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Evaluation of the Rock Burst Intensity of a Cloud Model Based on the CRITIC Method and the Order Relation Analysis Method

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Abstract
Rock burst has always been a major problem in deep underground engineering with high stress, and rock burst strength evaluation has become an important research topic. To effectively predict the rock burst hazard in underground rock mass engineering, a cloud model (CM) rock burst intensity evaluation method based on the CRITIC method and order relation analysis method (G1) was proposed in this paper. First, a rock's uniaxial compressive strength σ_c, tangential stress σ_θ, uniaxial tensile strength σ_t, ratio of uniaxial compressive strength to tensile strength σ_c/σ_t (brittleness coefficient), ratio of tangential stress to uniaxial compressive strength σ_θ/σ_c (stress coefficient), elastic deformation energy index W_{et}, and depth of cover H were selected as evaluation indices of rock burst intensity. Ninety-five groups of rock burst measured data at home and abroad were selected, and the objective weight and subjective weight of each index were calculated by using the CRITIC method and G1 method, respectively. The comprehensive weight was determined according to the combined weighting method of game theory, and the sensitivity of each evaluation index was analyzed. By utilizing a forward cloud generator, the membership degrees of different rock burst grades were calculated, and then the rock burst intensity grades of the samples were evaluated and compared with the evaluation results of the CRITIC-CM method and G1-CM method and the actual grades. Finally, the rock burst classification ability of the model was analyzed. To better verify the accuracy and reliability of this model, the rock burst case of the W39 line in the Chengchao Iron Mine was analyzed by using this model. The research results show that the rock burst evaluation results based on CRITIC-G1-CM are basically consistent with the actual rock burst grade, and the rock burst intensity grade evaluation model has good practicability and reliability.

Keywords Cloud model · Rock burst · Grade evaluation · Combination weighting · Sensitivity

1 Introduction

Rock burst refers to the redistribution of the stress field in a rock mass caused by unloading during excavation in rock engineering. This occurs when the accumulated energy in the deep, high-stress regions of the rock mass is greater than the energy consumed during rock failure. When hard or brittle rock experiences a sudden release of a large amount of energy [1, 2], rock burst occurs, and a large number of rock fragments from the rock are loosened, collapsed, or ejected, leading to a geological disaster [3, 4]. With the increase in underground mining depth in China, the problem of rock burst is becoming increasingly serious. Rock burst, which has a strong sudden and destructive nature, directly threatens the safety of operators and equipment and causes large amounts of economic and property loss [5]. Because there are many rock burst factors, which have the characteristics of randomness and fuzziness [6], quickly and effectively predicting the grade of rock burst has become the main problem.

Many experts and scholars at home and abroad have conducted much exploration and research on the mechanism of rock burst from different research directions. First, the strength, energy, brittleness, and critical depth of rock were studied. By analyzing the relationship between rock strength and rock burst and surrounding rock stress, the theoretical

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criterion of rock strength was proposed [6–9]. It was found that energy is closely related to the occurrence of rock burst, and the criterion of energy theory was proposed [10, 11]. The brittleness coefficient of rock and the ratio of total deformation to permanent deformation before rock burst were analyzed, and the brittleness index criterion was proposed [12, 13].

As research progressed, it was found that the relationship between the evolution and formation of rock burst and its influencing factors is nonlinear. Therefore, artificial intelligence methods such as applied mathematics, big data, and deep learning have been adopted to comprehensively predict and study rock burst. Zhou et al. [14] used different bullseye distances of the gray target theory to represent the corresponding rock burst grades and established a rock burst intensity evaluation model based on the gray target decision theory and the idea of variable weight synthesis. Li et al. [15] studied a new application of a Bayesian network (BN) and applied it to the prediction and classification of rock burst. Xue et al. [16] used rough set theory to study the rock burst index combined with extension theory to evaluate rock burst and established a rock burst prediction model. Shukla et al. [17] used XGBoost, a decision tree and a support vector machine to predict rock burst in underground engineering and evaluated the performance of these three machine learning methods. The above methods have been used to predict and evaluate the rock burst tendency from different angles and have achieved good results. However, the influencing factors of rock burst are random, variable, and fuzzy. Therefore, the above methods are not suitable for all rock burst predictions. The cloud model can solve the randomness and fuzziness of evaluation indices and measured data well. Therefore, many scholars have introduced cloud models into rock burst grade evaluation. Li et al. [18] proposed a new rock burst evaluation and analysis method by combining the gray correlation method, principal component analysis, and cloud theory. Lin et al. [19] used three machine learning algorithms combined with a cloud model to predict rock burst and compared the performance of the three methods. Zhou et al. [20] used the entropy cloud model to predict rock burst grade. Wang et al. [21] used the CRITIC method combined with a cloud model to predict the rock burst level. It is undeniable that these methods have achieved good results in real-life examples. However, the key to a rock burst prediction method based on the cloud model lies in the determination of the index weight. All the above methods use a single weighting method to determine the index weight, which is inevitably influenced by objective factors or subjective factors.

Aiming at the problem of improving the single empowerment of the cloud model evaluation method, in this paper, an objective empowerment method and a subjective empowerment method are naturally combined, and a new cloud model rock burst evaluation method based on combined weighting is proposed. First, the combined weighting method of game theory is adopted to optimize the weight combination of the CRITIC method and G1 method, and the objective and subjective factors are fully considered to make the weighting more reasonable. By considering the difference between the unilateral interval distribution of the traditional cloud model and the actual rock burst classification, the unilateral interval of the traditional cloud model is improved. Then, the rock burst evaluation method based on combination weighting of the cloud model is established. Finally, the model is used to analyze the rock burst case of Line W39 in the Chengchao Iron Mine. The validity and rationality of the model, as applied to the prediction of rock burst intensity, are verified. Thus, a new method for the study of rock burst prediction problems is provided.

2 Theoretical Basis

2.1 Cloud Model Definition

The cloud model is a mathematical model first proposed by Li et al. [22] in 1995. Based on classical stochastic theory and fuzzy set theory, the cloud model solves the transformation uncertainty between a qualitative concept, and its quantitative numerical representation and uses the membership degree to solve the correlation between randomness and fuzziness. This model has been successfully applied to many fields [23–26], such as data mining, decision analysis, and image processing, and has achieved good results.

The cloud model is a two-way uncertainty transformation model between a qualitative concept and its quantitative representation expressed by linguistic values. Let X be a quantitative set expressed by exact numerical values, $X = \{x\}$. Then, X is referred to as the A domain (one-dimensional, two-dimensional, or multidimensional), and A is a qualitative concept (fuzzy set) of X. In X, any element x and each corresponding $x \in A$ has a stable random number in the mapping μ: $x \rightarrow \mu(x)$, a random number with a stable tendency in $\mu(x) \in [0, 1]$. The random number $\mu(x)$ is referred to as the certainty of x to the qualitative concept A or the membership degree. The distribution of the degree of certainty $\mu(x)$ on X is referred to as a cloud [27]:

$$\mu : X \rightarrow \left(A_{\min}, A_{\max}\right), A \subseteq X, \forall x \in A, x \rightarrow \mu(x) \tag{1}$$

The mapping between X and its corresponding qualitative concept A is not a one-to-one mapping in the traditional fuzzy membership function but a one-to-many mapping [28]. The distribution of $\mu(x)$ on fuzzy set A is referred to as the cloud, and each point $[x, \mu(x)]$ is referred to as a cloud drop. $A_{\min}$ represents the minimum value of the fuzzy level interval, which is generally 0, and A_{max} represents the maximum value of the fuzzy horizontal interval. In the case that the maximum value of the fuzzy horizontal interval is infinite, that is, the horizontal interval of the fuzzy edge is (A_{min}, + ∞), we generally consider the upper and lower limits of the data to determine the boundary value. The normal cloud model is the most basic cloud model and has universality and adaptability. In a large number of studies, the expected curves of cloud models with qualitative knowledge approximately follow normal or semi-normal distributions. The traditional normal cloud model is shown in Fig. 1. Each point in the figure corresponds to a cloud drop. The abscissa represents the value x corresponding to the fuzzy set in the domain of discourse, and the ordinate represents the determination of x to the fuzzy set, with the value range [0,1].

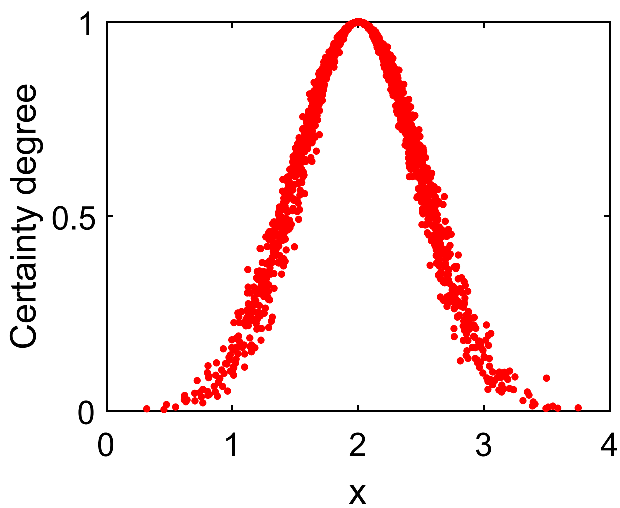


Fig. 1 Traditional cloud model

2.2 Digital Features of the Cloud

The digital characteristics of the cloud model reflect its qualitative concept and quantitative characteristics, which are usually represented by three values: expectation E_x, entropy E_n, and super entropy H_e [29]. The specific meanings of the three digital features are shown in Fig. 2, where expectation E_x is the central value of the dataset of cloud drops in the domain space and the random value corresponding to $\mu(x) = 1$ in the cloud model. Entropy E_n is a measure of the fuzziness and randomness of the qualitative concepts and determines the range of cloud drop values in the domain space. The larger the entropy is, the larger the width of the cloud model, and the value range of cloud drops, and the greater the fuzziness and randomness of the qualitative concepts. The super entropy H_e, which is defined as the entropy of the entropy, represents the uncertainty of the entropy and is represented as the thickness of the cloud in the cloud image. The larger the super entropy is, the thicker the cloud.

If every cloud drop in the cloud model satisfies $x \sim N\,(E_x, {E'_n}^2)$, where $E'_n \sim N\,(E_n, H_e^2)$, then the certainty of x for A is as follows [30]:

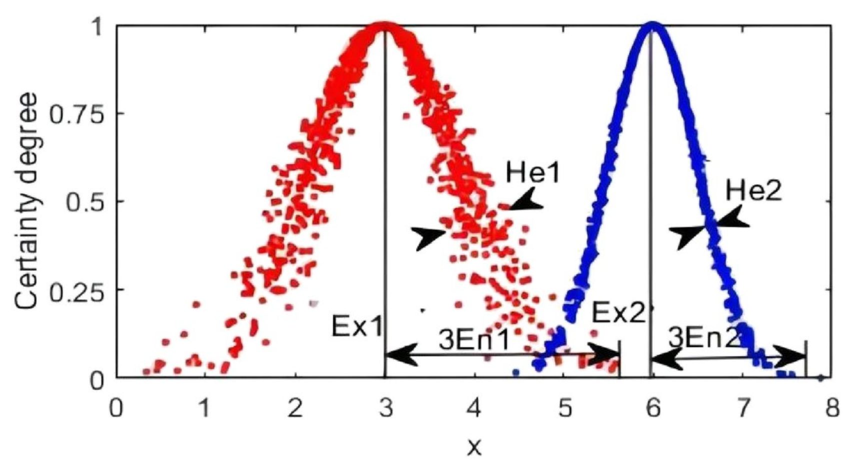
$$\mu(x) = \exp\left[\frac{-\left(x - E_x\right)^2}{2E_n'^2}\right] \tag{2}$$

According to the concept of the cloud model, the cloud digital characteristics of a certain grade standard of rock burst intensity can be calculated according to Formula (3) [31]:

$$\begin{aligned} E_x &= \tfrac{C_{max}+C_{min}}{2} \\ E_n &= \tfrac{C_{max}-C_{min}}{6} \\ H_e &= K \end{aligned} \tag{3}$$

where C_{max} and C_{min} are the maximum boundary value and the minimum boundary value of the corresponding grade standard, respectively, and K is a constant, which can be adjusted according to the fuzzy threshold of the variable.

Fig. 2 Digital feature representation of the cloud model



The value of K will only determine the thickness of the cloud and has no effect on the final result. In this paper, K is uniformly set as 0.01. For an interval with unilateral bounds, such as $[C_{\min}, +\infty]$, the missing boundary parameters can be determined according to the upper and lower limits of the variables, and then the numerical characteristics of the cloud model can be calculated according to Eq. 3.

2.3 Cloud Generator

A cloud generator is an algorithm used to realize the transformation between qualitative concepts and quantitative data in cloud models, and it is the key to applying cloud models in practice. The cloud generator includes a forward cloud generator and a reverse cloud generator. The forward cloud generator is a mapping from qualitative to quantitative, which visualizes cloud digital features to generate cloud images.

The reverse cloud generator realizes the transformation from a quantitative value to a qualitative concept and transforms the realized cloud image into an accurate cloud digital eigenvalue [32]. The computing flow of the cloud generator is shown in Fig. 3

Since the evaluation of rock burst intensity is a qualitative to quantitative study, the forward cloud generator is adopted in this paper. According to the digital characteristics of the cloud ($_{Ex,\ En,\ He}$), N cloud drops are generated in fuzzy set A to form the cloud image. In this paper, N is set as 1000. For the rock burst intensity grading interval, there are fuzzy edge level intervals such as $(0, C_{\max})$ and $(C_{\min}, +\infty)$. At this time, x no longer follows a normal distribution. Instead, it follows a uniform distribution with a certainty of 1. For the two different distribution situations of the nonedge level interval and edge level interval, the specific algorithm for determining the degree $\mu(x)$ is as follows:

$$\begin{cases} \mu(x) = 1 & x \in (0, E_{x(\min)}) \cup (E_{x(\max)}, +\infty) \\ \mu(x) = \exp\left[\frac{-(x-E_x)^2}{2E_n'^2}\right] & x \in \text{otherrval} \end{cases} \tag{4}$$

where $E_{x(\min)}$ and $E_{x(\max)}$ correspond to the minimum expected value and maximum expected value of different rock burst intensity classification intervals under the same index, respectively.

The half-ascending cloud model and half-descending cloud model are usually used to describe the fuzzy edge interval, as shown in Fig. 4. Curve 1 and curve 2 represent the half-descending cloud model and half-ascending cloud model, respectively. The left and right edges of the two curves follow uniform distributions.

2.4 Index Selection and Grading Criteria

The evolution process and occurrence mechanism of rock burst are very complex, and there are many influencing factors. Therefore, the selection of the evaluation index of rock burst is a key step in the prediction of rock burst intensity grade. Too many indicators will complicate the prediction process because it is too difficult to obtain some indicator values, and too few indicators will make the prediction process too one-sided, resulting in the prediction results being inconsistent with the actual results [6]. Therefore, the selected evaluation indicators should be scientific, independent, and representative. The influencing factors of rock burst can be divided into internal factors and external factors. In the high-stress environment of deep underground rock masses, the stress distribution of the surrounding rock changes due to excavation and unloading is an external factor. Hard brittle rocks are more prone to rock burst because of the mechanical properties of the rock mass itself, which are internal factors [33, 34]. According to the characteristics and causes of rock burst, the internal and external factors of rock burst are synthesized in this

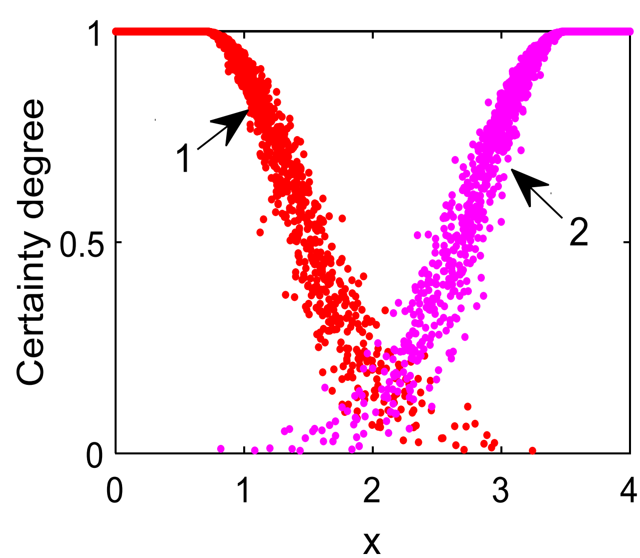
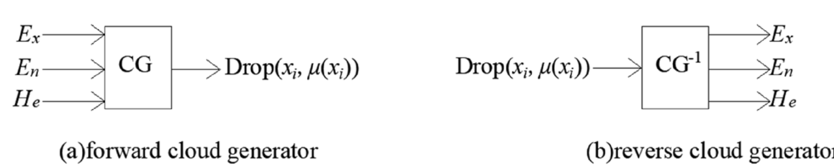


Fig. 4 Half-descending and half-ascending cloud models

Fig. 3 Cloud generator



paper. A rock's uniaxial compressive strength σ_c, tangential stress σ_θ, uniaxial tensile strength σ_t, ratio of uniaxial compressive strength to tensile strength σ_c/σ_t (brittleness coefficient), ratio of tangential stress to uniaxial compressive strength σ_θ/σ_c (stress coefficient), elastic deformation energy index W_{et}, and depth of cover H are used as evaluation indices for the prediction of rock burst intensity level.

According to the relevant research and classification standards of rock burst intensity grade, the rock burst intensity grade can be divided into four grades: grade I (no rock burst), grade II (slight rock burst), grade III (medium rock burst), and grade IV (strong rock burst). With reference to the relevant research results of rock burst criteria and classification [16, 20, 35], a specific rock burst intensity classification standard is established, as shown in Table 1.

2.5 Determination of Evaluation Index Weight

2.5.1 CRITIC Method

The CRITIC method is a kind of objective weight assignment method that uses the variability and conflict between different evaluation indicators to assign weights and can comprehensively measure the evaluation indicators [36]. This method is mainly used to calculate a weight vector of different importance indices, assemble it into a weight evaluation matrix, and establish a comprehensive evaluation model to carry out weight assignment. The detailed operation steps are as follows:

Step 1: Construct the indicator sample matrix
Assuming that there are m evaluation samples and n evaluation indices for rock burst intensity grade evaluation, the index sample matrix is as follows:

$$A=(a_{ij})_{m\times n} \tag{5}$$

where a_{ij} is the corresponding value of the jth ($j=1, 2,\ldots, n$) index of the ith ($i=1, 2,\ldots, m$) evaluation object.
Step 2: Construct the normalized matrix
Due to the differences in the nature and dimension of each evaluation index, the weight of the evaluation index will shift. To avoid this problem, it is necessary to normalize the index sample matrix to map the evaluation index value in the interval of [0,1], eliminate the influence of different variables, and obtain the normalization matrix X. If the evaluation index is a benefit-type index, the calculation formula is as follows:

$$x = \frac{a_{ij} - \min(a_{ij})}{\max(a_{ij}) - \min(a_{ij})} \tag{6}$$

If the evaluation index is a cost-type index, the calculation formula is as follows:

$$x = \frac{\max(a_{ij}) - a_{ij}}{\max(a_{ij}) - \min(a_{ij})} \tag{7}$$

where x is the normalized treatment value and $\max(a_{ij})$ and $\min(a_{ij})$ are the maximum and minimum values of a certain evaluation index, respectively.
Step 3: Calculate the coefficient of variation
Since the evaluation index has been standardized, the coefficient of variation can be expressed by the standard deviation σ_j of the evaluation index, and the calculation formula is as follows:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{n}\left(x_{ij} - \overline{x}_j\right)^2}{n-1}} \tag{8}$$

where $\overline{x}_j$ is the average of the jth evaluation index and n is the total number of jth evaluation index.
Step 4: Calculate the correlation coefficient r of the evaluation index and obtain the correlation coefficient matrix R. The formula is as follows:

$$r = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}} \tag{9}$$

$$R = \left(r_{kl}\right)_{n\times n} \tag{10}$$

where r_{kl} is the correlation coefficient between the kth ($k=1,2\ldots n$) index and the lth ($l=1,2\ldots,n$) index.
Step 5: Calculate the evaluation index conflict coefficient ηj, and the formula is as follows:

Table 1 Classification standard of rock burst intensity

Rock burst grade	Evaluation index						
	σ_c(MPa)	σ_θ(MPa)	σ_t(MPa)	σ_c/σ_t	σ_θ/σ_c	W_{et}	H(m)
I	(0, 80)	(0, 24)	(0, 5)	(40, +∞)	(0, 0.3)	(0, 2)	(0,50)
II	(80, 120)	(24, 60)	(5, 7)	(26.7, 40)	(0.3, 0.5)	(2, 3.5)	(50, 200)
III	(120, 180)	(60, 126)	(7, 9)	(14.5, 26.7)	(0.5, 0.7)	(3.5, 5)	(200, 700)
IV	(180, +∞)	(126, +∞)	(9, +∞)	(0, 14.5)	(0.7, +∞)	(5, +∞)	(700, +∞)

$$\eta_j = \sum_{m=1}^{n} \left(1 - r_{kl}\right) \tag{11}$$

Step 6: Calculate the weight coefficient Cj of the total information of each evaluation index. The formula is as follows:

$$C_j = \sigma_j \times \eta_j \tag{12}$$

Step 7: Calculate the weight coefficient w_j of the total information of each evaluation index. The formula is as follows:

$$w_j = \frac{C_j}{\sum_{j=1}^{n} C_j} \tag{13}$$

2.6 Sequence Relation Analysis Method (G1 Method)

The sequence relation analysis method is a subjective weighting method, which is an improvement of the analytic hierarchy process (AHP). Compared with the AHP, there is less calculation, and there is no need to carry out a consistency test [37]. The specific calculation steps are as follows:

Step 1: Determine the ordering relationship
For n evaluation indices (x_1,x_2,…,x_n) in descending order of importance to determine the order relationship:

$$h_1 > h_2 > \cdots > h_n \tag{14}$$

Step 2: Determine the importance of adjacent indicators
The ratio D_{k-1}/D_k of the importance of the underlying indicators h_{k-1} and h_k is defined as:

$$B_k = \frac{D_{k-1}}{D_k}(k = 2, 3, \cdots, n) \tag{15}$$

The B_k assignment is shown in Table 2.

Step 3: Determine the weight w_n. The formula is as follows:

$$w_n = \left(1 + \sum_{k=2}^{n} \prod_{i=k}^{n} B_k\right)^{-1} \tag{16}$$

$$w_{k-1} = B_k w_k \tag{17}$$

Step 4: Determine the comprehensive weight
To ensure that the evaluation index is more objective and comprehensive, S experts ($S \geq 1$) are hired to conduct the evaluation. Let the assignment given by the mth expert in name S be denoted as $B_{(m)k}$, and the weight of $B_{(m)k}$ can be obtained as $w_{(m)k}$ according to Eqs. 16 and 17. Then, the comprehensive weight w_k of each index is as follows:

$$w_k = \frac{1}{S} \sum_{m=1}^{s} w_k^m \tag{18}$$

2.7 Combined Weighting Method of Game Theory

To avoid the information loss caused by a single weighting method and improve the accuracy of the weights, the combined weighting method of game theory is used to optimize the weights obtained by the two weighting methods, determine the consistency among them, and obtain the optimal weight [38]. The specific steps are as follows:

Step 1: Assuming that L weighting methods are used to weight n evaluation indicators, the basic weight matrix is as follows:

$$\mathrm{W} = \left(w_{kp}\right)_{L \times n} \tag{19}$$

where w_{kp} is the weight value corresponding to the pth ($p = 1, 2, \ldots, n$) evaluation index of the kth ($k = 1, 2, \ldots, l$) weighting method.
Step 2: Linear combination of each weight vector:

$$w_i = \sum_{k=1}^{L} \alpha_k w_k^T \tag{20}$$

where α_k is the linear combination coefficient of the kth weighting method.

Table 2 B_k assignment table

B_k assignment	Description of B_k assignment
1.0	The indicator h_{k-1} has the same importance as the indicator h_k
1.2	The indicator h_{k-1} is slightly more important than the indicator h_k
1.4	The indicator h_{k-1} is significantly more important than the indicator h_k
1.6	The indicator h_{k-1} is strongly more important than the indicator h_k
1.8	The indicator h_{k-1} is extremely important than the indicator h_k

The value of B_K is obtained from the consultation of experts in relevant fields

Step 3: To minimize the deviation between w_i and w_k, L linear combination coefficients α_k are optimized to obtain the optimal w_i, namely:

$$\min\left\| \sum_{k=1}^{L} \alpha_k w_k^T - w_k \right\|_2 \quad (k = 1, 2, \ldots, L) \tag{21}$$

By using matrix differential properties, Eq. 22 is equivalently transformed into a linear system of equations under optimal first derivative conditions, namely:

$$\begin{bmatrix} w_1 \cdot w_1^T & \cdots & w_1 \cdot w_L^T \\ \vdots & \ddots & \vdots \\ w_L \cdot w_1^T & \cdots & w_L \cdot w_L^T \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_L \end{bmatrix} = \begin{bmatrix} w_1 \cdot w_1^T \\ \vdots \\ w_L \cdot w_L^T \end{bmatrix} \tag{22}$$

Step 4: Calculate the optimized linear combination coefficient α_k according to Eq. 23, normalize it by $\alpha_k^* = \alpha_k / \sum_{k=1}^{L} \alpha_k$, and finally obtain the combination weight w^*. The formula is:

$$w^* = \sum_{k=1}^{L} \alpha_k^* w_k^T \tag{23}$$

2.8 Generation of the Rock Burst Evaluation Cloud Model Based on Combination Weighting

The process of the CRITIC-G1-CM rock burst intensity grade evaluation model is shown in Fig. 5. The red dotted box is based on cloud model theory, and the rock burst intensity classification standard is regarded as a qualitative concept and mapped into a cloud model. The black dotted box is the combination of subjective and objective weights, and the optimal combination of weights is obtained through game theory. Finally, by considering the cloud model and combined weights, the determination degree of each sample belonging to a certain rock burst intensity level is obtained. The specific process is as follows:

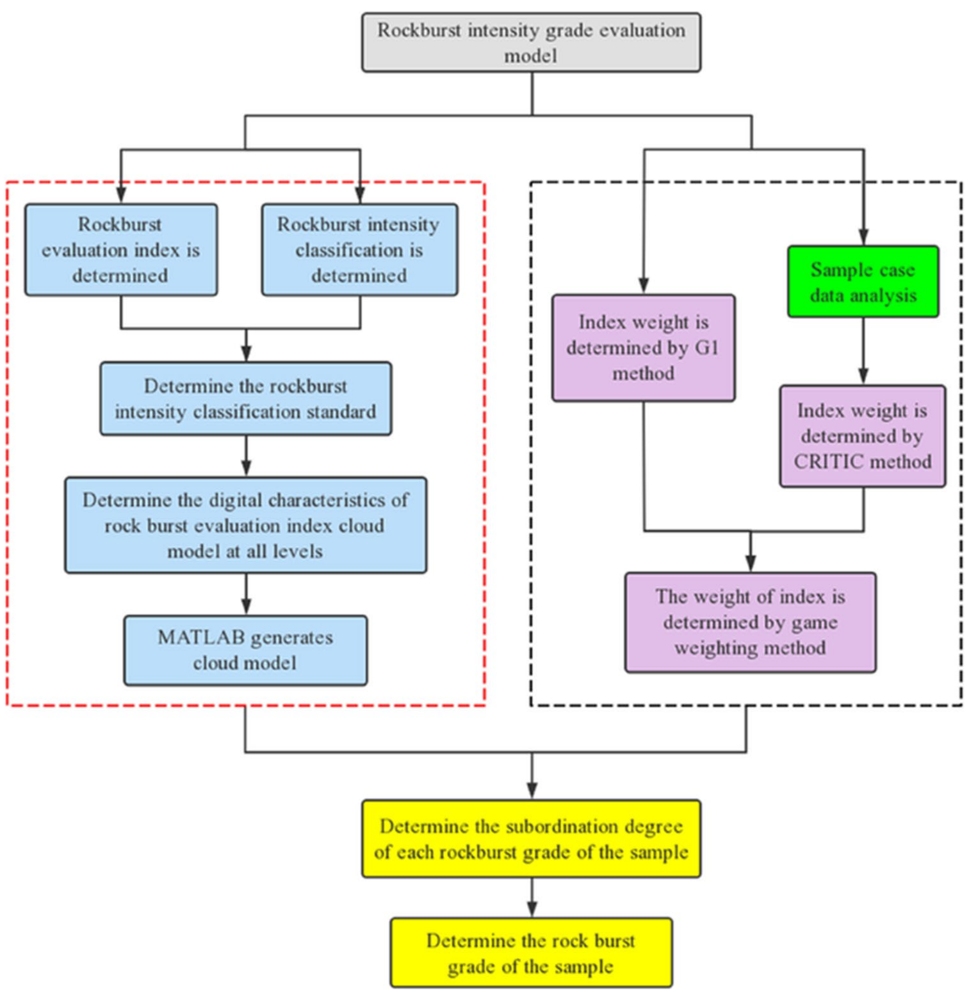


Fig. 5 Evaluation process of obtaining the CRITIC-G1-CM rock burst intensity grade

Step 1: In considering the related research and classification standards of rock burst, determine the evaluation index and establish the specific classification standards of rock burst intensity.
Step 2: Using the established rock burst intensity classification standard, obtain the numerical eigenvalues of cloud models with different intensity levels of different evaluation indices through Table 1 and Eq. 3. Specific values are shown in Table 3.
Step 3: With the digital characteristic values of the evaluation index cloud model in Table 3, use the MATLAB software to generate the cloud image of each evaluation index and calculate the degree of certainty $\mu(x)$ of each evaluation index through the forward cloud generator and Eq. 4, as shown in Fig. 6.
Step 4: Calculate the weight value of each evaluation index using the CRITIC method (Eqs. 5–13) and G1 method (Eqs. 14–18), and then use the combined weighting method of game theory (Eqs. 19–23) to calculate the optimal combination weight.
Step 5: From the obtained optimal combination weight and the determination degree of each evaluation index, determine the degree of certainty of different intensity levels of rock burst. The calculation formula is as follows:

$$\mu_k = \sum_{j=1}^{n} w_j \cdot \mu_{kj} \tag{24}$$

where μ_k represents the determination degree of the k-level rock burst intensity of the sample. w_j represents the combined weight of the jth evaluation index of the sample. μ_{kj} indicates the determination degree of the jth evaluation index of the k-level rock burst intensity of the sample.
Step 6: Compare the determination degree of the rock burst intensity at all levels of the samples to determine the rock burst intensity of the samples.

Figure 6 represents the grade I to grade IV rock burst cloud maps for each evaluation index. The horizontal coordinate is the value of the evaluation index, and the vertical coordinate is the corresponding certainty. In this figure, the evaluation indices except σ_c/σ_t are all cost indicators. For these indices, the smaller the value is, the smaller the corresponding rock burst grade. The curves from left to right represent grade I rock burst, grade II rock burst, grade III rock burst, and grade IV rock burst. σ_c/σ_t is a benefit-type index. The larger the value is, the smaller the corresponding rock burst grade, and the curve is opposite from left to right.

3 Evaluation and Analysis of the Sample Rock Burst Intensity Grade

3.1 Sample Data Collection and Analysis

To verify the rationality and feasibility of the rock burst intensity grade evaluation model in this paper, the rock burst intensity grade was evaluated by combining 95 groups of rock burst instance data at home and abroad from references [39–41] and [42]. The matrix scatter plot of the rock burst instance dataset is shown in Fig. 7a. There is no obvious correlation between the evaluation indices. The boxplot of the rock burst instance dataset is shown in Fig. 7b. To better display all evaluation indices, evaluation indices σ_c and σ_θ are divided by 10, σ_θ/σ_c is multiplied by 10, and H is divided by 20. In this figure, the median of most evaluation indicators is not in the center of the box, indicating that the data of most evaluation indicators are asymmetrically distributed. In addition, some of the evaluation indicators have individual outliers, which belong to

Table 3 Calculation results of digital characteristics of cloud models of rock burst indicators

Rock burst grade	Digital features	Evaluation index						
		σ_c(MPa)	σ_θ(MPa)	σ_t(MPa)	σ_c/σ_t	σ_θ/σ_c	W_{et}	H(m)
I	E_x	40	12	2.5	60	0.15	1	25
	E_n	13.33	4	0.83	6.67	0.05	0.33	8.33
	H_e	0.01	0.01	0.01	0.01	0.01	0.01	0.01
II	E_x	100	42	6	33.35	0.4	2.75	125
	E_n	6.67	6	0.33	2.22	0.033	0.25	25
	H_e	0.01	0.01	0.01	0.01	0.01	0.01	0.01
III	E_x	150	93	8	20.6	0.6	4.25	450
	E_n	10	11	0.33	2.03	0.033	0.25	83.33
	H_e	0.01	0.01	0.01	0.01	0.01	0.01	0.01
IV	E_x	250	163	19.5	7.25	0.85	12.5	1350
	E_n	23.33	12.33	3.5	2.42	0.05	2.5	216.67
	H_e	0.01	0.01	0.01	0.01	0.01	0.01	0.01

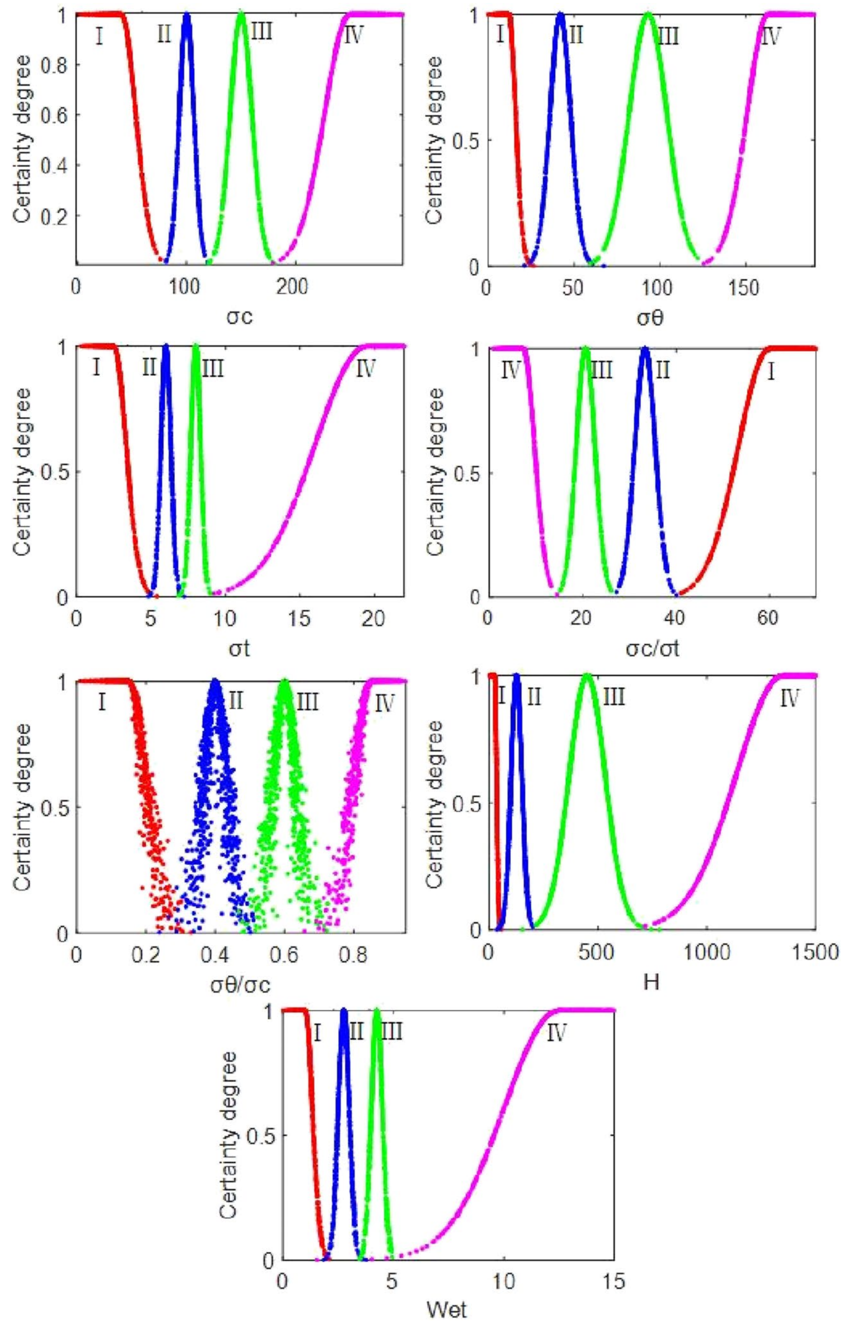


Fig. 6 Cloud model for each evaluation index

normal conditions, indicating that the rock burst instance data used in this paper are reasonable. The straight square distribution diagram of the rock burst instance dataset is shown in Fig. 8. All indicator data of rock burst are random variables. According to relevant research results [17], many random variables in natural sciences basically or approximately follow a normal distribution. As shown in Fig. 8, the histograms of the 7 evaluation indicators all decrease sequentially from the highest straight square column to both sides. Although affected by the small sample size, some individual straight square columns in σθ and σc/σt violate this rule. However, the degree of deviation is small, and it is a normal situation. Therefore, the 7 evaluation indices basically obey or approximately follow the normal distribution. Therefore, it is reasonable to adopt the cloud model to determine the rock burst intensity level in this paper.

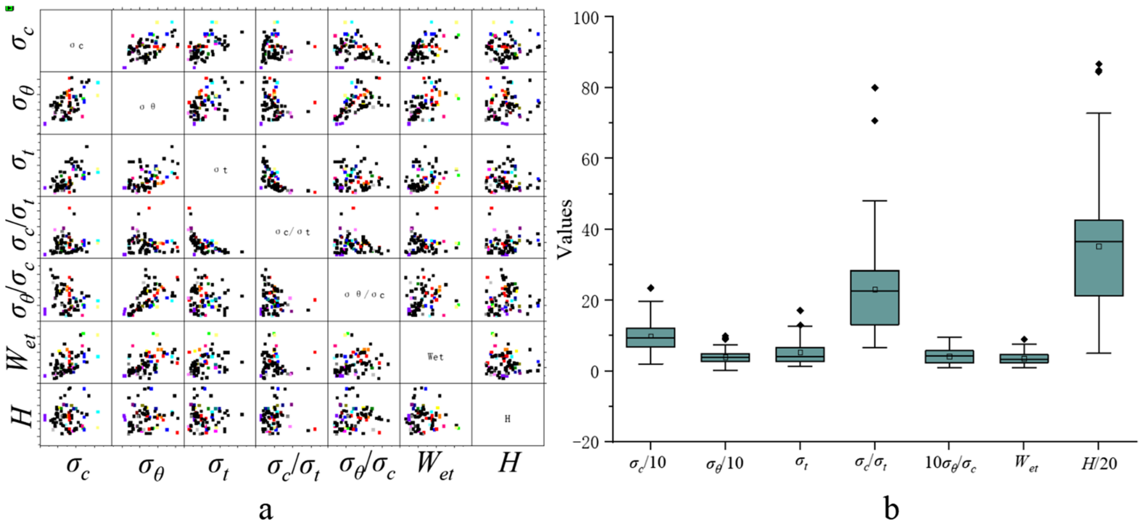


Fig. 7 Rock burst data analysis (**a**) scatter plots of the rock burst index matrices; **b** boxplots of the rock burst indicators)

3.2 Weight Determination and Sensitivity Analysis

In this paper, the combined weighting method of game theory is used to combine the weights of indicators obtained by the CRITIC method and G1 method. The CRITIC method mainly uses the variability and conflict between different evaluation indicators to assign weights, and the weight values of the evaluation indicators can be obtained through Eqs. 5–13. When treating the relation analysis method as a subjective weighting method, three experts are invited to score the importance of the evaluation indices, and the weight value of each evaluation index is obtained through Eqs. 14–18. Finally, the weight obtained by the CRITIC method and G1 method is optimized by the combined weighting method of game theory (Eqs. 19–23), and the optimal combination weight value is obtained. The weight values obtained by the three weighting methods are shown in Fig. 9

The analysis of the sensitivity of the evaluation index is helpful to determine the importance of the evaluation index so that appropriate measures can be taken to prevent rock burst. According to the weight obtained by combination weighting in Fig. 9, the index σ_θ/σ_c has the largest weight, indicating that this index has a greater impact on the occurrence of rock burst than other indicators and is the most important. The depth of cover H is the second most important index, with a weight of 0.1582, followed by σ_t, W_{et}, σ_c/σ_t, σ_θ, and σ_c in descending order. Therefore, for rock burst, the stress coefficient σ_θ/σ_c is the most sensitive index, and the other indices are H, σ_t, W_{et}, σ_c/σ_t, σ_θ, and σ_c.

The stress coefficient σ_θ/σ_c and depth of cover H, with sensitivity rankings of first and second, respectively, reflect the rock stress conditions. Therefore, we find that the occurrence of rock burst is closely related to the rock stress conditions. Liu et al. [3] selected indices σ_c, σ_t, σ_θ, σ_c/σ_t, σ_θ/σ_c, and W_{et} to predict the intensity grade of rock burst and believed that σ_θ/σ_c was the most important factor for the occurrence of rock burst.

4 Result Prediction and Analysis

The numerical characteristic results of the cloud model obtained in Table 3 and the combined weight values in Fig. 9 are substituted into Eq. 4 and Eq. 24 in turn to calculate the comprehensive certainty of the rock burst intensity grade of the sample, determine the rock burst intensity grade of the sample, and compare the evaluation results of the CRITIC-CM and G1-CM rock burst evaluation models. The results are shown in Table 4 (because there are too many samples, only some results are listed). According to the evaluation results, the accuracy of the CRITIC-G1-CM rock burst evaluation model can reach 94.7%, which is higher than that of the CRITIC-CM rock burst evaluation model of 90.5% and the G1-CM rock burst evaluation model of 91.6%, as shown in Fig. 10. This also shows that the core of the rock burst evaluation model based on the cloud model lies in the determination of the weight of the evaluation index, and it also shows that the CRITIC-G1-CM rock burst evaluation model effectively integrates the objectivity of the CRITIC method and the subjectivity of the G1 method and improves the accuracy of the weight.

The F1 score is widely used to evaluate the discrimination ability of classification models. It has two evaluation indices: precision and recall. The F1 score contains four basic concepts: TP (evaluation is positive, fact is positive), FP

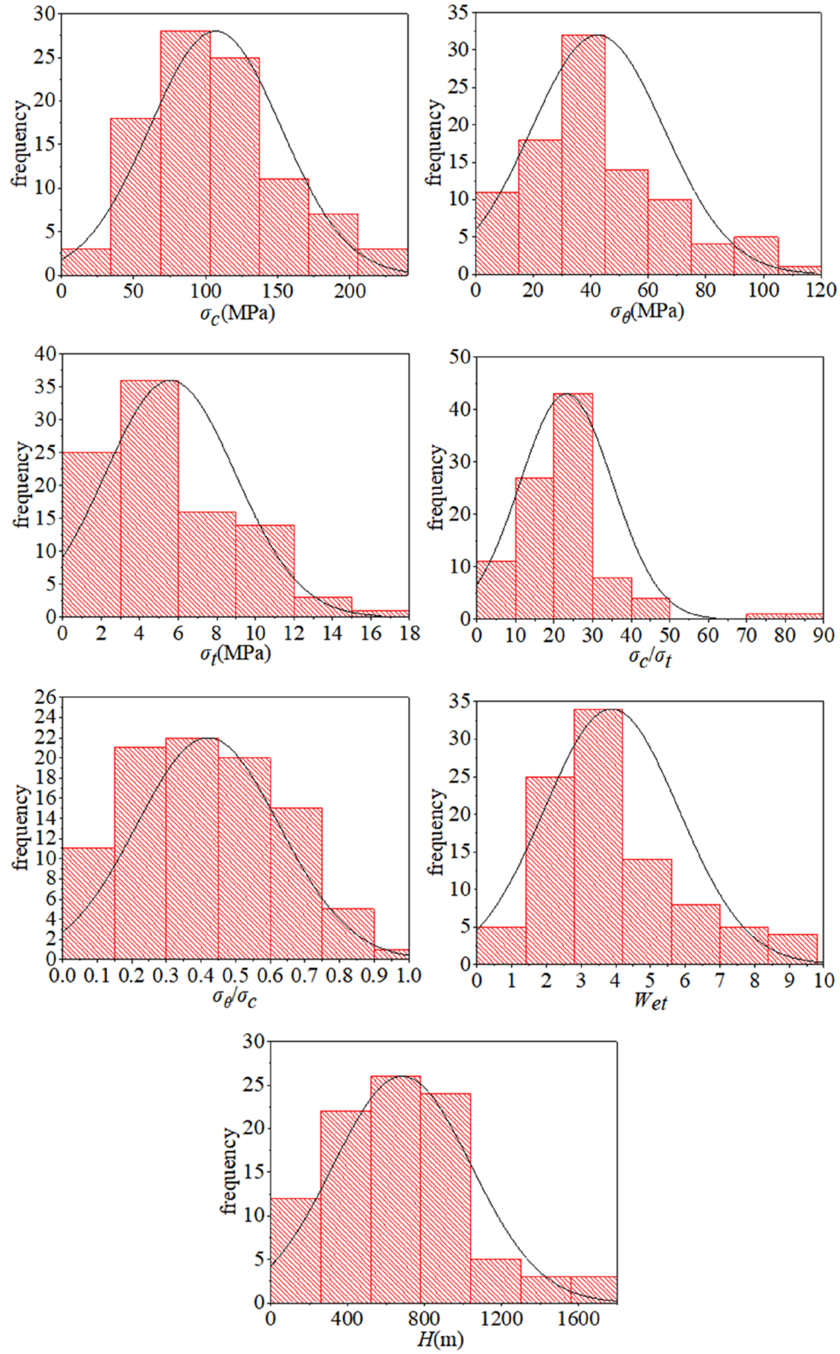


Fig. 8 Histograms of the rock burst indicators

(evaluation is positive, fact is negative), TN (evaluation is negative, fact is negative), and FN (evaluation is negative, fact is positive).

Precision refers to the proportion of samples determined to be positive by the evaluation model that are actually positive. The calculation formula is as follows:

$$Precision = \frac{TP}{TP + FP} \tag{25}$$

Recall is the proportion of positive samples determined to be positive by the evaluation model to the total positive samples, and the calculation formula is as follows:

$$Recall = \frac{TP}{TP + FN} \tag{26}$$

The F1 score is the weighted average of the precision value of the evaluation model and the recall, calculated by the formula:

Fig. 9 Weight values obtained by the three weighting methods

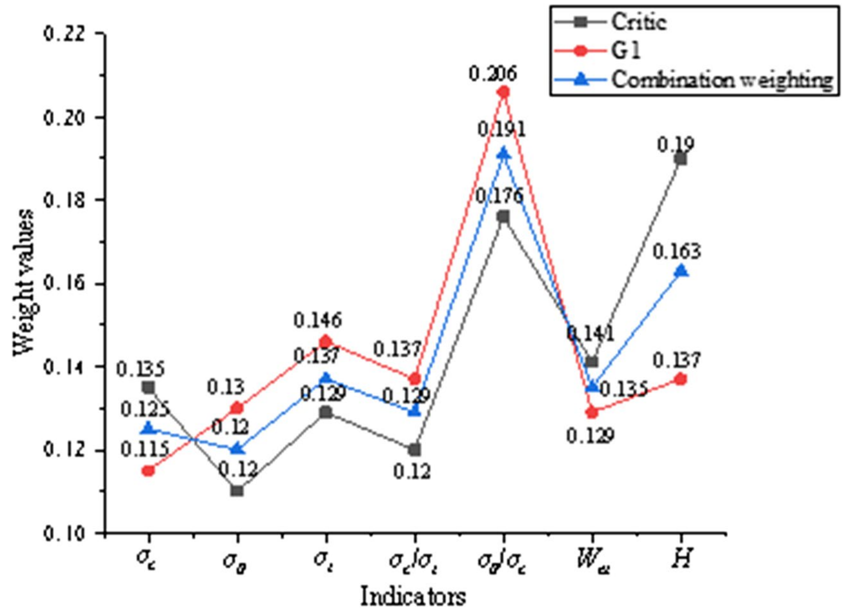


Table 4 Evaluation results of rock burst intensity grade

Sample number	Degree of synthetic certainty				CRITIC-G1-CM	CRITIC-CM	G1-CM	Actual grade
	μ(I)	μ(II)	μ(III)	μ(IV)				
1	0	0.2135	0.1479	0.0692	II	II	II	II
2	0	0.0011	0.1668	0.0599	III	III	III	III
3	0.0001	0.1522	0.0844	0.0522	II	II	II	II
4	0	0.0453	0.2128	0.1582	III	III	III	III
5	0.142	0.1639	0.0108	0	II	II	I ~ II	II
…	…	…	…	…	…	…	…	…
94	0.0059	0.1995	0.1126	0.1601	II	IV*	II	II
95	0.3168	0.0778	0.0245	0.002	I	I	I	I

* indicates the evaluation error

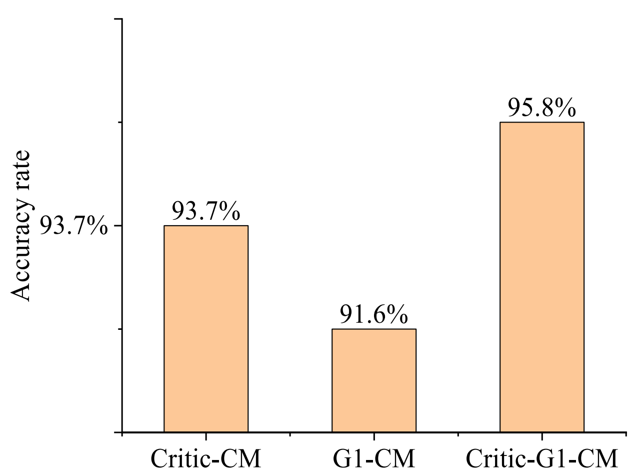


Fig. 10 Comparison of the evaluation accuracy rates of the three models

Table 5 Values of the classification indicators

	Precision	Recall	F1
I	0.963	0.963	0.963
II	1	0.938	0.968
III	0.971	0.971	0.971
IV	0.5	1	0.667

$$F1 - score = 2 \times \frac{Precision \times Recall}{Precision + Recall} \tag{27}$$

The calculation results of the precision, recall and F1 score are shown in Table 5. As observed from the data in the table, the precision, recall, and F1 score of all four categories are very good. In terms of class IV, the exact value and F1 score of the evaluation model are lower than those of classes I, II, and III, which is caused by the lack of strong rock burst sample data and insufficient cardinality in the calculation. In general, the model is reasonable and feasible in the evaluation of rock burst intensity.

5 Engineering Application

Chengchao Iron Mine is located in Ezhou City, Hubei Province, on the south bank of the middle reaches of the Yangtze River. It is the third largest iron mine in China. After years of mining, the mining site has reached the deep ore body, i.e., Chengchao Iron Mine W39 line ore body V. The surrounding rocks of the ore body hanging wall are mainly marble, granite porphyry, quartz feldspar porphyry, diorite porphyrite, and diorite, and granite, diorite, and skarn are the main surrounding rocks of the ore body footwall. With increasing mining depth, the ground stress in the mining operation area becomes increasingly larger, the hardness and brittleness of the rock mass become increasingly higher, and the risk of rock burst becomes increasingly stronger. Therefore, it is necessary to predict rock burst to take corresponding measures to ensure safety.

In this paper, based on the measured rock burst data of the No. V ore body on the W39 line of the Chengchao Iron Mine, with a depth of cover of 430 ~ 700 m given by Xu et al. [13], the data of five typical ores are selected for analysis. The data are shown in Table 6.

The rock burst grade of five kinds of ores is predicted by the CRITIC-G1-CM rock burst intensity evaluation model. The evaluation results shown in Table 7 show that the rock burst tendency of magnetite is the strongest and belongs to grade IV, followed by granite porphyry and quartz porphyry, which belong to grade III, and diorite and granite, which belong to grade II. The prediction results in this paper are completely consistent with the actual rock burst grades, which shows the effectiveness and reliability of the CRITIC-G1-CM rock burst intensity grade evaluation model in the classification prediction of rock burst intensity.

6 Conclusion

Rock burst is a complex nonlinear change process, and all its index data are random variables that approximately follow a normal distribution. Therefore, this paper proposes a cloud model based on combination weighting to analyze and evaluate the rock burst intensity level. The main conclusions are as follows.

1. A total of 95 groups of measured rock burst data at home and abroad are selected as samples, the evaluation index system of the rock burst intensity level is established by selecting σ_c, σ_θ, σ_t, σ_c/σ_t, σ_θ/σ_c, W_{et}, and H, and the weight of each index is calculated by weighting the CRITIC, G1, and CRITIC-G1 game combinations.
2. The sensitivity of each evaluation index of the CRITIC-G1-CM rock burst evaluation model is analyzed, and the order of sensitivity is $\sigma_\theta/\sigma_c > H > \sigma_t > W_{et} > \sigma_c/\sigma_t > \sigma_\theta > \sigma_c$. The stress coefficient σ_θ/σ_c and depth of cover H, with sensitivity ranking of first and second, reflect the stress condition of rock, indicating that reducing the stress of the rock can be identified as the main measure to reduce the risk of rock burst.
3. The evaluation index weight and cloud model are used to evaluate the rock burst intensity grade. The results show

Table 6 Measured data of rock burst in Chengchao Iron Mine

Rock type	Evaluation index						
	σ_c(MPa)	σ_θ(MPa)	σ_t(MPa)	σ_c/σ_t	σ_θ/σ_c	W_{et}	H(m)
Diorite	130.5	44.6	10.9	11.97	0.34	4.6	530
Granite	126.8	55.9	10.2	12.43	0.44	8.1	560
Granitic porphyry	120.5	72	10.2	11.81	0.60	2.5	580
Quartz porphyry	106.8	68	8.69	12.29	0.64	7.2	600
Magnetite	95.6	83.9	6.56	14.57	0.88	3.7	630

Table 7 Rock burst evaluation results of the Chengchao Iron Mine

Rock type	Degree of synthetic certainty				Predicted results	Actual grade
	μ_1	μ_2	μ_3	μ_4		
Diorite	0.0007	0.1414	0.0719	0.0262	II	II
Granite	0	0.0891	0.0094	0.0463	II	II
Granitic porphyry	0	0.0857	0.2043	0.0251	III	III
Quartz porphyry	0	0.0767	0.1072	0.0303	III	III
Magnetite	0	0.1373	0.0956	0.1857	IV	IV

that the accuracy of the CRITIC-G1-CM rock burst evaluation model is 94.7%, which is higher than that of the CRITIC-CM and G1-CM rock burst evaluation models of 90.5% and 91.6%, respectively. The CRITIC-G1-CM rock burst evaluation model effectively integrates the objectivity of CRITIC model and the subjectivity of the G1 model and improves the rationality and accuracy of the weight. The rock burst classification ability of the evaluation model is analyzed by the F1 score, and the result shows that the CRITIC-G1-CM evaluation model has a good rock burst classification ability.

4. The rock burst grade of five kinds of ores with a depth of cover of 430 ~ 700 m in the W39 line of the Chengchao Iron Mine is predicted by the CRITIC-G1-CM rock burst evaluation model. The prediction results are consistent with the actual rock burst grade, which verifies the effectiveness and accuracy of the model and indicates that the rock burst evaluation model of CRITIC-G1-CM has certain practical application value.

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Data Availability The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Competing Interests The authors declare that they have no conflicts of interest. No conflict of interest exists in the submission of this manuscript, and the manuscript has been approved by all authors for publication. The authors declare that except for the preprint published in the research square, it has been published elsewhere. All the authors listed have approved the manuscript that is enclosed.

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