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SHORT COMMUNICATION

Bipolar fuzzy attribute implications

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Abstract
Recently, bipolar fuzzy concept lattice has received much attention among the research communities. In this process, a problem was addressed while dealing with a large number of bipolar fuzzy concepts. In this case, bipolar fuzzy attribute implications can be helpful for multi-decision process based on user-required chosen attributes. To achieve this goal, the current paper focuses on introducing a method for bipolar fuzzy attribute implications and its measurement using accuracy function with an illustrative example.

Keywords Attribute implication · Fuzzy implications · Bipolar fuzzy set · Bipolar information · Bipolar fuzzy implication

1 Introduction

Recently, handling the data with bipolar fuzzy attributes is considered as one of the major tasks by the researchers (Singh, 2019a; Singh, 2019b). The bipolar information contains positive $(0, 1)$ and negative $(-1, 0)$ membership values to represent the attributes like known-unknown and day-night shown in Fig. 1. It is based on mathematical paradigm of Ying-Yang theory (Zhang, 1994). The positive membership value $(0, 1)$ represents that the object somehow satisfies the corresponding properties, whereas the negative membership value $(-1, 0)$ represents that the object contains implicit counter property. The 0 membership value means that the object does not have the given property or irrelevant property. In this way, data with bipolar fuzzy attributes can be represented precisely using a defined bipolar fuzzy space $[-1, 0] \times [0, +1]$ rather than a unipolar space [0, 1]. The property of bipolar fuzzy set was introduced by Zhang (Zhang, 1994) motivated from Taoism theory (Welch, 1957). Later, it is applied for data analysis and representation tasks by Bloch (Bloch, 2011) based on applied lattice theory (Wille, 1982). Singh and Kumar (Singh & Kumar, 2014a; Singh & Kumar, 2014b) introduced bipolar fuzzy graph representation of concept lattice and noted the bipolar implications as a future research. Recently, some of the researchers focus on bipolar fuzzy set (Wei et al., 2018) and its applications using score and accuracy function (Singh, 2019). These researches motivated the author to work on bipolar fuzzy attribute implications. The objective is to investigate some useful bipolar fuzzy implication in case of knowledge discovery without bipolar concept lattice.

Association rule mining provides us the frequent items which are applied in various fields for knowledge processing tasks (Agrawal & Srikant, 1993; Agrawal et al., 1993). It is useful in case a large number of concepts are generated and its concept lattice visualization becomes complex (Kumar & Srinivas, 2010). Due to this, property implications and its mathematical paradigm are applied in various fields for multi-decision process (Kumar, 2012; Ganter, 1999). The problem arises when fuzzy attributes contain the uncertainty and vagueness (Belohlavek & Vychodil, 2005a). Due to which, attribute implications in fuzzy setting were established by various researchers (Belohlavek & Vychodil, 2005b; Belohlavek & Vychodil, 2006; Glodeanu, 2012). In this case, the implications $A \Rightarrow B$ consider A and B as fuzzy attributes (Belohlavek & Vychodil, 2005b). It represents that the objects have the attributes from A (to at least the degree a) and, then, they also have the attributes from B (to at least the degree b) (Glodeanu, 2012; Singh & Kumar, 2017; She & Zhai, 2008). In this process, a problem arises in case bipolarity exists in the given fuzzy attributes (Singh & Kumar, 2014b; Singh & Kumar, 2017). It is required when the expert needs to deal with known-unknown, action-reaction, and love-hate and its implications for knowledge processing

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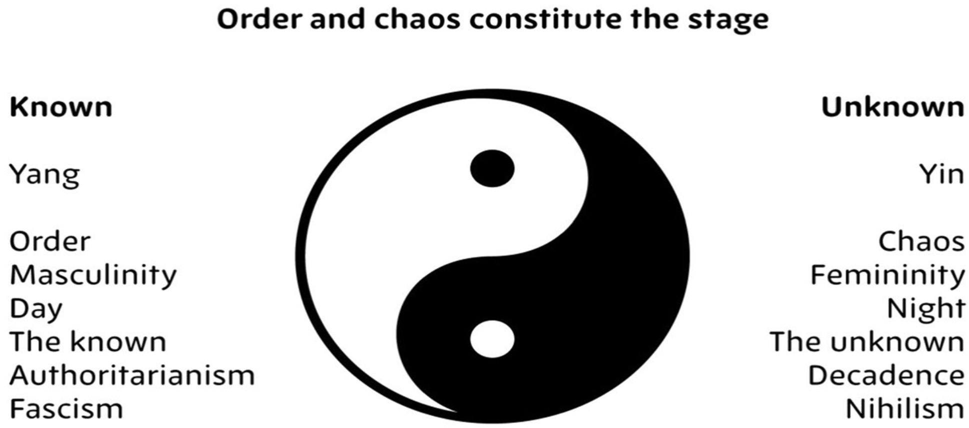

Fig. 1 The necessity of bipolar fuzzy set for dealing known-unknown and its implications

tasks as shown in Fig. 1. Another reason of exploring bipolar fuzzy attribute implications becomes necessary while dealing with the equilibrium among quantum information and its security as shown in Fig. 2 (Zhang, 2018). Same time it motivated to deal with human (Singh, 2021; Akram et al., 2021) and its changes (Singh, 2020). The problem arises while dealing with a large number of bipolar fuzzy attributes considered one of the most crucial tasks (Ali et al., 2019; Ali et al., 2020; Riaz & Therim, 2021). Singh and Kumar (Singh & Kumar, 2017) tried to generate the fuzzy attribute implication using Armstrong's rules, which is somehow helpful in multi-decision process. Motivated from these studies, the current paper focuses on exploring the bipolar fuzzy attribute implications. The support and confidence of bipolar implications are computed using the algebra of bipolar fuzzy sets with its accuracy measurement for adequate analysis. The obtained results are compared with their concept lattice (Singh & Kumar, 2014b) and distance metric (Singh, 2019) for validation of knowledge.

The rest of the paper is organized as follows: Section 2 provides a background about data with bipolar fuzzy attributes and attribute implication. Section 3 contains the proposed method with its illustration in Section 4. Section 5 contains the conclusions followed by acknowledgements and references.

2 Data with bipolar fuzzy attributes

In this section, we provide some basic definition of bipolar fuzzy contexts, residuated lattice, and fuzzy implication, which is given to understand the proposal:

Definition 1: (Bipolar fuzzy formal context) (Singh & Kumar, 2014a): A bipolar fuzzy formal context is a triplet $K = (X,\ Y,\ \tilde{R})$ where X is set of objects, Y is set of bipolar fuzzy attributes, and $\tilde{R}$ is a bipolar fuzzy relationship among them as follows: $X \times Y \rightarrow \tilde{R}$. The bipolar fuzzy relation $\tilde{R}$ represents at which level the object (X) has the attribute (Y) in $[-1,0] \times [0,+1]$. The membership value in (0, 1] means the object somehow possesses the given attribute, whereas the membership values [−1, 0) mean the implicit counter property. The membership value 0 means there is no relationship among them. This membership can be mapped for any $x \in X$, $y \in Y$, and $\tilde{R}(x,y) \in L$ to which object x has attribute y (L is a support set of some complete residuated lattice L).

Definition 2: (Residuated lattice). A residuated lattice $L = (L, \wedge, \vee, \bigotimes, \rightarrow, 0, 1)$ is the finite structure of truth values of object and its properties. L is complete residuated lattice iff:

(1) $(L, \wedge, \vee, 0, 1)$ is a complete lattice.

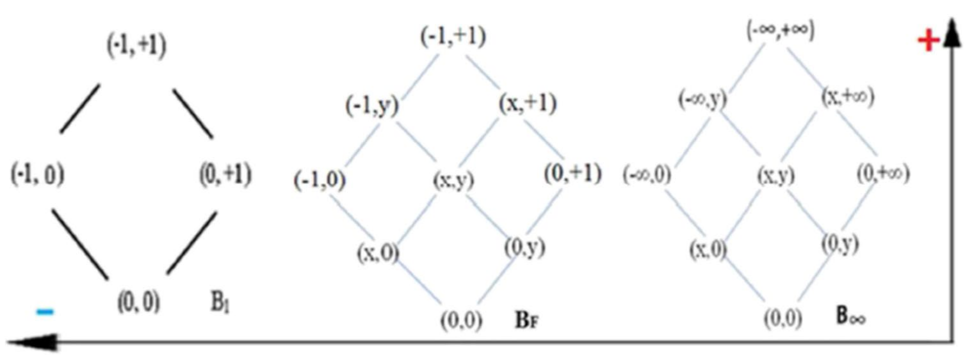

Fig. 2 The application of proposed method while dealing with quantum data and its implications

(2) $(L, \otimes, 1)$ is commutative monoid.
(3) $\otimes$ and $\rightarrow$ are binary operations called multiplication and residuum, respectively, i.e., $a \otimes b \leq c$

iff $a \leq b \rightarrow c$ for any $a, b, c \in L$.

The operators $\otimes$ and $\rightarrow$ are defined distinctly by Lukasiewicz, Gödel, and Goguen as given below:

Lukasiewicz: $a \otimes b = \max(a + b - 1, 0)$, $a \rightarrow b = \min(1 - a + b, 1)$.

Godel: $a \otimes b = \min(a, b)$, $a \rightarrow b = \begin{cases} 1 & \text{if } a \leq b \\ b & \text{otherwise} \end{cases}$

Goguen: $a \otimes b = a.\, b$, $a \rightarrow b = \begin{cases} 1 & \text{if } a \leq b \\ b/a & \text{otherwise} \end{cases}$

This helps in finding the attribute implications in fuzzy setting [17-20].

Definition 3: (Attribute implications) (Carpineto & Romano, 2004; Ganter, 1999): A (fuzzy) attribute implication (over Y) *is an expression* $A \Rightarrow B$, *where* $A, B \in L^Y$ (A *and* B *are fuzzy sets of attributes*). The intended meaning of the expression is "if it is true that an object has all attributes from A, then it has also all attributes from B". The problem arises while dealing with data with bipolar fuzzy attributes and its representation.

Definition 4: (Bipolar fuzzy set representation of object) (Singh & Kumar, 2014b): Each of the object $x \in X$ in a given bipolar fuzzy formal context K can be represented by a bipolar fuzzy set (A) as follows: $A(x_i) = \left\{ \frac{\left(\mu^P_{\tilde{R}(x_i,y_1)}, \mu^N_{\tilde{R}(x_i,y_1)}\right)}{y_1} + \frac{\left(\mu^P_{\tilde{R}(x_i,y_2)}, \mu^N_{\tilde{R}(x_i,y_2)}\right)}{y_2} + \cdots + \frac{\left(\mu^P_{\tilde{R}(x_i,y_m)}, \mu^N_{\tilde{R}(x_i,y_m)}\right)}{y_m} \right\}$

where $\{y_1, y_2, \ldots, y_m\}$ is bipolar fuzzy attributes and $\left(\mu^P_{\tilde{R}(x_i,y_m)}, \mu^N_{\tilde{R}(x_i,y_m)}\right)$ is the bipolar fuzzy membership defined among ith object (x_i) and mth bipolar fuzzy attribute (y_m) in the given bipolar fuzzy context (K). In a similar way, the attributes can be represented as $B(y)$.

The problem arises while finding implications among the bipolar fuzzy attributes for knowledge processing tasks in case of large number of generated concepts (Singh & Kumar, 2014b). To accomplish this task, the current paper tried to introduce bipolar fuzzy attribute implications motivated from recent studies (Ganter, 1999; Belohlavek & Vychodil, 2005a; Belohlavek & Vychodil, 2005b; Belohlavek & Vychodil, 2006; Glodeanu, 2012; Singh & Kumar, 2017; She & Zhai, 2008). The proposed method is shown in the next section.

3 Proposed method

The proposed method is based on finding some of the bipolar fuzzy implications using support and confidence of bipolar fuzzy set. The computed support and confidence based on bipolar fuzzy set are measured via accuracy function. Figure 3 represents the graphical abstract of the proposed method to investigate the bipolar fuzzy implications.

Step 1: Let us suppose, the data with fuzzy or bipolar fuzzy attributes as triplet $K = (X, Y, \tilde{R})$ where X is set of objects, Y is set of bipolar fuzzy attributes, and $\tilde{R}$ is a bipolar fuzzy relationship among them as follows: $X \times Y \rightarrow \tilde{R}$.

Step 2: Let us suppose, the number of objects (i.e., transaction) is n and number of attributes is m.

Step 3: A bipolar fuzzy implication $\frac{\left(\mu^P_{\tilde{R}(x_i,y_1)}, \mu^N_{\tilde{R}(x_i,y_1)}\right)}{y_1} + \frac{\left(\mu^P_{\tilde{R}(x_i,y_2)}, \mu^N_{\tilde{R}(x_i,y_2)}\right)}{y_2}$ can be generated using the subset of attributes (2^m) where $\frac{\left(\mu^P_{\tilde{R}(x_i,y_1)}, \mu^N_{\tilde{R}(x_i,y_1)}\right)}{y_1} + \frac{\left(\mu^P_{\tilde{R}(x_i,y_2)}, \mu^N_{\tilde{R}(x_i,y_2)}\right)}{y_2} \subseteq Y$.

Step 4: The implications can be selected based on defined support and confidence on the bipolar fuzzy attributes as follows:

$$Support(y_j) \frac{\left(\sum_{i=1}^{n} \min\ \mu^P_{\tilde{R}(x_i,y_j)}, \sum_{i=1}^{n} \max\ \mu^P_{\tilde{R}(x_i,y_j)}\right)}{|X|}$$

where y_j is the subset of attributes and $|X|$ is the total number of transactions.

Step 5: The confidence can be computed as follows:

$$Confidence\left(\frac{\left(\mu^P_{\tilde{R}(x_i,y_j)}, \mu^N_{\tilde{R}(x_i,y_j)}\right)}{y_j} \rightarrow \frac{\left(\mu^P_{\tilde{R}(x_i,y_k)}, \mu^N_{\tilde{R}(x_i,y_k)}\right)}{y_k}\right)$$
$$= \frac{\left(\sum_{i=1}^{n} \min\ \mu^P_{\tilde{R}(x_i,y_k)} \wedge \mu^P_{\tilde{R}(x_i,y_k)}, \sum_{i=1}^{n} \max\ \left(\mu^N_{\tilde{R}(x_i,y_k)} \vee \mu^N_{\tilde{R}(x_i,y_k)}\right)\right)}{\sum_{i=1}^{n} \min\ \mu^P_{\tilde{R}(x_i,y_j)}, \sum_{i=1}^{n} \max\ \mu^N_{\tilde{R}(x_i,y_j)}}$$

Step 6: The confidence can be redefined using the accuracy function as follows:

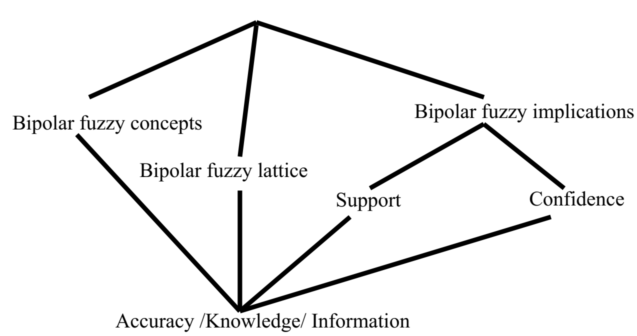

Fig. 3 The graphical understanding of bipolar fuzzy implications

Table 2 A bipolar fuzzy context

	y_1	y_2	y_5
x_1	(0.4, −0.5)	(0.5, −0.5)	(0.7, 0.0)
x_2	(0.6, −0.3)	(0.3, −0.1)	(0.5, −0.3)
x_3	(0.8, −0.2)	(0.4, −0.4)	(0.6, −0.3)
x_4	(0.5, −0.2)	(0.7, −0.2)	(0.4, −0.4)

$$Accuracy\left(\frac{\left(\mu^P_{\underset{\sim}{R}(x_i,y_j)},\ \mu^N_{\underset{\sim}{R}(x_i,y_j)}\right)}{y_j}\right) = \frac{\left(\sum_{i=1}^{n}\mu^P_{\underset{\sim}{R}(x_i,y_j)} - \sum_{i=1}^{n}\mu^N_{\underset{\sim}{R}(x_i,y_j)}\right)}{2}$$

Step 7: In this way, the support and confidence of bipolar fuzzy attributes can be represented as a single value for precise analysis.

Armstrong's rule are discussed (Singh & Kumar, 2017). This paper focused on generating the bipolar fuzzy implications for multi-decision process. To achieve this goal, a method is proposed in Section 3. In this section, the proposed method is illustrated with an example for further development and applications in various fields. To fulfil this need, a bipolar fuzzy context shown in Table 2 is considered as discussed in (Singh, 2019). The table represents set of cars $\{x_1, x_2, x_3, x_4\}$ and the opinion of people to select or reject the car based on $\{y_1 =$ Cost, $y_1 =$ Beautiful; y_5=Luxurious$\}$ among all the attributes. The opinion of people based on collected feedback after sell of car is collected and represented using the bipolar fuzzy context as shown in Table 2. The bipolar fuzzy membership values $\frac{(0.4,-0.5)}{(x_1,y_1)}$ represent that the car x_1 is selected by 40% people due to its beautiful look, whereas 50% people rejected for the same reason. In this way, other bipolar fuzzy relations shown in Table 2 can be interpreted. The problem arises while generations of implications among bipolar fuzzy attributes.

Example 1: Let us suppose the implication $y_1 \to y_2$ and the

$$\frac{((\min(0.4,0.5)+\min(0.6,0.3)+\min(0.8,0.4)+\min(0.5,0.7)),\max(-0.5,-0.5)+\max(-0.3,-0.1)+\max(-0.2,-0.4)+\max((-0.2,-0.2)))}{4}$$
$$= \frac{((0.4+0.3+0.4+0.5),(-0.5-0.1-0.2-0.2))}{4} = \frac{(1.5,-1.0)}{4} = (0.375,-0.25) = \frac{(0.375,-0.25)}{2}(Accuracy) = 0.31$$

Step 8: The user can define the level for the support and confidence to refine the implications as shown in Table 1.

Step 9: Write down all the bipolar fuzzy implications for knowledge processing tasks.

Table 1 A proposed algorithm for attribute implications in crisp setting

Complexity: Let us suppose that there are m number of bipolar fuzzy attributes and n number of object. In this case, the proposed method shown in Table 1 takes (2^m) time complexity to generate the subset which leads overall O $(m.2^m)$ time complexity. In case the user is expert having background knowledge about the given implications (Ganter, 1999) as well as threshold for support and confidence (Singh & Kumar, 2017), the proposed method will be suitable for those users. Otherwise in large context, it is not feasible. In this way, the proposed method will be helpful for experts working in handling the data with bipolar fuzzy attribute.

4 Illustrations

Recently, some of the authors focused on data with bipolar fuzzy attributes and its visualization for knowledge processing tasks. In this process, a problem was left in (Singh & Kumar, 2014b) for generation of bipolar fuzzy implications. In this direction, attribute implications in fuzzy setting considering

$Support(y_1y_2)$ can be computed as follows:

$$\text{Confidence} = \frac{(0.375-0.25)}{(0.575,-0.3)} = \frac{0.31}{0.43} = 0.72$$

It shows that the attribute y_1 and the attribute y_2 appear more than 31% while selling the car, whereas 72% time customer selects attribute y_1 considers the attribute y_2.

Example 2: Let us suppose the implication $y_1 \to y_5$ and the $Support(y_1y_5)$ can be computed as follows:

$$= \frac{(1.9,-0.7)}{4} = (0.475,-0.175) = 0.33(Accuracy)$$

$$\text{Confidence} = \frac{0.33}{0.43} = 0.77$$

Table 3 The bipolar fuzzy attribute implications generated from Table 2 and its analysis

	Support	Confidence	Analysis
$y_1 \to y_2$	0.31	0.72	Cost not issue
$y_1 \to y_5$	0.33	0.77	Second preferred
$y_2 \to y_5$	0.28	0.74	Cost not issue
$y_1y_2 \to y_5$	0.25	0.81	Maximum sell based on these attributes

Table 4 A comparison of the proposed method with recent methods on bipolar attributes

	Bipolar concept lattice (Singh & Kumar, 2014a; Singh & Kumar, 2014b)	Fuzzy implications (Singh & Kumar, 2017)	The proposed method
Uncertainty measurement	Yes	Yes	Yes
Bipolar fuzzy attributes	Yes	No	Yes
Implications	No	Yes	Yes
Method	Concept lattice	Armstrong rule	Support and confidence using accuracy
Graphical representation	Yes	No	No
Applications	Bipolar information	Market basket analysis	Quantum information and its security
Time complexity	Based on subset	Based on subset	Based on subset

It shows that the attribute y_1 and the attribute y_5 appear more than 33% while selling the car whereas 77% chance that customer selects attribute y_1 considers the attribute y_5.

Example 3: Let us suppose the implication $y_2 \rightarrow y_5$ then:

$$Support\left(y_2 \rightarrow y_5\right) = (0.4, -0.15) = 0.28(Accuracy)$$

$$\text{Confidence}\frac{0.28}{0.38} = 0.74$$

It shows that the attribute y_2 and the attribute y_5 appear more than 28% while selling the car whereas 74% chance that customer selects attribute y_2 considers the attribute y_2.

Example 4: Let us suppose the implication $y_1 y_2 \rightarrow y_5$ then:

$$Support\left(y_1 y_2 y_5\right) = (0.375, -0.125) = 0.25(Accuracy)$$

$$\text{Confidence}\frac{0.25}{0.31} = 0.81$$

It shows that the attributes $y_1 y_2$ y_5 appear more than 25% while selling the car whereas 81% time customer selects attributes y_1 and y_2 considers the attribute y_5.

Table 3 represents that the implication $y_1 y_2 \rightarrow y_5$ has higher confidence and less support.

It means users prefer car (y_5) due to its beautiful (y_1) or luxurious (y_5) facility rather than its cost (y_2). In this case, the car x_3 will be preferred more by customer due to its beautiful and luxurious reason. The obtained results echo with its concept lattice structure (Singh & Kumar, 2014a; Singh & Kumar, 2014b) and granulation method (Singh, 2019) which validates the obtained results. Figure 3 shows that the proposed method is distinct than any of the available approaches on data with bipolar fuzzy attributes in various aspects. It is believed that the current method can be useful for dealing with the quantum data intelligence and its security purpose (Zhang, 2018).

In the near future, the author will focus on reducing the bipolar fuzzy attributes (Ali et al., 2019; Ali et al., 2020; Riaz & Therim, 2021) and its implications for dealing with the quantum of information and its graphical visualization (Singh, 2021; Akram et al., 2021; Singh, 2020).

5 Conclusions

This paper puts forward effort to compute the bipolar fuzzy attribute implications for knowledge processing tasks as shown in Fig. 2. To achieve this goal, a method is proposed to compute the support and confidence among chosen subsets of bipolar fuzzy attributes with its accuracy function in Section 3. It is shown that the obtained results resembled with its concept lattice structure (Singh & Kumar, 2014a; Singh & Kumar, 2014b) and information granulation method (Singh, 2019). In this way, the proposed method provides an alternative way while large number of bipolar concepts is generated from the given context. It is one of the most significant outcomes of the proposed method. In the near future, the author will try to focus on introducing other metrics to deal with the data with bipolar fuzzy attributes beyond m-polar fuzzy space (Singh, 2021; Akram et al., 2021; Singh, 2020).

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Declarations

Conflict of interest The author declares no competing interests.

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