optimal invariant assignments can generate at least suboptimal designs based on heuristic methods, and the key idea is to assign the component with high reliability to the position with large IM. For CAP in linear consecutive- k -out-of- n systems, the reliability pattern should match the IM pattern. The initial assignments of the two heuristics should be generated randomly. Heuristic 1 (ZKA) assigns the components with low

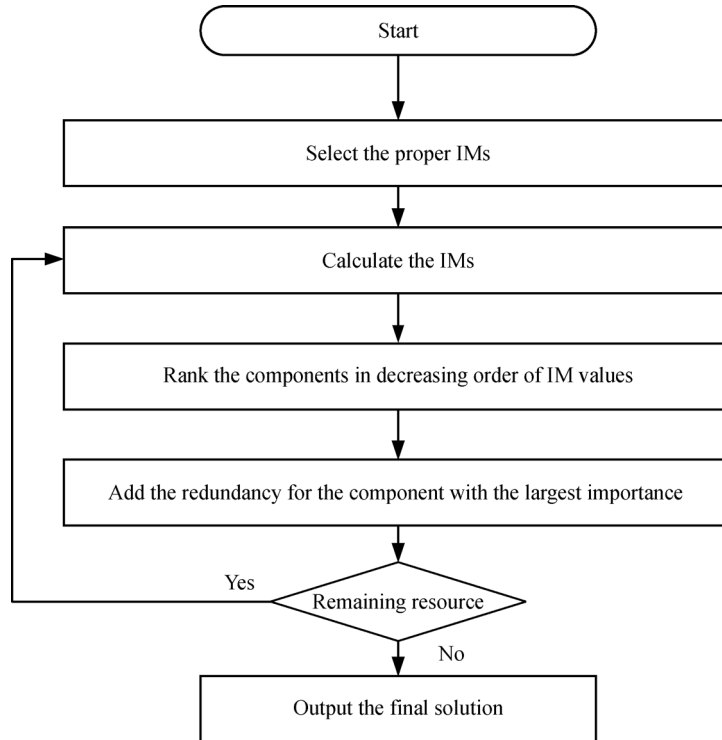


Fig. 3 Procedures of ranking-based optimization rules for RAP.

Table 2 Process of the MAD-based heuristic

| Steps | Description |
|-------|---|
| I | Evaluate the I^{MAD} of each component by simulation with the max-flow min-cut algorithm |
| II | (a) Determine the number of redundant components based on the cost per unit increase in the value of I^{MAD} (b) Update each binary minimal cut vector |
| III | Judge the stopping rules, if the rules are not satisfied, the process goes to Step I; otherwise, stop this heuristic |

reliability to the position with low IM. Heuristic 2 (ZKB) assigns the components with high reliability to the position with high IM. If the component reliability pattern does not match with the IM pattern, the component should exchange with other components, thereby making these two patterns consistent and improving the system reliability after the exchange. The ZKA heuristic process is summarized in Table 3.

A greedy heuristic method called LKA heuristic is developed to search for an ideal assignment according to

reliability importance (Lin and Kuo, 2002). If an optimal invariant design is present, the optimal assignment can be determined by the ranking of the component reliability, and the optimal assignment generated by the LKA heuristic is the same with the optimal assignment. LKA heuristic arranges the components with the lowest reliability to all positions as the initialization. Then, it assigns the available components with the highest reliability to the unassigned positions with the largest IM iteratively. The LKA heuristic process is shown in Table 4.

Table 3 Process of the ZKA heuristic

| Steps | Description |
|-------|---|
| I | Generate an initial arrangement randomly, $\pi = (\pi_1, \pi_2, \dots, \pi_i, \dots, \pi_n)$ |
| II | Calculate $I_{CAP}^{BM}(i)$ for all positions from position 1 to position n by Eq. (7) |
| III | For $k = 1$ to $n - 1$, do the loop (a) Find positions m and r such that $\pi_m = k$ and $\pi_r = k + 1$ (b) If $I_{CAP}^{BM}(m) > I_{CAP}^{BM}(r)$ and $R(P, \pi) > R(P, \pi(m, r))$, exchange the assignments of components π_m and π_r . |
| IV | If there is no exchange in Step III, output the final assignment; otherwise, go to Step II |

Table 4 Process of LKA heuristic

| Steps | Description |
|-------|--|
| I | Assign component 1 to all positions that are set $P_{i1} = P_{11}$ and $\pi_i = 1$ for $i = 1, 2, \dots, n$ |
| II | Let $S = \{1, 2, \dots, n\}$, which is the set of available positions that could receive other components |
| III | For $k = n$ to 2, do the loop (a) Calculate $I_{CAP}^{BM}(i)$ for all $i \in S$ by Eq. (7) (b) Find the position $m \in S$, which meets that $I_{CAP}^{BM}(m) = \max_{i \in S} I_{CAP}^{BM}(i)$ (c) Let $S = S/\{m\}$, assign component k to position m |
| IV | If there are no components in S , output the final assignment; otherwise, go to Step II |

The Birnbaum importance is used to design heuristics for solving CAP. The ZKA and ZKB heuristics start pairwise exchanges from the least reliable component, while the ZKC and ZKD heuristics start pairwise exchanges from the most reliable component with the revised Alternatives 1 and 2, respectively (Yao et al., 2011). The new Alternative 1 in ZKC heuristic compares the component $\pi_m = k$ with the component $\pi_r = k - 1$, and the new Alternative 2 in ZKD heuristic compares component $\pi_m = k$ with the component with the largest Birnbaum importance among the positions whose reliabilities are lower than P_{i1} . The differences between ZK-type heuristics are listed in Table 5.

Three new LK-type heuristics are proposed by modifying the initialization and/or the assignment rule of the LKA heuristic (Yao et al., 2011). The LKB heuristic assigns the component according to the increasing order of component reliability and initializes the components with the lowest reliability in all positions. The LKC heuristic has the same initialization as the LKA heuristic but uses a different assignment rule. That is, the assignment of position with the smallest Birnbaum importance is retained, and the least reliable unassigned component is allocated to all other positions. The LKD heuristic uses the same assignment rule as the LKC heuristic, but it iterates from the component with high reliability to the component with

the lowest reliability. The differences between LK-type heuristics are listed in Table 6.

The Birnbaum importance-based two-stage (BIT) approach is proposed by integrating the simulation results of ZK- and LK-type heuristics for solving the CAP (Yao et al., 2011). First, we generate two initial assignments by using both LKA and LKB heuristics. Second, we select the ZKB heuristic if all the reliabilities of components are less than 0.2; otherwise, we select the ZKD heuristic. Finally, we select the better one as the final solution. The procedures of the BIT heuristic are summarized in Table 7.

A single type of component is used in CAP, and this problem is extended to a multi-type CAP by dividing the components into different types of components that should be assigned to the corresponding type of position (Zhu et al., 2017). The BIT method for the multi-type CAP (BITSM) is developed after the simulation based on the extended ZK- and LK-type heuristics. BITSM can be summarized as follows. In the first stage, we generate two initial assignments based on the parallel iterative assignment. In the second stage, we perform the parallel pairwise exchange separately on these two assignments. In the third stage, we should improve the solution.

Uncertainties are inevitable in many real engineering projects because of insufficient data and complex relationships. The extension of the LKA heuristic using the

Table 5 Differences between ZK-type heuristics

| Heuristics | Step III | Step III(a) | Step III(b) |
|------------|--------------|---|-------------------------------------|
| ZKA | 1 to $n - 1$ | $\pi_r = k + 1$ | $I_{CAP}^{BM}(m) > I_{CAP}^{BM}(r)$ |
| ZKB | 1 to $n - 1$ | $I_{CAP}^{BM}(r) = \min_{i: P_{i1} > P_{m1}} I_{CAP}^{BM}(i)$ | $I_{CAP}^{BM}(m) > I_{CAP}^{BM}(r)$ |
| ZKC | n to 2 | $\pi_r = k - 1$ | $I_{CAP}^{BM}(m) < I_{CAP}^{BM}(r)$ |
| ZKD | n to 2 | $I_{CAP}^{BM}(r) = \max_{i: P_{i1} < P_{m1}} I_{CAP}^{BM}(i)$ | $I_{CAP}^{BM}(m) < I_{CAP}^{BM}(r)$ |

Table 6 Differences between LK-type heuristics

| Heuristics | Step I | Step III | Step III(b) | Step III(d) |
|------------|--------------------------------|--------------|--|-----------------------------------|
| LKA | Component 1 to all positions | n to 2 | $I_{CAP}^{BM}(m) = \max_{i \in S} I_{CAP}^{BM}(i)$ | Component k to position m |
| LKB | Component n to all positions | 1 to $n - 1$ | $I_{CAP}^{BM}(m) = \min_{i \in S} I_{CAP}^{BM}(i)$ | Component k to position m |
| LKC | Component 1 to all positions | 2 to n | $I_{CAP}^{BM}(m) = \min_{i \in S} I_{CAP}^{BM}(i)$ | Component k to positions in S |
| LKD | Component n to all positions | $n - 1$ to 1 | $I_{CAP}^{BM}(m) = \max_{i \in S} I_{CAP}^{BM}(i)$ | Component k to positions in S |

Table 7 Process of the BIT heuristic

| Steps | Description |
|-------|---|
| I | Generate two initial arrangements by both LKA and LKB heuristics |
| II | (a) Select the ZKB heuristic if all the components have low reliability; otherwise, select ZKD heuristic (b) Stop by giving the final arrangement with higher system reliability |

evidential network and interval-valued IM solves CAP under uncertain parameters and models (Qiu et al., 2018). The procedures of the proposed heuristic method are similar to those of the original LKA heuristic, but the relations of interval order are introduced to compare the bounded system reliability and the interval-valued Birnbaum importance. The system reliability and the Birnbaum importance are interval-valued when the parameter is uncertain, and the interval order should be conducted in Steps III(a) and III(b) in the LKA heuristic process.

3.4 Summary of the optimization rules

(1) Optimization rules by IM-based ranking methods

By analyzing the optimization rules of ROPs, RAPs, and CAPs, all the ROPs and almost all the RAPs are solved by the IM-based ranking methods, but none of the CAPs are solved using the IM-based ranking methods. The general procedures of the IM-based ranking methods for ROPs are similar to those of RAPs. We choose the proper IMs to analyze the optimization model at first. Subsequently, we rank the components in decreasing order according to IM values. Then, we improve the reliability or redundancy of the component with the largest importance. Finally, we check the remaining resource. If no resources remain, we output the final solution; otherwise, we return to the previous step to rank the components.

(2) Optimization rules by IM-based heuristic methods

Some RAPs and all the CAPs are solved by the IM-based heuristic methods with the consideration of the complexity of the corresponding problems by analyzing the optimization rules of ROPs, RAPs, and CAPs. For RAPs, redundancy should be assigned to the component with the highest IM, whereas the highest (lowest) reliable component should be assigned to the position with the highest (lowest) IM. The general procedures of IM-based heuristic methods can be summarized as follows. First, we generate an initial solution. Second, we assign the resources to the components with the highest IM. Third, we update the results. Finally, we check the remaining resources and stop the IM-based heuristic methods.

4 IM-based optimization algorithms for system reliability optimization

System optimization problems consistently minimize the cost subject to the requirement of system reliability or

maximize the system reliability under resource constraints. Considering the complexity of the system ROPs, many scholars have proved that the system ROPs are NP (non-deterministic polynomial)-hard (Lin and Chen, 1997). Many problems are complicated. Thus, obtaining the optimal solution by using the enumeration method is difficult. However, IM-based optimization algorithms can generate an effective solution. On the one hand, IM-based rules are combined with evolutionary algorithms to improve the performance of the proposed algorithms. On the other hand, the IMs are used to screen the critical factors to simplify the optimization problems. Both approaches are IM-based optimization algorithms, and they are known as the IM-based local search and simplification methods. The references for optimization algorithms based on IMs are summarized in Table 8 by considering the problems, system states, IMs, and algorithms.

4.1 Optimization algorithms for ROP

To solve the ROP, Birnbaum importance and Δ -importance are introduced into the genetic algorithm to obtain the optimal solution with constraints on cost (Wang et al., 2018). The optimization algorithm combines the local search method with the advantages of IM. The mechanism of IMs is that the components with larger IM should focus on improving its reliability. Genetic algorithms based on Birnbaum importance or Δ -importance are proposed to illustrate the effectiveness of I^{BM} and I^{Δ} for solving the ROP. The performance of the genetic algorithm based on Δ -importance is better than that based on Birnbaum importance via the comparison of the generations of convergence and system reliability.

Considering the limited design resources, increasing the reliability of some components can maximize system reliability. I^{GB} can evaluate the contribution of component reliability to system reliability considering the boundary of component reliability and the difficulty of increasing component reliability (Si et al., 2019). The GBIM-based genetic algorithm is presented to solve the ROP by increasing the reliability of the component with the highest I^{GB} during local search. The GBIM-based genetic algorithm could find a better near-global solution with a faster convergence speed compared with genetic algorithms that are based on Birnbaum importance and Δ -importance, and the effectiveness of the proposed algorithm is illustrated by a mixed system with 17 components.

Table 8 Reference analysis of optimization algorithms based on IMs

| References | Problems | Systems | IMs | Algorithms |
|----------------------------|----------|------------|---------------------------|----------------|
| Wang et al. (2018) | ROP | Binary | I^{BM} and I^A | Local search |
| Si et al. (2019) | ROP | Binary | I^{GB} | Local search |
| Zio and Podofillini (2007) | ROP | Any states | I^B | Simplification |
| Cai et al. (2018) | ROP | Continuous | I^U | Local search |
| Xiong et al. (2017) | RAP | Binary | I^{PRW} | Simplification |
| Shojaei and Mahani (2019) | RAP | Binary | I^{BM} | Simplification |
| Zhao et al. (2019c) | RAP | Binary | I^{BM} | Local search |
| Yao et al. (2014) | CAP | Binary | I^{BM} | Local search |
| Cai et al. (2016) | CAP | Binary | I^{BM} | Local search |
| Zhang et al. (2019) | CAP | Binary | I^{BM} | Local search |
| Dui et al. (2018) | CAP | Binary | I^{opt} | Simplification |
| Zhao et al. (2019b) | CAP | Binary | I^{BM} | Simplification |
| Nguyen et al. (2017) | CP | Binary | I_S^{BM} | Simplification |
| Du et al. (2019) | CP | Binary | I^{c-IM} and I^{p-IM} | Simplification |
| Xing and Dugan (2002) | CP | Multistate | I^{BS} | Simplification |
| Li et al. (2015) | CP | Binary | I_k^{PM} | Local search |
| Wu and Wu (2017) | CP | Binary | I_C^{PR} | Local search |
| Wu et al. (2018) | CP | Binary | I^{GC} | Simplification |

The performance improvement of multistate systems with multistate components is a CP because system performance depends on the combinations of components' performance levels and is related to the component performance and their positions. Balanced IM I^B is considered an objective in the mathematical model for the multi-objective ROP (Zio and Podofillini, 2007), which can simplify the model because maximizing the balanced IMs can make the system fully balanced to avoid bottlenecks or overly reliable components.

Continuous systems can describe component situations more accurately than discrete-state systems. However, solving optimization problems with continuous systems using conventional optimization algorithms is complicated. The performance improvement problem for a continuous system is to arrange the resources optimally to maximize system performance after the maintenance, which can be solved by the performance improvement-based genetic algorithm (Cai et al., 2018). The local search method based on I^U is proposed to rearrange the maintenance time of components. When the maintenance information of other components is known and identical, the components with the largest I^U should provide the optimal value, and the maintenance time of other components is balanced based on the proportion of previous maintenance time.

4.2 Optimization algorithms for RAP

Many scholars have investigated the optimization algorithms for RAP because some extended RAPs are developed by establishing complicated mathematical models. The RAPs are extended by considering the abnormal external failures when designing a system that maximizes the system reliability considering the uncertain abnormal external failures under normal and worst cases (Xiong et al., 2017). A multicomponent IM can identify the amounts of abnormal external failures for the worst case when the system configuration is provided. As proven, the effect of component failures on the system can be simplified by considering the effect of component failures on the subsystems, which can determine the system reliability under worst cases with different amounts of the abnormal external failures and simplify the complexity of calculating the objective. This extended RAP is a multi-objective optimization problem that maximizes system reliability under normal and abnormal external failures and minimizes cost. I^{PRW} can be used to qualify the importance of component combinations with arbitrary amounts based on the effect of their failures on system reliability. Therefore, I^{PRW} can solve the optimization model of RAPs.

This multi-objective nonlinear RRAP maximizes system

reliability and minimizes the variance of IMs (Shojaei and Mahani, 2019). The variance of IMs in the latter objective can evaluate the dependent degree of the system. The system is more dependable when the variance of IMs for subsystems decreases, indicating that the IMs of each component is very close. The variance of IMs is used to simplify the objective functions of the RRAP.

A multi-objective particle swarm optimization algorithm that combines IM and harmony search is developed to deal with multi-objective RRAP for serial parallel-series systems considering system reliability and design cost (Zhao et al., 2019c). Two interesting IM-based local searches are developed on the basis of two reliability adjustment strategies, namely, increasing component reliability after the redundancy decrement and decreasing component reliability after the redundancy increment. Birnbaum importance can evaluate the improvement of system reliability. The IM-based local search method can improve the effectiveness of solving the multi-objective RRAP, and the procedures are summarized in Table 9.

4.3 Optimization algorithms for CAP

A Birnbaum importance-based genetic local search (BIGLS) algorithm is proposed to solve the CAP by combining an IM-based local search method with the genetic algorithm (Yao et al., 2014). The idea of the Birnbaum importance-based three-way exchange is to adjust the position of three components to make their component reliability consistent with their importance. If the exchange is not beneficial for increasing system reliability, then the assignment does not change. The effectiveness of the BIGLS algorithm is compared with those of BIT and the general genetic algorithm.

Linear consecutive- k -out-of- n systems are a type of typical system of CAP. A Birnbaum importance-based genetic algorithm (BIGA), which combines a genetic algorithm with BIT-based local search method, is proposed to obtain the near-global optimal solution of CAP (Cai et al., 2016). The BIGA is applied to deal with the system reliability optimization of the circular consecutive- k -out-of- n systems (Zhang et al., 2017). Birnbaum importance-based quantum genetic algorithm (BIQGA) is developed by combining the quantum genetic algorithm with the Birnbaum importance-based local search method to solve the CAP efficiently and accurately based on the advantages of quantum computing and IMs (Zhang et al., 2019).

The changes in the optimal assignment for linear consecutive- k -out-of- n systems are analyzed by considering the changes in component reliability and IM of components during the system's lifetime (Dui et al., 2018). These results show the relationships between the changes in component reliability and IM with the changes in the optimal assignment. The importance of a specific component represents the improvement of the largest system reliability when the component reliability changes. The proposed importance can simplify the optimization process by considering the relationship between the optimal arrangement and the changes in component reliability.

An integrated method that combines the rearrangement method with the replacement method, is proposed to maximize the reliability of the reconfigurable system and minimize the cost during the reconfiguration (Zhao et al., 2019b). The BIT heuristic is used to generate the current optimal assignment, which is essential in measuring the optimal system reliability and the reconfiguration cost. Therefore, the BIT heuristic can be used to evaluate the system reliability when the reconfigurable cost is the largest to simplify the calculation of the fitness value.

4.4 Optimization algorithms for CP

Advanced technology in optimization theory can solve the system ROPs for complex systems in different fields (Coit and Zio, 2019). Network designs are used to determine the redundant configurations and select different available components to form the system structure. For fixed network topology, the reliability of a network system is improved by allocating redundancy or adjusting the links' reliability between node pairs with the constraints on a budget (Marseguerra et al., 2005). Sometimes, obtaining the optimal configuration directly in case the network structure is complex is difficult. Multiple phases are widely used in many practical applications, and many practical systems are phased-mission systems (PMS), which are multiple, consecutive, non-overlapping phases (Levitin et al., 2012). The PMSs need to accomplish some specified tasks during each phase. The features of system performance and the component degradation may change from phase to phase, in which the relationships between components in different phases are complicated. Therefore, the systems of the CP can be classified into two categories: Systems with complex structures, such as network systems (Compare et al., 2019; Xiao et al., 2018);

Table 9 Procedures of the IM-based local search method

| Procedures | Description |
|------------|---|
| I | Select the component modules with the highest $I^{BM}(i)$ |
| II | Perform the reliability adjustment strategy 1 or 2 randomly (1) Strategy 1: Increasing component reliability after decreasing the component redundancy (2) Strategy 2: Decreasing component reliability after increasing the component redundancy |
| III | Identify the solution after the adjustment |

and systems with a complicated relationship, such as PMS (Levitin et al., 2017a; 2017c).

A joint PM and inventory strategy of systems with complex structures can minimize the cost rate (Nguyen et al., 2017). The Birnbaum structure importance of components is evaluated on the basis of predictive reliability, which is used in deciding the ordering of spare parts and PM at a regular time. The structure importance decreased the number of decision parameters to three, regardless of the number of components.

When the component failure follows the counting process, two IMs are proposed to evaluate the effect of one edge on network failure or working (Du et al., 2019). These IMs depend on the structure of the network and the failure distribution of edges instead of the probabilistic information of the individual edge. The ranking of the proposed IMs can be potentially used to arrange resources of networks during the design stage and maintenance process.

An analysis method for a generalized PMS is proposed to calculate the probability of mission success with high computation efficiency (Xing and Dugan, 2002). The Birnbaum sensitivity measure I^{BS} can be used to estimate the effect of parameter-change on the result without considering a full model-solution as all parameters change. I^{BS} over the entire phased mission is a weighted sum of sensitivity in each phase. The more unreliable a phase is, the more sensitive the component in the phase should be. The proposed sensitivity measure can upgrade suitable candidates who can improve the system performance effectively to simplify the objective of the optimization problem.

For multi-mission networked avionics, a mission reliability allocation method is proposed to maximize avionic mission reliability (Li et al., 2015). The proposed allocation principles are summarized as follows: (1) adjusting the complex component with low reliability and (2) assigning the important component with high reliability. The IM-based heuristic algorithm is regarded as the local search method combined with the algorithm provided by the Advisory Group on the Reliability of

Electronic Equipment (AGREE) to solve the problem with the network structure.

A hybrid algorithm that combines IM-based heuristic with the genetic algorithm is presented to solve the RRAP in PMS (Wu and Wu, 2017). The cost-based parallel redundancy importance is proposed by combining the Birnbaum importance with the cost function. The IM-based heuristic method increases the reliability of components with higher I_C^{PR} and decreases the reliability of components with lower I_C^{PR} , thereby possibly reducing the total system cost. The hybrid algorithm takes advantage of the higher global search ability of the genetic algorithm and the local search ability of the proposed heuristic.

Considering the uncertainty of components' parameters caused by the changing environments in each phase and the insufficient information, a reliability allocation model is introduced to incorporate component uncertainty during the reliability allocation process (Wu et al., 2018). The global component importance I^{GC} can evaluate the importance level of a component for mission reliability because components' reliabilities vary randomly. The improved procedure based on global importance is that decreasing the MTBF of the component with the highest cost and setting the new MTBF based on the global importance value. I^{GC} is an effective way to generate the initial feasible solution, which can simplify optimization.

4.5 Summary of the optimization algorithms

(1) Optimization algorithms by IM-based local search methods

The IM-based optimization algorithms vary because the optimization rules of the local search methods are proposed on the basis of the ranking of IM or the IM-based heuristic algorithms. Therefore, the optimization algorithms based on the local search methods are analyzed in Table 10 to illustrate the differences among different optimization algorithms.

The general procedures of optimization algorithms by IM-based local search methods based on the analysis of the procedures of the previous research are proposed in Fig. 4.

Table 10 Reference analysis of the optimization algorithms by IM-based local search methods

| References | Problems | Systems | Algorithms | Optimization rules |
|---------------------|----------|------------|--------------------------|--------------------|
| Wang et al. (2018) | ROP | Binary | Genetic algorithm | Ranking |
| Si et al. (2019) | ROP | Binary | Genetic algorithm | Ranking |
| Cai et al. (2018) | ROP | Continuous | Genetic algorithm | Ranking |
| Zhao et al. (2019c) | RAP | Binary | Particle swarm algorithm | Heuristic |
| Yao et al. (2014) | CAP | Binary | Genetic algorithm | Heuristic |
| Cai et al. (2016) | CAP | Binary | Genetic algorithm | Heuristic |
| Zhang et al. (2019) | CAP | Binary | Genetic algorithm | Ranking |
| Li et al. (2015) | CP | Binary | AGREE method | Heuristic |
| Wu and Wu (2017) | CP | Binary | Genetic algorithm | Heuristic |

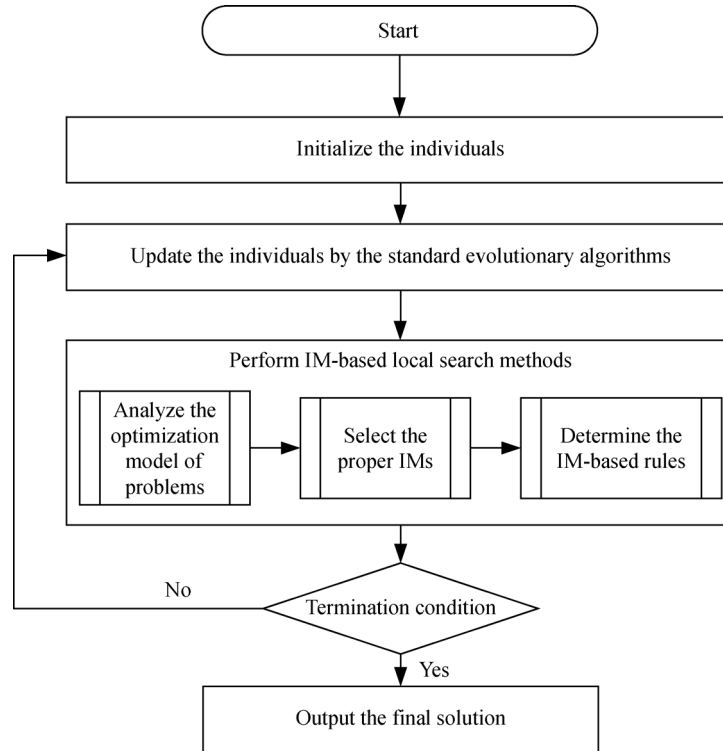


Fig. 4 General procedures of optimization algorithms by IM-based local search method.

The termination conditions are used to stop the optimization algorithms, which are optimization algorithm parameters used to determine whether the algorithms should be terminated. Thus, the conditions can be the maximum generations, the running time, or the convergence conditions, and they depend on the optimization algorithms, the requirements of the practical engineering, and the accuracy of the solution.

(2) Optimization algorithms by IM-based simplification methods

The optimization algorithms by the IM-based simplification method mainly use IM in the mathematical models or the optimization process to simplify the objectives or screening the critical factors to simplify the optimization problems. The analysis of the simplification method

focuses on the sub-process to illustrate where the IM can be used to simplify the optimization models. Therefore, we summarized the usage of IMs during simplification in Table 11, and the general procedures of optimization algorithms using IM-based simplification method is proposed in Fig. 5.

5 Optimization framework driven by IMs

Various IMs clarify the effect of one component’s performance on the overall system performance from different perspectives to distinguish the weak link of the systems. If the weak link is found, then giving additional resources to the weakest component can improve the

Table 11 Reference analysis of optimization algorithms by IM-based simplification methods

| References | Problems | Systems | Sub-processes | Strategies |
|----------------------------|----------|------------|---------------------|---|
| Zio and Podofillini (2007) | ROP | Any states | Objective | Use importance as the objective |
| Xiong et al. (2017) | RAP | Binary | Objective | Simplify the solving method |
| Shojaei and Mahani (2019) | RAP | Binary | Objective | Use importance as the objective |
| Dui et al. (2018) | CAP | Binary | Decision variable | Obtain the solution effectively |
| Zhao et al. (2019b) | CAP | Binary | Fitness | Simplify the complexity of the objective calculation |
| Nguyen et al. (2017) | CP | Binary | Decision parameters | Use the importance ranking to screen critical factors |
| Du et al. (2019) | CP | Binary | Decision variable | Obtain the solution effectively |
| Xing and Dugan (2002) | CP | Multistate | Objective | Evaluate the objective effectively |
| Wu et al. (2018) | CP | Binary | Initialization | Obtain the initial feasible solution |

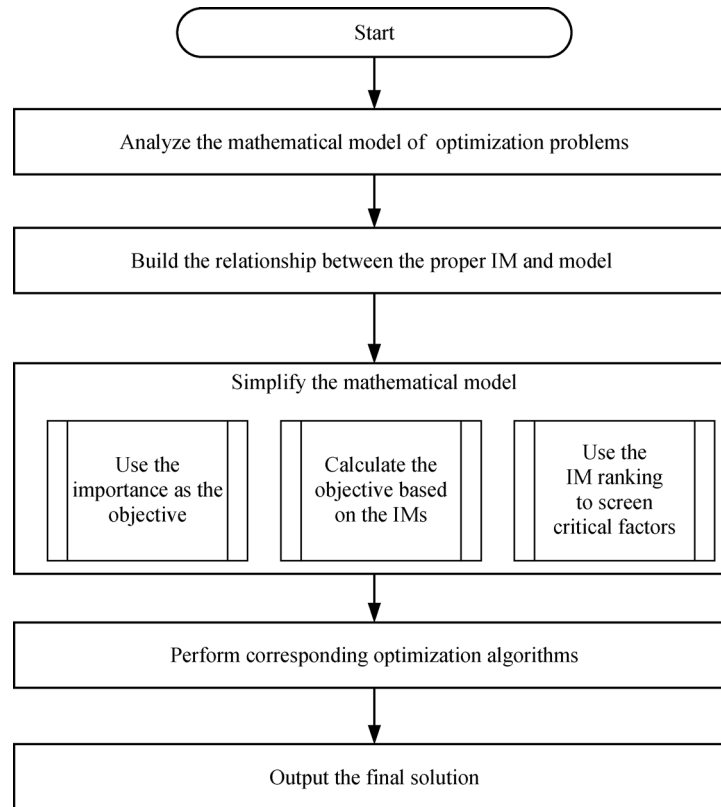


Fig. 5 General procedures of optimization algorithms by IM-based simplification methods.

system performance significantly. Sometimes, this approach is a shortcut for improving system performance through IM ranking. Thus, the merits of IMs are to find the weakest parts in the system, possibly improving system performance cost-effectively.

The previous solutions of the ROPs focused on the mathematical methods or algorithms of the operation research, but the contributions of IMs are ignored. The weakest link in the system is the premise and basis of the optimization problems; improving the component performance based on IM is an effective method to improve the system performance. The IM-based methods can summarize some optimization rules easily to solve the optimization problems effectively. This simplification is the benefit of practical engineering. Moreover, many future research topics are related to IMs. We will discuss these methods in Section 6.

According to the state-of-the-art about the system reliability optimization based on the IMs in Sections 3 and 4, the general optimization framework driven by IMs is summarized in Fig. 6. Clearly, the selection of solving methods mainly depends on the optimization problems and the characteristics of IMs.

For the mentioned optimization problems (ROP, RAP or RRAP, CAP, and CP), the IMs can maximize the system performance by arranging resources for the components

with higher IM. The solving method should be proposed by analyzing the complexity of problems, choosing the proper IM, and checking the effectiveness of the IM rankings. After the analysis of the system ROP and IMs, if the IM rankings are useful in solving the problem, then the IM-based rules should be selected to deal with the system optimization problem; otherwise, the IM-based algorithm is selected.

The solving methods based on IMs include the IM-based optimization rules and the IM-based optimization algorithms. The IM-based rules are proposed on the basis of the following principles: (1) improving the component performance with the highest importance by the IM-based ranking methods to maximize the system performance with the constraints on the limited resources and (2) assigning the resources to the components by the IM-based heuristic methods for maximizing system performance. The IM-based optimization algorithms are introduced to generate an improved solution for CPs with a large-scale system or complicated tasks in the system. The IM-based optimization algorithms consistently combine the IM-based optimization rules with the evolution algorithms. Sometimes, IM-based optimization algorithms also use the IM-based rules to simplify the mathematical model by using the IMs to replace the objectives or screen the critical factors.

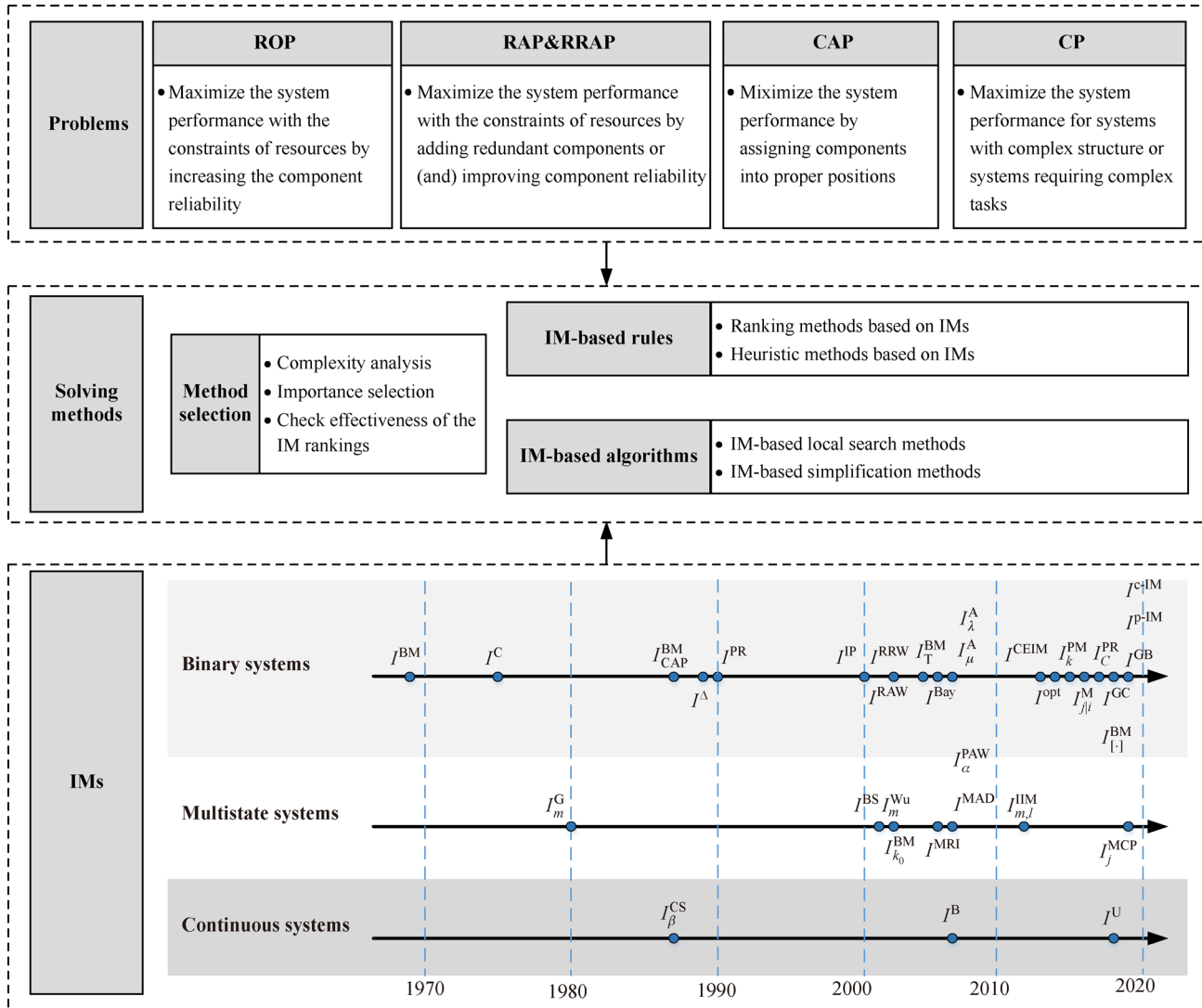


Fig. 6 General optimization framework driven by IMs.

6 Future research directions

Some challenges on the system reliability optimization driven by IMs remain for contemporary researchers. Therefore, the following problems should be a focus in the future.

(1) How can the mechanism of IMs in system ROPs be summarized? The mechanism of IMs can be used for solving system ROPs effectively.

(2) How be the IMs defined with the consideration of the relationship between different objectives and constraints in the optimization model? The proper definition of IMs is the basis of a suitable heuristic method for solving the system ROPs.

(3) How can the matching degree between an IM and a specific optimization problem be evaluated? Such a matching degree could be valuable for engineers to deal with the system ROPs driven by IMs.

(4) How can the IMs of nodes in the complex networks be evaluated? This problem is one of the most complicated problems in many practical engineering projects for considering the transition mechanism and the structure feature comprehensively. The results would simplify the networks' control, deposition, and defense by identifying some critical nodes during optimization.

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