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REVIEW

Research on Vibrating Screen Screening Technology and Method Based on DEM: a Review

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Abstract
Particulate materials are prevalent in the natural and engineering fields, and the screening of particulate materials is constantly improving with the development of industrial needs. New and efficient screening equipment is endless. Discrete element simulation plays a vital role in the design and development of vibrating screens, which improves the design speed of new screening machines and reduces the research and development cost. The purpose of this paper is to collect the literature published in recent years on the research development and application of discrete elements, with the expectation of providing a relatively comprehensive and advanced literature review on the application of discrete components in the field of vibratory screening, which includes the introduction of particle models, the setting of crucial simulation parameters, discrete elements in screening optimization. Meanwhile, the results of many researchers in the field of vibratory screening simulation in recent years are summarized to provide readers with references on the use of the discrete element method in the screening simulation process and to provide a cutting-edge summary for subsequent research.

Keywords DEM simulation · Particle model · Parameter selection · Screening performance · Target optimization

1 Introduction

Since the beginning of the industrial society, particulate matter types and applications have become more extensive. They are commonly found in natural and industrial fields such as agriculture, pharmaceuticals, environment, and mining [1]. Particle systems form a huge system with a complex and varied media composition, but with the same basic characteristics. In routine manufacturing processes, it is often necessary to separate particulate solids into sub-products with different particle size ranges [2]. Hundreds of millions of tons of particulate materials are subjected to industrial screening every year, ranging from traditional mining engineering to food and pharmaceutical engineering [3]. Screening is a widely used technique for separating discrete materials based on particle characteristics such as size and shape.

The screening process of particles is affected by many factors, in addition to the vibration operation coefficients such as vibration trajectory, vibration frequency, vibration amplitude, and feed rate, which will have a significant impact on the screening efficiency [4]; through the role of the excitation force also leads to complex interactions between the particles and their surroundings, and particulate matter can exhibit a variety of complex motion phenomena: e.g., particle size separation, particle deflection, and mixing motion [5]; meanwhile, the nature of the material, such as particle size and shape, material properties, and moisture content, also has a greater degree of influence on the sieving process [6]. The traditional analytical approach is to use the mechanics of continuous medium, to consider the particle population as a whole to view the research process using primarily phenomenological models, and to use probabilistic and kinetic formulas to analyze and study the sieving process; the main limitations of these methods are: the research is mainly based on physical experiments, and there is no experimental method to analyze the principles in microscopic aspects, and the particles within the particle population cannot be investigated [7, 8]. The discrete element method (DEM) is a numerical simulation method proposed by CUNDALL to deal with the problem of discontinuous media, and advanced computational techniques have pushed the improvement of the discrete element method so that it is used to solve the

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engineering problems of granular systems and is widely used in the study of vibrating screens [9].

After nearly 40 years of development, the discrete element method has been successfully applied in many areas of particulate matter treatment. There is a plethora of simulation software based on the discrete element method, which can provide detailed information on the sieving process and help tremendously in studying the sieving mechanism. At present, the more widely used commercial software are PFC2D, PFC3D, EDEM, and so on, among which EDEM has more outstanding stability advantages in large-scale particle motion calculation and is most widely used in vibrating screening. Commercial discrete element software is more convenient to apply. Still, the core computational code is difficult to modify by personal means, so some open-source DEM software is often used for model development and research, such as YADE [10] and LIGGGHTS [11].

This review will focus on translating the research in three aspects: particle shape, contact model, and parameter setting, expecting to provide help for readers to choose a particle model that can improve the simulation efficiency and save the simulation time; at the same time, the existing application of the discrete element method will be summarized to summarize the application of discrete element simulation on vibrating screen and the research results at this stage.

2 Discrete Element Method

Discrete element modeling (DEM) is a computational technique to simulate particle-level dynamics within powder systems. A review of the extant research literature indicates that particle motion simulation is progressing from the completion of effective modeling to the drawing of reliable conclusions under simulation. This represents a promising development trend. This paper presents a discrete element simulation process based on the screening of a vibrating screen, which is divided into three parts: pre-processing, mid-processing, and post-processing. The corresponding flow chart is shown in Fig. 1.

The pre-processing stage is primarily concerned with the creation of particle models and mechanisms. The credibility of the simulation results is contingent upon the creation of an appropriate particle model. In accordance with the experimental objectives, the appropriate particle model is selected to simulate the real particle shape, and the contact model is chosen according to the characteristics of the

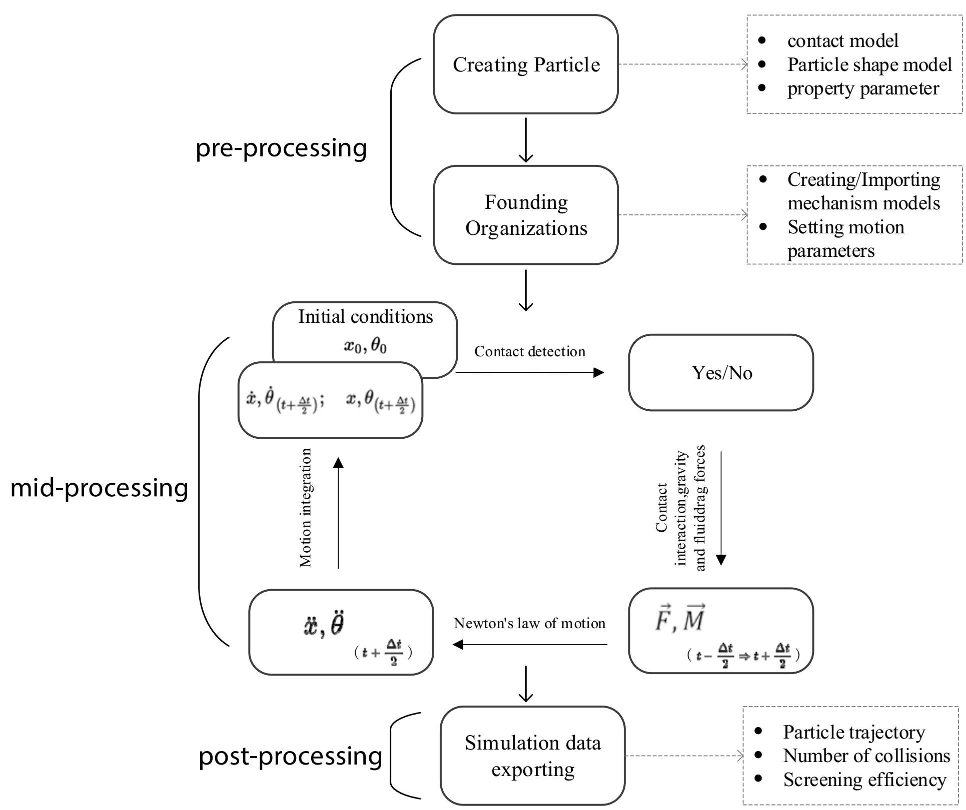


Fig. 1 Discrete element simulation process based on vibratory screening

material [12]. The model's property parameters are then set. DEM necessitates a comprehensive comprehension of the interrelationships between particles under diverse loads. The simulation of particle collisions is achieved through particle contact modeling calculations [13]. For different simulation objects, different contact models should be selected. In accordance with the research objectives of the simulation experiments and the selected contact model, the pertinent parameter sets of the particulate material are imported in order to ascertain the kinematic behavior of the particles. The screening mechanism is created in DEM software or imported through professional modeling software, and the relevant motion parameters of the screen plate and other components are given to simulate the working process of the vibrating screen.

The mid-processing procedure is concerned with the calculation of the motion of the particles during the simulation. Particles in collision are linked through a pair of equal-sized and opposite-direction forces. These forces are based on the following parameters: the amount of overlap at the point of contact, the physical properties of the contacting particles, the associated impact velocities, and the previous time-step contact information. In the brief time interval Δt, the acceleration and velocity are held constant, and all forces acting on the particle are considered, including those due to gravity, interactions with neighboring particles, and forces resulting from boundary conditions. By summing all forces and applying Newton's second law of motion, a new particle acceleration is derived. Subsequently, these accelerations are integrated numerically in order to derive the displacement x and the rotational increment θ [3]. These quantities are then used to determine the new particle position. The trajectories of the particle population can be obtained by implementing the aforementioned steps in each iteration of the simulation calculation.

Post-processing is the process of reading and analyzing the data obtained during mid-processing. The analysis of particle trajectories recorded in the DEM software allows for the acquisition of a comprehensive understanding of the movement characteristics of the particle population on the sieve plate at any given time and location. This includes the trajectory, movement speed, degree of accumulation of particles per unit area, and other essential data. This is of significant benefit to the design of optimized vibrating screens and the research and development of new equipment.

3 Particle Model

The motion of the particle group under the working condition of the screening machine is affected by many factors, and it is the main purpose of the calibration particle motion simulation to complete the simulation test reflecting the basic characteristics of the motion of the particles within the range of the computational capacity, to guide the optimization of the parameter configuration and to carry out the study of the screening mechanism. In this section, the research and development status of the key steps of particle modeling in DEM-based vibrating screen simulation will be presented in terms of the contact model as well as the shape model of the particles.

3.1 Contact Model

Depending on the contact mode, there are two main types of hard particle contact and soft particle contact in discrete elements [14]. Among them, hard particle contact is a highly desirable collision model that simplifies the computation of the collision process by neglecting details such as deformation or overlap during particle contact and considering only instantaneous collisions between particles [15] [16]. The hard particle model is more applicable to the high-speed movement of particles, or the case of sparse particles [14, 17]. However, its application scenario does not fit with the background of large-scale particle movement of vibratory screening. Moreover, it is almost not applied in the vibratory screening simulation at the present stage. Consequently, the hard particle contact model will not be discussed in detail in this paper.

Unlike hard particle contact, the soft contact particle model uses elasticity and damping coefficients to simplify the contact force between particles, with the parameters assumed to remain constant during the collision [18]. Allow particle collisions to occur with deformation or overlap between contact points due to collision, calculate the contact force based on the amount of overlap between particles, which is very small compared to the radius of the particles, and allow the particle collisions to last for a certain time so that collisions of multiple particles can be considered at the same time. The soft particle contact model can incorporate numerous contact models and has advantages in simulating the motion of large numbers of particles [19]. In the field of vibratory screening, it is usually necessary to face a vast number of particle motion simulations, so this review focuses on describing the application of soft particle models in the field of vibratory screening simulation. Figure 2 briefly illustrates the main differences between these two particle modeling approaches.

The energy loss mechanism of particles during collision is very complex, and for larger-sized particles, the usual contact effects include viscoelastic, elastic–plastic, and coulomb friction. In the viscoelastic model, the energy dissipation caused by particle collisions is mainly reflected in the viscosity or internal friction of the particle material; in the elastoplastic model, the energy loss is mainly reflected in the deformation of the particles [20]. The particulate

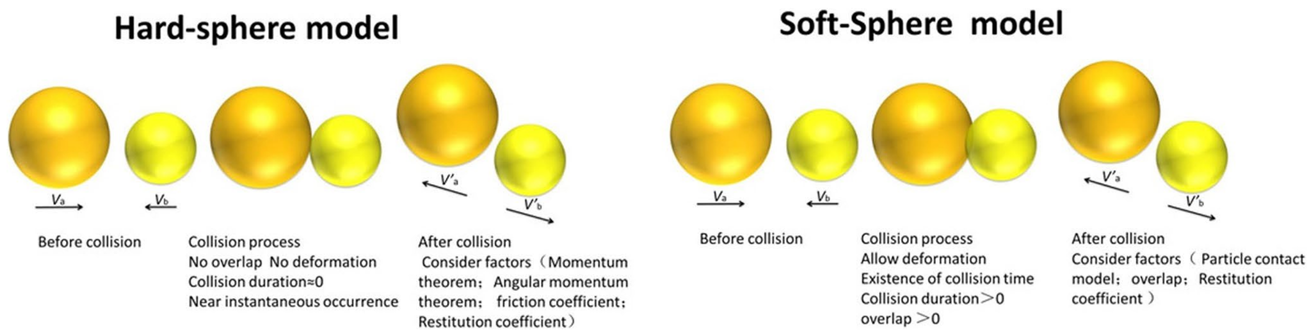


Fig. 2 Brief illustration of hard and soft particle model

materials involved in the field of industrial screening are usually screened with low-velocity motion, such as sand grains and rock minerals. The elastic deformation produced by the materials in the screening process is much larger than the plastic deformation, so the viscoelastic contact model is very common in the simulation application in the field of vibratory screening. Hertz [21] provides an excellent elastic particle model for solving the purely elastic collision problem of particles, which provides an important reference for subsequent contact model development.

The collisions occurring in the actual particle contact process are incomplete elastic collisions, and under the influence of the deformation of the particles and other factors, energy loss is bound to occur, and Hertz's model is not applicable to simulate complex particle collision processes. The variety of contact models employed in DEM, and the disparate methodologies utilized in contact force calculations, notwithstanding, the underlying calculation principles remain consistent. The forces and moments applied to the particles in the simulation are determined by the results of the selected contact model, which is essentially the result of the elasticity analysis of the contact mechanics of the granular solid in the quasi-static state. The most widely used models in vibratory screening simulation are contact models developed based on spherical particles, which mainly include the Linear Spring Dashpot (LSD) model, the Hertz-Mindlin (HM) model [22, 23], and the Hertz-Mindlin JKR bonding model [24, 25]; the formula for the above contact model is shown in Table 1.

3.1.1 Linear Spring Dashpot Model

The principle of the linear spring damping model is to convert the collision process of particles into a simple mechanical structure to represent it, which consists of a spring, a damper, and a slide plate. In the linear model, the contact force is linearly related to the relative displacement of the particles and the contact stiffness, and the linear model has the advantages of simple calculation, simple model construction, and easier programming [20]. Li et al. [26, 27] simulated the particle motion of spherical particles on a vibrating screen based on a spring-damper contact model

Table 1 Mechanical equations for the contact model

Contact model	Mechanical formula	Parameter meaning	Source
Linear spring damping model	$F_n = -k\Delta x_n + Cv_n$ $F_t = -k\Delta x_t + Cv_t$	Normal particle displacements Δx_n, tangential particle displacements Δx_t, v_n, and v_t are relative velocities, the spring stiffness k damping coefficient C	[43]
Hertz-Mindlin no-slip contact modeling	$F_n = \frac{4}{3}E^*\sqrt{R^*}\delta_n^{\frac{3}{2}}$ $F_t = -S_t\delta_t$ $E^* = 2G^*(1 + v^*)$ $S_t = 8G^*\sqrt{R^*\delta_n}$	Normal overlap δ_n, tangential overlap δ_t, tangential stiffness S_t, equivalent Young's modulus E^*, equivalent radius R^*, equivalent shear modulus G^*, and equivalent Poisson's ratio v^*	[44]
Bonded particle model (BPM)	$\Delta F_b^n = -k_b^n A_b v^n \Delta t$ $\Delta F_b^t = -k_b^t A_b v^t \Delta t$ $\Delta M_b^n = -k_b^t J\omega^n \Delta t$ $\Delta M_b^t = -\frac{1}{2}k_b^n J\omega^t \Delta t$	Shear stiffness k_b^n, normal bonding stiffness k_b^t, normal velocity v^n, tangential velocity v^t, normal angular velocity ω^n, tangential angular velocity ω^t; A_b is the area of the contact region,J is the moment of inertia	[45]
Hertz-Mindlin JKR Linear adhesion contact modeling	$F_{JKR} = -2\sqrt{2\pi WE^*a_0^3} + \frac{4E^*a_0^3}{3R^*}$ $\delta_n = \frac{a_2^2}{R^*} - \sqrt{\frac{2\pi Wa_2}{E^*}}$	Cohesion W, contact radius for adhesion a_0, equivalent modulus of elasticity E^*, equivalent radius R^*, normal overlap δ_n	[46]

under two-dimensional modeling conditions, demonstrating the unique advantages of DEM in studying and analyzing the field of vibratory screening. Jiao et al. [28] employed a spring-damper contact model to investigate the interaction of particles with the sieve plate. They constructed a mathematical model of the particle collision process and developed a program for the motion of a single particle on the sieve plate using VC++. As contact modeling theory continues to evolve, the linear spring-damper contact model is being gradually supplanted in vibrating screen simulations.

3.1.2 The Hertz-Mindlin Model

In practical microscopic experiments, particles are subjected to inter-particle interaction forces during collisional deformation, and the particles are deformed from an initial point contact to a surface contact in a very short time. At the present time, the nonlinear contact model, as represented by the Hertz-Mindlin contact model, remains the most widely utilized approach for simulating particles on vibrating screens[29–33]. In the Hertz-Mindlin contact model, the normal contact force is determined in accordance with the principles of the Hertz contact theory, while the tangential contact force is determined in accordance with the Mindlin-Deresiewicz contact theory. Figure 3(a) shows the schematic diagram of particle contact collision based on the Hertz-Mindlin contact model.

The adhesion force between moist particles is primarily attributable to the liquid bridge force between particles. The conventional Hertz-Mindlin contact model considers only the elastic contact between particles and is predominantly employed for the analysis of dry particle movement. The JKR contact theory is founded upon the Hertz contact theory and its calculation of the normal elastic contact force is based on the Johnson-Kendall-Roberts theory, which accounts for the expansion of the contact area caused by surface attraction and describes the nature of the attraction between the contact surfaces [34]. The JKR model is a widely utilized tool that can be employed not only to consider van der Waals forces but also to simulate liquid bridge forces. Nevertheless, due to the considerable complexity involved in the numerical implementation of the model, simplified versions are employed with greater frequency [35]. The model is schematically illustrated in Fig. 3(b). Zhou et al. [25] calibrated the parameters of wet gravel particles of different shapes based on the JKR contact model. They then employed two distinct contact models for vibrating screen simulation, which demonstrated that the JKR model yielded more precise results than the Hertz-Mindlin model in simulating the sieving of particles with adhesive forces. At this juncture, when examining the impact of moisture-laden particles on screening efficacy, researchers typically elect to employ the JKR model for the simulation of adhesive particles.

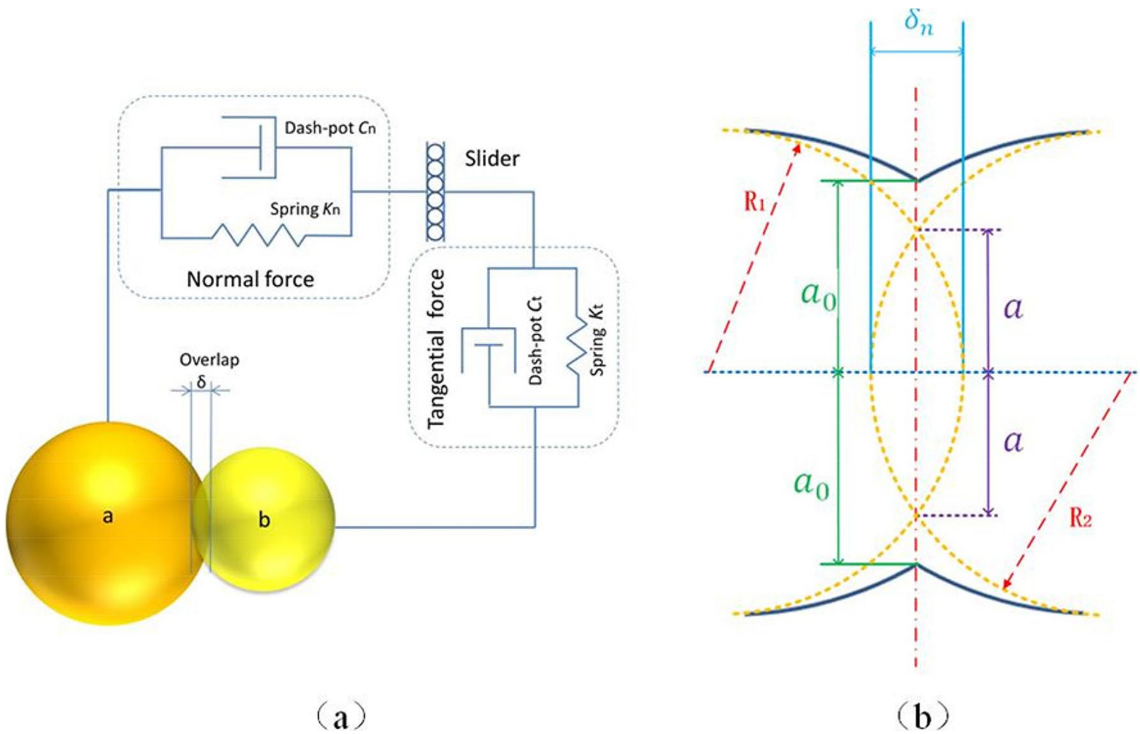


Fig. 3 Schematic of Hertz-Mindlin-based particle collision. **a** Hertz-Mindlin model; **b** Hertz-Mindlin JKR model

The contact models for multi-sphere particles are usually directly adopted from the contact algorithms for spherical particles without introducing new algorithms and thus are equally computationally efficient [36, 37]. In the actual screening simulation process, the deformation caused by particle collision is very small compared to the particle volume, to ensure the stability of the simulation calculation, the step size of the simulation process should be set to be very small, to obtain a more reliable and accurate particle contact relationship, but it should be pointed out that the smaller step size will also lead to an increase in the simulation computation time, which may cause some trouble for the screening simulation, which already has a huge amount of computation.

3.1.3 Other Models

In addition to the aforementioned contact models based on spherical particles, non-spherical particle contact models have been an important object of study in discrete metaparticle systems. Compared to spherical particles, ellipsoidal and hyperellipsoidal particles have more complex surfaces, such as intersection algorithms [38], geometric potential algorithms [39], and generalized normal algorithms [40]. The more widely used algorithm is the geometric potential algorithm, which is similar in basic principle to the contact modeling algorithm for spherical particles [41]. When two super ellipsoids overlap in contact, the deepest point connecting the overlapping parts of the two particles, the midpoint of the connecting line is determined as the contact point, and the direction of the contact point pointing to the deepest point is the direction of the contact force. The contact modeling algorithm for polyhedral particles is also based on the overlapping volumes between the contacting particles, which satisfies the principle of energy conservation for elastic collisions between particles. When two polyhedra are in contact, there are many different types of contact, such as point-to-point, point-to-opposite, edge-to-opposite, and so on. By calculating the area and unit normal vector of the overlapping polyhedra, the center of mass of the overlapping polyhedra is determined, and finally the volume of the overlapping block is obtained [42]. Hyperellipsoidal particle and polyhedral particle models are currently used in a few applications in the vibratory sieving industry, so more detailed modeling will not be presented in this paper.

3.2 Particle Shape Mode

Non-spherical particulate materials have significantly different macroscopic properties than spherical particulate materials, and the shape of the particles to be screened is a key influencing factor when evaluating the efficiency, and performance of a vibrating screen performed [47]. The conventional evaluation of particle shape is routinely presented in the form of three dimensionless parameters, namely sphericity, angularity, and roughness [48]. However, in the existing discrete element software, to facilitate the simulation of large-scale particles, irregularly shaped particles are generally simplified into symmetric spherical particles during particle modeling as a way to facilitate computer calculations. With the development of computer technology and the improvement of computing power, in the process of the application of the discrete element method gradually matured, many researchers constructed various particle models to improve the accuracy of particle modeling [41, 49, 50], such as the sphere model [51], multi-sphere model [52], ellipsoid model [41, 53], and polyhedral model [42].

3.2.1 Spherical Particle Model

Typically, the preferred choice for particle modeling is the sphere model, which is a simpler way to introduce particle shapes, with advantages such as easy contact detection and simple particle arithmetic. The sphere model has been developed and applied for many years and has been generally recognized by researchers for its simulation stability and accuracy [54–57]. Numerous researchers have applied spherical particles to the vibratory screening process, and a great deal of research has been conducted on the engineering and operational parameters of the screening process (Table 2). Many researchers have conducted extensive studies on the vibratory screening process using spherical particles. Different types of screens such as inclined screen and banana screen have been simulated, and the effects of vibration parameters [58], vibration mode [59–61], inclination angle of screen plate [56], length of screen plate [43], structure of screen holes [55], and number of layers of screen plate [62] on the screening results have been analyzed according to the simulation results. It is also applied to the development of new vibrating screens, such as oscillating vibratory screen [63], vibrating flip-flow screen [64], and the particle flow mechanism model is developed based on the spherical particle model.

In the current field of discrete element vibratory screening, a large number of particle operations are often faced which seriously affects the efficiency and time of the simulation, while the simplicity of the contact of the sphere particles makes the model quite computationally cost-effective.

With the development of discrete element technology, researchers' requirements for simulation accuracy gradually increased, and the spherical model was gradually replaced by other models, but due to the better stability of the spherical particle model, which is characterized by a high computation rate, it is also used in the research and development of new vibrating screens and screen model validation studies [64–66].

Table 2 Research and application of spherical particle modeling in screening

Category	Particle size distribution	Research contents	Result	Source
Test screen	Two sizes of spherical particles (5.25 and 3.50 mm)	Two-dimensional and three-dimensional particle sieving models were developed, and the effects of frequency, amplitude, and direction of vibration on particle sieving were analyzed. The efficacy of DEM simulation in vibratory sieving is corroborated	The DEM was initially employed in screening, which eliminated the constraints of traditional experiments in operating vibration conditions. The screening efficiency equation based on vibration condition parameters was derived, and its validity was validated through comparative experiments	[67]
Inclined stationary screen	Two sizes of spherical particles(bimodal mixing)	The collisions and motions of particle populations on a stationary inclined screen are investigated. The relationship between the definition of aperture boundaries in the simulation process, the motion of discrete particles along different regions of the sieve and the physical mechanisms inherent in the solids separation process, and their determining effect on the sieving efficiency are mainly discussed	The screening efficiency of all stages of the screen in the simulation is in accordance with the experimental results. The simulation results demonstrate that the proximity to pore-size particles has a detrimental impact on the screening operation, and that segregation in the material layer can have a significant effect	[27]
Multi-deck banana screen	Seven sizes of spherical particles	Simulations were carried out to investigate three and five deck screens, and the effects of vibration amplitude, vibration frequency and sieve plate geometry on the trajectory of the particle population with respect to the screening rate were discussed by varying a single factor	Screening performance can be improved by reducing the amplitude and frequency of vibration, replacing linear vibration with cyclic vibration, and reducing the angle of inclination of the screen mesh. It was confirmed that a five-deck screen provides better screening results under similar operating conditions	[62]
Inclined screen	Two sizes of spherical particles (bimodal normal distribution)	The effects of vibration parameters such as amplitude, frequency and direction of vibration on particle screening efficiency were investigated	The optimum vibration angle of 45° for the inclined screen was obtained. A mathematical model about screening efficiency and vibration parameters was established	[68]
Inclined screen	Fixed ratio (2–15 mm)	The linear, circular, and elliptical vibrations of the screen mesh were investigated, and the trajectories and penetration of particles on the screen plate were analyzed in detail for different vibration modes	The motion trajectories of the particle population on the screen plate under different vibration trajectories were revealed. The circular mode was found to have the largest incremental screening efficiency, while the linear mode had the smallest incremental screening efficiency. The vibration mode with the optimal screening efficiency was determined	[61]
Vibrating flip-flow screen	Five sizes of spherical particles (4—8 mm)	A continuous elastic sieve mat was discretized into multiple cells, and the displacement signals of each cell were analyzed by Fourier series to establish an elastic sieve plate model. The flow and separation of particles of different sizes in VFFS were investigated	Flexible motion simulation of sieve mats was realized. The motion characteristics of particles of different sizes on the sieve mat were observed. Directions are provided for the DEM of the flexible sieve	[64]

3.2.2 Multi-sphere Model

Most particles encountered in industry are non-spherical, such as ellipsoids, cubes, plates, or other irregular shapes, and it is widely recognized by researchers that the shape of the particles has a significant effect on the dynamics of the particle system. In particle–particle interactions, the shape of the particle plays a crucial role in determining the interaction forces as well as analyzing the particle trajectory. Figure 4 compares the sieving of spherical particles with non-spherical particles on the screen plate. Therefore, for the simulation of these particle models, the discrete element method based on spherical particle models is not sufficient to obtain satisfactory experimental data [69]. At low feed rates, the small and medium-sized screening efficiencies of spherical particles are in high agreement with experimentally obtained data, but in the screening of particles of near pore sizes, the spherical particle model has higher screening efficiencies than the experimental irregular particles [70]. This is caused by the fact that the spherical particle model is too idealized to accurately reproduce the particle shape.

Multi-spheres are the most widely used non-spherical models in discrete element applications. Multi-spheres refer to the combination of a certain number of spherical models to simulate the shape of real particles by changing the number, size, and overlap of the spherical particles in a local area. Figure 5 demonstrates the modeling of multi-spherical particles. Irregular particles can only pass through the screen holes in specific orientations and angles, whereas spherical particles can pass through the screen holes in any way which results in a higher probability of screen penetration for spherical particles [32, 71, 72]. A significant effect of shape on particle motion has been shown in many studies on particle mobility [73, 74]. These studies are also applicable in the field of screening, where the screening efficiency is strongly influenced by the size and shape of the particles, especially in the near-pore size range, and decreases for various particles in the order of spheres, symmetrically shaped non-spherical particles, and vertebral cones [75]. In the discrete element simulation, the motion velocity of spherical particles on the screen plate is high compared to the experimental data, while the motion velocity of non-spherical particles is low [35]. Another researcher noted the finding that the effect of particle shape on grain shape on sieving efficiency is quantitative, and the trend of screening efficiency per unit time for different shaped particles is the same [69].

The use of multi-spheres allows for a more accurate representation of particles with complex shapes, allowing for more accurate particle motion simulation [32]. Li et al. [76] used the Hertz-Mindlin model with JKR cohesion to simulate the screening process of moist multi-spheres on a multi-layer linear vibrating screen, and the optimal combinations of vibration parameters of the vibrating screen under different working conditions were obtained through a large number of numerical simulations. In agricultural screening simulations, the superposition and combination of multiple spheres in the multi-spheres model facilitate the restoration of seed and fruit shapes, thereby providing more accurate data for simulation. Fu et al. [77] designed a variable slope screen with a pore structure for screening corn grain mixtures, applied a multi-sphere model to accurately model different types of corn seeds, corn cobs, and stover fragments, discussed in depth the minimum dimensions required for particles, such as corn kernels, to pass through the sieve holes smoothly in various attitudes, and verified the effectiveness of the new screen. The advantage of the multi-sphere model is that simple spherical contact detection algorithms can be retained even for complex particle shapes [37]. The combined forces and moments of the multi-sphere particles due to inter-particle contact or collision are the sum of the forces and moments acting on each subsphere, which are then accrued to the center of mass of the multi-sphere for subsequent calculation of the particle motion [36]. The

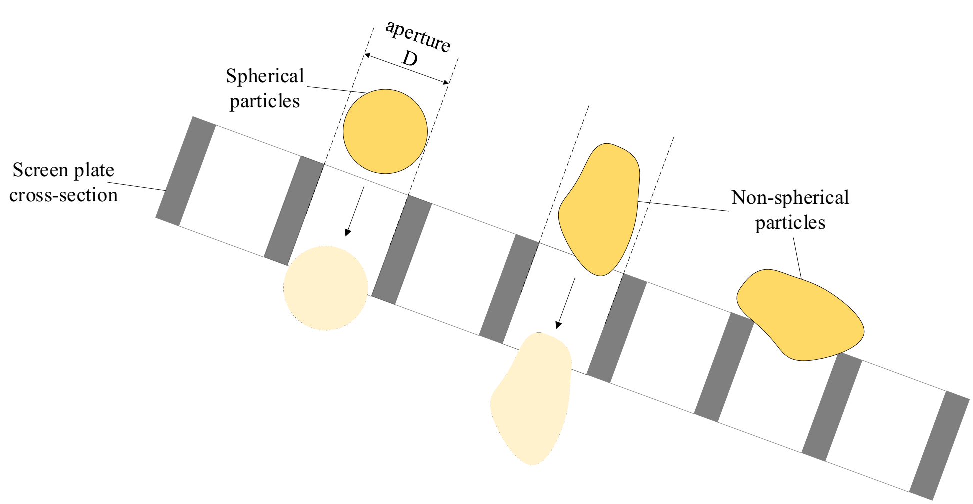


Fig. 4 Movement of particles on the screen

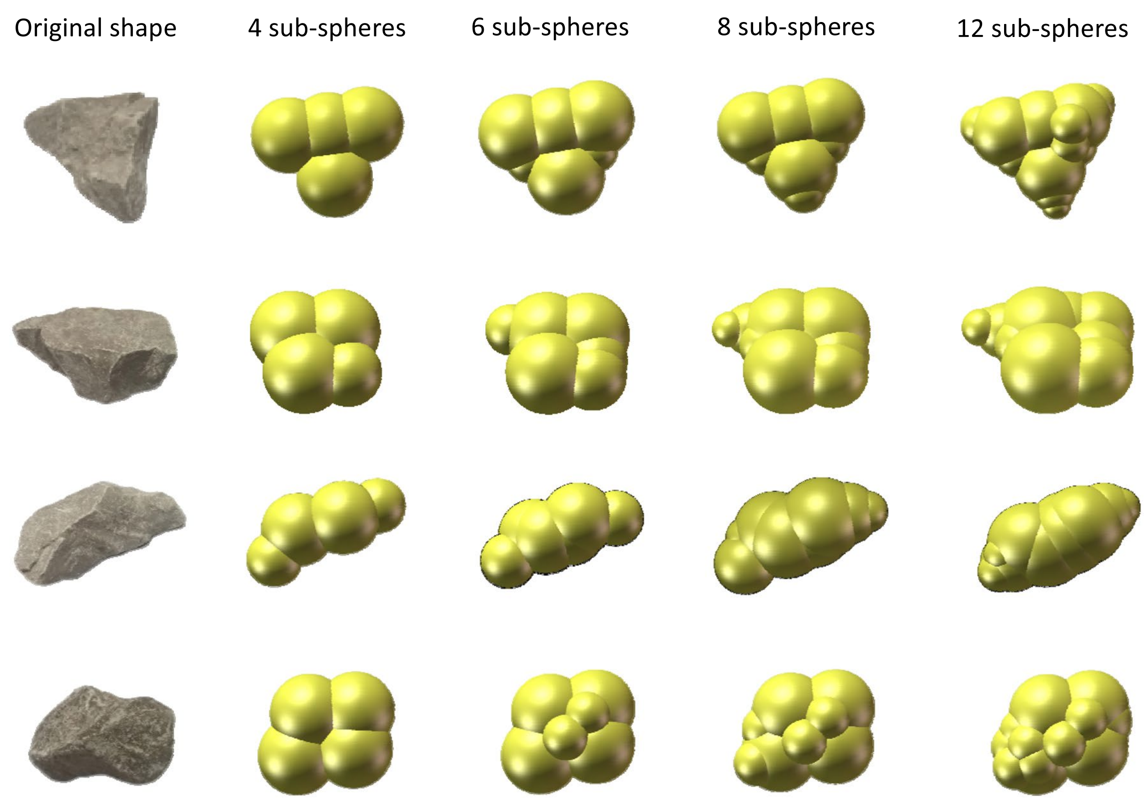


Fig. 5 Multi-spherical particle model

contact detection of multi-sphere particles is determined according to the spheres on the body that are in contact with other particles or screen bodies, following the contact model of spherical particles, which has high computational efficiency and stability. In summary, it is shown that the multi-sphere should be the most commonly used discrete element model in the field of vibratory screening at present.

It has been shown that while multi-spheres can simulate a more realistic particle shape, the total particle mass and inertial properties can be biased and are not easily modified [78]. Multi-sphere model belongs to non-convex non-smooth particles, spherical particles that overlap each other under the positional relationship between the surface of the model will still exist in the combination of local gaps, affecting the overall smoothness of the surface of the particles, want to simulate the smooth particles need to be added in the establishment of the model of the sphere to fill the gaps to restore the true shape of the particles, to improve the accuracy of simulation. The multi-sphere approach has limitations when used for the simulation of approximate spheres and therefore faces difficulties when applied to other arbitrary shapes [36]. Although the accuracy of the model can be increased by increasing the number of spheres, the calculation of the screening process using polymers as the particle model follows the solution method of collision and friction between spherical particles, and its computational cost is usually much higher than that of the sphere model, and an excessive increase in the number of spheres will greatly increase the computational difficulty. Therefore, when using the multi-sphere model to represent irregular solid particles, the number of spheres in the multi-sphere model needs to be rationally analyzed to strike a balance between determining accuracy and simulation efficiency.

3.2.3 Other Particle Models

In addition to the abovementioned particle models that are more commonly used in the field of discrete elements, this paper will introduce two models that have a very high potential in the field of vibratory screening: the hyperellipsoid model and the polyhedral model. Particle models based on hyperellipsoids originate from hyperquadratic methods originally applied to computer graphics and are extended by quadratic equations. This model was first used by Williams and Pentland in a 2D DEM. Later, Cleary introduced them in three dimensions [79]. They are introduced in three dimensions. The standard formula for hyperquadratic equations in three dimensions can be written as:

$$f(x,y,z) = \left(\left| \frac{x}{a} \right|^{s_2} + \left| \frac{y}{b} \right|^{s_2} \right)^{\frac{s_1}{s_2}} + \left| \frac{z}{c} \right|^{s_1} - 1 = 0 \quad (1)$$

where *a*, *b*, and *c* respectively represent the semi-major axis length of the particle along its main axis. The shape indices s1 and s2 determine the curvature of the particle edges, and the sharpness of the curvature increases with the shape index. Using the hyperquadratic equation, various shapes can be generated effortlessly by adjusting the lengths of the three half-length axes (*a*, *b*, and *c*) and the two shape indices (s1 and s2). The shape of the hyperellipsoid is characterized by the fact that the aspect ratio can be varied arbitrarily, and the angular curvature of the particles varies as a function of the parameters in the hyperellipsoid equation [42].

The hyperellipsoid model is mainly used in the simulation of various particles with smooth curved surfaces. The ellipsoid can represent a variety of shapes, ranging from flat to elongated, and all types of particulate matter have this shape, which is applied to the modeling and simulation of particles with different aspect ratios and sphericities, Fig. 6 shows a common model of a superellipsoid. Many researchers have made a lot of efforts to model ellipsoids in discrete elements [80–83]; the experiments and corresponding DEM simulations of the related theoretical studies demonstrate that the DEM simulation results using the superellipsoid model are capable of modeling and predicting the properties and motions of non-spherical discrete particle systems [84, 85]. In multi-sphere simulations, when a large number of constituent spheres are used to approximate ellipsoidal particles, the values of the volume solid fraction are also close to the experimental results, but too large a number of spheres can seriously affect the simulation efficiency. When fewer spheres are used to approximate ellipsoidal particles, the accuracy of the simulation decreases. The results of the discrete element experiments show that the use of hyperellipsoids to represent ellipsoidal particles is more advantageous than the use of multi-sphere models [41].

Fan et al. [86] modeled coated fuel particles with superellipsoids and investigated the separation characteristics of superellipsoids on an inclined vibrating plate (IVP). Zhao et al. [87] proposed a new method for three-dimensional discrete elemental modeling of granular media using a superellipsoid and verified its generality and practicality. Mori et al. [88] developed a new superellipsoidal DEM/SDF model to simulate powder mixing of non-spherical particles in an industrial mixer and provided new ideas to solve the problems inherent in the simulation of non-spherical particle systems. At present, the hyperellipsoidal particle model is less used in the field of industrial vibrating screens, but with the theoretical research on the hyperellipsoidal model and the further improvement of the contact model, the advantages of the hyperellipsoidal model in the field of sphere-like simulation will be reflected.

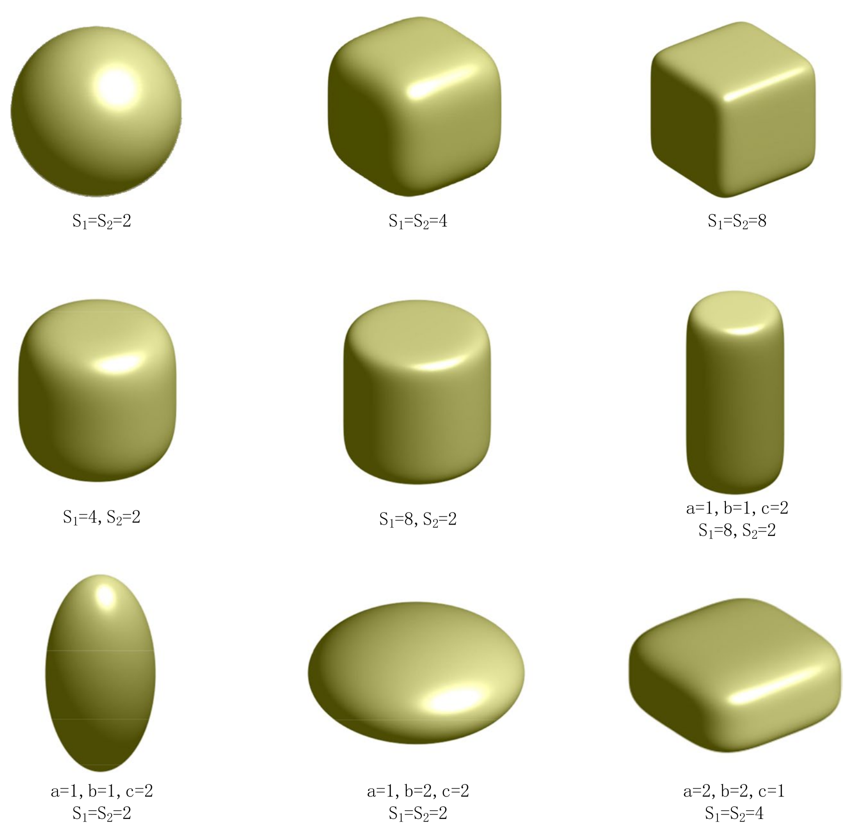


Fig. 6 Hyperellipsoidal model

Most of the mineral particles in the real screening process are irregular polyhedral particles with sharp angles, Angles are a geometric property of particles that resist the relative rotational motion of the particles in the assembly due to the interlocking effect of the particles. This geometric feature can significantly affect the macroscopic behavior of the particle system. There are various ways to represent the shape of polyhedra and common convex polyhedra are usually categorized into sharp-edged and smooth-edged polyhedra. Smooth polyhedra are often simplified in place of irregular polyhedral particles using either spherical, polyspherical models [89], or ellipsoids [90], in which rounded polyhedra are used in place of the edges of polyhedral particles by applying smaller spherical particles. The sharp-edged polyhedral particle model is illustrated in Fig. 7.

Wachs et al. [91] proposed a new discrete element method to simulate the flow dynamics of four particle models: spherical, cylindrical, cubic, and tetrahedral, and verified that the numerical code Grains3D provides reliable and fairly accurate computational solutions for both spherical and non-spherical particles. The geometry of particles may also have other profound effects on the physical behavior of particle systems in many different ways. At this stage, there are still some problems in the numerical simulation of the screening process using the polyhedral particle model, the number of particles in the vibration screening process is huge and there is a lack of accurate and efficient non-spherical particles computational model, and the huge amount of computation in the simulation process is always troubling the researchers, which has led to the current researchers focusing more on how to accelerate the efficiency of the simulation under the premise of steadily improving the accuracy of the model.

3.3 Particle Fragmentation Model

The phenomenon of adhesion and fragmentation of particles is prevalent in industrial and mining engineering, especially in the field of coal processing, where many of the materials to be separated may be wet due to sprinkling of water to reduce dust, seepage of water from the coal seam, etc., which leads to particles with small gaps being more prone to adhesion to each other in clusters, forming large-sized particles well above the size of the screen aperture. In industrial vibrating screen work, to effectively screen a large number of particles so that the particles can effectively pass through the screen surface, the screen plate is usually high-frequency vibration to produce far more than gravity several times the peak of the acceleration [92]. Under the action of a large load particle material will be subjected to greater impact and force, the collision between particles in the screening process; the collision with the screen plate may cause the particles to produce deformation, which in turn produces the effect of crushing, resulting in a change in the shape of the particles, affecting the mobility of the particle group in the screen plate; large particles produced by the crushing of the small particles will further affect the particles in the screening efficiency of the screen plate [93, 94]. Some researchers have treated the moist material as a flowing slurry and used a multi-phase discrete element fluid model to study the motion of the particles on the screen plate [95]. However, in most cases, the moisture content of the particles to be separated is not sufficiently high for the water to move independently of the particles, so the discrete element fluid model does not represent the screening process well [92]. Robertson et al. [96] proposed for the first time an adhesion model for particles, which provided a new idea for discrete meta-studies of particle fragmentation.

Among the particle fragmentation models currently used in discrete elements, there are three main types: the bonded particle model (BPM) [97], the particle replacement model (PRM) [98], and the fast breakage model (FBM) [45, 99]. Although each of the three models has its advantages in application, PRM itself is a simpler model and does not accurately show the complex movement of particles on the screen plate, and there is a strong subjectivity in the shape of particle crushing in the simulation of particle crushing, leading to a large discrepancy between the crushing process and the results and the reality; FBM has limitations in describing the forced deformation profile measured by single-particle fragmentation as well as the size distribution

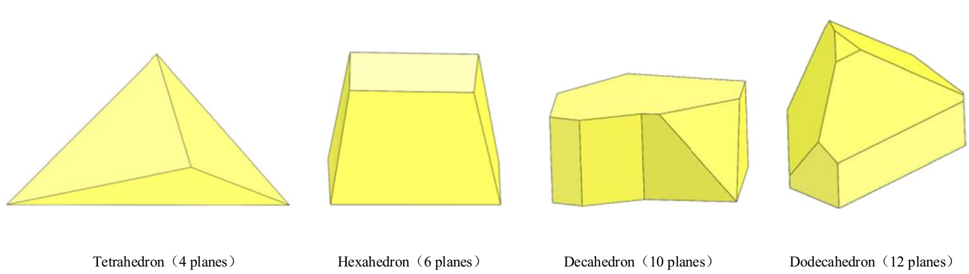


Fig. 7 Irregular polyhedral model for simulating ore particles

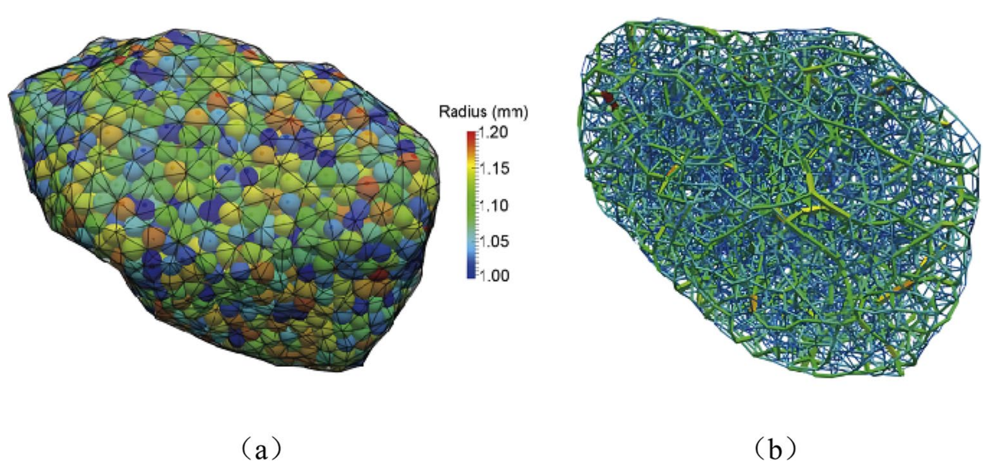
of the fragmented particles; The parameters of the BPM are directly related to the mechanical properties of the particulate material, such as critical tensile and shear stresses, and are able to characterize multiple aspects of the particle fragmentation process, providing more accurate simulation results [100]. In light of the aforementioned considerations, the particle crushing model employed in the context of vibrating screening simulation at this stage is primarily BPM. However, due to the large number of particles involved in the adhesive particle model, the computational time to be consumed is much more massive [101]. Facing this problem, a large number of researchers use CPU and GPU parallel computing to improve the simulation rate [102–104].

The adhesive particle model is similar in modeling to the multi-spherical model in that a certain number of spheres are used to connect them, and again the size and position of the local spheres can be varied according to the irregular particles to be modeled, allowing for overlap between the spheres [105]. The bonded sphere model can be used directly to determine the position using a contact detection algorithm for spherical particles, and in the field of vibratory screening, the BPM model is currently the most used, and the particle model is shown in Fig. 8. Previous models of spheres and multi-sphere model applied to vibratory screening have viewed the particle itself as a rigid being, the bonded sphere model is significantly different from them [106]. In the bonded-particle model utilizing bonding bonds that can be broken to link each particle in the particle, the motion of each particle within the bonded-particle during the simulation will be based on the forces and moments acting on each particle. In this process, different particles in the model of the adhesive sphere may produce relative displacements due to different forces, thus deforming the adhesive sphere, and when the pressure received exceeds the critical level, the bonds between the particles will be broken, and particle fragmentation occurs.

The research of particle crushing is a very popular topic nowadays, but compared with the three particle models mentioned above, the research of the adhesive sphere model is still in a starting state, and the current research results are more at the theoretical level, and many studies are based on numerical experiments of single-particle crushing to explore the particle crushing [103, 107, 108]. Single-particle crushing tests are inconsistent with real-world engineering situations in which particles are subjected to multiple contact loads, and the complex stress fields within the particles caused by multiple contacts make it difficult to determine which factors promote crushing. Fu & Zhou et al. [109, 110] noted that particle shape significantly affects the fracture mode and crushing strength of particles. In the context of screening, when particles are influenced by other particles due to adhesive forces, this may adversely affect the penetration of particles through the vibrating screen deck screening mechanism. In contrast, strong collision and shear forces due to bed shear and vibratory motion of the screen plates may break the cohesive bonds between particles, thereby inhibiting particle mobility.

Cleary et al. [92] used the cohesive particle model to investigate the effect of inter-particle cohesion on particle flow and separation efficiency on banana screens, and found that at high levels of cohesion, large lumps of cohesive particles severely impeded the screening process, causing a buildup of particles on the screen decks; for intermediate levels of cohesion, the behavior of the material rapidly changed from being viscous and difficult to flow to one in which the particles could be appropriately handled by the screen; and for lower levels of cohesion, the screening performance became independent of

Fig. 8 Bonded particle modeling of rock blocks. **a** Spherical particle assemblage model; **b** bond-bond connection model



the level of adhesive force. In addition, discrete element simulation based on the model of cohesive particles is also used in the development and testing of new vibrating screens: when it comes to the agglomeration of particles that are easily affected by the environment, special screens such as the new Rigid-Flexible-Rod Vibrating Screen (R-FRVS) [111], and the Cantilever Vibrating Screen (CVS) [112] with open apertures can effectively solve the problem of wet particles clogging the apertures of the screen.

At the present stage, the application of the cohesive sphere model simulation in the field of vibratory screening is less; the main reason may be because the crushing simulation of the cohesive particle model requires higher arithmetic power compared with the three particle models mentioned above, and the lack of the relevant theoretical support, which results in the large number of the use of the cohesive particle model in the field of vibratory screening to study the crushing of the particles on the screen plate, is a difficult thing. However, it is undeniable that the model of bonded balls has very great potential in the field of vibratory screening, especially in the simulation of special materials or special environments to provide a new program of screening, and also a new direction for the future study of particle kinematics. With the improvement of particle crushing theory and modeling, the bonded sphere model is more often used in vibratory screening simulation.

4 Contact Parameters

In industrial production, the main parameters to measure the mechanical properties of ores are modulus of elasticity, Poisson's ratio, density, compressive strength, shear strength, and so on. It has been proposed that the primary challenges associated with the industrial implementation of the DEM method pertain to the input parameters of the granular material [113].In discrete element modeling, the required material parameters can be divided into two main categories: material properties and interaction properties [114]. Other material properties that need to be set in discrete element modeling are elasticity, shear modulus, Poisson's ratio, and yield strength. In addition to the material's properties, particle–particle, and particle-boundary interactions significantly affect particle motion [114].

In a DEM simulation, model parameters can only be set at the microscopic level, such as particle volume and shape, particle density, elastic modulus, friction coefficient, damping coefficient, and so forth. Conversely, macroscopic attributes, such as bulk density, internal friction angle, expansion angle, and angle of repose, among others, cannot be set in a DEM [115]. The accurate representation of these macroscopic features requires the overall tuning of the microscopic parameters to make the kinematic behavior of the particle population in the simulation consistent with that in the physical material experiments [116]. It is possible that different contact models may contain different microscopic parameters. Furthermore, different contact models may not require the same model parameters when chosen to represent the same macroscopic results.

The accurate modeling of the size and shape of particles in large industrial equipment, such as vibrating screens, is a challenging endeavor. Consequently, field measurements or laboratory experiments are employed to assess the overall properties of a specific material. The experiments were replicated digitally in accordance with the laboratory or field setup and procedures, with the utmost fidelity to the original conditions. Subsequently, the DEM parameter values were altered in a iterated manner until the predicted volumetric response was found to be in accordance with the measurements. One potential issue with this methodology is that the volumetric response of numerical experiments may be influenced by a multitude of parameters.

4.1 Modulus of Elasticity and Poisson's Ratio

Depending on the selected particle material and contact model, the corresponding mechanical parameters can be measured. For example, contact stiffness, elastic modulus, or shear modulus can be introduced when considering the elastic effect of particle contact. The elastic parameters affect not only the deformation behavior of the particles, but also the time step of the particles (Rhodes, Wang, Nguyen, Stewart, & Liffman, 2001). The modulus of elasticity E and shear modulus G are linearly related to each other through Poisson's ratio v [117].

Many studies [118–120] have pointed out that Young's modulus, Poisson's ratio, and compressive strength are crucial for simulating the behavior of simulated rock particles. Through discrete element simulation, the experimenters used the bonded particle model to reproduce the macroscopic properties of the minerals on the particle assemblage, reproduced the crushing and fracturing of the ore samples in the simulation, and discussed the effect of the particle size on the measured elastic modulus, Poisson's ratio, and various mechanical parameters [45, 121]. Although the application of the discrete element method for the simulated reduction of mineral particles has been recognized by researchers at this stage, there is still a discrepancy between the elastic modulus of particles used in discrete elements and that measured in the laboratory [122].

The modulus of elasticity, in addition to having a large impact on the simulation accuracy, has a significant impact on the computational efficiency and simulation time, by reducing the modulus of elasticity of the particles to make the simulation process a larger step size per unit of time, the total computation time becomes smaller as a result [123]. Compared to the approach of parallel computing with the GPU by optimizing the collision detection model of the particles, it is more convenient and easier to implement the approach of increasing the time step by appropriately decreasing the elastic modulus of the material and thus reducing the computational load. In order to provide the reader with a reference when simulating, Table 3 presents a summary of the particle mechanics parameters that have been utilized by numerous researchers in vibratory screening simulations.

4.2 Damping and Recovery Coefficients

The coefficient of restitution (COR, also denoted by e) refers to the ratio of the final relative velocity to the initial relative velocity of two particles after collision. The coefficient of restitution is influenced by many factors such as impact velocity, geometry, particle elasticity, and temperature [127–129]. The recovery coefficients obtained during particle collision experiments are higher than in normal physical experiments [130], Melo et al. [131] proposed three empirical equations for the coefficient of recovery based on impact velocity, which provide a new tool for predicting the coefficient of recovery of particles with high modulus of elasticity under low velocity motion. In discrete element simulations of vibrating screens, a constant recovery factor is usually chosen based on the particle material, and the constant is usually modified in an iterative manner until the results obtained match the experimental data. However, the constant coefficient of recovery does not accurately represent the real system because the real system has different coefficients of recovery at different locations. In order to reduce the influence of particle properties on the simulation results, another researcher proposed to use parameter-dependent recovery coefficients for discrete element simulation, and proposed and validated a recovery coefficient model based on the change of particle state [132].

Some of the kinetic energy of the particles in motion is dissipated in plastic deformation or converted into thermal and acoustic energy; to account for this in the discrete element, a damper system is added to the contact model of the particles. The equation for the relationship between the contact damping force and the damping coefficient varies in many different contact models, which also leads to the fact that the kinematic behavior of colliding particles may be very different and strongly dependent on the chosen values of the contact parameters [127].

4.3 Coefficient of Friction

The friction coefficient is an important factor affecting the mechanical properties of particle materials; at this stage, there are many scholars through theoretical research, experimental analysis, simulation, and simulation of the way particle friction on the particles of the movement behavior of the particles impacts a large number of studies, which verified that the mechanical behavior of the particle material has an important impact [133–136]. There are two main types of friction coefficients considered in discrete elements; one is the particle-to-particle friction coefficient and the other is the particle-to-boundary friction coefficient. Based on the research of many researchers on the particle friction coefficient, the current research on the particle friction coefficient on discrete elements is gradually mature, in addition to the study of the friction coefficient of spherical particles [137], the effect of different particle shapes, particle velocity,

Table 3 Elastic characterization of particles in vibratory screening simulation applications

Materials	Elastic modulus (Mpa)	Shear modulus (Mpa)	Poisson's ration *n*	Density (kg/m^3)	Source
Sand	——	23	0.3	2678	[43]
Grit	50	——	0.2	2600	[76]
Rock	24	——	0.3	2500	[124]
Iron ore	——	1.8	0.25	3150	[125]
Broken rock	50	——	0.45	2700	[32]
Sintered ore	2160	——	0.13	3000	[126]
Coal particles	10*	——	0.3	1400	[62]
Coal particles	5*	——	0.45	1400	[2]
Coal particles	1200	——	0.25	1430	[65]
Coal particles (damp)	20000	——	0.2	1100	[66]
Coal particles (damp)	1000	——	0.3	1400	[31]

Some of the * symbols are deliberately modified modulus of elasticity, which is much smaller than the true modulus of elasticity (~ 10 MPa) contact model, and other factors on particle friction [134, 138–140]. The movement of particles on a vibrating screen contains not only a large amount of inter-particle friction but also friction between the particles and the screen plate, which also significantly influences the sieving motion of the particles [141].

Although friction between particles and walls and between particles plays a crucial role in particle motion, the lack of friction coefficients makes it almost impossible to quantitatively analyze the effect of friction on experimental results. To address this issue, some researchers have provided a way to measure a suitable friction coefficient for discrete element simulation by measuring the motion of experimental particles to derive the friction coefficient [142], and others have proposed new calibration methods to calibrate the boundary parameters by using the impact force in the particulate streams and the packing density in the stacking as the experimental reference results [143]. The friction coefficient of particles is a very well-defined material parameter, while in most of the literature, the discussion is about the friction mechanism and modeling between particles, and no precise friction parameters are provided to support the experimental study. Table 4 summarizes the values of the coefficients of friction in volume for the selected particulate materials for the reader's reference.

The representation of particle shape is a computationally expensive attribute when modeled in a DEM, and in the face of interactions involving millions or even hundreds of millions of particles with complex shapes on an industrial vibrating screen, the computational time required to be spent without simplification is unacceptable. In addition to the aforementioned methods of increasing arithmetic power through GPU parallelism, some researchers propose that by simplifying the particle model, uniformly using spherical particles instead of particles with complex shapes, and introducing the concept of rolling friction, this discrete meta-model is quite computationally cost-effective [144]. The idea behind this treatment is that since the shape of the non-spherical particles will impede the particle movement to some extent, the average velocity of spherical particles will usually be higher than that of non-spherical particles on the screen plate, and the movement of the spherical particles is determined by the friction between the particles and the particles or the screen plate [144]. Therefore, adding additional rolling friction to the model of spherical particles is equivalent to applying additional torque at the point of contact, which to some extent reduces the motion of non-spherical particles on the screen plate [145, 146]. The introduction of rolling resistance into the contact model significantly improves the efficiency of the DEM in simulating the real behavior of granular materials. Chen et al. [147] investigated the effect of rolling friction on spherical particles during vibratory screening. By combining the results of simulations and experiments, polyhedral particles and spherical particles with different rolling friction coefficients are compared, and it is verified that spherical particles can exhibit the kinematic

Table 4 Reference table of friction coefficients for particle models

Material	Contact object	Restitution coefficient e	Damping coefficient ζ	Static friction coefficient	Coefficient of rolling friction	Source
Sand particles	Particle—particle	0.1	——	0.545	0.01	[58]
	Particle—screen	0.2	——	0.5	0.01	
Grit particles	Particle—particle	0.0003	——	0.44	0.01	[76]
	Particle—screen	0.03	——	0.5	0.002	
Rock particles	Particle—particle	0.2	——	0.6	0.01	[124]
	Particle—screen	0.6	——	0.45	0.01	
Iron ore particles	Particle—particle	0.05	——	0.3	0.1	[125]
	Particle—steel	0.08	——	0.35	0.25	
	Particle—rubber	0.05	——	0.71	0.05	
Sintered ore particles	Particle—particle	0.18	——	0.45	0.12	[126]
Coal particles	Particle—particle	——	0.00005	0.3	0.01	[62]
	Particle—screen	——	0.0002	0.3	0.01	
Coal particles (damp)	Particle—particle	0.5	——	0.35	0.01	[66]
	Particle-steel	0.5	——	0.154	0.01	
	Particle—screen	0.5	——	0.35	0.01	
Coal particles (damp)	Particle—particle	0.5	——	0.6	0.05	[31]
	Particle—screen	0.5	——	0.4	0.05	
Coal particles	Particle—particle	0.5	——	0.45	0.05	[65]
	Particle—screen	0.5	——	0.63	0.05	

properties of polyhedral particles to a certain extent when the rolling friction of spherical particles is set in an appropriate range. The concept of rolling friction is currently widely used in DEM modeling to improve the accuracy of simulations, especially in the field of dynamic simulation [146].

5 Application of DEM in the Field of Vibrating Screen

The screening process is controlled by multidisciplinary principles ranging from physics to applied fluid dynamics. Particle motion during screening is influenced by a variety of factors, and these variables generally fall into three broad categories: screen geometry, such as screen aperture size, shape, and screen length; particle properties, such as shape, modulus of elasticity, and coefficient of friction; and screen operating conditions, such as vibration frequency and inclination angle [8]. Because there are so many variables, the screening principles of vibrating screens are complex and difficult to model, and the interactions between the variables are so complex that there are no convincing methods to evaluate and predict the screening process and to study the motion of the particles, which seriously affects the efficient operation of industrial vibrating screens and the design and development of new vibrating screens [148–151]. In the last two decades, numerical simulation has been increasingly used in the screening process, and the discrete element method has played a significant role in studying the screening process [152]. This section summarizes the various current studies and applications of discrete elements in the field of vibrating screens and discusses the existing studies in three separate areas based on the influence of various variables in the screening process; the flow is shown in Fig. 9.

5.1 Revealing the Trajectories of Particle Populations

The motion of particles on the screen plate will be affected by various factors, which we have discussed in the above sections, and how the trajectory of the particle population will change under the influence of different conditions has been the focus of researchers. The movement of the particles on the screen plate will directly determine the particle perforation rate, which in turn determines the screening efficiency of the vibrating screen; at the same time, the properties of the particles themselves also have an impact on the movement, and these issues are to be more in-depth study. The particle flow of particles during sieving depends on the interaction between individual particles and between particles

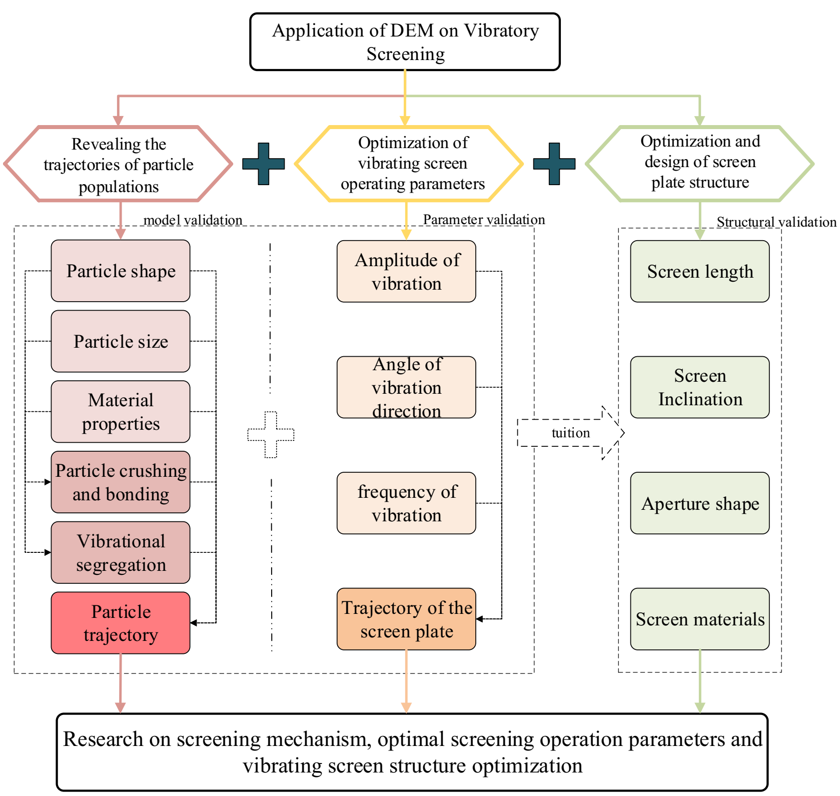


Fig. 9 Flow chart of vibrating screen optimization design

and screen plates, which is difficult to quantify experimentally [153]. Recent studies have used empirical validation and discrete element modeling to validate the screening efficiency of vibrating screens, but these studies are usually limited to the efficiency analysis of different parameters for a specific screen plate screen, and no more general model has been developed for predicting screening efficiency [2, 32]. Understanding the motion of the particles on the screen plate is a crucial part of obtaining a more generalized prediction model.

In addition to the accurate prediction of overall efficiency and product size, the most significant advantage of the discrete element method is that it can reveal the motion of the particle population. This allows for a more in-depth analysis of the screening process, such as tracking the trajectory of the particles, the stratification and segregation of particles in the sieve plate, and collision between the particles. This provides researchers with valuable insights [76]. The process of separating mixed particles due to differences in density, particle size, and shape under vibration is called vibratory segregation [154]. The particle group under the action of vibration usually produces three different segregation modes: large particles move up and small particles sink; large particles sink and small particles move up; large particles and small particles have no obvious relative displacement changes, and all of these changes will have a great impact on the efficiency of the vibrating screen [155]. A number of theories have been put forth by researchers to account for this phenomenon. These include void filling [156], global convection [157], and density-driven theories [158]. The dissociation model is shown in Fig. 10. The three most important material properties that affect segregation are particle shape, size, and density [159]. In DEM simulations, researchers have also made an in-depth study of the effects produced by the properties of granular materials [160–164]. The particle motion occurring at varying stages of the sieving process can be quantified on a particle scale, which is crucial for a comprehensive analysis of the separation of materials. Soldinger [151] identified three primary factors that influence the stratification parameters: the size distribution of the particles, their relative size in relation to the surrounding material, and the layer thickness. The behavior of particles of different sizes in terms of stratification and passage on the screen is discussed. In addition to the aforementioned influences, Zhao et al. [89] conducted an investigation into the segregation of spherical and cubic moist particles by introducing a linear cohesive contact model. Their findings indicated that cohesion also exerted a significant influence on the segregation of particles. Vibration conditions also exert a considerable influence on dissociation. Qiao et al. [165] employed binary spherical particle mixtures to investigate the vibrational dissociation behavior of particles, thereby elucidating the impact of vibration amplitude and frequency on the dissociation of particle mixtures.

The bonding and crushing of particles also have a great impact on the work of the vibrating screen, the actual industrial screening process, because of the working environment, environmental protection, dust reduction, and screened material characteristics of their reasons, the water content of the particles will rise sharply, when the particles are in the wet state, the particles of the static and dynamic characteristics will change [166]. When the particles of cohesion of particles due to the elevation of water content reach a certain limit, often wet material agglomeration phenomenon, the particles of the agglomeration of the agglomeration will lead to fine particles of the throughput rate is reduced, and even block the screen hole to affect the movement of the subsequent particles; the principle is shown in Fig. 11 [167]. Some scholars have proposed to break the bonding bonds of agglomerated particles by adding more water and simulated wet sieving on banana screens using a single coupling of discrete elements and smooth particle hydrodynamics (SPH), which provides a new research direction for the sieving of

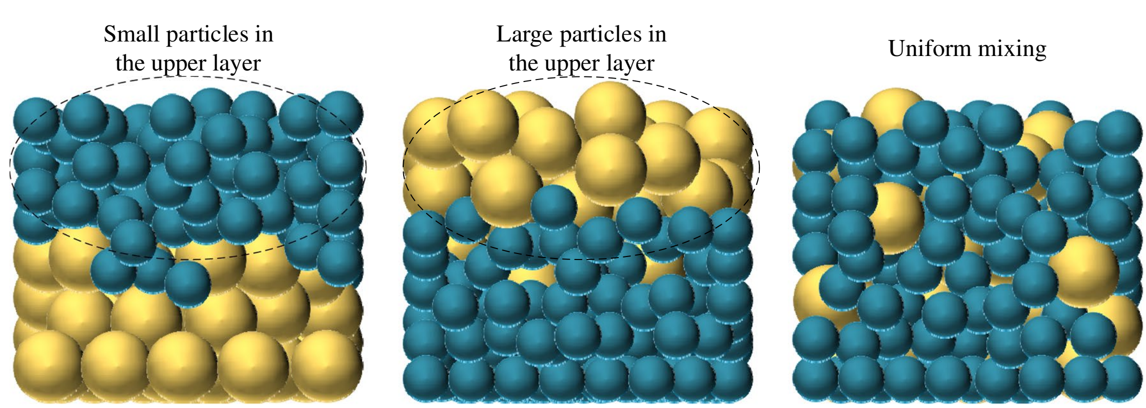


Fig. 10 Results of vibrational segregation of binary mixed particles

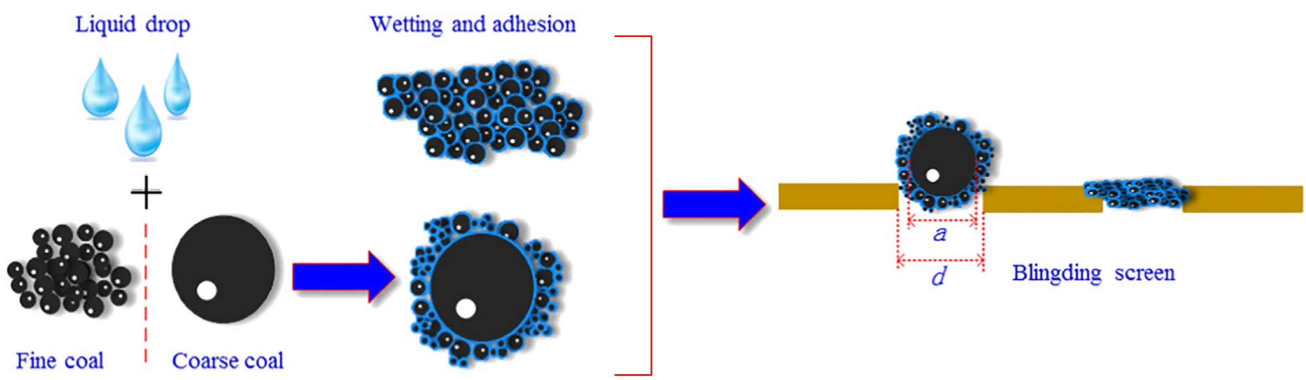


Fig. 11 The process of particle adhesion and pore blockage

agglomerated particles [95]. Other scholars have chosen to use trommel screens to address the effects of particle agglomeration on screening, and have concluded that particle shape and cohesion have little effect on the screening process in trommel screens [31].

5.2 Optimization of Vibration Operating Parameters

To achieve effective particle separation, the particles to be screened must have many opportunities to collide with the screen plate and pass through the screen holes in the correct manner. In the screening process, the movement of the particles on the screen is mainly affected by the vibration of the screen plate, causing the relative movement of the particles on the surface of the screen plate. The vibration movement of the screen plate is also one of the main factors affecting the screening efficiency; the screening efficiency of the vibrating screen, particle handling capacity, and other performance indicators mainly depend on the amplitude of the screen plate, frequency, vibration angle, angle of inclination, and other vibration parameters; the determination of these parameters for the entire process line is critical [30, 61]. Discrete element numerical simulation can get the influence of different factors on the sorting effect, which is valuable for the analysis and research of the mechanism of the vibration sorting process.

Most of the early studies were considering mathematical modeling of the factors affecting screening efficiency [149, 168, 169], But the traditional phenomenological experimental analysis has many drawbacks, not only need to spend a huge test cost, and the sieving process is difficult to observe, cannot reveal the mechanism of particle movement, even with the use of high-speed cameras cannot accurately locate the particles in the screen plate movement rules; Especially for studying the effect of a single vibration parameter on screening, smaller parameter changes can lead to more imperceptible changes in experimental results. With the development of the discrete element method, more and more scholars have applied discrete elements to study the effect of vibration parameters on screening, simulating the screening of particles with different vibration modes. By maintaining the vibration frequency, amplitude, and inclination angle of the screen plate parameters constant, the linear, circular, and elliptical three modes of vibration of the linear screen plate were analyzed. Additionally, the movement and penetration of the particles on the screen plate were analyzed. The particles exhibited the fastest movement and penetration in the linear vibration mode, which also exhibited the lowest screening efficiency. Conversely, the highest screening efficiency was achieved in the circular mode [61] The impact of screen length on particle screening efficiency under varying single-parameter conditions, including frequency, amplitude, vibration angle, and screen inclination, will be investigated [43]; Effect of different amplitudes, frequencies, and vibration directions on particle screening efficiency [68]. The progress of computer big data technology and the use of computer artificial intelligence to analyze discrete metadata research methods have gradually emerged, through the establishment of the least squares support vector machine (LS-SVM) and adaptive genetic algorithm intelligent models will be a large number of nonlinear simulation data for fitting to assist in the prediction of the screening efficiency of the vibrating screen is also an emerging research method [170, 171].

The discrete element method has unique advantages in the mechanism research vibration parameter research and optimization of the new vibrating screen. There are all kinds of connections between the vibration parameters of the screen plate in the traditional prototype test, and it is difficult to ensure the consistency of the other parameters accurately under the circumstance of changing one parameter [30]. Not only can discrete element simulation accurately control the variation of a single vibration parameter without affecting other parameters, but more efficient data processing can also help researchers validate design concepts faster. The Equal

Amplitude Vibrating Screen (EAVS) uses a centralized excitation system to maintain a constant amplitude of the screen panels from the feed end to the discharge end during the screening process [172]. On this basis, the researchers proposed variable amplitude equal thickness screen (VAETS) [172] and variable elliptical vibrating screen (VEVS) [167]; used the EDEM software to numerically simulate the new variable amplitude equal thickness screen; divided the screen model into four segments and set up four groups of different vibration parameters; simulated the amplitude from the feed end to the discharge end of the phenomenon of linear decrease; and investigated the effect of the center amplitude, the amplitude gradient, the projectile angle, the angle of inclination, and other factors on the performance of the variable amplitude equal thickness vibrating screen; the particle motion model is shown in Fig. 12.

5.3 Design and Optimization of a Screen Plate Structure

Limited by the influence of computer arithmetic, there are few industrial large vibrating screen models for direct DEM simulation using the discrete element method for screening process simulation. In the structural design and optimization of the vibrating screen, it is obvious that the screen plate structure is the key factor affecting the screening performance of the vibrating screen. In the process of material classification, the screen surface is an important part of the vibrating screen that is in direct contact with the screened particles. On the one hand, the screen surface collides with the particle group for energy exchange, so that the particle group obtains kinetic energy; on the other hand, the target particles in the process of movement through the screen holes on the screen surface to complete the screen through the screen, the screen holes on the non-target particles to impede the large particles, and to achieve the grading of the material. The design of the screen panels will vary depending on the application site and the screening objective. Often it is necessary to adjust the structure of the screen to cope

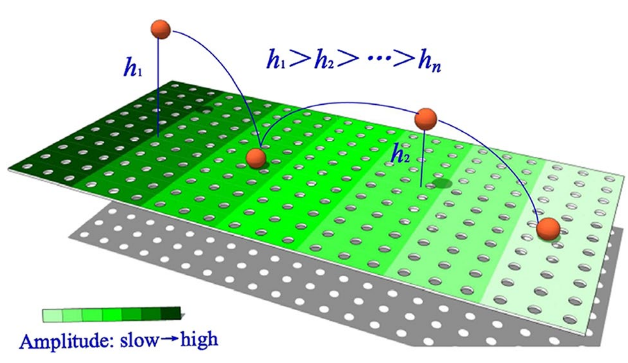


Fig. 12 Energy distribution and particle trajectories on the VEVS

with a change in the screening objective, such as changing the size or shape of the aperture [173].

Traditional prototype tests can provide experience and optimization targets for the design of new vibrating screens, but such an approach is both time-consuming and risky for testing experiments. To minimize this risk, the use of the discrete element method for the study of vibrating screens is a very suitable choice. The main structural parameters affecting screening efficiency include screen width, screen length, aperture shape, and screen inclination. Cleary and Sawley [174, 175] first proposed to use of the discrete element method to construct a three-dimensional planar screen to simulate the process of vibrating screen separation, but there is no in-depth discussion on the operation process. Many scholars have analyzed the relationship between screen length and screening efficiency based on experimental and empirical studies, and some experts have pointed out that an increase in screen length does not affect the probability of particles penetrating through the screen mesh density [176]; The selection of screen length needs to consider the particle size distribution of the particles; the screening of fine particles usually does not need too long screen plate. Theoretically, increasing the length of the screen mesh can ultimately make the screening efficiency reach 100%, but the screening efficiency and the length of the screen plate obey the decreasing exponential distribution, too long a screen plate cannot produce the actual production efficiency but will cause unnecessary waste of energy [43]. In the discrete element simulation mesh structure of the screen plate is often simplified to plate structure, the woven mesh screen plate structure is accurately modeled [177]; the screen hole shape and layout of the same are the focus of the study of the screen plate structure of the part of the different screen hole shapes [177] and materials will have a different impact on the final screening efficiency [55, 178].

Theoretically, increasing the length of the screen mesh can ultimately make the screening efficiency reach 100%, but the screening efficiency and the length of the screen plate obey the decreasing exponential distribution, too long a screen plate cannot produce the actual production efficiency but will cause unnecessary waste of energy. The researchers applied the discrete element method to study the effect of screen length on particle screening efficiency, and established a functional relationship between screening efficiency and screening length [43]; in discrete element simulation, mesh screen panels are often simplified to plate-like structures, the woven mesh screen panel structure has been accurately modeled, and the motion of the interactions between the particles on the woven mesh screen panels has been observed [177]; rectangular aperture screen plates with aspect ratio and orientation were constructed, and the effect of the aperture shape of the screen plates on particle flow and separation during the screening process was investigated [55]; Constructed screen plates with different aperture shapes and materials to study the screening efficiency per unit length of the plate [178]. In addition to the linear screen plate, many scholars have investigated the design of other complex screening mechanisms through the discrete element method, such as the study of factors related to the screening efficiency of banana screens [179]. Shen et al. [56] used a step-by-step optimization method to design the screen plate structure, using a new curved surface sieve in which the screen plate is divided into five layers with different inclination angles and used discrete element simulation to simulate and validate each layer of screen plate with different inclination angles to ensure that each layer of the screen plate is the optimal angle of inclination, and proved that the screening performance of the new plate is better than that of the traditional banana sieve, and optimization of the design process is shown in Fig. 13.

Conventional rigid screen surfaces in the screening process generated by the deformation of small, low-speed operation of the particles of the agglomeration problem will affect the screening efficiency and are difficult to resolve. Resilient screens can provide greater deformation and higher acceleration during the screening process, and further studies have shown that resilient screens are effective in overcoming agglomeration and clogging problems with wet materials [167]. For the variable frequency vibrating screen, the vibration response of each position of the elastic screen plate is different. Yu et al. [180] restored the motion state of the screen by setting multiple discrete rigid motions on the elastic screen mesh of the vibrating flip-flow screen (VFFS), used accelerometers to test the amplitude response of the elastic screen plate at different positions, and reproduced the motion process of the agglomerated particles on the elastic screen mesh, and the simulation model is shown in Fig. 14.

With the gradual maturity of the application of the discrete element method in the field of vibratory screening, researchers began to try to compatible coupling finite element method, computational fluid dynamics, etc., with discrete element simulation, and to complete the study of some more complex problems through parallel computational research, for example, the motion model of the particle group on the elastic screen plate [66], the screening motion of the particles in the environment with the influence of the flow field [181], and the stress exerted by the particle group on the screen plate in the screening process [182]. Xu et al. [65] simulated the flexible screening process based on the coupled method of finite element method and discrete element method, and the coupled simulation flow is shown in Fig. 15; the effects of vibrating screen operating parameters on particle velocity, screening efficiency, and screen plate stress were analyzed through single-factor and composite experiments; multi-objective optimization was ultimately achieved, and mathematical models were fitted to describe the relationship between evaluation

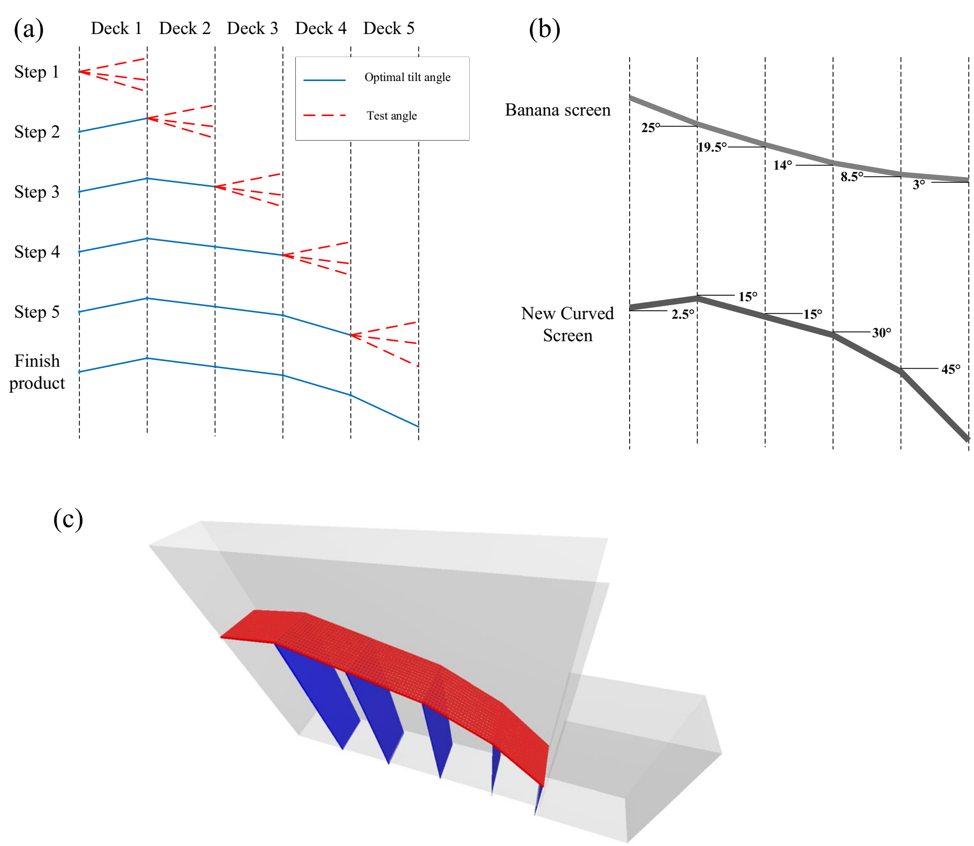


Fig. 13 Design principle and simulation model of five-layer surface screen: **a** The design process of the stepwise optimization method; **b** comparison of screen plate structures

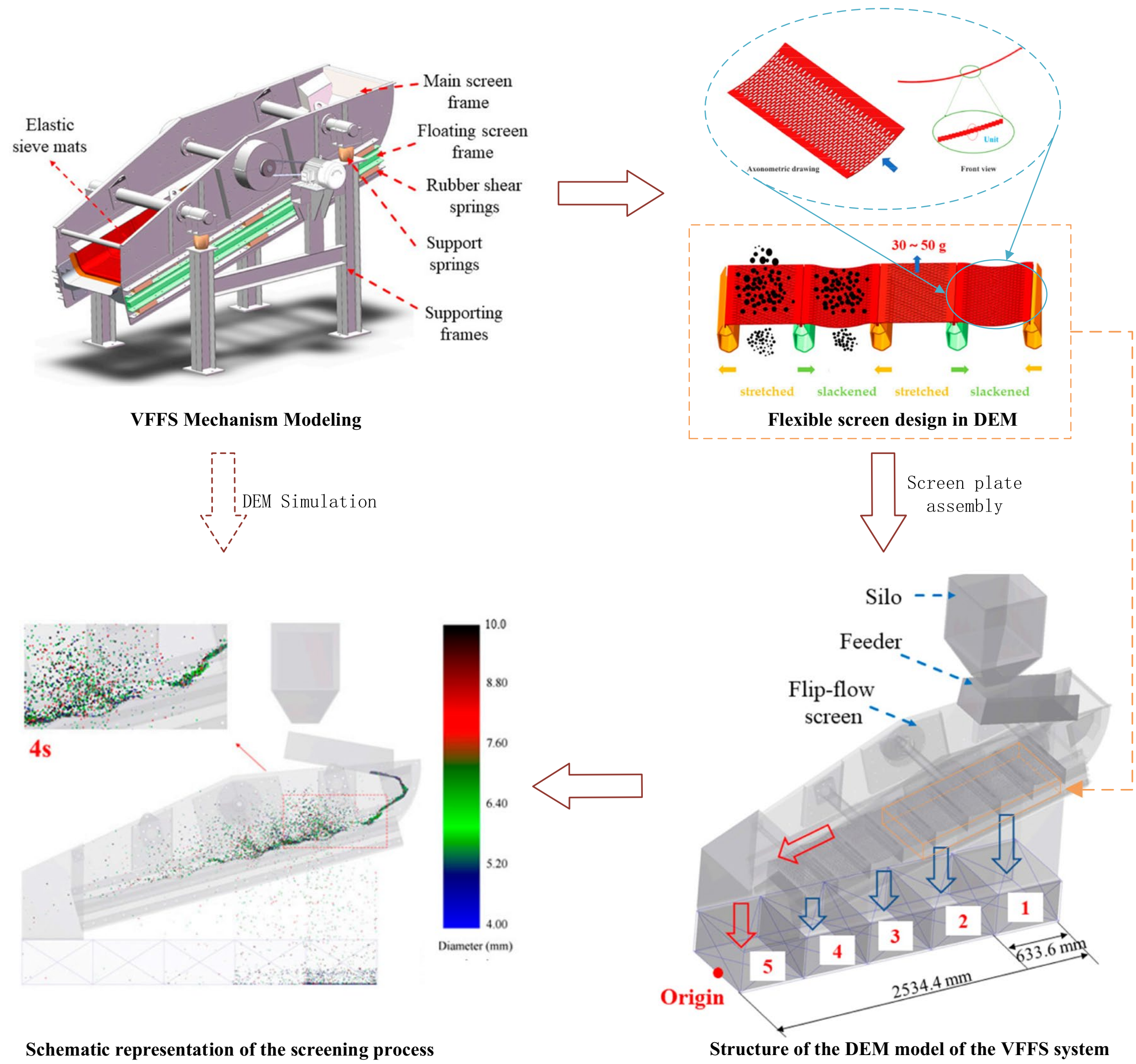


Fig. 14 DEM simulation flow and model validation of VFFS

indexes and vibration parameters. Zhao et al. [66] established the motion model of the particle group on the elastic screen plate, studied the motion characteristics of the mixed particles in different regions on the elastic screen plate, and made an in-depth analysis of the motion mechanism of the materials on the elastic screen surface of the vibrating overturning flow. Table 5 summarizes the application of the discrete element method to the study of the screening efficiency of vibrating screens and the optimization and innovation of screen plate structure.

6 Conclusions and Outlook

At present, the research on the application of the discrete element simulation method on vibrating screens has become more mature and achieved certain results. The discrete element method obtains the accurate results of the interaction between particles by calculating the motion of each particle in the particle group individually, and the shape of the particles, the contact model of the particles, and the property parameters of the particle materials are the key input data to ensure the high-efficiency simulation and the accurate simulation results. In these simulation models, the combination of parameter selection will directly determine the accuracy of the simulation results and the operational efficiency of the simulation of discrete element simulation data accuracy is mainly dependent on the accuracy of the simulation modeling and simulation time, more detailed modeling will directly affect the accuracy of the final data, but this will give the computer to bring a greater burden to greatly extend the time required for the simulation, which in turn, reduces the advantages brought by the simulation. A proper balance of simulation modeling accuracy and simulation time is important for production needs.

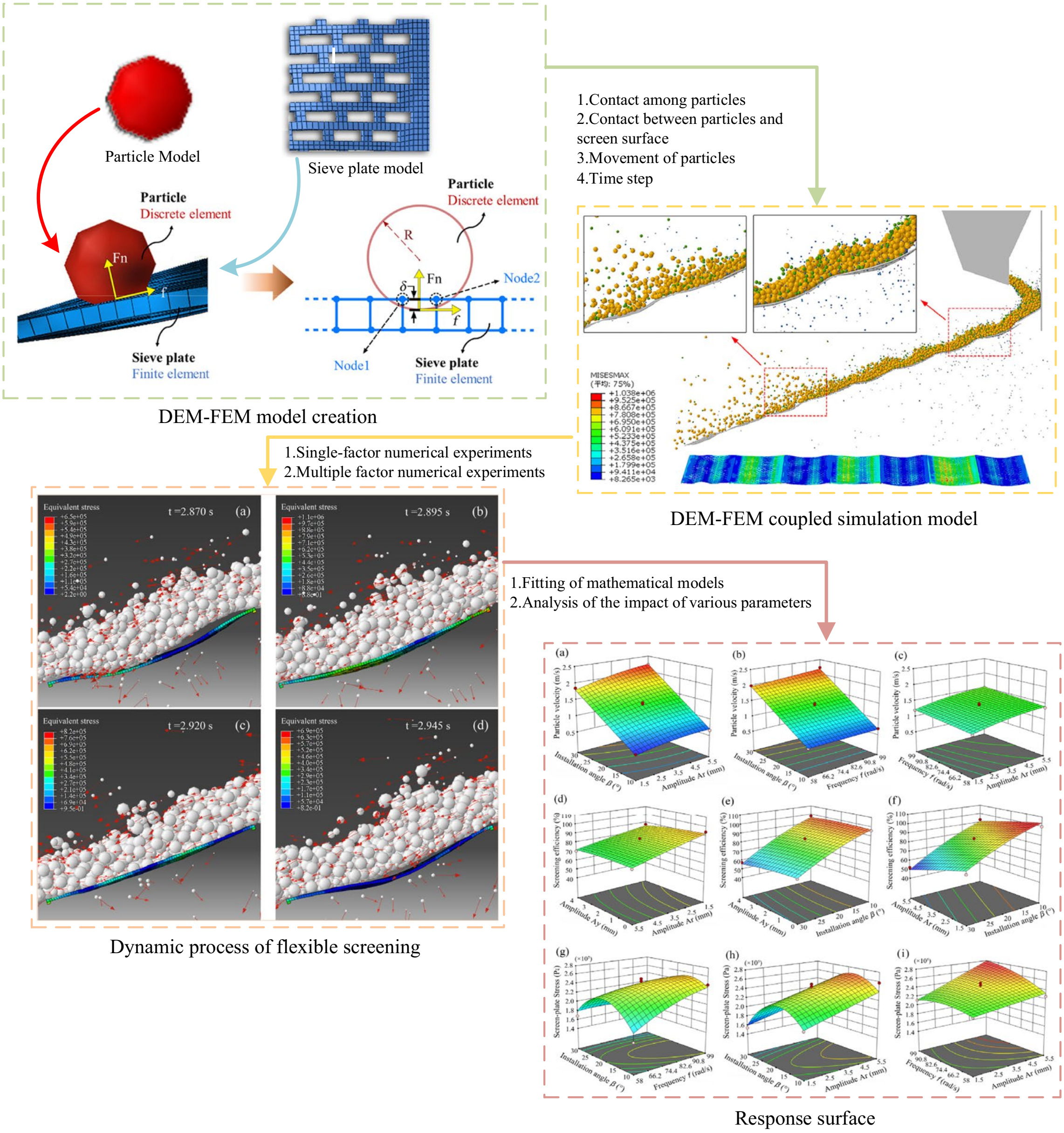


Fig. 15 Coupled DEM-FEM simulation analysis (adapted from [65], Copyright 2024, with kind permission from Elsevier)

To provide and more accurate and efficient numerical simulation of the screening process, this paper summarizes and reflects on the existing research and proposes the development trend of vibratory screening simulation.

(1) Upgrading and optimization of the existing particle model. The shape of the particles has a significant impact on the screening process, the most widely used in the field of vibratory screening simulation is the multisphere model, and other types such as hyperellipsoid and polyhedral models still need more theoretical research support; the contact model determines the mechanical condition of the particles colliding in the simulation, and the most commonly used is the nonlinear viscoelastic model, such as the Hertz- Mindlin model. There is still a need to develop more contact models that can quickly and accurately calculate microscopic forces such as adhesion forces and liquid bridge forces in the simulation process.

(2) Coupling discrete elements with other simulation software for joint simulation. Existing discrete element vibration simulation technology has gradually matured, with the vibrating screen optimization and innovation; researchers will focus on a wider range of applications, through the combination of other simulation software will be more screening factors into the screening process, to get in more different conditions of the screening pro-

Table 5 Parameter selection and research applications of DEM on vibrating screen design

Object	Purpose	Particle shape mode	Vibration mode	Contact model	Simulation software	Source
Multi-deck banana screen	Study of the different effects of vibration parameters on industrial and laboratory screening	Spherical particle model	Linear vibration	Hertz–Mindlin model	LIGGGHTS	[2]
Inclined screen	The effect of vibration intensity on the screening process was verified using an orthogonal test approach	Multi-sphere model	Circular vibration	Hertz–Mindlin model	EDEM	[30]
Inclined screen	Comparison of the effect of particle shape on simulation results at different vibration parameters	Spherical particle model and Multi-sphere model	Linear vibration	Hertz–Mindlin model	LIGGGHTS	[32]
Vibrating flip-flow screen	The motion of particles on the vibrating flip-flow screen plate was studied and the optimum excitation frequency was determined	Spherical particle model	Linear vibration	Hertz-Mindlin model	FEM-DEM coupling	[66]
Inclined screen	Applying the concepts of particle rheology to the study of particulate media on vibrating screens overcomes the extreme dependence of vibrating screens on empirical models of specific machines	Spherical particle model	Sinusoidal translation	Hertz-Mindlin model	EDEM	[183]
Roller screen	Study of the optimum operating parameters for industrial scale drums and validation of the effect of particle shape on screening	Multi-sphere model	Roller rotation	Hertz-Mindlin linear cohesion contact model	EDEM	[31]
Multi-layer vibrating screen	The particle penetration rate, the number of collisions, and the distribution of the particles under 23 combinations of structures and vibration parameters were investigated	Spherical particle model and multi-sphere model	Linear vibration and elliptical vibration	Hertz-Mindlin JKR cohesion model	EDEM	[76]

cess, for example, discrete element method (DEM) and computational fluid dynamics (CFD) joint simulation, the study of particles and fluids in the mixed slurry on the screen plate screen process; will be discrete elements of the method of the joint operation with the finite element method (FEM), the study of the screening process of the flexible screen mesh.

(3) Combining discrete metamethods with artificial intelligence machine learning. In the screening simulation process, there are a large number of factors affecting screening exist, the existing screening process model is difficult to consider the huge control variables and complex interparticle interactions, and currently more research on the impact of a single or a small number of factors on particle movement and screening efficiency. The progress of computer technology has greatly improved the data analysis ability of the computer, the use of artificial intelligence technology to develop a machine learning model about the efficiency of vibratory screening; by importing a large amount of simulation data into the computer, you can get a more effective model to evaluate the performance of vibratory screening.

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Author Contribution Yufei Yang and Zhiping Xie conceived and designed the study. Yufei Yang, Zhiping Xie, Junhao Wang, Siqian Wang, and Wenxin Feng conducted data gathering. Yufei Yang, Xinyue Hou, and Yuelong Yu performed statistical analyses. Yufei Yang and Xie Zhiping wrote the article.

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Data Availability All data and materials used to support the findings of this study are included within the article.

Declarations

Ethical Approval Ethics approval was not required for this research.

Conflict of Interest The authors declare no competing interests.

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