ORIGINAL ARTICLE

Persistence in daily returns of stocks with highest market capitalization in the Indian market

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Abstract

The study in this paper emphasizes the presence of long memory or persistence observed in the Indian stock market. The analysis is performed on the daily returns of stocks with highest market capitalization listed in the national stock market index NIFTY. Empirically, persistence is quantified by the values obtained through calculating the Hurst exponent, H and further analysis like Detrended Fluctuation Analysis (DFA), Multifractal Detrended Fluctuation Analysis (MFDFA) and the multifractal spectrum analysis are carried out to determine the observed degree of fractality. Further, the observed multifractality present is analysed by plotting Hurst surfaces through Multiscale Multifractal Analysis (MMA). It is observed that the return series of the prices of stock with the highest market capitalization shows multifractal characteristics and indicates the presence of long-range dependence in the Indian stock market. The results of our analysis provide are statistically significant to contradict the validity of the Efficient Market Hypothesis (EMH) in Indian stock returns.

Keywords Multifractal · Long-range dependence · Hurst surface · Persistence · Indian stock returns

JEL Classification C12 · C58 · G14 · G19

1 Introduction

In social science, it is observed that events in the past influence the present and affect the future. This compounding effect in a financial market that is not cancelled by ongoing market operations and contradictory to the Efficient Market Hypothesis

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(EMH) (Fama, 1970) is termed as long memory or long-range dependence. The traditional methods that are based on EMH may not be able to accurately describe complex dynamic stock markets involving multiple agents affected by multiple factors where the changes are non-linear Kwapień and Drożdż (2012). There are many implications for both portfolio and risk management that stem from long-range correlation characteristics in stock returns and volatility. The Fractal Market Hypothesis (FMH) (Mandelbrot & Ness, 1968) was developed to modify the practicality of the EMH theory.

Long-range dependence in time series is observed over a wide range of disciplines, namely, natural sciences, DNA sequences, cardiac dynamics, internet traffic (Willinger et al., 1995) and finance (Lo, 2002). The Hurst exponent (Hurst, 1951) denoted by H, provides a measure for long memory, and the dimension of the time series D, can be calculated to check for fractality. On the empirical side, it has been shown that many irregularities in the financial time series cannot be elucidated by the EMH and the FMH (Peters, 1994). There are two different theoretical backgrounds underlying these two competing theories. The concept of multifractality (Mandelbrot & Taylor, 1967) is perceived as a challenge to the EMH. Peters (1994) attested that the fractional Brownian motion (FBM) results in a more accurate forecasts of financial market behavior since it accounts for some well evidenced irregularities including non-linearity, long-range dependence, fat tails, asymmetry, self-similarity and so on. In the multi-fractal model of asset returns (MMAR), Mandelbrot and Taylor (1967), the main financial models with their scaling properties are discussed the concept of multifractality in economics is introduced.

Studies on financial markets reveal the detection of significant fractal patterns within realized returns, consequently, the validity of the EMH is debatable. The FMH arose from the detection of significant persistence patterns in asset returns, replacing standard Brownian motion through a fractal generalization (Mandelbrot & Ness, 1968). Many different methods based on the FMH theory were utilized to characterize the multifractal behaviors and dynamics of financial markets. The FMH was tested for European (Onali & Goddard, 2011) and global equity markets (Kristoufek & Vosvrda, 2013, 2014). The FMH has been investigated for crude oil markets (Shao, 2020) and for foreign exchange markets (Shahzad et al., 2018). In these studies, multifractal properties have been empirically analyzed via several methods, namely, partition functions (Jiang & Zhou, 2008; Guoxiong & Ning, 2008), generalized Hurst exponents (Garcin, 2019), wavelet methods (Conlon et al., 2018; Oral & Unal, 2019), and Multifractal Detrended Fluctuation Analysis (MFDFA) (Li et al., 2020), as used in this study. Some researchers reported that the observed multifractal singularity spectrum has predictive power for price fluctuations (Wei & Huang, 2005; Zhi-Yuan et al., 2009). In the context of multifractal volatility (Wei & Wang, 2008), the observed multifractal singularity spectrum acts as a measure of market risk, and can be used to quantify the inefficiency of markets (Zunino et al., 2008).

More recently, there are studies examining the presence of long memory in the Indian stock market indices (Samadder et al., 2013) and public sector enterprises (Charutha et al., 2020). In this paper, we attempt to examine the fractal properties of asset returns for stocks with highest market capitalization within the NIFTY index. Our study focuses on the top ten of those companies' stock, so we are analysing the stocks

with highest market value which is assumed to be influential in the sector to which it belongs. The methods used to detect presence of long range dependence, scaling and multifractality among the returns series are: Detrended Fluctuation Analysis (DFA), MFDFA, Multiscale Multifractal Analysis (MMA).

This study contributes to the literature in the following ways: First, it helps to fill the void in empirical research analyzing the long memory present the individual stocks of an index in an emerging market, say India. Second, the memory present in the stock market traditionally measured by the Hurst parameter, H and the fractal dimension are calculated to observe the nature of the generative process. Finally, this paper employs the DFA, MFDFA and MMA methods to examine the presence of long-memory in asset returns of stocks in NIFTY index. The rest of the paper is organized as follows: Sect. 2 describes the methodology used in this study; Sect. 3 presents the empirical results and discusses the implications; Sect. 4 concludes the study.

2 Methodology

The choice of stocks in our analysis is under the assumption that, for any stock to be included in the index, it has to have the market power to influence public opinion, credibility amongst stakeholders, higher and steady returns, therefore making it the top performer amongst the sector to which it belongs (Gompers & Metrick, 2001). Within the NIFTY index, we have chosen ten stocks with highest market capitalisation as they are perceived to possess the aforementioned characteristics. This will be useful to understand market dynamics and investor behaviour. The asset returns of stocks in NIFTY index used in this study are: ICICI BANK LTD., BHARTI AIRTEL LTD., WIPRO LTD., HERO MOTO CORP LTD., NMDC LTD., CIPLA LTD., CAIRN INDIA LTD., UNITED SPIRITS LTD., POWER FINANCE CORPN. LTD. and OIL INDIA LTD. The daily returns of stocks over a period of ten years from 2005 to 2015 are used in the study. The returns are calculated as the differences in log price and are used for the following analyses.

In our preliminary analysis, we use the Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979) to check the presence of a unit root in the returns series for all the stocks used in this study. Further, the DFA is carried out to determine the statistical self-affinity or self-similarity of the series, the MFDFA to provide a generalization of the multifractality using variable order of moments, the multifractal spectrum to examine the degree of multifractality and the MMA to generate the surface of the fluctuation function to understand the variation of fluctuation function for changes in scaling window and order of moments, for the asset returns of NIFTY index stocks. A brief summary of the methods used in this study are given in the following subsections.

2.1 Augmented Dickey–Fuller test

The ADF test is performed to check the stationarity of the returns series of the NIFTY stocks under analysis, the steps involved in the ADF test are elaborated as follows:

$$\Delta \rho_{it} = \alpha_0 + \alpha_1 t + \rho_0 \rho_{(it-1)} + \sum_{i=1}^{q} \rho_{it} \rho_{(it-1)} + \epsilon_{it} \tag{1}$$

where, ρ_{it} denotes the price of the ith NIFTY stock at time t, and

$$\Delta \rho_{it} = \rho_{it} - \rho_{(it-1)} \tag{2}$$

where, ρ_{it} are coefficients to be determined, q is the number of lagged terms, t is the trend term, α_1 is the estimated coefficient for the trend, and α_0 is a white noise constant.

2.2 Detrended fluctuation analysis

DFA is the conventional method used to analyze time series that appear to be generated by a long-memory process, and to express the nature of long-range correlations (Peng et al., 1993). These processes have a divergent sum of autocorrelations, that is, there is certain power law decay in the autocorrelation function or $1/f$ noise for the time series, where f is the frequency of the fluctuation function. DFA has two advantages over traditional methods for analyzing fluctuations, namely, it is smoother and numerically more robust for short data sets and the influence of simple trends can be removed easily (Höll et al., 2019).

DFA is a method to estimate the generalized Hurst exponent $H(q)$ from a stationary series, where q is the order of exponent in the fluctuation function. The presumption that follows is that, the second moment of the fluctuations of the input series, after being averaged over a time scale s, is a power law function of s. This attribute is called scale invariance, or just scaling. When examining the conditions for applying DFA, the scaling assumption suggests that the fluctuation function $F(s)$ of DFA conforms to a power law.

Moreover, the fractal dimension denoted by D of the time series can be calculated from the following expression:

$$D = \frac{\log(d)}{\log(s)} \tag{3}$$

where, d is the number of segments of the integrated time series used in analysis, and s is the length of each segment.

Technically, D and H are independent of each other. The fractal dimension is a local property, while Hurst exponent is a global property, which is used to characterize the long-memory dependence in a time series. For self-affine processes, local properties are reflected in global ones, which lead to the relationship $D + H = n + 1$ between D and H for a self-affine surface in n-dimensional space.

In our study, we tabulate the calculated values for D and H for asset returns of the NIFTY stocks in Table 2. The fluctuation function $F(s)$ for the NIFTY stocks is plotted to check if the time series data is mono-fractal or multifractal (interested readers may refer to Appendix 1). If the plot conforms to a horizontal straight line then the time series is said to mono-fractal, and can be explained using a single exponent

called Hurst exponent $H = H(q)$ (Hurst, 1951; Feder, 2013) whereas if the plot of the fluctuation function is a curve with humps, then the time series under analysis is said to be multifractal (West & Scafetta, 2003).

2.3 Multifractal detrended fluctuation analysis

MFDFA is one of the most popular methods for studying the multifractal characteristics of the financial market (Hasan & Mohammad, 2015; Ali et al., 2018). The MFDFA is an alternate technique that is derived from generalization of the DFA method. The execution of this new method is not tedious when compared to the conventional DFA, as it includes only one additional step, a q dependent averaging procedure, is employed. The order of fluctuation q focuses on analyzing the variation in magnitudes of the observed fluctuations, i.e., mostly for $q < 0$, small fluctuations are analyzed, while for $q > 0$, large fluctuations are analyzed. The power law scaling function in the form

$$F_s(q) = s^{H(q)} \qquad (4)$$

with $H(q)$ as the generalized Hurst exponent. The scalar $H(2)$ equals the monofractal Hurst exponent H, values of $H(2) > 0.5$ relate to a trending and positively auto-correlated behavior; similarly, $H(2) < 0.5$ characterizes anti-persistence. If the series is monofractal and $n = \infty$, then the scaling behavior of the variance within each segment is identical for all scales s such that $H(q)$ will be independent of q. If $H(q)$ is dependent on q, the time series will exhibit multifractality. Thus, the higher the variability of $H(q)$, the greater the degree of multifractality (Shahzad et al., 2018).

The scaling exponents determined via the MFDFA method are identical to those obtained by the standard multifractal formalism based on local fluctuation functions. The MFDFA (Kantelhardt et al., 2002) analyzes the scaling properties of fluctuations by calculating a set of multifractal fluctuation functions, $F_s(q) = F(., q)$, for each fixed s the fluctuation function varies with the moment order q. The q-order scaling exponent, or q-order mass exponent, $\tau(q)$ is one among the variety of scaling exponents by which the multifractal structure of time series is parametarized.

2.4 Multifractal spectrum

The multifractal series is characterized using the singularity spectrum using the Legendre transform, that is, the q-order singularity dimension is characterized by α Feder (2013). For a multifractal time series, the q-order mass exponent, $\tau(q)$ has a curved plot for q-dependency which leads to a decreasing singularity exponent, α. The multifractal spectrum is the plot of $H(q)$ versus $f(\alpha)$, where $f(\alpha)$ is denoted as $D(q)$ in the results following (Ihlen, 2012). The plot of the multifractal spectrum has a large arc, and the width of the spectrum is the difference between the maximum and minimum values of the q-order singularity dimension.

For an infinite monofractal series, for all $q : \alpha = $ constant, that is the singularity spectrum has a width of zero. In the presence of multifractality, the spectrum takes

the shape of an inverted parabola. Consequently, a wider spectrum corresponds to stronger multifractality; hence, it is common to study the spectrum width given by

$$\Delta\alpha = \max(\alpha) - \min(\alpha) \tag{5}$$

as another proxy for multifractal strength. The symmetry of the spectrum allows one to obtain deeper insights into the fractal structure. If the spectrum is skewed negatively, then large fluctuations exhibit stronger multifractality than small fluctuations. According to Drożdż and Oświcimka (2015), a negative skew in the singularity spectrum is commonly observed in financial data. With a negative skew, it also follows that the spectrum will be truncated on the right side.

2.5 Multiscale aultifractal analysis

More recently, has been demonstrated many times that the fractal properties vary from point to point along the series and the different scaling exponents are usually required for different parts of the series. Therefore, it is not adequate to illustrate the internal dynamics of signals by using the DFA or MFDFA method. In order to avoid errors due to improperly predefined scaling ranges, and to obtain all information among the entire timescales, Gierałtowski et al. (2012) calculated a multifractal spectrum with variable scale ranges. The scales s are varied continuously and the generalized Hurst surface $H(q, s)$ is generated. This technique allows us to investigate not only the multifractal properties but also dependence of these properties on the timescale. The generated surface surface is further studied for identifying crossover phenomena, i.e. the changes in the series from persistent to anti-persistent and vice-versa. However, this paper focuses on validating the existence of long memory in NIFTY stock returns.

In the MMA method after calculating the q- order fluctuation functions as in MFDFA, a moving fitting window sweeps through all range of the scale s along the $F_s(q)$ plot and a series of overlapped windows is obtained. In this way, the quasi-continuous changes of the $H(q)$ dependence versus the range of scale s is visualized by the Hurst surface and the points on the surface represent the generalized dependence $H(q, s)$. The Hurst surface is presented using three linear axes, i.e., order axis q, scale axis s, and scaling exponent axis $h(q, s)$, and the points of the Hurst surface graph are connected to form a colored surface. For interested readers, the plots of the generalised Hurst surface for asset returns of NIFTY stocks are included in Appendix 4.

In our study, the methods explained above were carried out using MATLAB R2018b version and the code for the same is available upon request to the authors.

3 Results and discussion

Stocks are included in the index based on the criteria that companies must rank within the top 200 rank based on free-float market capitalisation. While existing studies analyse the indices and sectors as a whole, this present study examines the

asset returns of top ten stocks in the NIFTY index with highest market capitalisation. The indexes are comprised of these stocks and analysing their statistical properties in detail is interesting to understand the market dynamics and other factors or anomalies in the market. Therefore, we expect our results to provide more insight into market behavior through the methodology that is briefly mentioned above. Existing literature focuses on long memory behavior in the index as a whole, whereas we examine the behavior of asset returns of top ten stocks in the index, which constitute the index.

The returns of historical closing prices of the NIFTY stocks are used in our analysis. Table 1 shows the NIFTY index stocks in order of largest market capitalization.

3.1 Results from preliminary analysis

3.1.1 ADF test

The ADF test was carried out using daily returns of the 10 NIFTY stocks under analysis. All three forms of the test equation were performed on the returns from the NIFTY stocks. The results of the ADF test for NIFTY stocks show that the *p*-values in all three forms of the test equation are significant at the 5% level. The *p*-values are less than 5% level; hence, the null hypothesis can be rejected at the 5% significance level and the returns series do not have a unit root. This means that the return series is stationary, and could be used to estimate future price changes. A similar study using the ADF test in the context of the Indian stock market also rejected the null of unit root presence Gupta et al. (2007).

3.2 Results from DFA

The results of the DFA performed on the asset returns of top ten NIFTY index stocks with highest market capitalisation are given in Table 2.

Table 1 The top ten NIFTY stocks listed in order of largest market capitalization in INR (Cr.)

S. No	Company name	Symbol	Market capitalization
1	I C I C I BANK LTD.	ICICIBANK	2044986.884
2	BHARTI AIRTEL LTD.	BHARTIARTL	1409883.016
3	WIPRO LTD.	WIPRO	1370619.864
4	HERO MOTOCORP LTD.	HEROMOTOCO	620359.172
5	N M D C LTD.	NMDC	574883.82
6	CIPLA LTD.	CIPLA	502949.938
7	CAIRN INDIA LTD.	CAIRN	450991.242
8	UNITED SPIRITS LTD.	MCDOWELL-N	404541.572
9	POWER FINANCE CORPN. LTD.	PFC	397860.268
10	OIL INDIA LTD.	OIL	346494.764

The DFA was performed for fixed scaling window sizes of width $s = 10, s \in (5, 1000)$. Table 2 displays the results of DFA (i.e.: scaling parameter and exponent) for each of the stock. The values of H usually vary from 0 to 1. The nature of the generative process of the time series can be determined from values of H as follows, $H = 0.5$, corresponds to a completely uncorrelated differenced series, and is the characteristic of the standard Brownian motion; $0 < H < 0.5$, the autocovariances at all lags are negative and the time series process is called anti-persistent, indicating that the time series belongs to a mean-reverting process; $0.5 < H < 1$, the autocovariances are positive at all lags and the time series process is called persistent, corresponding to a process that is not mean-reverting and follows a trend.

From the values of H in Table 2, we see that for all stocks except Cairn and OIL India have $H \geq 0.5$. This suggests that the asset returns are fractal, the EMH is not confirmed, the series are persistent, returns are highly correlated, there is black noise and a trend in the market. The two stocks Cairn and OIL India have $H < 0.5$, this suggests the data are fractal, the EMH is not confirmed, the series are anti-persistent, returns are negatively correlated, there is pink noise with frequent changes in the direction of price movements, trading in the market is riskier for individual participants. The results from DFA suggest that the returns series are not mean-reverting and the asset prices do not follow a random Brownian motion and there is memory present in the asset returns series.

In furtherance to this study, the fluctuation functions, $F(s)$ for all window lengths, s are plotted on logarithmic axes. Usually the relationship between $F(s)$ and s has a linear form in a double logarithmic coordinate system across many sizes of s. The slope of the least-squares line in this graph is termed as the DFA exponent or scaling exponent or self-similarity parameter α. This slope is a direct estimation of the Hurst parameter and indicates the strength of the temporal correlations present in the time series (Hardstone et al., 2012). If the values of the DFA exponent lies within [0.5, 1.0], it indicates a presence of persistent temporal correlations. From the values of α in Table 2, we see that there are strong long-range temporal correlations in asset returns of NIFTY stocks expect for Airtel, Cipla and OIL India, where the asset returns are anti-persistent.

S. No	Stock	Fractal dimension (D)	Hurst exponent (H)/ Scaling exponent (α)
1	ICICI	1.4726	0.5274
2	Airtel	1.5653	0.4347
3	Wipro	1.4105	0.5895
4	Hero	1.4837	0.5163
5	NMDC	1.4459	0.5441
6	Cipla	1.4482	0.5274
7	CAIRN	1.5350	0.4650
8	United spirits	1.4455	0.5545
9	Powerfincorp	1.4378	0.5622
10	OIL India	1.5911	0.4089

Table 2 The fractal dimension and Hurst exponent/ scaling exponent for asset returns of NIFTY stocks estimated using DFA

Therefore, to examine the fluctuation function, $F(s)$, we plot the fluctuation function of asset returns of NIFTY stocks on a logarithm scale with base 10. $F(s)$ is plotted for all window sizes s on log–log axes. If a time series is characterized by long-range temporal correlations, then $F(s)$ increases with window size s according to the power law $F(s) \propto s^{H(q)}$. In Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 in Appendix 1, the fluctuation functions $F(s)$ are represented using log–log plots. It is evident from the plot that for each stock, the plot of the fluctuation function, $F(s)$ is a curve with humps and not a horizontal straight line. This shows that the asset returns of the of top ten NIFTY index stocks with highest market capitalisation are multifractal. For interested readers, the plots are included in Appendix 1.

3.3 Results from MFDFA

The MFDFA was carried out for variable scaling window, s and fixed order parameter q. Similar to Ghazani and Khosravi (2020), q is chosen in the range of [10, 10]. We chose this approach because q should take on positive and negative values to capture multifractality for both large and small fluctuations. Further, q should be sufficiently large to cover as much of the singularity spectrum as possible. However, an excessively large mod q destabilizes the estimation of the singularity spectrum, $f(\alpha)$; thus, we limit q by ± 10.

Table 3 displays the q-order Hurst exponents computed using the MFDFA.

It follows from Table 3, that the q-order Hurst exponent values are not constant. $H(q)$ is depends on q, this implies that the time series is multifractal. As the returns series are multifractal, the scaling behavior of the variance within each segment for all scales s is not identical. It follows that the asset returns series of NIFTY stocks are multifractal. In a multifractal time series, local fluctuation denoted by $F_q(s)$ within each window, s, will have an extremely large magnitude for s within the time

Table 3 The values of generalised Hurst exponent, $H(q)$ for asset returns of NIFTY stocks and the order parameter, q, estimated using MFDFA

S. No	Stock	Generalised Hurst exponent, $H(q)$						
		Order parameter, q						
		−5	−3	−1	0	1	3	5
1	United Spirits	0.6208	0.5871	0.5511	0.5331	0.5122	0.4561	0.3946
2	Airtel	0.5331	0.5098	0.4842	0.4704	0.4554	0.4181	0.3724
3	Cipla	0.5644	0.5400	0.5108	0.4945	0.4773	0.4414	0.4080
4	NMDC	0.6346	0.5995	0.5722	0.5652	0.5590	0.5329	0.5042
5	Wipro	0.6077	0.5794	0.5539	0.5446	0.5382	0.5241	0.4966
6	Powerfincorp	0.5318	0.5219	0.5139	0.5085	0.5013	0.4809	0.4565
7	OIL India	0.5848	0.5528	0.5143	0.4938	0.4729	0.4308	0.3908
8	Hero	0.5500	0.5206	0.4866	0.4685	0.4496	0.4095	0.3694
9	CAIRN	0.5267	0.4858	0.4569	0.4509	0.4464	0.4037	0.3292
10	ICICI	0.5650	0.5461	0.5277	0.5189	0.5088	0.4743	0.4290

periods of large fluctuations and extremely small magnitude for s within time periods of small fluctuations. To investigate the multifractal characteristics of the asset returns of NIFTY stocks, the regression of q-order Hurst exponent $H(q)$ on order parameter q computed by MFDFA is plotted in Figs. 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 in Appendix 2. For interested readers, the plots are included in Appendix 2.

The regression line is constructed for varying values of moment q for each segment N_s of the returns data, the corresponding fluctuation function, $F_q(s)$ is calculated for each a fixed segment N_s for varying moment q, and a polynomial of degree one is fitted. In the generalized $H(q)$ plot spectrum, the regression line is the compression of N_s lines along the order parameter q, compressed using polyfit function in MATLAB and each point shows the relationship $F(s, q) = sH(q)$. The q-order Hurst exponent can be defined as the slopes of the regression line for each q-order Root Mean Squares (RMS) fluctuation. Figures 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 in Appendix 2, highlight the q dependency of the slopes of the regression line which are typical of multifractal time series. The increasing resemblance between q-order RMS fluctuation of the time series with expanding segment sample size brings about a decrease in the value for $H(q)$ for multifractal time series. Figures 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 in Appendix 2 show decreasing values for $H(q)$ and account for a multifractality present in the time series under analysis. However, the segments in plots of Figs. 15, 16, 18, 19 and 20 in Appendix 2, exhibit persistence for values of the order parameter, $q > 1$ and generalized Hurst exponent, $H(q) > 0.5$. Therefore, the MFDFA gives a better understanding of the fractal properties of the time series under analysis when compared to DFA.

3.4 Results from multifractal spectrum

The distribution of the multifractal spectrum can be visualized by plotting the q-order singularity dimension $f(\alpha)$ also known as $D(q)$ following (Ihlen, 2012) versus the q- order singularity exponent α. For interested readers, the plots are included in Appendix 3. The spectra of the asset returns take on the form of a smooth inverted parabola, which validates that $H(q)$ exponents are estimated consistently over q. Also, an inverted parabola indicates the presence of multifractality in the time series. Ihlen (2012) noted that the local Hurst exponent H, which basically measures the current degree of anti-persistence, will exhibit greater fluctuations if $H(q)$ has a q-order dependence. Thus, if a time series shows multifractal behavior, its persistence will exhibit stronger variations over time. From economic intuition, one could argue that times of crises are accompanied by greater planning uncertainty; thus, investors will react more sensitively to news, which could cause overreacting behavior. Overreacting behavior will be recognized as anti-persistence within the market sentiment. Following this logic, times of financial recovery will be less exposed to extreme risks; hence, investors may feel more confident and be less sensitive/overreacting, and the sentiment will be more stable.

The width of the multifractal spectrum represents the deviation from the average fractal structure for segments with large and small fluctuations. The shape of

the multifractal spectrum need not be symmetric. It can have a long left tail when the time series is insensitive to the local fluctuations with small magnitudes and long right tail when the time series is insensitive to the local fluctuations with large magnitudes. Thus, the width and shape of the multifractal spectrum aids in the classification of invariant structures of time series. From the plots Figs. 21, 22, 23, 24, 25, 26, 27, 28, 29 and 30 in Appendix 3, we can see that the multifractal spectra for the stocks Airtel, Cipla, CAIRN, ICICI, Power Fin Corporation, and OIL India, have a long left tail and the spectra are truncated on the right. This means that the spectra are skewed negatively, consequently, large fluctuations exhibit stronger multifractality than small fluctuations. Therefore, the time series for these stocks are insensitive to local fluctuation with small magnitude. The spectrum for NMDC has a slightly long-right tail, so the time series of this stock is insensitive to local fluctuation with large magnitude. However, the multifractal spectra for the other stocks are relatively symmetrical.

3.5 Results from MMA

The generalised Hurst surface $H(q, s)$ is generated for varying scaling window s and order parameter q. Compared with the standard MFDFA, the Hurst surface $H(q, s)$ contains abundant information in which the results of standard MFDFA can be represented by a single line, that is, a cross section of $H(q, s)$ at a constant s.

For interested readers, the plots are included in Appendix 4. In the $H(q, s)$ plots for asset returns of the NIFTY stocks, the window length, $s \in (20, 700)$ and the range of order parameter, $q \in (-10, 10)$. It is observed that for smaller window lengths flatter surfaces were observed whereas, for wider window lengths, the surface showed more fluctuation. A variable fitting window sweeps across the entire range of the scales s in the $H(q, s)$ plot. Moreover, in this analysis, the values of the generalized Hurst exponent, $H(q)$ are calculated over fine subdivisions of q; therefore, it can be said that scale of $H(q, s)$ is non-homogeneous with this regard. It consequently, leads to the formation of a generalized Hurst surface, $H(q, s)$ where $H(q)$ is the best fit from the cross-sections of $H(q, s)$ as in the MFDFA plot Figs. 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 in Appendix 3 over the range of the scale s.

Further, it is important to note that, the Hurst surface plots for asset returns of stocks like Airtel, Cipla and Hero, are relatively flat as compared to the the Hurst surface plots of other stocks in the analysis. This suggests that there is no uniformity in the multifractality in the asset returns of the NIFTY stocks. In all plots of the Hurst surface, it is observed that the generalized Hurst surface predominantly lies in the range where $H \in (0.5, 1)$. This shows that the stock returns are persistent time series where the direction of movement of the next value from the current value depends on its previous increments. Therefore, the results from both the methods employed for analysis supports the presence of persistence in the time series under analysis and that the NIFTY stock returns are affected by the long memory present of the market.

4 Conclusion

The presence of long memory properties in the asset returns of top ten NIFTY index stocks with highest market capitalisation represents evidence in favour of FMH. It implies that historical time series can give information about future returns and be effective. It also means that asset pricing models not allowing for long memory will be missing out on a key feature of the data and not perform particularly well.

The results of our analysis are coherent with the existing study which analyses the multifractal nature of the Indian stock market indices. We note from Table 2, that the value of H for stocks like Airtel, CAIRN and OIL India are less than 0.5, which indicate that the series are anti-persistent, and the results from DFA plots show that they are multifractal, also the Hurst surface for these stocks are unvaried, when compared to stocks that have H value greater than 0.5. A closer examination on the nature of the series is shown through the tabulated values of $H(q)$ in Table 3. The varying nature of the fluctuation function with respect to the different values of the order parameter indicate the presence of long memory in the returns among stocks with highest market capitalisation in the Indian stock market. It was observed that the behavior of stock return prices does not have independent increments as assumed but showed dependency, evident from fractal characteristics such as multifractality and scaling. This empirical study analyses the presence of multifractality (Kantelhardt et al., 2002; Saichev & Sornette, 2006), nonlinear correlations and time scaling in the asset returns of stocks in NIFTY index, evidenced by the results of DFA, MFDFA, multifractal spectrum plots and MMA analysis.

It is inferred from the results of our study that, the movement of the stock returns in the index is not random, so the validity of EMH cannot be confirmed. Our analyses examines the fractality of each stock, this way the individual behavior of the stock is accounted for and not cancelled out. Our analysis examines the property of long-range dependence of stock returns and the results suggest the presence of multifractality in in the asset returns of stocks in NIFTY index. Consequently, there may be the scope for earning abnormal returns in the short run. Such evidence against the random walk behaviour implies predictability in market dynamics which is inconsistent with EMH.

The long memory present in the asset returns of top ten NIFTY index stocks with highest market capitalisation provides additional evidence in favour of the Adaptive Market Hypothesis (AMH) and partially explains why profitable trading strategies become unprofitable and vice versa, and also the necessity of switching between trading strategies. Therefore, our study presents statistically significant results to confirm the invalidity of EMH with respect to asset returns of NIFTY stocks. Moreover, a deeper understanding of market sentiments, multifractal properties and the related causes of time-varying persistence are likely to be of interest for economic policies, striving to stabilize markets.

Appendix 1: Log–log plots of DFA fluctuation function for the asset returns of top ten NIFTY index stocks

See Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.
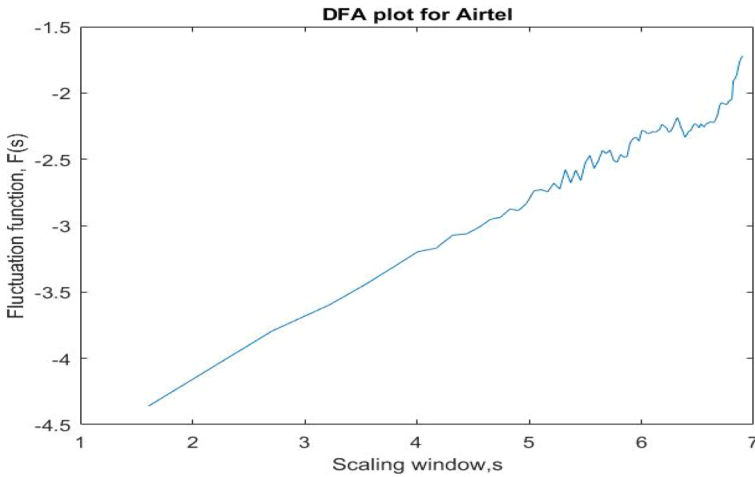
Fig. 1 Log–log plot of fluctuation function for Airtel
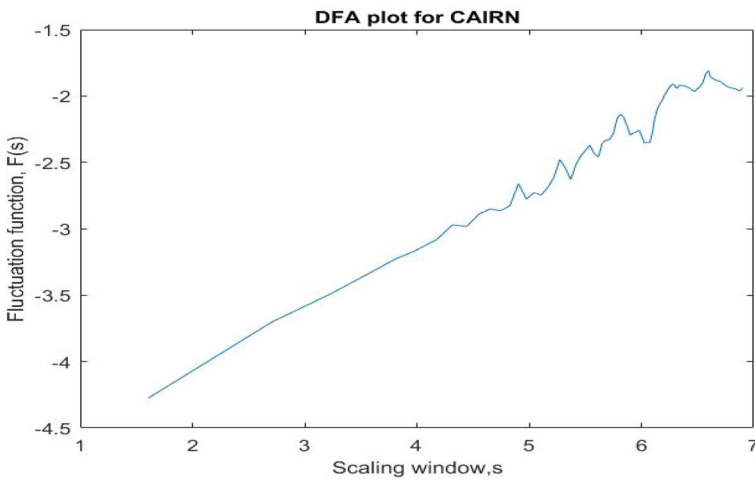
Fig. 2 Log–log plot of fluctuation function for CAIRN
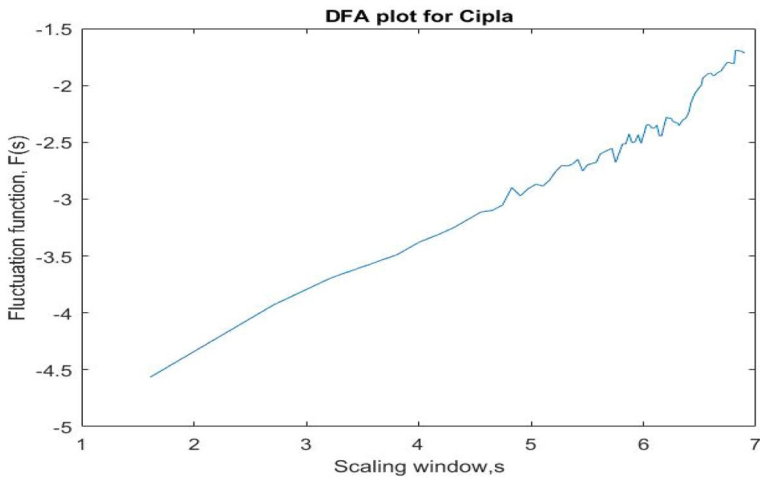
Fig. 3 Log–log plot of fluctuation function for Cipla
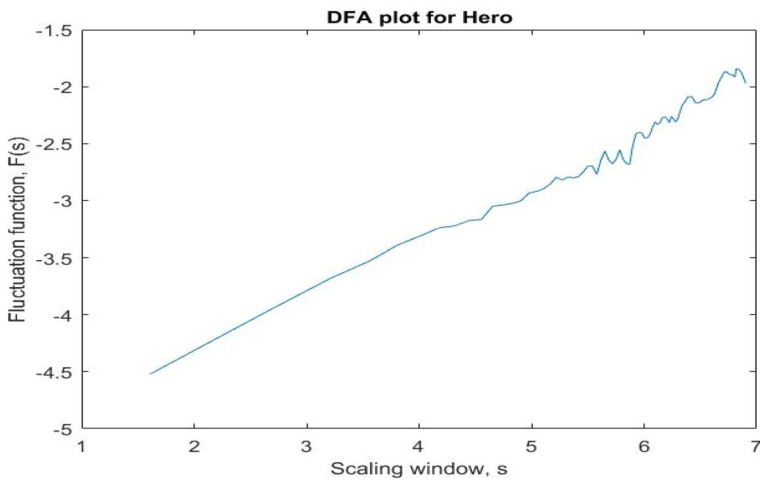
Fig. 4 Log–log plot of fluctuation function for Hero
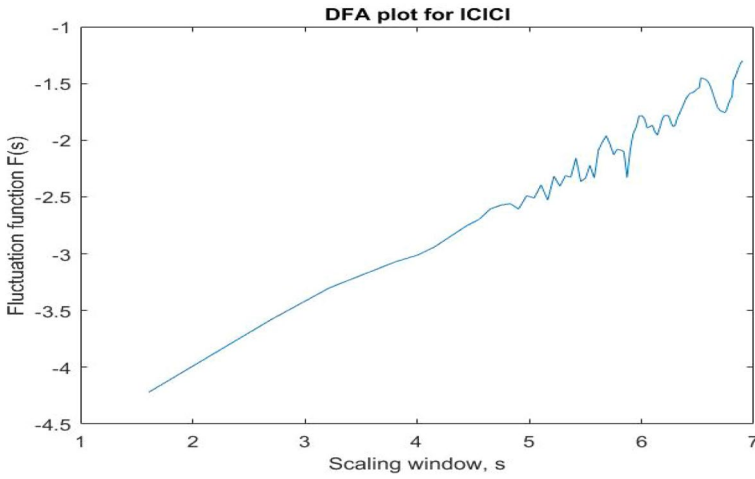
Fig. 5 Log–log plot of fluctuation function for ICICI
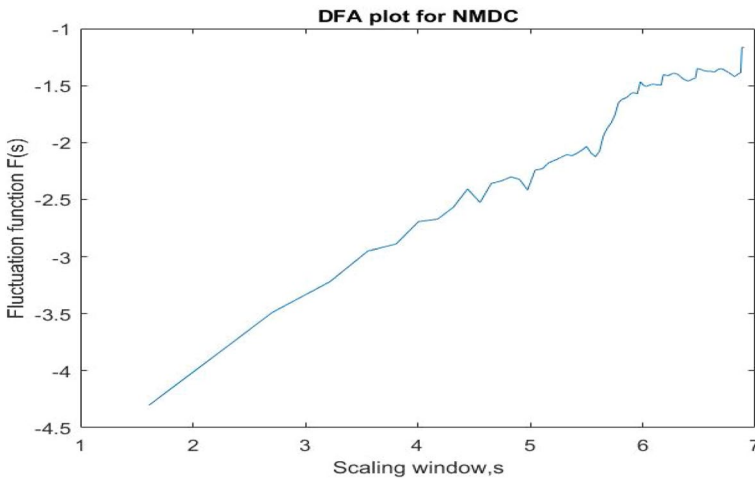
Fig. 6 Log–log plot of fluctuation function for NMDC
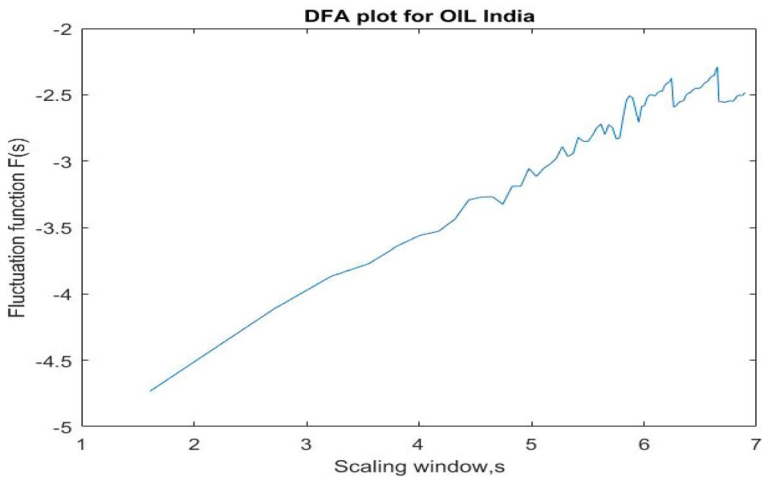
Fig. 7 Log–log plot of fluctuation function for OIL India
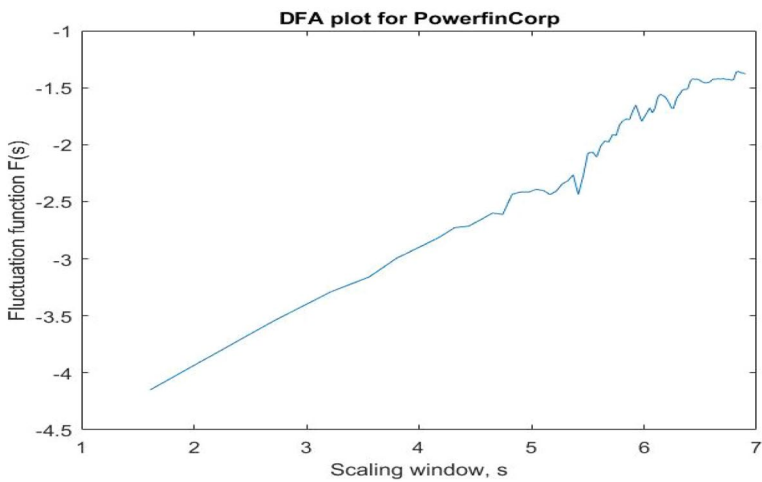
Fig. 8 Log–log plot of fluctuation function for Power Fin Corp
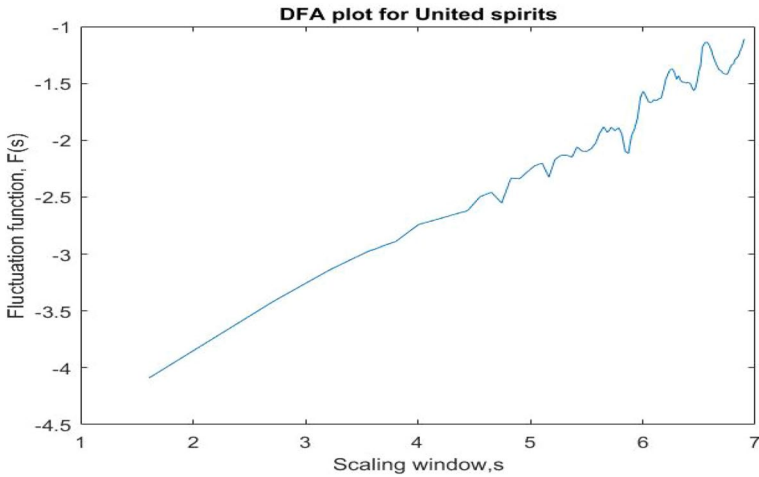
Fig. 9 Log–log plot of fluctuation function for United Spirits
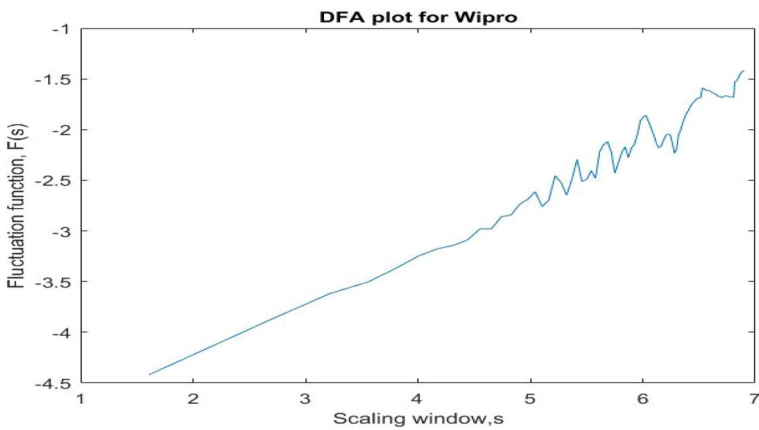
Fig. 10 Log–log plot of fluctuation function for Wipro

Appendix 2: *q*- order Hurst exponent *H*(*q*) on order parameter *q* computed by MFDFA is plotted for the asset returns of top ten NIFTY index stocks

See Figs. 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20.
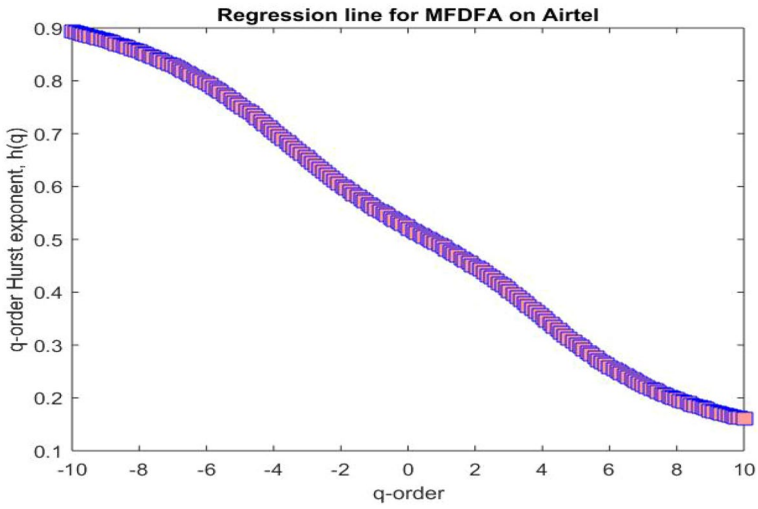
Fig. 11 Plot of q-order Hurst exponent, $H(q)$ for Airtel
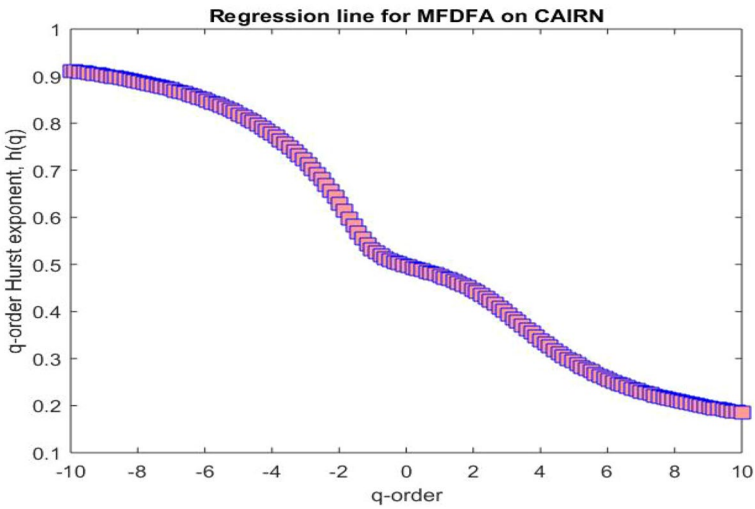
Fig. 12 Plot of q-order Hurst exponent, $H(q)$ for CAIRN
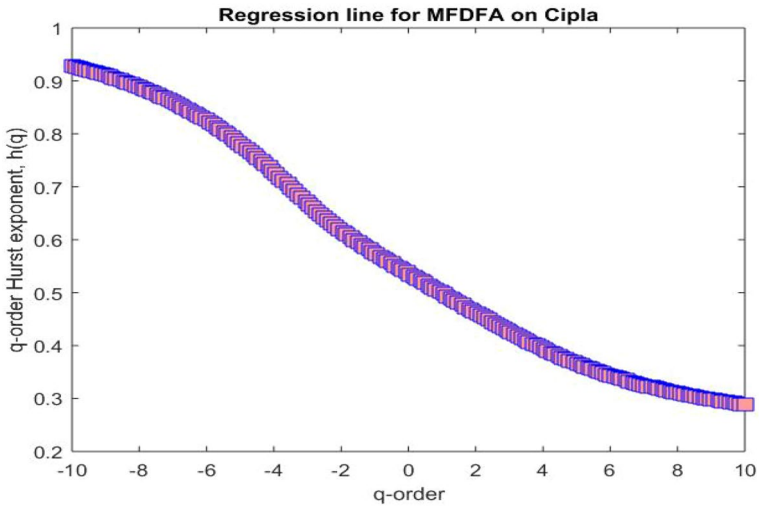
Fig. 13 Plot of *q*-order Hurst exponent, *H*(*q*) for Cipla
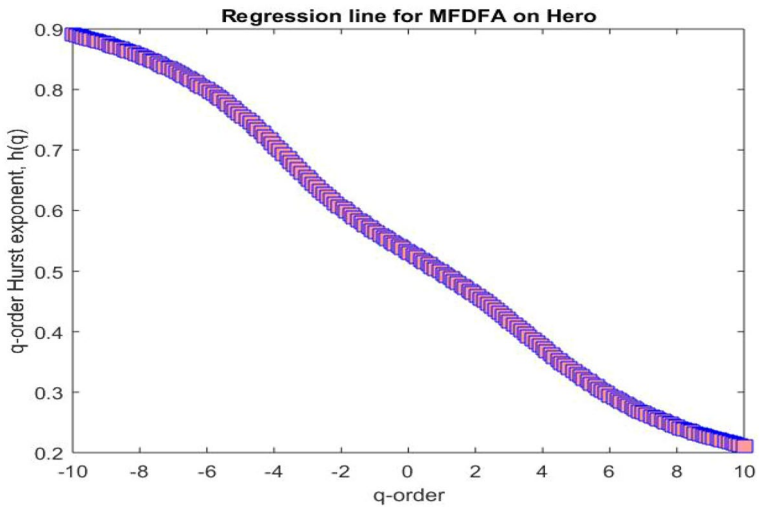
Fig. 14 Plot of *q*-order Hurst exponent, *H*(*q*) for Hero
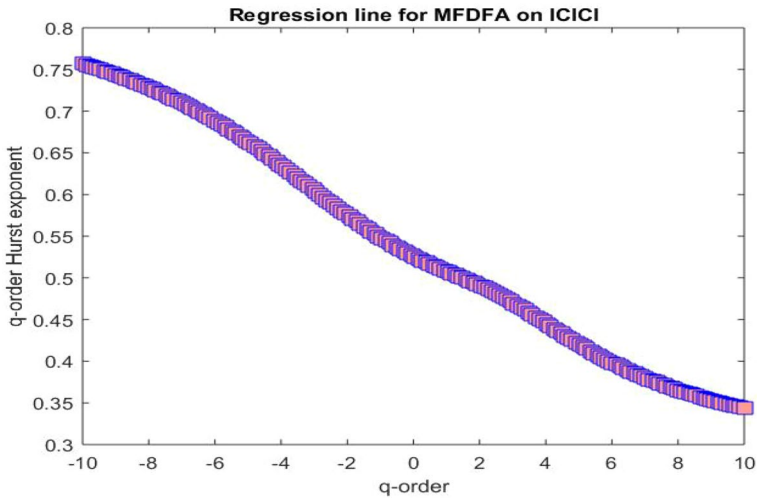
Fig. 15 Plot of *q*-order Hurst exponent, *H*(*q*) for ICICI
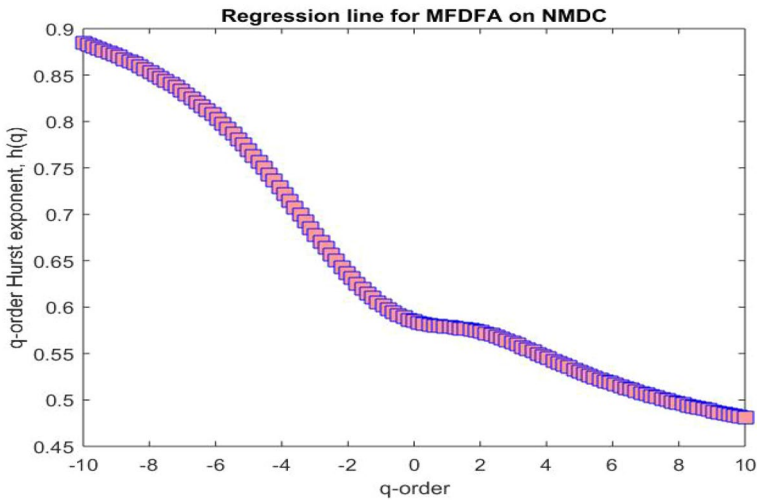
Fig. 16 Plot of *q*-order Hurst exponent, *H*(*q*) for NMDC
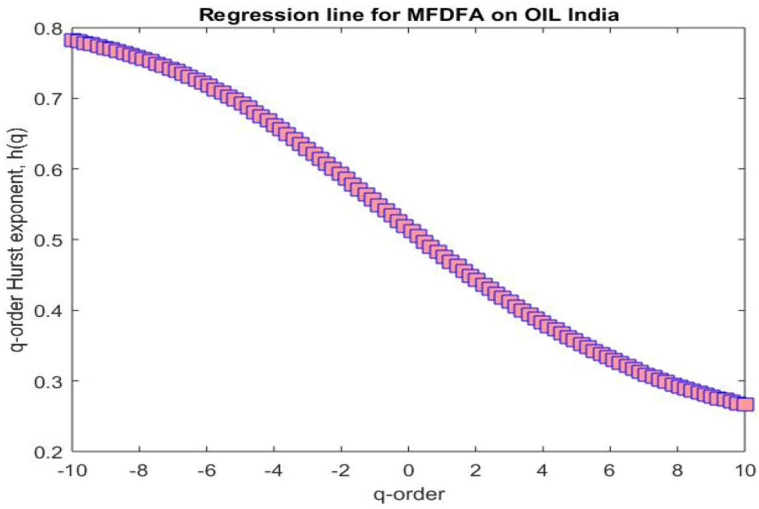
Fig. 17 Plot of q-order Hurst exponent, $H(q)$ for OIL India
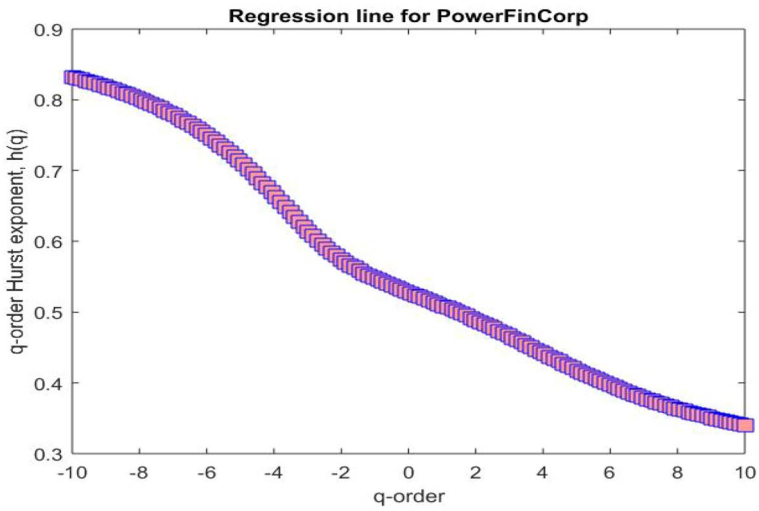
Fig. 18 Plot of q-order Hurst exponent, $H(q)$ for Power Fin Corp
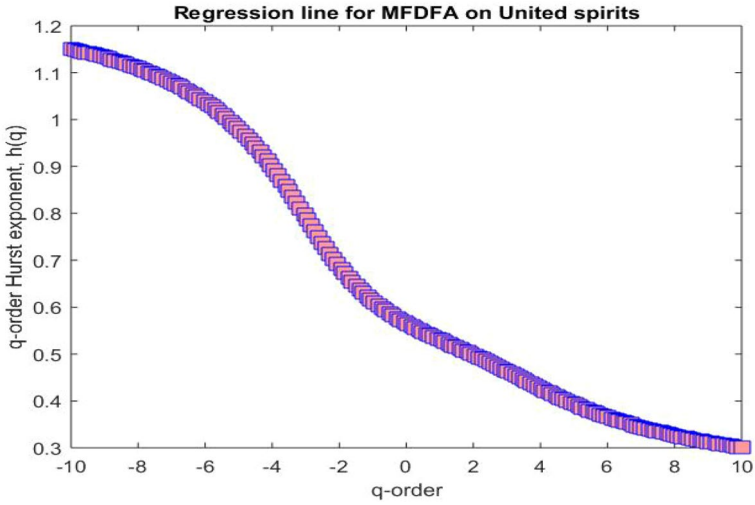
Fig. 19 Plot of *q*-order Hurst exponent, *H*(*q*) for United Spirits
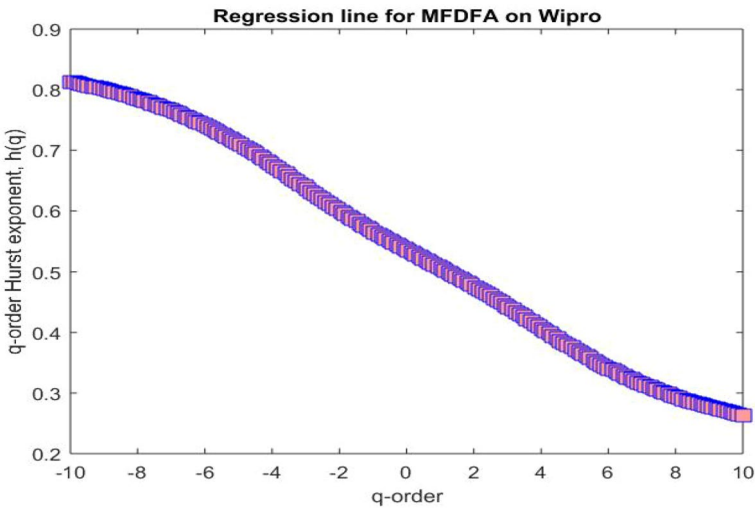
Fig. 20 Plot of *q*-order Hurst exponent, *H*(*q*) for Wipro

Appendix 3: Multifractal spectra plots for the asset returns of top ten NIFTY index stocks

See Figs. 21, 22, 23, 24, 25, 26, 27, 28, 29 and 30.
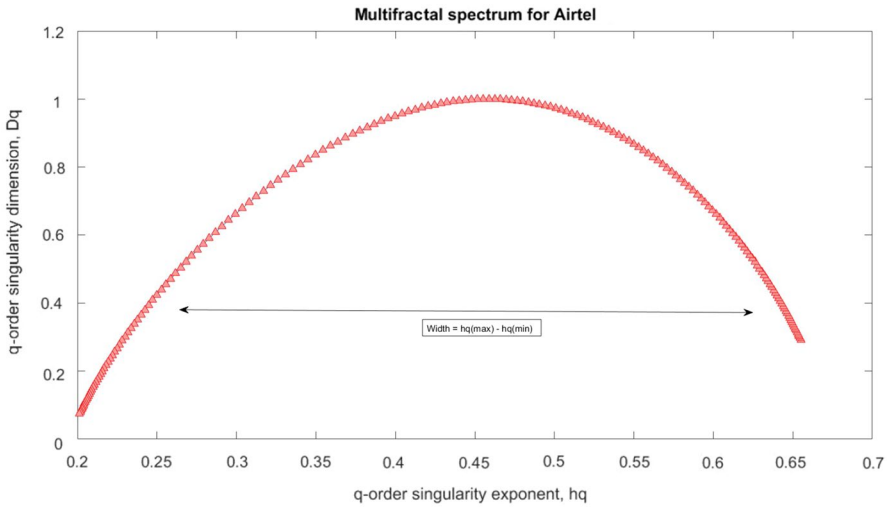
Fig. 21 Plot of multifractal spectrum for Airtel
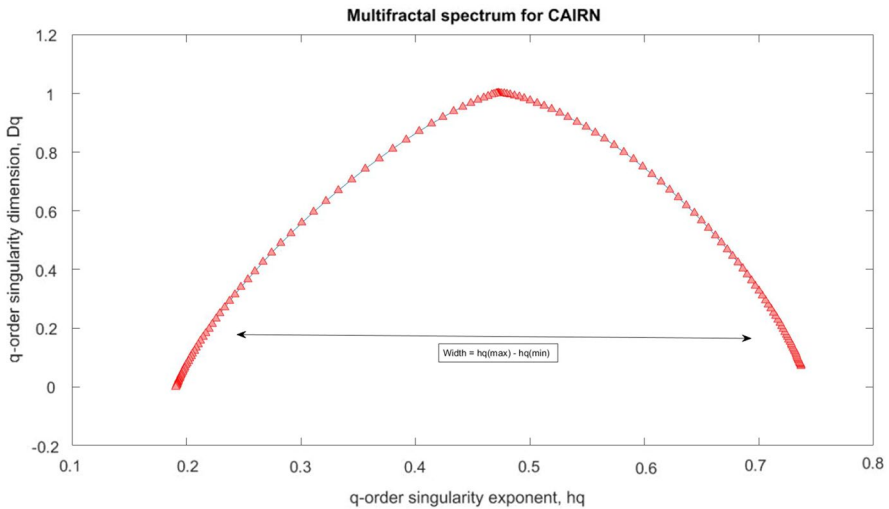
Fig. 22 Plot of multifractal spectrum for CAIRN

Multifractal spectrum for Cipla

Fig. 23 Plot of multifractal spectrum for Cipla
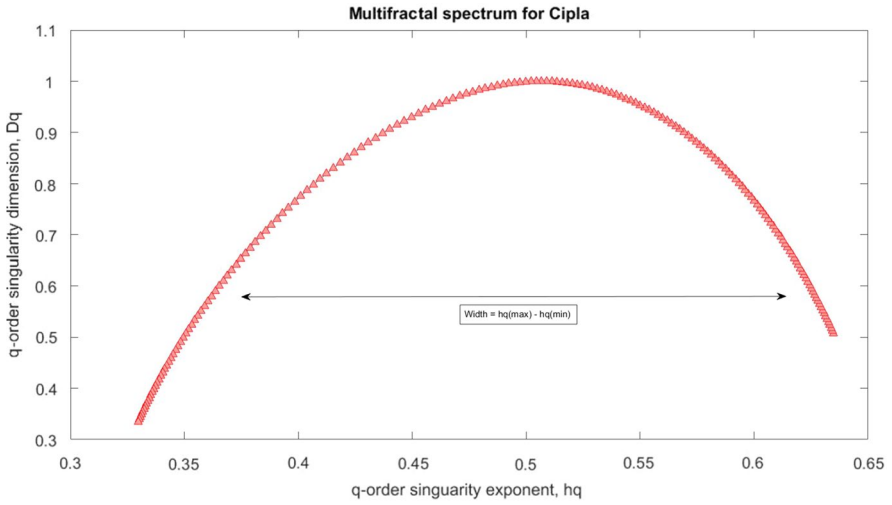
Multifractal spectrum for Hero
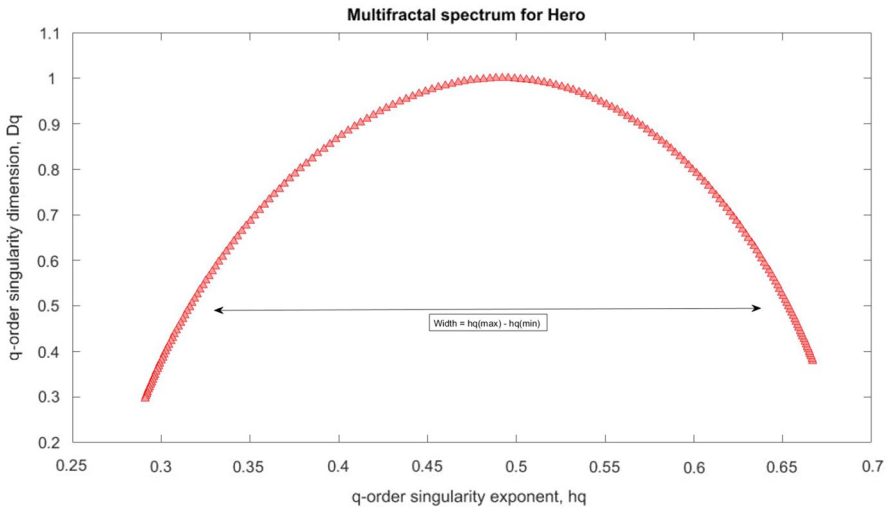
Fig. 24 Plot of multifractal spectrum for Hero
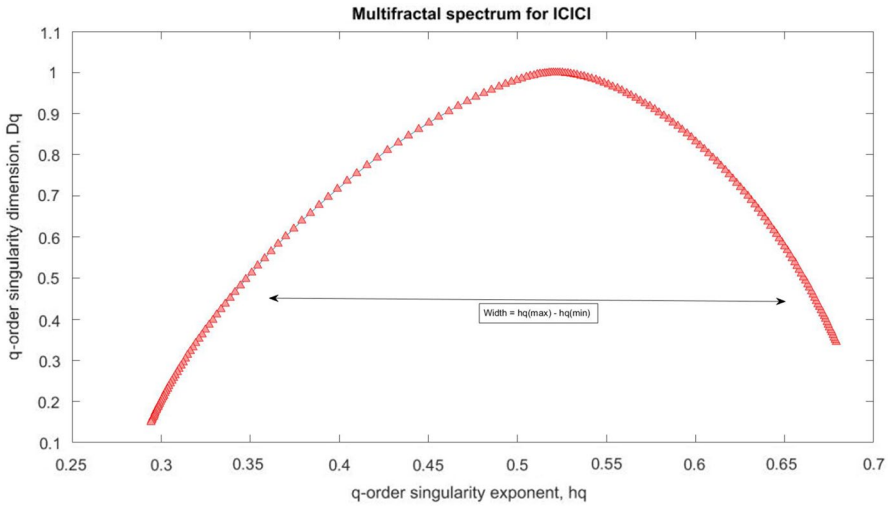
Fig. 25 Plot of multifractal spectrum for ICICI
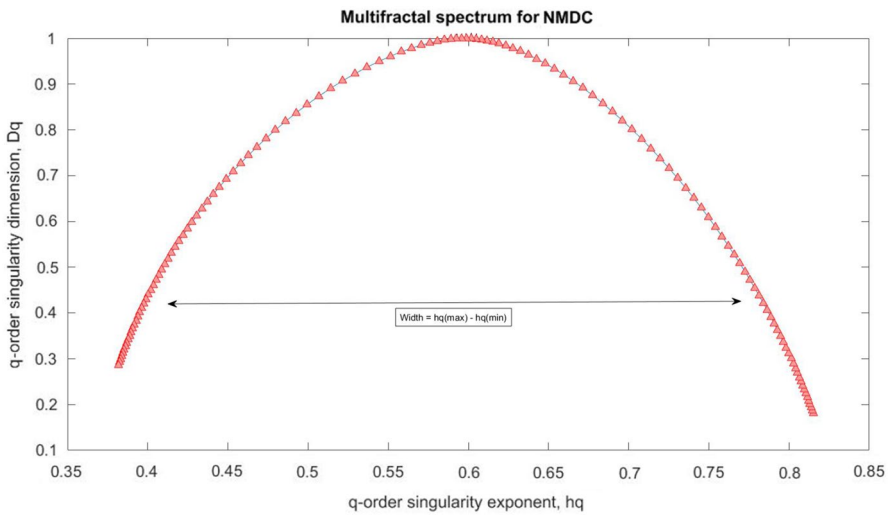
Fig. 26 Plot of multifractal spectrum for NMDC
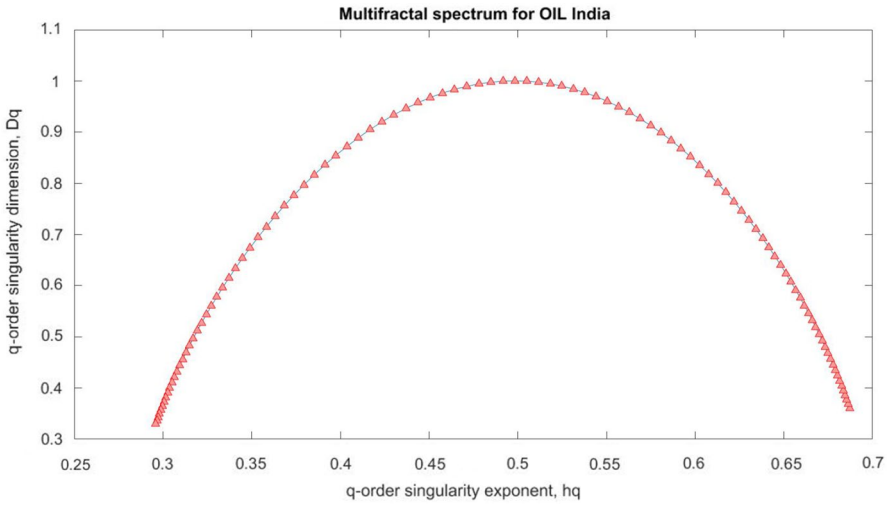
Fig. 27 Plot of multifractal spectrum for OIL India
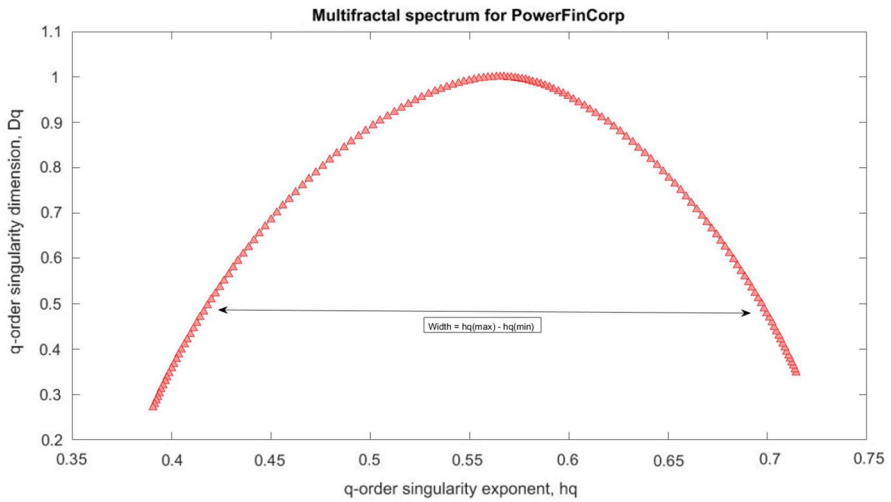
Fig. 28 Plot of multifractal spectrum for Power Fin Corp
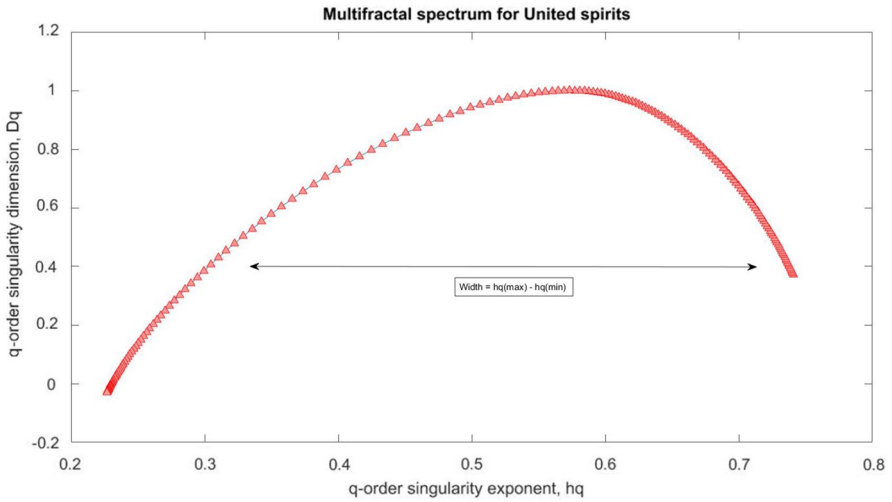
Fig. 29 Plot of multifractal spectrum for United Spirits
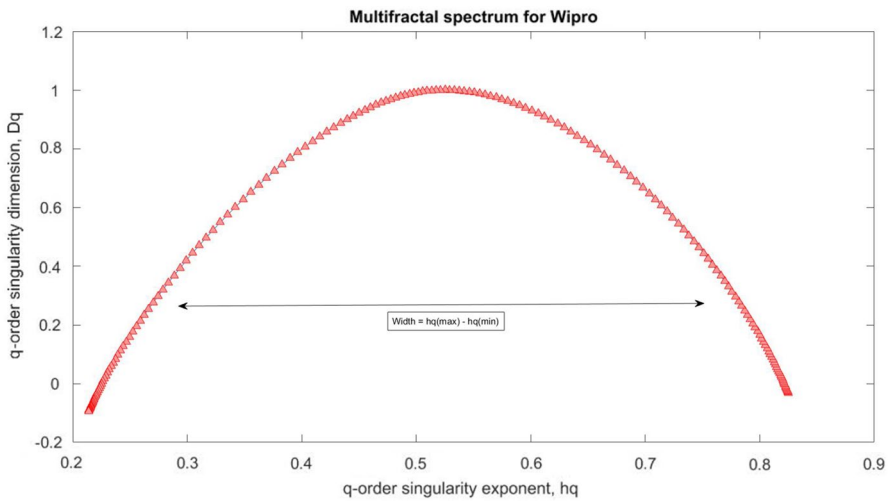
Fig. 30 Plot of multifractal spectrum for Wipro

Appendix 4: Generalised Hurst surface $H(q, s)$ for the asset returns of top ten NIFTY index stocks

See Figs. 31, 32, 33, 34, 35, 36, 37, 38, 39 and 40.
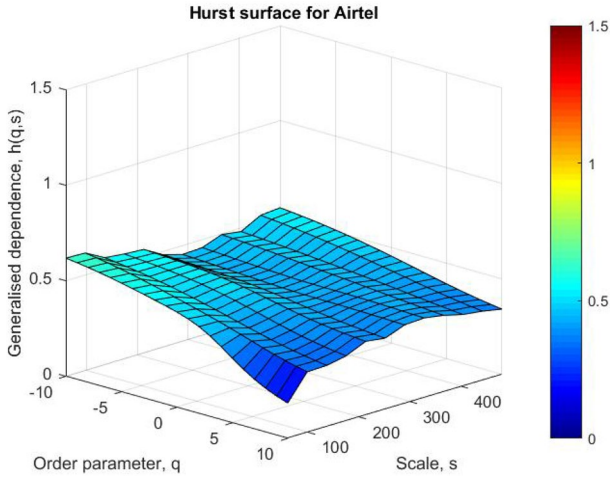
Fig. 31 Plot of the generalised Hurst surface, $H(q, s)$ for Airtel
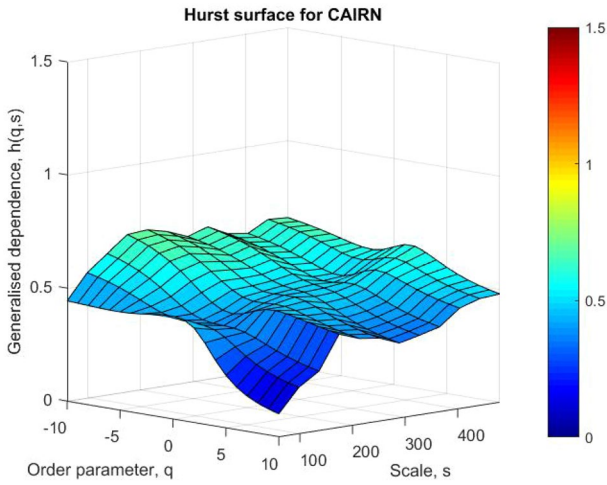
Fig. 32 Plot of the generalised Hurst surface, $H(q, s)$ for CAIRN
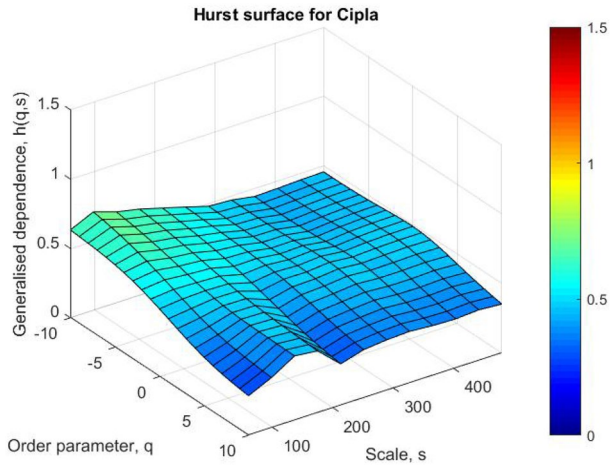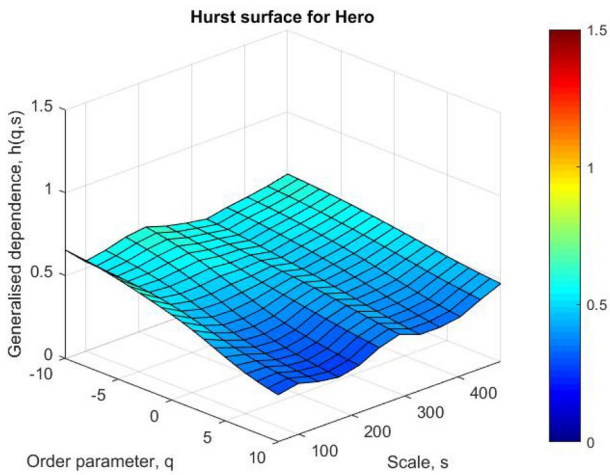
Fig. 33 Plot of the generalised Hurst surface, $H(q, s)$ for Cipla

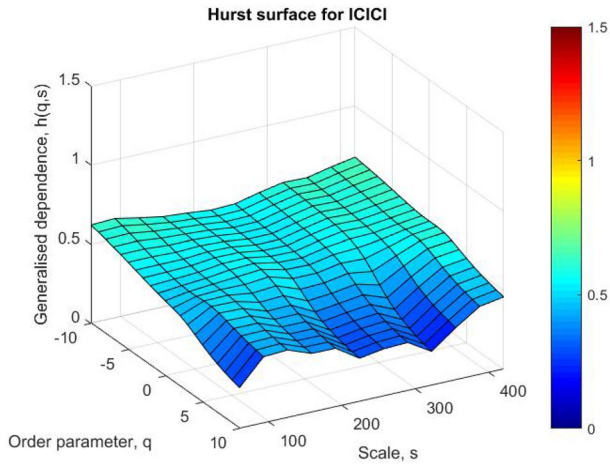**Fig. 34** Plot of the generalised Hurst surface, $H(q, s)$ for Hero

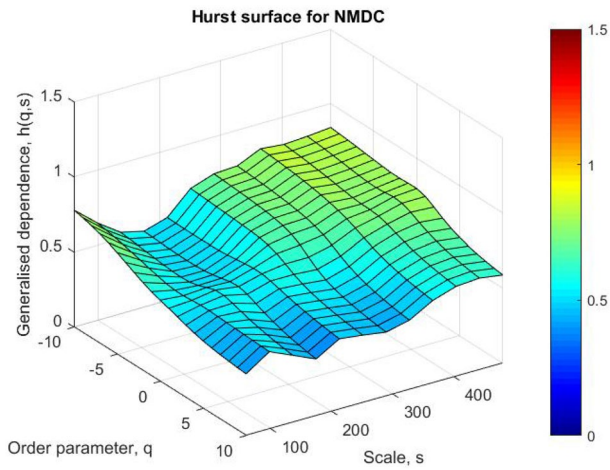**Fig. 35** Plot of the generalised Hurst surface, $H(q, s)$ for ICICI

Fig. 36 Plot of the generalised Hurst surface, $H(q, s)$ for NMDC
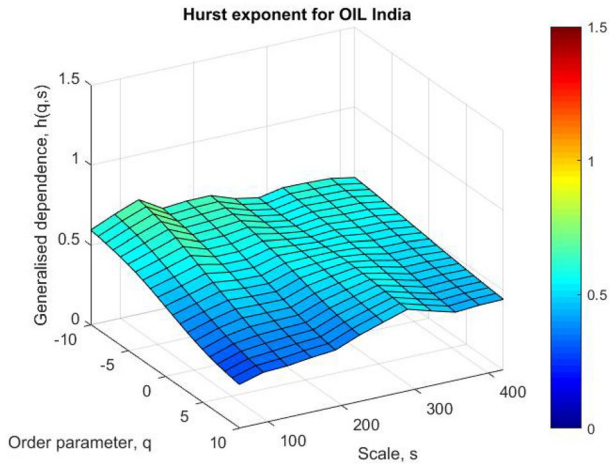
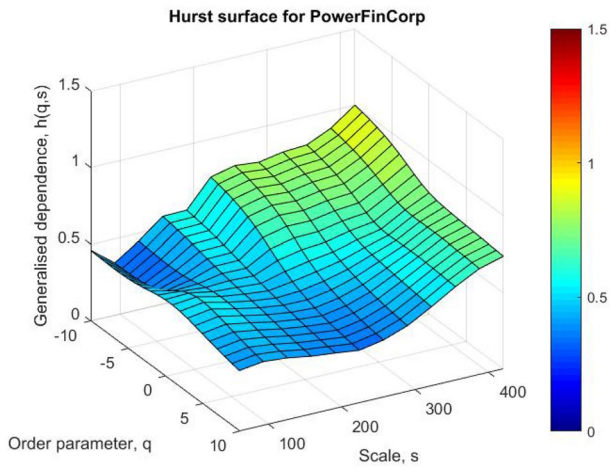**Fig. 37** Plot of the generalised Hurst surface, $H(q, s)$ for OIL India

Fig. 38 Plot of the generalised Hurst surface, $H(q, s)$ for Power Fin Corp
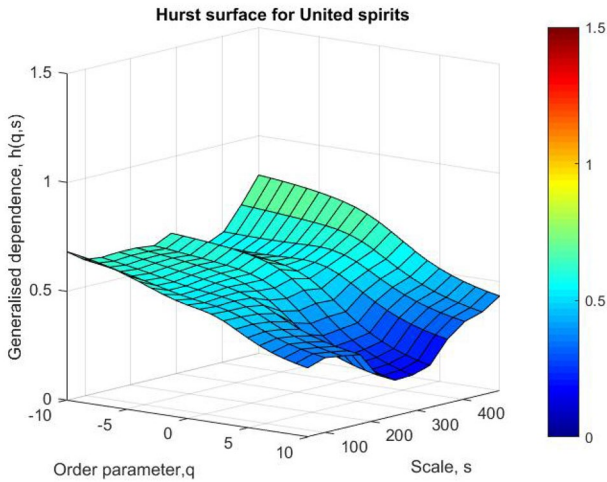
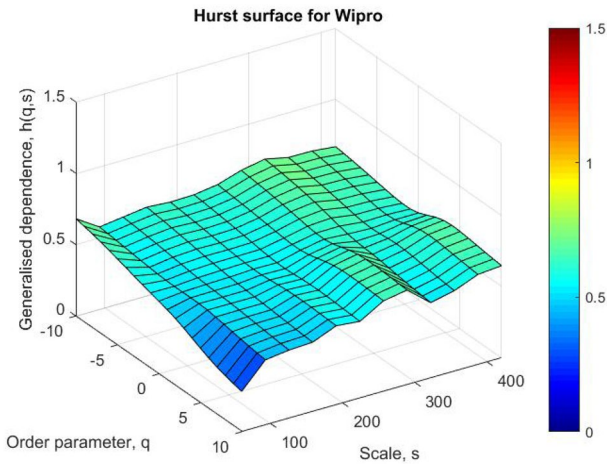**Fig. 39** Plot of the generalised Hurst surface, $H(q, s)$ for United spirits

Fig. 40 Plot of the generalised Hurst surface, $H(q, s)$ for Wipro

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