REVIEW

Computer Simulation of Protein Materials at Multiple Length Scales: From Single Proteins to Protein Assemblies

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Abstract

Computer simulation of protein materials and their dynamic or mechanical behavior is of high signifcance, as proteins perform their functions through their structural changes in response to a force (or stimulus). The computer simulation enables the detailed insight into the structure and behavior of proteins at atomistic resolution, which is inaccessible with experimental toolkits such as single-molecule experiments. With the advancement of computing resources, the computer simulation has recently played a vital role as a virtual microscopy in understanding and characterizing the structure and behaviors of protein materials at atomistic resolution. For examples, computer simulations allow for gaining insight into how some protein domains can exhibit the remarkable mechanical properties and functions. In this article, we would like to address the current state-of-arts of computer simulations that have been employed for studying the structure and properties (or behaviors) of proteins at multiple length scales ranging from single proteins to protein assemblies. Specifcally, we summarize various computational modeling techniques, ranging from atomistic models to coarse-grained models and continuum models, applicable for modeling protein structures at multiple length scales from single proteins to protein assemblies. This paper discusses how such various computational modeling/simulation techniques can be employed for studying the structure and properties of protein materials at multiple length scales. This paper sheds light on computer simulations, which are able to unveil the hidden, complex mechanisms related to the structure and properties of protein materials at multiple length scales.

Keywords Computer simulation · Protein materials · Multiple length scales · Atomistic simulation · Coarse-grained modeling · Protein mechanics

Introduction

It is of high importance to study the dynamic (or mechanical) behavior of a single protein molecule, as a protein molecule performs its function through its structural changes upon a chemical stimulus (e.g. ligand binding) and/or a force. For example, hemoglobin transports the oxygen when the oxygen is bound to the hemoglobin. The ability of hemoglobin to bind with oxygen is attributed to the existence of the hemoglobin's various conformations (structures), few of which may correspond to the conformation of oxygenbound hemoglobin. In other words, a protein molecule can explore many conformations, some of which corresponds to the ligand-bound state, because the energy landscape of a protein is very rugged in that there are multiple local minima (or equilibria), and that the energy barrier between these minima is comparable to the amount of thermal energy $[1-3]$ $[1-3]$. This unique dynamic feature of a protein molecule enables it to perform its function such as ligand binding or unbinding. Moreover, some protein domains are able to perform remarkable mechanical functions and to exhibit excellent mechanical properties. For instance, an immunoglobulin (Ig) domain in a muscle protein chain titin has a high resilience against a mechanical force in such a way that hydrogen bonds formed between the β-strands of an Ig domain are able to sustain a mechanical force $[4-7]$ $[4-7]$ $[4-7]$. In addition, the high resilience of a protein domain is also attributed to its unfolding behavior, that is, a force-driven unfolding behavior of a protein folded domain results in its energy dissipation, and consequently, the high mechanical resilience [[8](#page-22-4)]. These two representative examples suggest the necessity of studying

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the dynamic (or mechanical) behavior of proteins in order to gain a fundamental insight into their functions.

Protein assemblies have recently received signifcant attention due to their important role in disease pathology $[9-12]$ $[9-12]$ as well as their applicability in bionanotechnology [\[13–](#page-22-7)[15\]](#page-22-8). As a representative example, amyloid fibrils have been highlighted due to their role in the pathogenesis of various diseases ranging from neurodegenerative diseases [\[16,](#page-22-9) [17\]](#page-22-10) to type 2 diabetes [[18](#page-22-11)] and cardiovascular diseases [\[19\]](#page-22-12). The amyloid fbrils are formed through self-assembly process, that is, amyloidogenic proteins (e.g. denatured proteins) are aggregated so as to form the fibril structure $[10]$ $[10]$. Once amyloid fbrils are formed by self-assembly process, they are quite rigid and structurally stable in a physiological condition, resulting in the formation of amyloid plaques. These fbrils have recently suggested to play a role in disease pathologies due to their remarkable mechanical properties. Specifcally, the elastic modulus of amyloid fbrils is in a range of $1-10$ GPa $[20-22]$ $[20-22]$, which is a few orders of magnitude higher than the elastic modulus of cell membrane. The growth of an amyloid fbril on a cell membrane results in the mechanical disruption of cell membrane [[23](#page-22-16)] due to the diference between elastic moduli of amyloid fbril and cell membrane [[24](#page-22-17)]. In addition, a spider silk crystal, whose molecular structure is similar to that of amyloid fibril, possesses anomalous mechanical properties whose elastic modulus is in the order of $1-10$ GPa $[25]$ $[25]$. The remarkable properties of these protein assemblies (e.g. amyloid fbril, and spider silk crystal) led researchers to study the underlying principles showing how the excellent mechanical properties of protein assemblies can be determined.

For last two decades, single-molecule experimental techniques [[26\]](#page-22-19) based on atomic force microscopy (AFM) [\[27–](#page-22-20)[29](#page-22-21)] and/or optical tweezer [\[30\]](#page-22-22) have played a central role in probing the properties and functions of protein molecules. As an example, AFM-based single-molecule pulling experiments have allowed for gaining a quantitative insight into the response of a protein domain to a mechanical force. Specifcally, AFM-based pulling experiments are able to capture the force-driven unfolding event of a protein domain based on measuring the force-extension (equivalent to force–displacement) relationship of a protein domain during its unfolding process. Despite the ability of singlemolecule experimental techniques to quantitatively probe the properties of protein molecules, they are very restrictive in that they are unable to provide how the molecular structure of a protein molecule is changed (e.g. deformed) due to a force. In addition, these experimental techniques cannot unveil the underlying principles showing how the properties and behaviors of protein materials are determined. Due to these limitations of single-molecule experimental techniques, computer simulations can play an alternative role in revealing the underlying mechanisms of the dynamic (or mechanical) behaviors and properties of protein materials at atomistic resolution. In particular, computer simulations based on atomistic models are also able to provide a quantitative insight into the relationship between the molecular structure of protein materials and their properties [[31,](#page-22-23) [32](#page-22-24)]. Furthermore, computer simulations are capable of quantitatively understanding how the molecular structure of protein materials (e.g. protein assemblies) can be determined. The computer simulations can be implemented based on diferent modeling techniques at diferent length scales, ranging from atomistic model to coarse-grained model and continuum model. Here, we note that this paper excludes the details of a continuum mechanics-based simulation, which has recently been applied for studying the dynamics of proteins [[33,](#page-22-25) [34](#page-22-26)], while we briefy review the continuum model that is useful in analyzing the result of computer simulations based on atomistic or coarse-grained models.

In this work, we address the current state-of-arts of computational simulations of protein materials at multiple length scales, ranging from a protein domain to a protein assembly, for unveiling the hidden, complex mechanisms of how the remarkable properties (or functions) of protein materials are determined. In particular, we describe the current state-ofarts of computational simulations at multiple length scales based on atomistic model, coarse-grained model, and/or continuum modeling, which are applicable for understanding the structure–property-function relationship of protein materials at diferent length scales ranging from a single protein domain to protein assembly. In addition, we present the computer simulation-based analyses of single proteins such as their unfolding mechanics and/or conformational changes. Moreover, we introduced the computer simulationbased characterization of the structures and properties of protein assemblies such as amyloid fbrils and viral capsids. Finally, we conclude our paper with some remarks and outlooks for future works in computational modeling of protein materials.

Computational Methodologies

This Section demonstrates the fundamental principles of computational methodologies that have been applied for modeling protein materials. Here, we present the computational simulation methodologies at diferent length scales, based on atomistic model or coarse-grained model.

Atomistic Modeling/Simulation

Molecular Dynamics Simulations

The atomistic molecular dynamics (MD) simulation had been dated back to 1950s, when Alder and Wainwright [[35\]](#page-22-27)

frst considered the atomistic modeling for studying the phase transition of hard-sphere gas system. From 1950s to 1960s, the MD simulations were extensively employed for studying the phase transition of gas and solution. Not until 1970s were the MD simulations employed for modeling biological molecules such as DNA, RNA, and protein. In 1977, Karplus and colleagues [\[36\]](#page-22-28) had utilized the MD simulations for studying the conformational dynamics of bovine trypsin inhibitor (that is a very small protein molecule). In 1970s, Karplus [[37\]](#page-22-29), Levitt [[38\]](#page-22-30), and Warshel studied the dynamics of biological molecules (that is related to their function) by using the MD simulations, which led them to become a pioneer in the MD simulation-based study of biological molecules and their dynamics. The pioneering works of Karplus, Levitt, and Warshel allowed them to win a Nobel prize in Chemistry 2013 [\[39\]](#page-22-31). Since 2000s, the MD simulations have been extensively used for gaining a quantitative insight into the structure, dynamics, properties, and functions of biological molecules such as proteins, DNAs, and RNAs [\[40](#page-22-32)[–43](#page-22-33)].

The fundamental principle of atomistic MD simulation is to numerically solve the Newton's equation of motion for a molecular system composed of atoms. Specifcally, the position vector of each atom, which is a component of a molecular system, is numerically obtained as a function of time based on the numerical integration of the Newton's equation of motion with computing forces acting on each atom. Here, the forces acting on each atom originate from inter-atomic interaction, thermal energy, and interaction between the molecular system and an environment (e.g. water molecules) surrounding the molecular system. The equation of motion for the molecular system is written as [[44\]](#page-22-34)

$$
m_i \partial_t^2 \mathbf{r}_i = -\nabla_i V(\mathbf{r}_1, \dots, \mathbf{r}_N)
$$
 (1)

where \mathbf{r}_i is the position vector of an *i*-th atom in a molecular system, m_i is the mass of the *i*-th atom, *N* is the total number of atoms for the molecular system, *V* is the potential energy of the molecular system, and a symbol ∇*ⁱ* represents a spatial gradient with respect to position vector **r***ⁱ* . With a given initial position vector $\mathbf{r}_i(0)$ for the molecular system, the timedependent position vector $\mathbf{r}_i(t)$ of the molecular system can be obtained by the numerical integration of the equation of motion given by Eq. ([1\)](#page-2-0). The details of numerical integration scheme are well described in ref [[44\]](#page-22-34). The dynamic behavior of a biological molecule (e.g. protein) can be analyzed based on calculating the fuctuation matrix **Q** that consists of block matrices \mathbf{Q}_{ii} defined as [\[45](#page-22-35)]

$$
\mathbf{Q}_{ij} = \left\langle \left[\mathbf{r}_i(t) - \left\langle \mathbf{r}_i(t) \right\rangle \right] \otimes \left[\mathbf{r}_j(t) - \left\langle \mathbf{r}_j(t) \right\rangle \right] \right\rangle \tag{2}
$$

where a symbol ⊗ indicates a tensor product, and an angle bracket represents an ensemble average. The root meansquare fuctuation (RMSF) of an *i*-th atom can be measured as follows:

$$
\xi_i = \sqrt{tr \mathbf{Q}_{ii}} = \sqrt{\langle \left[\mathbf{r}_i(t) - \langle \mathbf{r}_i(t) \rangle \right] \otimes \left[\mathbf{r}_i(t) - \langle \mathbf{r}_i(t) \rangle \right] \rangle}
$$
\n
$$
= \sqrt{\langle \left[x_i(t) - \langle x_i(t) \rangle \right]^2 + \left[y_i(t) - \langle y_i(t) \rangle \right]^2 + \left[z_i(t) - \langle z_i(t) \rangle \right]^2 \rangle}. \tag{3}
$$

Here, x_i , y_i , and z_i represent the components of a position vector $\mathbf{r}_i(t)$ in the *x*-, *y*-, and *z*-directions, respectively. The RMSF of a protein is very useful in not only understanding the fuctuation motion of the protein but also validating the atomistic model by ftting the RMSF obtained from the MD simulation to that acquired from experiments based on nuclear magnetic resonance or X-ray crystallography. In addition, the correlated motion of a protein can be quantitatively described using a correlation factor (CF) defned as

$$
c_{ij} = tr\mathbf{Q}_{ij} / \sqrt{tr\mathbf{Q}_{ii} \cdot tr\mathbf{Q}_{jj}}.
$$
\n(4)

It should be noted that the value of CF is in a range of -1 to 1. The value of CF being +1 indicates that two atoms *i* and *j* move collectively (i.e. together) in the same direction, while the value of CF being -1 means that these two atoms move in the opposite direction, and the value of CF being zero implies that the motion of an atom *i* is not correlated with that of other atom *j*.

For a quantitatively analysis of the mechanical response of a protein to a force (as can be obtained from single-molecule pulling experiment), the potential feld should be modifed by including the strain energy stored in a force probe that is used to pull the protein.

$$
U = V(\mathbf{r}_1, \dots, \mathbf{r}_N) + \frac{K}{2} (R_{ij} - vt)^2
$$
\n(5)

where *K* is the stiffness of a force probe, R_{ii} is a distance between two atoms, one of which is fxed and the other is pulled by a force probe, *v* is a pulling rate, and *t* is the time. The equation of motion for a protein that is pulled by a force probe is given by

$$
m_i \partial_t^2 \mathbf{r}_i(t) = -\nabla_i U(\mathbf{r}_1, \dots, \mathbf{r}_N, t).
$$
 (6)

The numerical simulation of solving the Newton's equation of motion given by Eq. (6) (6) for studying the mechanical response of a protein is referred to as steered molecular dynamics (SMD) simulations [[46](#page-22-36)]. The SMD simulations enables the acquisition of the force–displacement relationship of a biomolecule (e.g. DNA, RNA, and proteins), and consequently, the extraction of the mechanical properties of a biomolecule.

Normal Mode Analysis

Over the last two decades, normal mode analysis (NMA) has played a pivotal role in understanding the dynamic behavior of a protein structure [[47](#page-22-37)], as the NMA coupled with statistical mechanics theory has allowed for calculating the fuctuation matrix of the protein structure as long as its stifness matrix is evaluated. For an insight into a relationship between the fuctuation behavior of a protein and its stifness, we consider a simple molecular system composed of two identical atoms (i.e. diatomic molecule). The potential feld of a diatomic molecule is approximated as harmonic potential given by $V = (\gamma/2)u^2$, where γ is a force constant, and u is a distance between two atoms. The kinetic energy of the diatomic molecule is given as $L = (m/2)v^2$, where *m* is the atomic mass, and *v* is the velocity defined as $v = du/dt$. The statistical mechanics theory provides a probability distribution, $p(u, v)$, to find a specifc position and velocity (*u*, *v*) on the phase space given as

$$
p(u, v) = Z^{-1} \exp(-H/k_B T) = Z^{-1} \exp(-\{L + V\}/k_B T)
$$
\n(7)

where k_B and T are the Boltzmann's constant and absolute temperature, respectively, *H* is the Hamiltonian of the diatomic molecule, and *Z* is the partition function defned as

$$
Z = \iint\limits_{-\infty}^{\infty} \exp\left[-H(u,v)/k_B T\right] du dv.
$$
 (8)

The ensemble averages of the distance and the meansquare distance are obtained as follows:

$$
\langle u \rangle = Z^{-1} \iint\limits_{-\infty}^{\infty} u \cdot \exp\left[-\{L(v) + V(u)\}/k_B T\right] du dv = 0 \quad (9. a)
$$

$$
\langle u^2 \rangle = Z^{-1} \left[\int_{-\infty}^{\infty} \exp(-mv^2/2k_B T) dv \right].
$$

$$
\left[\int_{-\infty}^{\infty} u^2 \exp(-\gamma u^2/2k_B T) du \right] = \gamma^{-1} k_B T.
$$
 (9.b)

The statistical mechanics theory given by Eq. $(9.b)$ suggests that the fuctuation behavior of a molecular system is dependent on its stifness in such a way that the meansquare fuctuation of the system is inversely proportional to its stifness.

Now, we consider a molecular system that is composed of *N* atoms. For the dynamic motion of a protein near its equilibrium state, its potential feld can be approximated to a quasi-harmonic potential such as [\[48](#page-22-38)[–50](#page-22-39)]

$$
V = \frac{1}{2} \mathbf{r}^T \mathbf{K} \mathbf{r}
$$
 (10)

where **K** is the stifness matrix of a molecular system, **r** is the atomic coordinates of the system, and a superscript *T* indicates a transpose of a vector. With a spectral

decomposition of the stiffness matrix **K** such as $\mathbf{K} = \mathbf{P}^T \Lambda \mathbf{P}$, where Λ is a diagonal matrix consisting of eigenvalues λ_j for the stifness matrix and **P** is the modal matrix, the potential energy can be rewritten as

$$
V = \frac{1}{2} \mathbf{y}^T \Lambda \mathbf{y} = \frac{1}{2} \sum_{j=1}^{3N-6} \lambda_j y_j^2.
$$
 (11)

Here, **y** is a transformed coordinate defined as $y = Pr$. It should be noted that summation in Eq. (11) (11) excludes six zero normal modes (i.e. rigid body modes). The statistical mechanics theory provides a probability function $p(y)$ to find a specifc state **y** given by

$$
p(\mathbf{y}) = Z^{-1} \exp\left[-\frac{1}{2} \left(k_B T \right)^{-1} \sum_{j=1}^{3N-6} \lambda_j y_j^2 \right].
$$
 (12)

From a probability function $p(y)$ given by Eq. ([12\)](#page-3-2), the ensemble averages of quantities, y_i and $y_i \cdot y_j$, can be found as follows.

$$
\langle y_i \rangle = \iint\limits_{-\infty}^{\infty} y_i \cdot p(\mathbf{y}) d\mathbf{y} = 0
$$
 (13.a)

$$
\langle y_i \cdot y_j \rangle = \int_{-\infty}^{\infty} y_i \cdot y_j \cdot p(\mathbf{y}) d\mathbf{y} = \lambda_i^{-1} k_B T \cdot \delta_{ij}
$$
 (13.b)

where δ_{ij} is a Kronecker delta defined as $\delta_{ij} = 1$ if $i = j$, otherwise $\delta_{ii} = 0$. Here, we note that the repeated index in Eq. [\(13.b\)](#page-3-3) does not imply the Einstein summation rule. With using a transformation $\mathbf{r} = \mathbf{P}^T \mathbf{y}$, the fluctuation matrix **Q** of a molecular system can be written as

$$
\mathbf{Q} = \sum_{j=1}^{3N-6} \lambda_j^{-1} k_B T \mathbf{P}_j \otimes \mathbf{P}_j = \mathbf{P}^T \Lambda^{-1} \mathbf{P}
$$
 (14)

where P_j is the *j*-th column vector of a modal matrix P , that is, P_j is the *j*-th normal mode of the system.

Coarse‑Grained Modeling

Principle of Coarse‑Graining

For more than three decades, coarse-grained (CG) modeling techniques have been established and widely utilized for understanding the dynamic behavior of a large protein structures [\[51](#page-22-40)[–53](#page-22-41)]. Though a full atomistic MD simulations have recently allowed for gaining insight into the dynamics of a small protein (consisting of less than few hundreds of residues) at millisecond time scales [\[54](#page-22-42), [55\]](#page-22-43), the full atomistic

MD simulations are still computationally restrictive in characterizing the dynamic behavior of a large protein structures at long timescale [\[56](#page-22-44)]. In order to overcome the computational limitation of atomistic MD simulations, CG modeling approaches have been widely developed in such a way that a group of atoms, which move collectively, are replaced by a single bead (particle), and that the motion of such beads (that represent the groups of atoms) is numerically predicted based on Newtonian equation of motion while the interaction potential energies between beads have to be properly defned.

The process to construct a CG model from atomistic model is as follows: As briefy demonstrated above, a group of atoms are replaced with a single bead as long as such atoms undergo a similar (or mostly collective) motion. Specifcally, a group of atoms are selected based on a criterion given by Eq. (15) (15)

$$
\zeta^2 = \tau^{-1} \int_0^{\tau} \sum_{i,j \in \Omega} \left| \mathbf{r}_i(t) - \mathbf{r}_j(t) \right| < \varepsilon \tag{15}
$$

where Ω represents the set of atoms that are grouped, and τ is a short simulation time scale (e.g. \sim 1 ns). Here, all-atom MD simulation is used with a short timescale (e.g. \sim 1 ns) in order to measure a criterion given by Eq. [\(15](#page-4-0)). Once the group of atoms are replaced with a single bead, its position vector can be obtained from a following equation.

$$
\mathbf{R}_{j}(t) = \sum_{l=1}^{n_{j}} m_{l} \mathbf{r}_{l}(t) \Bigg/ \sum_{l=1}^{n_{j}} m_{l}. \tag{16}
$$

Here, subscript *j* indicates the *j*-th group of atoms, m_l and **r***l* are the molecular weight and position vector of the *l*-th atom, respectively, and n_j is the number of atoms in the *j*-th group. It should be noted that the position vector of a bead (representing a group of atoms) is equivalent to the center of mass for the atoms that are grouped. Once the groups of atoms are replaced with CG beads, the efective potential feld that can be prescribed to CG beads has to be defned. The effective potential field can be written as

$$
V = \sum_{j=1}^{N-1} \frac{k_D}{2} \left(D_j - D_j^0 \right)^2 + \sum_{j=1}^{N-2} \frac{k_D}{2} \left(\Omega_j - \Omega_j^0 \right)^2 + \sum_{j=1}^{N-3} \frac{k_D}{2} \left(\Phi_j - \Phi_j^0 \right)^2
$$

+
$$
\sum_{j=1}^{N-1} \sum_{k=j+1}^{N} X_{jk}^{-1} A Q_j Q_k + \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} 4E_0 \left[\left(Y/X_{jk} \right)^6 - \left(Y/X_{jk} \right)^{12} \right] \tag{17}
$$

where k_D , k_Ω , and k_Φ are the force constants for strain energies corresponding to the stretching of a pseudo-covalent bond, bending of a pseudo-bond angle, and twisting of a pseudo-dihedral angle, respectively, *N* is the total number of CG beads, D_j is the distance of a pseudo-covalent bond such as $D_j = |\mathbf{D}_j| = |\mathbf{R}_{j+1} - \mathbf{R}_j|$, Ω_j is a pseudo-bond angle defined as $cos\Omega_j = D_j \cdot D_{j+1}/D_j D_{j+1}$, Φ_j is a pseudo-dihedral angle

given as $cos\Phi_j = (\mathbf{D}_j \times \mathbf{D}_{j+1}) \cdot \mathbf{D}_{j+2} / ((\mathbf{D}_j \times \mathbf{D}_{j+1}) \cdot \mathbf{D}_{j+2}$, Q_j is an effective charge acting on a *j*-th CG bead, X_{ik} is a distance between two CG beads *j* and *k* such as $X_{ik} = |\mathbf{R}_i - \mathbf{R}_k|$, E_0 is the energy scale of van der Waal's interaction between CG beads, and Y is a length scale for van der Waal's interaction. The major procedure of CG modeling is to determine the parameters of effective potential field given by Eq. (17) (17) . The effective potential field can be validated by comparing the results of CG model-based simulations with those of atomistic MD simulations.

Once CG model is constructed as described above, the dynamics of a CG model can be analyzed based on either NMA or MD-like simulation. In particular, the NMA can be applied to the CG model while the stifness matrix of CG model is estimated based on the position vector of CG beads. Here, the stifness matrix of CG model (**K**) is composed of (off-diagonal) block matrices \mathbf{K}_{ii} given by

$$
\mathbf{K}_{ij} = -\nabla_i \nabla_j V(\mathbf{R}_i, \dots, \mathbf{R}_N).
$$
 (18)

Here, gradient ∇_i is defined with respect to the coordinates of CG beads, such as $\nabla_i = \partial/\partial \mathbf{R}_i$. As a consequence, once the eigen-values and their corresponding normal modes are computed, the statistical mechanics theory given by Eq. [\(14\)](#page-3-4) enables the quantitation of the fuctuation matrix **Q** for the CG model.

On the other hand, with empirical potential feld prescribed to the CG model given as Eq. (17) (17) (17) , the Newtonian equation of motion for the CG model can be described as follows.

$$
M_i \partial_t^2 \mathbf{R}_i = -\nabla_i V(\mathbf{R}_i, \dots, \mathbf{R}_N).
$$
 (19)

Here, M_i is the effective molecular weight of the *i*-th CG bead. The time-dependent trajectories of CG beads are obtained by numerically integrating the Newtonian equation of motion given by Eq. (19) (19) . This method is referred to as coarse-grained MD (CG-MD) simulations.

Go‑Like Model

In late 1970s, Go and coworkers [[57](#page-22-45)[–59\]](#page-22-46) had developed a CG model in such a way that the alpha carbons of a protein structure are selected to represent the CG beads of the CG model, while the other atoms are neglected. The empirical potential feld prescribed to CG beads is defned as follows.

$$
V = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[\left\{ \frac{k_1}{2} \left(d_{ij} - d_{ij}^0 \right)^2 + \frac{k_2}{4} \left(d_{ij} - d_{ij}^0 \right)^4 \right\} \delta_{j,i+1} + 4e_0 \left\{ \left(\sigma / d_{ij} \right)^{12} - \left(\sigma / d_{ij} \right)^6 \right\} \left(1 - \delta_{j,i+1} \right) \right]
$$
(20)

where *N* is the total number of CG beads, d_{ij} is the distance between two CG beads *i* and *j*, k_1 and k_2 are force constants for harmonic and quartic potential, respectively, for covalent bond, e_0 and σ are the energy parameter and length scale, respectively, for non-bonded interaction, superscript 0 indicates the equilibrium state, and $\delta_{i,j}$ is a Kronecker delta defined as $\delta_{i,j} = 1$ if $i = j$; otherwise, $\delta_{i,j} = 0$.

For recent decades, Go-like model has been extensively employed for studying the mechanical response of a protein molecule due to a force. For simulating the force-driven structural change of a protein molecule, the Langevin equation of motion was considered as follows [\[60](#page-22-47)].

$$
m_i \partial_t^2 \mathbf{r}_i = -\nabla_i U(\mathbf{r}) - \eta \partial_t \mathbf{r}_i + \mathbf{f}_i(t). \tag{21}
$$

Here, m_i and \mathbf{r}_i are the mass and position vector of an *i*th CG bead, respectively, η is a friction coefficient, $\mathbf{f}_i(t)$ is a Gaussian random force acting on the *i*-th CG bead, and *U*(**r**) is an effective potential field defined as $U = V + V_L$ with *V* and V_L being the potential field of the CG protein structure given by Eq. [\(20\)](#page-4-3) and a potential feld for a force probe, respectively. It should be noted that **f***ⁱ* (*t*) satisfes the fuctuation–dissipation theorem such as $\langle \mathbf{f}_i(t) \cdot \mathbf{f}_i(\tau) \rangle = 2\eta k_B T \delta(t - \tau)$), where k_B and *T* are the Boltzmann's constant and absolute temperature, respectively, an angle bracket \lt indicates an ensemble average (equivalent to the time average for the sufficient long-time simulation for an ergodic system), and $\delta(t)$ is the Dirac delta function. In addition, we note that the potential feld of a force probe is typically assumed to a harmonic potential given as $V_L = (K/2)(d_{1,N} - vt)^2$, where *v* is a pulling speed with which a protein is extended, and *K* is the stifness of a force probe. By numerically solving the Langevin equation given by Eq. (21) (21) , one is able to obtain the dynamic trajectory of a coarse-grained protein structure in response to a force-driven extension with a given pulling speed, which results in gaining the force-extension (i.e. force–displacement) curve for a protein structure, and consequently, its physical properties (e.g. elastic modulus).

Elastic Network Model

Though the Go-like model [\[57–](#page-22-45)[59\]](#page-22-46) is able to depict the dynamic and mechanical response of a protein structure, it is still computationally limited for studying the dynamic behavior of a large protein complex (or assembly), whose size is much larger than that of a single protein. In order to reduce a computational cost for simulating the dynamic behavior of large protein complex, Tirion [\[61](#page-22-48)] introduced an elastic network model (ENM), which assumes a harmonic potential for neighboring alpha carbon atoms. Bahar and colleagues [[62\]](#page-22-49) developed a Gaussian network model (GNM), in which a protein structure is described as a network of Gaussian chains that are widely used in rubber elasticity. Despite the inability of GNM to dictate the directionality of normal modes, GNM

was extensively utilized for studying the fuctuation dynamics and its correlated functions of large protein complexes. Atilgan et al. [[63\]](#page-22-50) introduced an anisotropic network model (ANM) which is able to depict the anisotropic fuctuation behavior of a protein structure. Here, we note that ANM is conceptually identical to ENM. The normal modes of ENM was found to be useful in predicting the conformational changes of a large protein complex due to a stimulus (such as ligand-binding).

As described above, the key concept of ENM modeling is to construct a harmonic potential feld prescribed to alpha carbon atoms such as $[61–65]$ $[61–65]$ $[61–65]$ $[61–65]$

$$
V = \frac{\gamma}{2} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(d_{ij} - d_{ij}^0 \right)^2 \cdot H \left(R_c - d_{ij}^0 \right) \tag{22}
$$

where γ is a force constant, R_c is a cut-off distance (usually given by $R_c = 7{\text -}10$ Å), and $H(x)$ is a Heaviside unit step function defined as $H(x) = 1$ if $x > 0$; otherwise, $H(x) = 0$.

In case of GNM, the Kirchhoff matrix of a protein structure is given by [[62\]](#page-22-49)

$$
\Gamma_{ij} = \frac{\partial^2 V}{\partial u_i \partial u_j}\bigg|_{\mathbf{r} = \mathbf{r}^0} = -\gamma H \Big(R_c - d_{ij}^0\Big) \cdot \left(1 - \delta_{ij}\right) + \delta_{ij} \sum_{k \neq i} \gamma H \Big(R_c - d_{ik}^0\Big). \tag{23}
$$

Here, u_i is the fluctuation of the *i*-th alpha carbon atom (i.e. CG bead) defined as $u_i = |\mathbf{r}_i - \mathbf{r}_i^0|$ with \mathbf{r}_i being the position vector of the *i*-th alpha carbon atom. Consequently, as described earlier, the fuctuation matrix of a protein structure can be obtained as follows.

$$
\mathbf{Q}_{ij}^{GNM} = \left\langle (\mathbf{r}_i - \mathbf{r}_i^0) \cdot (\mathbf{r}_j - \mathbf{r}_j^0) \right\rangle = \sum_{k=2}^N \lambda^{-1} k_B T \mathbf{q}_i^k \mathbf{q}_j^k \qquad (24)
$$

where λ_k is the *k*-th eigen-value of the Kirchhoff matrix, and \mathbf{q}_i^k is the *i*-th component of the *k*-th normal mode of the Kirchhoff matrix. With obtaining the fluctuation matrix from GNM, one is able to easily evaluate the RMSF and CF with using Eqs. (3) (3) and (4) (4) , which are useful in describing the fuctuation motion and correlated motion of a protein structure.

On the other hand, ANM (or ENM) assumes the anisotropic fuctuation behavior of a protein structure, while GNM presumes the isotropic fuctuation of the protein structure. The harmonic potential given by Eq. ([22](#page-5-1)) can be rewritten as [[63](#page-22-50)]

$$
V = \frac{1}{2} \sum_{i,j} \left(\mathbf{u}_i - \mathbf{u}_j \right)^T \mathbf{K}_{ij} \left(\mathbf{u}_i - \mathbf{u}_j \right)
$$
 (25)

where \mathbf{u}_i is the fluctuation vector of the *i*-th alpha carbon atom defined as $\mathbf{u}_i = \mathbf{r}_i - \mathbf{r}_i^0$ with \mathbf{r}_i and superscript 0 being the position vector of the *i*-th alpha carbon atom and equilibrium state, respectively, and \mathbf{K}_{ii} is the off-diagonal (3×3) stifness matrix given by

$$
\mathbf{K}_{ij} = -\gamma \left| \mathbf{d}_{ij}^0 \right|^{-2} H \left(R_c - d_{ij}^0 \right) \cdot \mathbf{d}_{ij}^0 \otimes \mathbf{d}_{ij}^0. \tag{26}
$$

Here, \mathbf{d}_{ii} is a distance vector (between two alpha carbon atoms *i* and *j*) defined as $\mathbf{d}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, and a symbol \otimes indicates a tensor product. Once the off-diagonal stiffness matrices are estimated, it is straightforward to obtain the (3 *N*×3 *N*) stifness matrix **K** for the protein structure. As mentioned earlier, the fuctuation matrix of a protein structure can be acquired as follows.

$$
\mathbf{Q} = \left\langle \left(\mathbf{r} - \mathbf{r}^0 \right) \otimes \left(\mathbf{r} - \mathbf{r}^0 \right) \right\rangle = \sum_{k=7}^{3N} \xi_k^{-1} k_B T \mathbf{v}^k \otimes \mathbf{v}^k \qquad (27)
$$

where **r** is the position vector of all alpha carbon atoms, defined as $\mathbf{r}=[\mathbf{r}_1, \dots, \mathbf{r}_N], \xi_k$ is the *k*-th eigen-value of the stiffness matrix \mathbf{K} , and \mathbf{v}^k is the *k*-th normal mode of the stiffness matrix **K**. Here, it should be noted that in Eq. (27) (27) , six zero modes corresponding to the rigid body motion of a protein structure are excluded for computing the fuctuation matrix **Q**.

Continuum Model

Though computer simulations based on atomistic and/or CG models are able to provide the fundamental insight into the dynamic and mechanical behavior of protein structures at molecular level, a continuum model is useful in analyzing the results of atomistic MD simulations of a large protein structure (e.g. protein fbril). For example, while the mechanical response of a protein fbril can be obtained from all-atom MD simulations or CG-MD simulations, its mechanical properties such as elastic modulus and fracture stress can be extracted from a continuum model such as Euler–Bernoulli beam model. This Section briefy reviews the continuum models that can be employed for characterizing the mechanical properties of protein structures ranging from a single protein to protein assembly.

Polymer Chain Statistics

Polymer chain model has been useful in characterizing the properties of one-dimensional biological structure such as fber-like structure [\[66](#page-22-52)]. Specifcally, polymer chain model allows one to measure the thermal fuctuation behavior of a biological structure resulting in the evaluation of the persistent length of the biological structure. Here, the persistent length is defined as $l_p = D/k_B T$, where *D* is the bending rigidity of the biological structure. Among polymer chain models, wormlike chain (WLC) model has been successfully able to capture the fuctuation behavior of a various biological molecules such as DNA and protein fbril. Statistical mechanics theory based on WLC model provides the mean-squared value of end-to-end distance (R) in the form of $[67]$ $[67]$

$$
\langle R^2 \rangle / l^2 = 4(l_p/l) \left[1 - 2(l_p/l) \left\{ 1 - \exp(-l/2l_p) \right\} \right]
$$

in two-dimensional space (28)

where *l* is the contour length of a biological structure. There are two ways to measure the persistent length of a biological molecule—the frst approach is to place a biological molecule on the surface, and then its end-to-end distance is measured as a function of the contour length by atomic force microscopy (AFM) imaging $[68]$ $[68]$. The alternative approach is to fix the one end of a biological molecule, while its other end is labeled with a fuorescence dye, and then the fuctuation behavior of the free end of the biological molecule is estimated [[69\]](#page-22-55).

For last two decades, single-molecule force spectroscopy based on AFM or optical tweezer has played a vital role in characterizing the mechanical response and properties of protein materials. For example, the mechanical behavior of a muscle protein titin, which is responsible for the elasticity of a muscle fber, has been analyzed based on a single-molecule force spectroscopy experiment [\[4](#page-22-2), [70–](#page-22-56)[73\]](#page-22-57). In addition, the mechanical response of a DNA molecule, which is associated with gene transcriptions, has been characterized by a single-molecule force experiment [\[30,](#page-22-22) [74](#page-23-0)[–76](#page-23-1)]. It is shown that when a protein is extended (with a small amount of displacement before the folded domain of a protein is denatured), the mechanical response of the protein is well dictated by WLC model, which provides a relation between the force and displacement such as [[77\]](#page-23-2)

$$
Fl_p/k_B T = (u/l) + \frac{1}{4}(1 - u/l)^{-1} - \frac{1}{4}
$$
\n(29)

where F is the force acting on the molecule, and u is the displacement (i.e. end-to-end extension) of the molecule. WLC model is very useful in extracting the mechanical property (i.e. persistent length) of a biological molecule when it is stretched by a single-molecule force spectroscopy.

Elastic Beam Model

Though the computer simulations based on atomistic or CG models are able to depict the mechanical response of a protein, a continuum elasticity theory is necessary in characterizing the mechanical properties of the protein. For example, the mechanical property of a one-dimensional structure such as protein fbril [[78\]](#page-23-3) (or nanoscale system [[79\]](#page-23-4)) can be measured by considering an elastic beam model such as Euler–Bernoulli beam model. In particular, the elastic beam model allows for evaluating the mechanical property (e.g. elastic modulus) of a protein fbril based on its vibrational characteristics (e.g. natural frequencies) or its mechanical deformation behavior (i.e.

force–displacement relation), which can be obtained from computer simulations based on atomistic or CG models.

The vibrational behavior of a protein fibril can be described by the equation of motion for Euler–Bernoulli beam model such as [[80](#page-23-5)]

$$
\rho A \partial_t^2 w(x, t) + \partial_x^2 \left[EI \partial_x^2 w(x, t) \right] = 0 \tag{30}
$$

where ρ , A , E and I represent the density, cross-sectional area, elastic modulus, and cross-sectional moment of inertia for a protein fibril, respectively, $w(x, t)$ is the transverse deflection of the fibril, x is a coordinate defined along the longitudinal direction of the fbril, and *t* is the time. The frequency of the fbril is written in the form of

$$
\omega = (\lambda/L)^2 \sqrt{EI/\rho A} \tag{31}
$$

Here, *L* is the length of the fibril, and λ is an eigenvalue that depends on the boundary condition. From Eq. (31) (31) , one is able to estimate the (bending) elastic modulus of a protein fbril as long as the frequency of the fbril is measured from computer simulations based on atomistic or CG models. The Euler–Bernoulli beam theory provides the relationship between the force and displacement for the fbril such as [[81\]](#page-23-6)

$$
w(x_0) = \alpha(x_0) \cdot (FL^3/EI) \tag{32}
$$

where x_0 is the location at which a force *F* is applied, and $\alpha(x_0)$ is a constant that is a function of x_0 and a boundary condition. If the force–displacement relation of a protein fbril is measured based on the bending deformation of the fbril with using computer simulations based on atomistic models, it is straightforward to evaluate the bending elastic modulus of the fbril based on Eq. ([32](#page-7-1)). In addition, for a recent decade, Timoshenko beam model has been highlighted in order to understand the role of the length of a protein fbril in its bending elastic property [\[69](#page-22-55)]. Timoshenko beam model provides the persistent length (l_n) of a protein fbril in the form of [\[69,](#page-22-55) [82](#page-23-7)]

$$
l_p = l_p^{\infty} \left[1 + (a/b) \left(c l_p^{\infty} k_B T / G_S A L^2 \right) \right]^{-1}
$$
 (33)

 l_p^{∞} is the persistent length for a very long fibril, G_S is the shear modulus of the fbril, and parameters *a* and *b* are the constants that depend on the boundary condition, and a parameter *c* is a constant that is dependent on the crosssectional shape of the fbril. Here, we note that the bending rigidity of a protein fibril is given by $EI = l_p k_B T$.

Mechanical Characterization of a Single Protein

For last two decades, there are a lot of efforts that have been made to characterize the mechanical response of a biological molecule such as DNA and/or protein, as its mechanical response has been associated with its biological functions. For instance, during the gene transcriptions, the unwinding of double-stranded DNA molecule is necessary. The underlying mechanisms of DNA unwinding have been revealed by a single-molecule force spectroscopy experiment [\[26](#page-22-19), [30,](#page-22-22) [83](#page-23-8)] and/or computer simulations [\[84\]](#page-23-9) based on atomistic or coarse-grained models. In addition, some proteins have recently been found to bear a mechanical loading, which endows such proteins to perform a mechanical function. As an example, protein domains found in a muscle protein titin, which is responsible for the passive elasticity of a muscle, have been found to resist a mechanical force even in the order of 100 pN, which is much larger than a thermal fluctuation force typically in the order of 1 pN (see Fig. [1\)](#page-8-0) [[4,](#page-22-2) [70–](#page-22-56)[72,](#page-22-58) [85](#page-23-10)]. This implies that for gaining a fundamental insight into the design principles of mechanically strong proteins, the mechanical response of such proteins has to be characterized. Here, we review the mechanical response of a protein, particularly the mechanical unfolding of a protein domain and the conformational change of a protein, with using computer simulations based on atomistic or CG models.

Mechanical Unfolding of Proteins

The study on the mechanical unfolding of proteins based on all-atom MD simulations or CG-MD simulations has been of high importance, as this study allows for not only fundamental insights into the mechanical stability, unfolding, and folding of protein molecules, but also validation of the physical theory such as reaction-rate theory.

The single-molecule force spectroscopy has allowed for characterizing the mechanical unfolding behavior of proteins such as immunoglobulin (Ig)-like domains [\[4](#page-22-2), [70](#page-22-56)–[72,](#page-22-58) [85](#page-23-10)] and fbronectin3 (fn3) domains [[6,](#page-22-59) [86,](#page-23-11) [87\]](#page-23-12) found in a muscle protein titin. It is shown that the force–displacement curve of Ig-like domains resembles the sawtooth-like curve such that force peaks in the sawtooth-like force curve correspond to the unfolding forces, at which each Ig-like domain unfolds (Fig. [1\)](#page-8-0). Before a protein domain is unfolded by a force, the force–displacement of the domain is well depicted by WLC model, i.e. Equation ([29](#page-6-1)). For more than two decades, steered molecular dynamics (SMD) simulation [[46\]](#page-22-36) has allowed for unveiling the underlying mechanisms of protein unfolding process. In particular, though a singlemolecule force spectroscopy experiment is able to actually measure the mechanical response of a protein (e.g. Ig-like domain), such an experiment does not provide any detailed insight into how some proteins exhibit remarkable mechanical properties. The underlying mechanisms of how a protein domain is able to resist a force can be unveiled by SMD simulations. Specifically, SMD simulations are able to show

Fig. 1 Atomic force microscopy (AFM)-based single-molecule force spectroscopy experiments of a protein chain made of a few immunoglobulin (Ig) domains. **a** Experimental results of force-extension curves for a protein chain composed of 8 Ig domains. **b** Six independent single-molecule force spectroscopy experimental results are superimposed. **c** Force-extension curves for a protein chain consisting of 4 Ig domains. **d** Six independent experimental results of forceextension curves are superimposed. **e** Force-extension curves for

a protein chain made of 4 Ig domains (top) or 8 Ig domains (middle), and native titin (bottom). The distance between two neighboring force peaks is measured as \sim 25 nm, which indicates that each force peak corresponds to the force, at which a single Ig domain is fully unfolded. **f** Schematic illustration showing the mechanical response of a protein chain made of Ig domains. Figure is adopted with permission from Ref. [\[4\]](#page-22-2). Copyright (1997) The American Association for the Advancement of Science

Fig. 2 Steered molecular dynamics (SMD) simulation on the mechanical unfolding of immunoglobulin-1 (I1) domain. **a** Forceextension curve for I1 domain that is extended with a pulling rate of 0.001 nm/s. **(b)** The molecular structures of I1 domain at initial state

(i), frst force peak (ii), and other force peaks (iii) and (iv). When I1 domain is stretched, hydrogen bonds (indicated by red-dashed lines) between β strands are ruptured so that I1 domain begins to be unfolded

the unfolding pathway of a protein, that is, the time evolution of the protein structure due to a force (Fig. [2](#page-8-1)) [\[7,](#page-22-3) [88](#page-23-13)[–90\]](#page-23-14).

Although all-atom SMD simulations are useful in understanding the unfolding mechanisms of a protein, they are computationally restrictive in analyzing the unfolding process of a large protein complex. In order to overcome this computational restriction, CG-MD simulations have been extensively employed, for more than a decade, for gaining insight into the unfolding mechanisms of a protein. Cieplak and coworkers [[60,](#page-22-47) [91](#page-23-15)] have utilized the Go-like model for simulating the unfolding process of a protein molecule. They showed that the force–displacement response of a protein domain obtained from CG-MD simulations based on Go-like model is comparable to that acquired from a single-molecule force spectroscopy, and that Go-like model is able to provide the unfolding pathway of a protein domain (Fig. [3a](#page-9-0)). Due to the computational efficiency of Go-like model, it has been widely considered for studying how the thermal or mechanical stability of a protein domain is determined. It is shown that, from the results of Go-like model, the thermal or mechanical unfolding pathway of a protein domain is governed by its native topology [[91\]](#page-23-15). Cieplak and colleagues [\[92](#page-23-16)] studied the mechanical unfolding responses of \sim 17,000 protein domains using CG-MD simulations based on Go-like model in order to gain a fundamental insight into the role of the topology of protein domains in their mechanical unfolding responses. Based on the simulations based on Go-like model with considering \sim 17,000 protein domains, Cieplak et al. [[92\]](#page-23-16) suggested the optimal topology of a protein structural motif, which maximizes the unfolding force of protein domain (Fig. [3](#page-9-0)b). In addition, they have also investigated the mechanical stability of a protein knot, which occupies $< 1\%$ of whole protein structures, using CG-MD simulations based on Go-like model [\[93](#page-23-17)]. They found that the presence of knot improves the stability of a protein domain in such a way that the knot increases the life time of intermediate unfolding states during the unfolding process of protein domain. A previous study by Sulkowska et al. [\[94\]](#page-23-18) reported that, by using Go-like model, the higher force applied to a knotted protein domain, the longer intermediate and metastable state, which is attributed to the high force-driven jamming of the knot resulting in the longer lifetime of intermediate unfolding states.

In recent years, Thirumalai and colleagues [[95–](#page-23-19)[97](#page-23-20)] developed a CG model, that is, self-organized polymer (SOP) model, whose potential feld is conceptually similar to that of Go-like model. Specifcally, SOP model considers the potential feld that consists of the energies associated with covalent bonds and native contacts, whereas the mathematical form of the potential feld of SOP is quite diferent from that of Go-like model. Thirumalai and coworkers [\[95](#page-23-19)] have shown that, by using CG-MD simulations based on SOP model, there are possible multiple unfolding pathways for a protein domain such that the alternative unfolding pathway of green fuorescence protein (GFP), which is inaccessible with conventional all-atom MD simulations, can be observed by SOP model. This is attributed to the fact that a pulling rate used in all-atom MD simulations is a few orders of magnitude larger than that employed in CG-MD simulations based on SOP model and/or single-molecule force spectroscopy experiments. The higher pulling rate used in all-atom MD simulations leads to the deterministic rupture of non-covalent bonds such as hydrogen bonds in a protein domain, while the lower pulling rate results in the stochastic rupture event of hydrogen bonds in the protein domain. In addition, a recent study by

Fig. 3 a Coarse-grained molecular dynamics (CG-MD) simulations based on Go-like potential model for mechanical unfolding of titin immunoglobulin (Ig) domain. The snapshot of titin Ig domain that is extended with 5 nm (top) or 15 nm (bottom) was shown from the results of CG-MD simulations based on Go-like model. Figures are adopted with permission from Ref. [[60](#page-22-47)]. Copyright (2002)

John Wiley and Sons. **b** Optimal structural motifs that are found in mechanically strong proteins are obtained from CG-MD unfolding simulations of $>17,000$ protein structures. Top five structural motifs that are responsible for high mechanical strength. Figures are adopted from Ref. [[92](#page-23-16)] under Creative Commons Attribution License

Thirumalai et al. [\[98](#page-23-21)] has interestingly shown the ability of SOP model-based simulations to unveil the refolding mechanisms of a protein domain under the quench of a mechanical force. Specifcally, the refolding dynamics of force-driven unfolded protein under the signifcant decrease of a force applied to the protein from stretching force (with amount of \sim 100 pN) to quenched force (around 10 pN) was studied with using CG-MD simulations based on SOP model. It is shown that refolding time for a unfolded protein is dependent on the amount of quenched force in such a way that the quench force allows the unfolded protein to explore the intermediate, metastable unfolded states during the refolding of the unfolded protein [[98\]](#page-23-21). In addition, SOP model-based CG-MD simulations enabled Thirumalai and coworkers [[96](#page-23-22)] to study the force-driven hopping of RNA hairpin between two intermediate states. The SOP model-based simulation results provide the force-dependent time evolution of end-to-end distance for RNA hairpin, which is comparable to that obtained from single-molecule force spectroscopy experiments. This SOP model-based simulations allowed for accurate description of the folding kinetics of RNA hairpin with dependence on its amino acid sequence.

Moreover, computer simulations such as all-atom MD simulations and CG-MD simulations based on Go-like model (or SOP model) are very useful in validating a physical theory, which has played an important role in interpreting the results of single-molecule force spectroscopy experiments. For instance, the kinetics of dissociation of protein complex such as antigen–antibody complex was experimentally observed by a single-molecule force spectroscopy [[99](#page-23-23)], and this kinetics was successfully analyzed by a reaction-rate theory (ranging from Arrhenius law to Kramers' theory). In particular, Bell [[100](#page-23-24)] revisited

the Arrhenius law [[2](#page-22-60)] in order to theoretically obtain the relationship between the kinetic rate of molecular dissociation (i.e. bond rupture) and a force applied to induce the molecular dissociation event. The theory developed by Bell for molecular dissociation was verifed by not only a single-molecule force spectroscopy but also all-atom MD simulation. In addition, the reaction rate theory of protein unfolding has recently been developed by considering the effect of force probe $[101]$. Previous study by Freund $[101]$ $[101]$ and Arya et al. [\[102\]](#page-23-26) revisited the Kramers' theory [\[103\]](#page-23-27) with including the strain energy of a force probe (which is used to stretch a protein molecule) in the total potential energy for a force-driven protein unfolding, and reported that the stifness of a force probe afects the kinetic rate of protein unfolding, and consequently, the force required to unfold the protein. This theoretical prediction was validated by CG-MD simulations based on Go-like model. Specifically, our previous study [[104](#page-23-28)] employed CG-MD simulations based on Go-like model in order to verify the robustness of the theory developed by Freund [[101\]](#page-23-25) and Arya et al. [\[102](#page-23-26)], and found that the dependence of unfolding force on the stifness of a force probe acquired from CG-MD simulations based on Go-like model is comparable to that predicted by the theory. In addition, our CG-MD simulations show the efect of the stifness of a force probe in the unfolding pathway of a protein (Fig. [4](#page-10-0)).

Conformational Change of Proteins

Proteins perform their biological functions through their conformational changes driven by a ligand (or inhibitor). For instance, the conformational change of protein kinases is responsible for their biological function such as signaling

Fig. 4 Mechanical unfolding pathway for ubiquitin using **a** a soft force probe or **b** a stif force probe. When a ubiquitin is extended by a soft loading device, the hydrogen bonds between β 1 and β 5 begin to be ruptured at an extension of 20.5 nm. However, for the stretching of

ubiquitin using a stiff loading device, the hydrogen bonds between $β1$ and β5 are fractured at an extension of 6.6 nm. Figures are adopted with permission from Ref. [[104\]](#page-23-28). Copyright (2012) AIP Publishing

pathway (that is relevant for cellular function or dysfunction resulting in disease pathologies such as cancer) [[105\]](#page-23-29). This highlights the importance of predicting (or analyzing) the conformational change of proteins (due to ligand-binding) for not only further understanding of their biological functions but also the efective design of novel drug molecules (that are able to regulate the conformational change of proteins) [\[106](#page-23-30)].

Though all-atom MD simulations have recently been extensively employed for simulating the long time-scale dynamic behavior of small proteins [\[54](#page-22-42), [107\]](#page-23-31), they are still unable to simulate the conformational change of proteins, as the time-scale relevant to the conformational change of proteins is still larger than that accessible with currently available all-atom MD simulations. For a recent decade, ENM has been widely utilized for computationally efficient analysis of the conformational transitions of proteins. One may ask why ENM is able to predict the conformational change of proteins despite the simplicity of ENM modeling. This is attributed to the important role of the native topology of proteins in their dynamics [[108\]](#page-23-32). A previous study by Teeter et al. [[48\]](#page-22-38) reports that the conformational dynamics of proteins is very insensitive to the details of potential felds, which implies that the conformational dynamics of proteins is not afected by the details of inter-atomic interactions. In a similar spirit, Lu and Ma [[109\]](#page-23-33) showed that the conformational fuctuation behavior of protein is unafected by perturbation of Hessian (stifness) matrix as long as the native contacts of a protein are maintained. In addition, it has been reported that the thermal fuctuation of an unbound protein structure can render it to explore the conformation of a ligand-bound protein structure because of the feature of free energy landscape for the protein structure in such a way that the free energy landscape has multiple metastable, intermediate conformations, some of which may correspond to the conformation of ligand-bound state [\[1](#page-22-0)]. These previous studies [[1](#page-22-0), [48](#page-22-38), [109](#page-23-33)] validates the robustness of ENM modeling in analyzing the conformational dynamics of proteins such as their fuctuation and/or their conformational changes.

For a recent decade, the low-frequency normal modes of a protein predicted from ENM have been accepted to predict the conformational change of a protein. For example, Bahar and coworkers [\[110](#page-23-34)–[112\]](#page-23-35) have analyzed the conformational change of a protein based on the low-frequency normal modes obtained from ENM. In particular, they showed that the conformational change of a protein can be predicted based on low-frequency normal modes, with which the protein structure is perturbed (Fig. [5\)](#page-11-0). As described in Ref. [\[110\]](#page-23-34), the displacement of a protein corresponding to its conformational change is represented in the form of $\mathbf{u}_i = \sum_{k=1}^n \alpha_k \mathbf{q}_i^k$, where \mathbf{u}_i is the displacement of the *i*-th residue of the protein, \mathbf{q}_i^k is the *k*-th low-frequency normal mode vector for the *i*-th residue, α_k is a fitting parameter, and *n* is the number of low-frequency normal modes that are utilized to describe the conformational change. They have also shown that the conformational change of protein kinases upon inhibitor binding can be also predicted using the low-frequency normal modebased perturbation of the protein structure $[111]$ $[111]$. These previous studies by Bahar and coworkers [[110](#page-23-34), [111](#page-23-36)] have supported the hypothesis of population shift model, which suggests that the conformation of ligand-bound state can be accessible based on the conformation of an unbound

Fig. 5 a The structure of leukocyte Ig-like receptor (LIR) in unbound state (red) or ligand-bound state (blue). The bound state is referred to as the complex formed with HLA-A2 (black). **b** The diference in the alpha carbon atoms of LIR between unbound and bound states (black). This diference can be inferred from the structural perturbation of LIR in unbound state along the 3rd lowest-frequency normal

modes obtained from ENM. This indicates that the low-frequency normal modes of a protein in its unbound state allows it to explore its ligand-bound structure. Figures are adopted with permission from Ref. [\[110](#page-23-34)]. Copyright (2005) National Academy of Sciences, U.S.A (color fgure online)

protein due to the ruggedness of the free energy landscape of the unbound protein [\[1](#page-22-0)]. In a similar spirit, Karplus and colleagues [[113\]](#page-23-37) have predicted the conformational change of myosin V based on a projection of displacement vector corresponding to the conformation change into the normal mode space. They have provided that the conformational change of myosin V can be predicted with using few lowfrequency normal modes. Furthermore, Wolynes and coworkers [[114\]](#page-23-38) had developed a theoretical model to predict the conformational change of proteins in such a way that the low-frequency normal modes of ENM were employed to generate the intermediate conformations of the protein during its conformational transition.

Moreover, Maragakis and Karplus [[115](#page-23-39)] developed a plastic network model (PNM) in order to predict the conformational transition pathway of proteins. Specifcally, PNM was developed in such a way that the harmonic potential feld of intermediate state is obtained by mixing the harmonic potential felds (i.e. the potential felds of ENMs) corresponding to two end states, that are, unbound and ligandbound states. The harmonic potential feld of intermediate state is assumed to be in the form of $E = (1/2)[E_1 + E_2 ((E_1 - E_2)^2 + 4\varepsilon^2)^{1/2}$, where E_1 and E_2 represent harmonic potential felds (i.e. the potentials of ENM) corresponding to two end states, and *ε* is a parameter related to the degree of mixing two harmonic potential felds. The concept of PNM by mixing two potential felds was inspired by a quantum mechanical coupling of two potential energy surfaces. In a similar manner, Takagi et al. [\[116](#page-23-40)] suggested a dual Go-like model (DGM) by mixing two potential felds corresponding to two end conformations, while the potential feld of a protein structure is assumed to be in the form of Go-like model, i.e. Equation [\(20\)](#page-4-3), rather than the ENM-like harmonic potential. Furthermore, Hummer and colleagues [[117\]](#page-23-41) had developed a mixed elastic network model (MENM) to predict the conformational transition pathway of proteins by mixing two ENM harmonic potentials at two end states. The harmonic potential feld of intermediate state is presumed to be written as $E = -k_B T \ln[\exp((E_1 + \varepsilon_1)/k_B T) + \exp((E_2 + \varepsilon_2)/k_B T)]$, where ε_1 and ε_2 are parameters that are related to the degree of mixing two potential energies.

In addition, Jernigan and coworkers [[118\]](#page-23-42) introduced an elastic network interpolation (ENI) to acquire the conformational transition pathways of proteins. In their ENI scheme, a displacement vector \mathbf{u}_i^k corresponding to a conformational transition at the *k*-th intermediate state is obtained by minimizing a cost function (i.e. strain energy for conformational transition) defined as $E^k = (1/2) \sum \left[\left\{ \left(\mathbf{r}_i^k + \mathbf{u}_i^k \right) - \left(\mathbf{r}_j^k + \mathbf{u}_j^k \right) \right\} - d_{ij}^k \right]^2$, where \mathbf{r}_i^k is the position vector of *i*-th residue at *k*-th intermediate state. Here, d_{ij}^k is an interpolated inter-residue distance given by $d_{ij}^k = \alpha_k d_{ij}^0 + (1 - \alpha_k) d_{ij}^F$, where d_{ij} is a distance between two residues *i* and *j*, superscripts 0 and *F* indicate initial (unbound)

state and final (ligand-bound) state, respectively, and α_k is a parameter to indicate the *k*-th intermediate state obtained by linear interpolation between two states. Moreover, they have extended ENI coupled to a rigid cluster model [[119](#page-23-43)], in which the rigid domains of a protein are modeled as a rigid body, while the interaction between the rigid domains is described by elastic springs. A recent study by Kim et al. [\[120](#page-23-44)] has suggested a normal mode-guided elastic network interpolation (NGENI) in such a way that low-frequency normal modes are used in the linear interpolation for obtaining an intermediate conformation. In their work, it is shown that NGENI is more computationally efficient when compared with ENI for predicting the conformational transition pathways of large protein complexes. Kidera and colleagues [\[121\]](#page-23-45) developed a theoretical model to understand the conformational change of protein structures based on a linear response theory with ENM modeling. In their work [\[121\]](#page-23-45), a displacement vector \mathbf{u}_i for the conformational change is written as $\mathbf{u}_i = (k_B T)^{-1} \sum_k \mathbf{q}_i^k \lambda_k \sum_k$ *j* $\mathbf{q}^k_j \cdot \mathbf{f}_j$, where \mathbf{q}^k_i is a *k*-th normal mode vector for *i*-th residue, λ_k is the *k*-th eigenvalue of a covariance (compliance) matrix, which is a pseudo-inverse of the stifness (Hessian) matrix of ENM, and **f***^j* is a force vector acting on *j*-th residue due to ligand-binding. In a similar spirit, Demirel and Lesk [\[122\]](#page-23-46) employed ENM for evaluating a force associated with ligand-receptor binding. Specifcally, a force related to ligand-receptor binding is obtained based on a linear response theory in such a way that the force f_i is given by $\mathbf{f}_i = k_B T \sum_l \mathbf{q}_i^l \lambda_l^{-1} \sum_l$ *j* $\mathbf{q}^l_j \cdot \mathbf{u}_j$, where \mathbf{u}_j is the displacement vector of *j*-th residue (for a receptor) corresponding to the conformational change from unbound state to ligand-bound state.

In recent years, there are a few efforts that have been made to couple ENM-based structural perturbation scheme and all-atom molecular simulations for quantitatively understanding the conformational change of proteins. It should be noted that intermediate conformations predicted by the low-frequency normal modes of ENM may be sometimes physically unacceptable, because of the simplicity of ENM, which does not have any information for residue-specifc interactions, side chain-dependent interactions, and so on. Bahar and coworkers [[123\]](#page-23-47) had coupled the ENM modeling and all-atom MD simulations for predicting the conformational transition of proteins. Specifcally, the all-atom MD simulation was used to obtain an equilibrium structure for a protein, while ENM modeling was employed to introduce a structural perturbation of the protein. The iterative computations based on ENM normal mode-based perturbation followed by all-atom MD simulations are able to provide an insight into the conformational transition of proteins. In a similar spirit, Doruker et al. [\[124\]](#page-23-48) had combined ENM normal mode-based structural perturbation scheme and all-atom Monte-Carlo (MC) simulations for studying the conformational transition of proteins. In their work, the structural perturbation of a protein based on normal modes of ENM

followed by all-atom MC simulation was iteratively implemented. These studies [\[123,](#page-23-47) [124](#page-23-48)] show the possibility of coupling two diferent scale modeling techniques for studying the conformational (structural) changes of proteins in such a way that a CG model (e.g. ENM) was used to induce the structural perturbation of proteins while all-atom MD simulation was considered to gain their equilibrium structures. In other words, two diferent scale modeling techniques can be coupled so as to develop a novel multi-scale simulation techniques, which can provide an insight into protein dynamics typically inaccessible with only all-atom MD simulations.

Characterization of Protein Assemblies

Understanding the formation and properties of protein assemblies is of high importance for gaining fundamental insight into the biological function of these assembles such as disease pathologies (or etiologies). For example, amyloid materials are formed by self-assembly process of protein aggregation resulting in the diferent structural forms of amyloids ranging from nanometer to micrometer (or even larger) scales [\[10](#page-22-13), [20\]](#page-22-14). These amyloid materials have been known to play a crucial role in the pathogenesis of various diseases from neurodegenerative diseases [\[17\]](#page-22-10) to type 2 diabetes [[18](#page-22-11)] and cardiovascular diseases [\[19](#page-22-12)]. This suggests the necessity of unveiling how the amyloid materials are formed as an assembled structure driven by protein aggregation. That is, it is very essential to characterize the structural feature and formation of amyloid materials for understanding the underlying mechanisms of disease pathologies driven by protein assembly. In addition, it has recently been suggested that the mechanical properties of protein assemblies are responsible for their biological functions. For instance, the formation of mechanically stif amyloid fbrils on a mechanically soft cell membrane results in the mechanical disruption of the cell membrane [[24\]](#page-22-17). This Section presents the structural, mechanical, and dynamic characterization of protein assemblies such as amyloid materials and viral capsids based on computer simulation techniques at diferent scales ranging from all-atom MD simulations to CG-MD simulations.

Structural Characteristics of Amyloid Materials

For a recent decade, there are efforts that have been made to understand the formation and structures of amyloid materials based on computer simulations. A previous study by Karplus and coworkers [[125\]](#page-23-49) reported MD simulations of dimerization for understanding the underlying mechanisms of the formation of amyloid dimers, which were formed by self-assembled aggregation of two monomers (Fig. [6a](#page-13-0)). They showed the ability of all-atom MD simulations to probe the structural properties of amyloid dimer isoforms, while their work $[125]$ is still computationally restricted in gaining insight into the kinetics of protein aggregation resulting in the formation of amyloid fbrils. A pioneering work

Fig. 6 a Equilibrium molecular dynamics (MD) simulations for the formation of amyloid dimers. Top panel shows the snapshots of two monomers at time $t=0$ and a dimer formed by aggregation of two monomers at time $t=1$ ns. Figures are adopted with permission from Ref. [[125\]](#page-23-49). Copyright (2004) National Academy of Sciences, U.S.A. **b** Replica exchange MD (REMD) simulation results show the domi-

nant structures of islet amyloid polypeptide (IAPP) monomers found in rat (top panel) or human (bottom panel). It is interestingly shown that rat IAPP is most likely to exist in the form of alpha helix, while human IAPP is more likely to exist in the form of β-hairpin, which is responsible for aggregation of IAPP monomers. Figures are adopted from Ref. [[132\]](#page-23-50) under Creative Commons Attribution License

by DeMarco and Daggett [\[126](#page-23-51)] reported that all-atom MD simulations may allow for predicting the structural conversion of cellular prion protein (PrP^C) into the scrapie isoform of prion protein (PrP^{SC}) , which is responsible for the formation of prion fbrils involving in various transmissible spongiform encephalopathies. This pioneering work [[126\]](#page-23-51) is still limited in quantitative understanding of the kinetics of structural conversion into dimerization (and subsequently, fbrillization). In recent years, with the advancement of computational resources, the all-atom MD simulations have been able to probe the more detailed information of the formation of amyloid materials. For example, Chong and Ham [[127\]](#page-23-52) have studied the dimerization of Alzheimer-β (Aβ) monomers using all-atom MD simulations. In their work [[127](#page-23-52)], they are able to unveil the underlying mechanisms of the dimerization process such that a water-mediated hydration force plays a crucial role in the formation of Aβ dimer. In addition, all-atom MD simulations with enhanced sampling methods such as replica exchange [[128](#page-23-53)] or metadynamics [\[129,](#page-23-54) [130\]](#page-23-55) have allowed for characterizing not only the formation of amyloid dimers but also the inherent structural feature of amyloid monomer, which may be correlated with self-assembly process of protein aggregation. For instance, a bias-exchanged metadynamics simulations have provided the pathway of the dimerization of islet amyloid polypeptide (IAPP) and the intermediate structural forms of IAPP dimers [[131](#page-23-56)]. A recent study by Shea and coworkers [[132\]](#page-23-50) have shown the ability of replica exchange MD (REMD) simulations to probe the inherent structural properties of IAPP monomers with the effect of sequence mutations. In their work [[132\]](#page-23-50), it is found that human IAPP (hIAPP) monomer is able to exhibit more contents of β-sheets, which are responsible for protein aggregation, than rat IAPP (Fig. [6](#page-13-0)b). Their fnding [[132\]](#page-23-50) is consistent with clinical observation [\[18](#page-22-11)] that though IAPP proteins are found in both human and rat, human suffers from type 2 diabetes due to aggregation of IAPP proteins, whereas the type 2 diabetes is not found in a case of rat. In addition, Chakraborty and Das [\[133](#page-23-57)] have recently employed REMD to probe the inherent structural properties of $A\beta$ monomer and the effect of inhibitors in the structural properties of Aβ monomers. They found that an inhibitor, which is a six-residue Aβ fragment (referred to as Aβ hexapeptide), makes a critical impact on the secondary structural propensity of Aβ monomers. In particular, Aβ hexapeptide reduces the contents of β-sheets in an Aβ monomer, which implies that $Aβ$ hexapeptide inhibits the aggregation of Λ β monomers. A recent study by Shea and colleagues [[134](#page-23-58)] have also considered REMD simulations for gaining insight into the role of osmolytes (e.g. urea or trimethylamine N-oxide, abbreviated as TMAO) in the structural propensity of tau fragment peptide. In their REMD simulations [[134](#page-23-58)], the osmolytes induces the shift of the populations of secondary structures for tau fragment peptide, which is consistent with experimental observation that the osmolytes inhibits the formation of amyloid fbrils but promotes the formation of helical amyloid oligomers. The previous studies reviewed here suggest the increasing role of all-atom MD simulations in revealing hidden, complex mechanisms that determine the structure and formation of amyloid materials at diferent length scales ranging from oligomeric structures to fbril structures.

Mechanical Properties of Amyloid Materials

Mechanical characterization of amyloid materials is a priori requisite for gaining insight into the molecular mechanism of disease pathologies, which is attributed to the fact that once amyloid fbrils are formed by self-assembly process (i.e. protein aggregation), these fbrils are quite stable and are unlikely to be denatured (or ruptured) in physiological conditions so as to deposit onto an organ (e.g. brain, pancreas, etc.) resulting in the disease pathologies. In addition, the mechanical properties of amyloid fbrils are directly linked to the amyloid-driven pathologies. For example, a recent study by Zewail and coworkers [\[24](#page-22-17)] suggest that the formation of mechanically stif amyloid fbrils (or protofbrils) on a mechanically soft cell membrane results in the disruption of the membrane. In addition, the fracture property (i.e. division rate) of prion fbrils are found to be correlated with prion infectivity, that is, cell-to-cell prion propagation [[135](#page-23-59)].

For a recent decade, there are a lot of attempts to measure the mechanical stability and properties of amyloid fbrils. For instance, we studied the dependence of mechanical stability and properties of hIAPP amyloid fbrils on their structural feature, that is, the pattern of cross-β structures made based on stacking of peptides, by using all-atom MD simulations [[136\]](#page-23-60). We have shown that the amyloid fbrils formed based on antiparallel stacking of hIAPP peptides are quite stable when compared with the fbrils constructed based on other stacking patterns (e.g. parallel stacking of peptides), which highlights the importance of the molecular structure (i.e. cross-β structure) of amyloid fbrils in their stability. In recent years, we have employed all-atom MD simulations in order to study how metal ions, which are found in patients suffering from neurodegenerative diseases, affect the formation and stability of Aβ structures at diferent length scales, such as amyloid oligomers and fbrils [[137\]](#page-23-61). We found that Zn^{2+} ions improve the stability of Aβ oligomers while these ions reduce the stability of Aβ fbrils, which is consistent with clinical observation that high concentration of Zn^2 + ions promotes the formation of Aβ oligomers that are known as toxic agent to functional cells (Fig. [7\)](#page-15-0). In addition, a recent study by MacPhee et al. [[138](#page-23-62)] reports the effect of protonation state in the stability of transthyretin (TTR) amyloid oligomers based on all-atom MD simulations. In their work [\[138](#page-23-62)], it is shown that the protonation state affects the

Fig. 7 Effect of metal ions on the mechanical stability of amyloid aggregates at diferent length scales such as oligomer and fbril based on all-atom molecular dynamics (MD) simulations. It is shown that metal ions make an amyloid oligomer be an ordered structure, while

they distorts the structure of amyloid fbril resulting in the decrease of its structural stability. Figures are adopted with permission from Ref. [[137\]](#page-23-61). Copyright (2018) Royal Society of Chemistry

stability of amyloid oligomers so as to result in the diferent form (e.g. decamer, hexamer, etc.) of amyloid oligomers. It is found that the decamer is stable regardless of the protonation state of TTR, whereas the protonation results in the stable formation of the hexamer, and de-protonation favors the formation of higher order oligomers. These previous studies [[136](#page-23-60)[–138](#page-23-62)] show the ability of all-atom MD simulations to characterize the mechanical stability and formation of amyloid structures at diferent length scales ranging from oligomers to protofbrils. It should be noted that, however, the all-atom MD simulations are still computationally prohibited for simulating the stability of a very long amyloid fbril whose length scale is larger than 100 nm. Until now, the all-atom MD simulations is computationally able to characterize the amyloid protofbrils whose length scale is in the order of 10 nm.

Recently, there are many recent works for studying the mechanical properties of amyloid fbrils. A pioneering work by Knowles et al. [[139](#page-23-63)] reports the mechanical properties of amyloid fbrils based on AFM experiments and ENM-based CG simulations. It is shown that the elastic modulus of amyloid fbrils is estimated in the order of 1 to 10 GPa, which is comparable to that of a mechanically strong protein material such as spider silk (Fig. [8](#page-16-0)). In their work [\[139\]](#page-23-63), it should be noted that the persistent length of amyloid fbrils (equivalent to their bending rigidity) is computed based on AFM images of the fbrils coupled with polymer chain statistics, i.e. Equation [\(28\)](#page-6-2). The remarkable elastic modulus of amyloid fbrils is attributed to interactions between neighboring β-sheets in the cross-β structure of amyloid fbrils. This hypothesis was verifed in a recent study by Buehler and coworkers [[25\]](#page-22-18), who showed

Fig. 8 Mechanical properties of amyloid materials in comparison with those of other materials. Blue region shows the mechanical properties of amyloid fbrils, while green region indicates the mechanical properties of other protein materials. It is shown that the mechanical property (e.g. elastic modulus) of amyloid fbrils exceeds that of other protein materials. The mechanical properties of amyloid fbrils were measured by atomic force microscopy (AFM) imaging experiments coupled with polymer chain statistics. Figures are adopted with permission from Ref. [\[139\]](#page-23-63). Copyright (2007) The American Association for the Advancement of Science

that the geometric confnement of hydrogen bonds between β-sheets gives rise to the remarkable elastic properties of β-sheet-rich protein materials such as amyloid materials and spider silk crystals. As the all-atom MD simulations are computationally limited for simulating large protein structures such as amyloid fibrils with their length of >10 to>100 nm, CG model-based simulations have been taken into account for characterizing the mechanical properties of long amyloid fbrils. For example, Buehler and coworkers [[140\]](#page-23-64) have employed ENM-based NMA simulations for measuring the elastic properties of Aβ fbrils. They have shown that the low-frequency vibration modes of Aβ fbrils correspond to the bending, twisting, and stretching deformation modes, and that the elastic moduli of $A\beta$ fibrils (with the twofold or threefold symmetries) are measured in a range of \sim 20 to \sim 30 GPa. Here, we note that the extraction of mechanical properties (e.g. elastic moduli) for amyloid fbrils is based on the Euler–Bernoulli beam theory, i.e. Equation (31) (31) , which relates the elastic moduli of the fbrils to their natural frequencies that were computed from ENM-based NMA simulations. It is found that the bending rigidity of Aβ fbrils (equivalent to their persistent length) is dependent on their length scales, which is ascribed to the length-dependent shear

efect on the bending deformation of the fbrils. Moreover, we studied the mechanical properties of hIAPP fbrils as a function of their structural features, i.e. stacking pattern of β-sheets, by using ENM-based simulations [[141](#page-23-65)]. We found that the antiparallel stacking of β-sheets renders the hIAPP fbril to exhibit the higher bending rigidity, which highlights the role of β-sheet stacking patterns in not only the mechanical stability of amyloid fbrils [[136](#page-23-60)] but also their mechanical properties [\[141](#page-23-65)]. We have also shown that the dependence of the bending rigidity of hIAPP fbrils on their length is well ftted to the Timoshenko beam theory, i.e. Equation ([33](#page-7-2)), which sheds light on the shear efect in the bending properties of amyloid fbrils. In addition, we have investigated the mechanical properties of prion fbrils using ENM-based NMA simulations $[142]$. It is interestingly found that the length-dependent mechanical (or vibrational) properties of prion fbrils provide an insight into the critical size of prion fbrils, at which their infectivity is maximized, and that the helical structure of prion fbrils determines their elastic properties. Furthermore, we measured the vibrational and elastic properties of hIAPP fbrils (with their length of \sim 10 nm) using all-atom MD simulations coupled with Euler–Bernoulli beam theory [[136\]](#page-23-60). In particular, the natural frequencies of hIAPP fbrils for their vibrational modes are obtained based on all-atom MD simulations, while the elastic moduli of the fbrils are extracted based on Euler–Bernoulli beam theory that relates the natural frequencies to the elastic moduli of the fbrils. The natural frequencies of hIAPP fibrils with their length of ~ 10 nm are critically dependent on their structural characteristics such as the stacking pattern (Fig. [9](#page-17-0)). We have shown that the antiparallel stacking of β-sheets results in the higher elastic moduli of hIAPP fbrils [\[136](#page-23-60)], which is consistent with our previous finding [\[141](#page-23-65)] based on ENM simulations. It is also found that low-frequency vibrational modes (corresponding to bending, torsional, and stretching deformation modes) mostly contribute to the thermal fuctuation of amyloid fbrils. Furthermore, a recent study by Na and colleagues [\[143](#page-23-67)] has reported that, using ENM-based NMA simulations, the low-frequency vibrational modes of triplet prion fbril are able to depict its conformational changes due to pH. This suggests that the low-frequency vibrational modes of amyloid fbrils play an important role in not only extracting their mechanical properties but also gaining insight into their conformational changes due to a chemical stimulus such as pH change. Recently, we have studied the mechanical properties of multi-stranded amyloid fbrils based on ENM-based simulations coupled with all-atom MD simulations [[144\]](#page-23-68). In our recent study [[144](#page-23-68)], all-atom MD simulations were employed to obtain the equilibrium structure of multi-stranded amyloid fbrils, while ENM-based simulations were used to extract the

Fig. 9 a Equilibrium molecular dynamics (MD) simulations of human islet amyloid polypeptide (hIAPP) protofbrils constructed based on eight diferent stacking patterns. MD simulations suggest that hIAPP protofbrils are mechanically stable as long as they are made based on antiparallel stacking of peptide chains. **b** Mode index for the bending, stretching, and torsional deformation modes of hIAPP protofbrils constructed based on 8 diferent structural

classes. **c** Natural frequencies corresponding to bending, torsional, and stretching deformation modes for hIAPP protofbrils are measured from MD simulations. **d** Contribution of bending, torsional, or stretching deformation modes to the thermal fuctuation motion of hIAPP protofbrils made based on 8 diferent structural classes. Figures are adopted from Ref. [\[136](#page-23-60)] under Creative Commons Attribution License

mechanical properties of the fbrils. We have shown that the dependence of persistent length (equivalent to bending rigidity) for multi-stranded amyloid fbrils is ftted to a scaling law of $l_p \propto [(nq)^2 - \sin^2(nq)]^{1/2}$ [[68](#page-22-54)], which was obtained from a helical model. It is also shown that a H31Y point mutation, where histidine residue locating in 31st residue from the N-terminus is changed into tyrosine residue, results in a change in not only the structure of multi-stranded amyloid fbrils but also their elastic moduli.

In recent years, all-atom SMD simulations have been extensively employed for studying not only the mechanical properties of amyloid fbrils but also their fracture mechanisms. Buehler and coworkers [[25](#page-22-18)] have interestingly reported that the fracture mechanism of β-sheet crystals is governed by their length scale in such a way that shear deformation dominates the force-driven fracture of short β-sheet crystals with their length of <3 nm, while the fracture of $β$ -sheet crystals with their length of > 3 nm is governed by bending deformation (Fig. [10](#page-18-0)). A recent study by Grater and coworkers [\[145\]](#page-23-69) have studied the mechanical deformation mechanisms and properties of β-sheet crystals, which are found in amyloid fbrils and/or spider silk, using all-atom SMD simulations. They have found that the direction of a force applied to the β-sheet crystals signifcantly afects not only their deformation (and fracture) mechanisms but also their elastic moduli. In particular, β-sheet crystal are able to efective resist a force applied to the perpendicular direction of the β-strand, whereas it becomes mechanically weak when

Fig. 10 Mechanical deformation mechanisms of β-sheet crystals with their length of $a L = 2.83$ nm or $b L = 6.56$ nm based on all-atom steered molecular dynamics (SMD) simulations. When a β-strand in the β-sheet crystal with its length of *L*=2.83 nm is pulled out, the fracture of the crystal is dominated by shear deformation-driven fracture. On the other hand, for a β-sheet crystal with its length of

a force is applied to the parallel direction of the β-strand. Recently, we have studied the mechanical deformation mechanisms and properties of hIAPP fbrils as a function of their length scales by using all-atom SMD simulations [[146\]](#page-23-70). It is shown that the dependence of bending elastic property and fracture mechanism for amyloid fbrils on their length scales is attributed to the length-dependent competition between shear and bending deformation modes, and that the length-dependent elastic property of the fbrils obtained from all-atom SMD simulation is well ftted to Timoshenko beam theory. We found that this length-dependent fracture mechanism for the fbrils is attributed to the lengthdependent rupture mechanisms of hydrogen bonds formed between β-sheets (in the cross-β structure of the fbril). In particular, for a short fbril, several hydrogen bonds formed between β-sheets are able to cooperatively resist a force so that at a critical value of a force, these hydrogen bonds are simultaneously ruptured. On the other hand, for a long fbril, the hydrogen bonds between β-sheets are fractured one-by-one, which results in the mechanical fragility of the long fbril. In recent years, we have investigated the fracture mechanisms of a helical prion fbril using all-atom SMD simulations [\[147](#page-23-71)]. The elastic moduli of an infectious prion fbril formed based on a left-handed β-helix and a non-prion fbril constructed based on a right-handed β-helical structure are measured as \sim 18 GPa and \sim 13 GPa, respectively. However, it is interestingly found that the fracture toughness of an infectious prion fbril is smaller than that of non-prion fibril $[147]$, which is consistent a recent finding $[135]$ that infectious prion fbril is more fragile. A previous study by

 $L=6.56$ nm, the pulling of a β -strand in the crystal leads to the bending deformation-driven fracture of the β-sheet crystal. These SMD simulations suggest the role of the length scale of β-sheet crystals in their fracture mechanisms based on a competition between shear and bending deformations. Figures are adopted with permission from Ref. [[25](#page-22-18)]. Copyright (2010) Springer Nature

Buehler and colleagues [\[148](#page-23-72)] has interestingly investigated the relationship between the molecular structure of amyloid fbrils and their mechanical (e.g. elastic) properties. It should be noted that amyloid fbrils can be formed based on a few types of structural models such as stacked β-sheets, β-helix, and stacked β-helices. They found that amyloid fibrils formed based on stacked β-helices exhibit the high fracture strength, while the fbrils constructed based on β-helix possess the high elastic moduli. The mechanical properties of amyloid fbrils are highly correlated with their molecular structure (i.e. structure type), which determines the rupture mechanisms of hydrogen bonds within the fbril.

Mechanical and Dynamic Properties of Viral Capsids

Viruses are nano-sized particles that are formed by selfassembly process. For a recent decade, there are eforts that have been conducted for characterizing the mechanical and dynamic properties of viral capsids [\[149\]](#page-23-73), as these properties are closely related to the biological function of viruses such as gene packaging. Viral capsids are mechanically strong so as to resist disassembly, so that they are able to contain a gene information (i.e. DNA or RNA molecules inside the virus). Here, we review computational efforts that have been made, for a recent decade, to characterize the dynamic and mechanical properties of viral capsids using CG and/or atomistic simulations.

Tama and Brooks [\[150](#page-23-74)] had utilized ENM-based NMA simulations for characterizing the dynamic behavior of **Fig. 11** Maturation dynamics of rice yellow mottle virus (RYMV) and southern bean mosaic virus (SMBV) was analyzed by elastic network model (ENM)-based normal mode analysis (NMA) simulations. The initial structures of **a** RYMV and **b** SBMV are shown. The structures of **c** RYMV and **d** SBMV in swollen state are obtained from ENMbased NMA simulations. Figures are adopted with permission from Ref. [\[150\]](#page-23-74). Copyright (2005) Elsevier

icosahedral viral capsids. They found that few low-frequency vibrational modes obtained from ENM are able to sufficiently depict the conformational changes of icosahedral viral capsids such as their shape change from spherical to faceted polyhedral. Their study [\[150](#page-23-74)] suggests that icosahedral viral capsids are able to adapt their shapes due to their fexibility (Fig. [11](#page-19-0)). A previous study by van Vlijmen and Karplus [\[151](#page-23-75)] reported the vibrational characteristics of icosahedral viruses based on all-atom NMA. In particular, the stifness of viruses is computed based on the second derivative of the potential energy prescribed to the virus structure with respect to its atomic coordinates, and then this stifness is used for NMA to fnd the natural frequencies of the virus and their corresponding vibrational modes. The lowfrequency vibrational modes of polivirus capsid are shown to be correlated with its shape adaptation. Rader et al. [[152\]](#page-23-76) studied the maturation dynamics of bacteriophage HK97 capsid using ENM-based NMA simulations. The authors found that the collective low-frequency vibrational motions of procapsid are highly correlated with its conformational change related to its maturation. This also confrms the role of fexibility for viral capsid in its shape adaptation. Kim et al. [\[119\]](#page-23-43) studied the maturation dynamics of HK97 virus capsid using rigid cluster ENI simulations. They showed the conformational transition pathways of HK97 virus capsid. These previous studies [[119,](#page-23-43) [150](#page-23-74)[–152](#page-23-76)] are still restricted for understanding the dynamic process of maturation (and its related to conformational change) for viral capsids, because the NMA-based studies do not consider the time-evolution dynamics of viral capsids but takes into account only their vibrational modes at equilibrium state. Arkhipov et al. [\[153\]](#page-23-77) had developed a CG model that is applicable for studying the time-evolution dynamics of viral capsids. Their CG-MD simulations show that some viral capsids such as an empty satellite plant virus (SMTV) collapse rapidly within 500 ns, while other capsids such as brome mosaic virus (BMV) are highly flexible but very stable for a long time of $>1 \mu s$. It is found that interlocking between the subunits of viral capsid plays a crucial role in its structural stability. Recently, Perilla and Schulten [\[154](#page-23-78)] have reported the mechanical and dynamic behavior of viral capsid (i.e. HIV-1 capsid) using all-atom MD simulations based on high-performance parallel supercomputing. In particular, the dynamic trajectories of 64 million atom-sized HIV-1 capsid for the timescale of

1 μs were obtained from all-atom MD simulations. Based on this long-time all-atom MD simulations, the stability of HIV-1 capsid was studied by measuring the size of HIV-1 capsid as a function of time, and the low-frequency vibrational motions of HIV-1 capsid were also investigated.

The mechanical properties of viral capsids have recently been probed by AFM indentation experiments [[149](#page-23-73)]. The elastic moduli of viral capsids were found to be 0.1 to 1.8 GPa with dependence on the Föppl-von Kármán (FvK) number, *γ*, which is defined as $γ = ER^2/D$, where *E*, *R*, and *D* are the Young's modulus, radius, and bending rigidity of the viral capsid. This FvK number, which quantifes the competition between bending energy and tension energy during the deformation of the viral capsid, determines the shape of the capsid, i.e. whether it exhibits spherical or icosahedral facetted shapes. In recent years, Widom and coworkers [\[155\]](#page-23-79) have considered ENM-based NMA simulations coupled with continuum elasticity theory, i.e. Lamé equation (for sphere). It is shown that the low-frequency vibration modes of viral capsid, obtained from ENM-based NMA simulations, were comparable to those predicted by Lamé equation, which suggests the robustness of a continuum elastic model in characterizing the mechanical properties of viral capsids. The Young's modulus (*E*) and Poisson's ratio (ν) of viral capsids were measured as $\nu = 0.2 \sim 0.3$ and E/γ =6.5 ~ 36.2 nm⁻¹, where γ is a force constant used in ENM (here, *γ*=~0.5 N/m), i.e. *E*=3.2~18.1 GPa. A previous study by Roos et al. [\[156](#page-24-0)] employed a continuum elastic model-based fnite element simulations for characterizing the squeezing (i.e. indentation) mechanics of viral capsids. Specifcally, the authors considered the fnite deformation continuum hyperelasticity (with a neo-Hookean constitutive law) in fnite element simulations to probe the indentation mechanism of viral capsids (Fig. [12](#page-20-0)). It is shown that the mechanical indentation behavior of viral capsids is critically

dependent on their shape that can be described by FvK number. For the capsids with their shape of $FvK < 150$, their indentation response is well dictated by a linear relation between indentation displacement and force. However, for the capsids with $150 < FvK < 800$, the indentation response becomes nonlinear, whereas the indentation behavior of capsids with Fv>800 is well depicted by a linear response as long as the indentation displacement becomes 40% of the radius of the capsid.

Conclusion

In this paper, we introduce recent efforts that have been made, for a last decade, to characterize protein materials at multiple length scales ranging from single proteins to protein assemblies, based on various simulation techniques (at diferent length scales) from atomistic MD simulations to CG simulations.

First, we have shown that CG-MD simulations (using Golike or SOP potential) have been extensively employed to understand the mechanical unfolding or refolding dynamics of a single protein molecule due to a force. Here, we note that though all-atom SMD simulations are able to provide a useful insight into protein unfolding (or refolding) mechanisms of a single protein, they are still computationally prohibited for directly interpreting the results of singlemolecule force spectroscopy experiments. Specifcally, the timescale accessible with all-atom SMD simulations is quite diferent from that used in a single-molecule force spectroscopy experiment such that a pulling rate used in SMD simulations is larger by six orders of magnitude than that considered in a single-molecule force spectroscopy experiment. This discrepancy between these timescales results in the extensive usage of CG-MD simulations for understanding

Fig. 12 Indentation mechanics of Hepatitis B virus (HBV) capsid, whose icosahedral geometry is given as $T=4$ with two- of three-fold symmetry axes. **a** Finite element simulation shows the mechanical deformation mechanisms of HBV capsids with their two- or threefold symmetry axes during the indentation. **b** Coarse-grained molecu-

lar dynamics (CG-MD) simulation results for the indentation mechanisms of HBV capsids with their diferent symmetries (left), and the force–displacement curve of indented HBV capsids with two-, three-, or fve-fold symmetry axes (right). Figures are adopted with permission from Ref. [\[156](#page-24-0)]. Copyright (2010) Elsevier

the unfolding (or refolding) mechanisms of single proteins. However, it should be noted that although CG-MD simulations are able to provide the unfolding dynamics of a single protein with a timescale comparable to that used in experiment, these simulations have to be cautiously used for interpreting the unfolding (or refolding) mechanisms of a single protein, because the CG models may not carry the all atomistic information of the protein. For example, a single point mutation (i.e. single amino acid substitution) may afect the structure, dynamic (e.g. unfolding and/or refolding), and properties of a single protein, which typically cannot be captured by CG models. Recently, with the improvement of computing resources as well as the development of enhanced sampling methods, all-atom MD (or SMD) simulations may be more extensively utilized for interpreting the results of a real single-molecule force spectroscopy experiment. For instance, metadynamics scheme has recently been developed and widely utilized to probe the dynamics of a protein with a timescale relevant to the experiment. Here, the principle of metadynamics is to apply a Gaussian penalty function to the potential feld of a protein in order to speed up the reaction rate of a protein related to its structural (conformational) changes due to a force or other stimulus (e.g. ligandbinding) [[129,](#page-23-54) [130\]](#page-23-55). Our recent studies report the unfolding mechanisms of ubiquitin [\[157\]](#page-24-1) and/or prion protein [\[158\]](#page-24-2) based on all-atom metadynamics simulations. In our recent studies [\[157,](#page-24-1) [158\]](#page-24-2), it is shown that all-atom model-based metadynamics simulations allow for unveiling the unfolding mechanics of a protein with a timescale relevant to a single-molecule force spectroscopy experiment. Despite few studies [[157](#page-24-1), [158\]](#page-24-2) of protein unfolding based on an enhanced sampling-based all-atom simulations (e.g. all-atom metadynamics), we believe that recent development of enhanced sampling techniques may enable all-atom MD simulations to be practically considered for interpreting the results of a single-molecule force spectroscopy experiment in near future.

For studying the conformational changes of a single protein molecule, we have discussed the extensive usage of CG models such as ENM. All-atom MD simulations have not been still applicable for revealing the underlying mechanisms of the conformational changes, as their available timescale is much shorter than that relevant to the conformational changes. This has led to the development and usage of CG models such as ENM for studying the conformational changes. The robustness of ENM in predicting the conformational changes is attributed to the fact that the conformational changes of a protein are correlated with its low-frequency vibrational modes. Nevertheless, ENMbased NMA simulations are unable to provide insight into the time-evolution dynamics of conformational changes for a protein. One possibility to overcome this restriction is the development of computational techniques based on coupling between CG model (i.e. ENM) and all-atom MD simulations. As discussed in a paper by Bahar and coworkers [[123\]](#page-23-47), the low-frequency vibrational modes obtained from ENM were used to explore conformational spaces relevant to the conformational changes, while all-atom MD simulations were utilized to obtain the equilibrium dynamics of a protein during its conformational transitions. We anticipate that the sampling based on the normal modes (obtained from CG models) may be coupled to all-atom MD simulations for gaining a proftable knowledge on the conformational transitions of a protein.

For simulating large protein assemblies such as amyloid materials and viral capsids, we have shown that CG or continuum model-based simulations are able to provide the fruitful insight into the structure, dynamics, and properties of these assemblies. Despite a recent study by Perilla and Schulten [\[154\]](#page-23-78) reporting the dynamics of viral capsids based on all-atom MD simulations, these atomistic simulations are still computationally unfavorable for predicting and characterizing the structure, dynamics, and properties of protein assemblies. We have presented many recent works on CG model (e.g. ENM) and/or continuum model-based simulations for characterizing the mechanical properties of protein assemblies such as amyloid materials and viral capsids, whose size is typically >100 nm. However, these simulations based on CG models (e.g. ENM) are unable to provide the time-evolution dynamics of mechanical behavior (e.g. fracture) for protein assemblies such as amyloid materials. Though continuum model-based fnite element simulations may provide the time-evolution of mechanical deformation for protein assemblies, they require a priori knowledge on a constitutive law (e.g. parameters related to material properties) for protein assemblies. One of routes to improve the simulation techniques for characterizing the protein assemblies is to combine all-atom MD simulations and CG modelbased simulations (or continuum model-based simulations such as fnite element simulation). Specifcally, as described in our recent work [[144](#page-23-68)], all-atom MD simulations are considered to obtain the equilibrium structure of structural segment for protein materials (e.g. amyloid materials), while simulations based on CG model (e.g. ENM) are applied to a whole protein assembly, which was constructed based on the equilibrium structure of structural segment, in order to characterize the properties of this assembly. Another avenue to improve the simulation techniques is that all-atom MD simulations are taken into account to compute parameters (e.g. parameters for a constitutive law) that are required for a fnite element simulation of protein assembly, while fnite element simulations are performed to depict the time-evolution of mechanical deformation for protein assembly [\[159](#page-24-3)].

In conclusion, computer simulations based on various models at multiple length scales, ranging from atomistic model to CG and continuum models, have recently played invaluable role in predicting, characterizing, and revealing

the hidden, complex, underlying mechanisms of proteins' dynamics and mechanics, which govern their biological functions. These simulations may help to not only interpret the experimental results (e.g. results obtained from singlemolecule experiments) but also provide new insights into proteins' dynamics and mechanics, which are still difficult to be observed in experiments due to their limited spatial and temporal resolutions. In addition, these computer simulations may also allow for providing an important design principles for developing a novel materials such as newclass protein materials [\[160](#page-24-4)[–162\]](#page-24-5) and nano-bio composite material $[163-165]$ $[163-165]$ (i.e. material formed by coupling protein material and nanomaterial).

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