



# Methodology for the Estimation and Classification of White Marble Reserves

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## Abstract

In the marble industry, ore reserves refer to the produced tonnage of orthogonal prismatic, defect-free blocks of commercial sizes and of marketable aesthetical quality. On the contrary, in the mining sector, ore reserves refer to the tonnage of ore with grade above some cut-off value. While in the mining sector ore reserves are reported according to existing standard guidelines, the marble industry has no standard method. The purpose of this paper is to present a method to estimate white marble reserves using drill coring or borehole wall logging and limited joint mapping on exposed marble surfaces. Data pertaining to fracture orientation, fracture frequency, size, and whiteness were collected. The primary phase of marble characterization is followed by the discretization of the orebody into a three-dimensional array of orthogonal prismatic blocks, where the marble whiteness and fracture intensity are attribute values. Both these variables are considered to obey continuous and not discrete probability distributions. The discretization is performed using computer aided design and the Block Kriging techniques. An outline to improve the description of a given marble ore deposit at the exploration stage is presented for the prediction of marble block size distribution from joint count measurements along diamond drill cores. An example case of a white dolomitic marble quarry is used to demonstrate these methods.

**Keywords** White marble · Drill coring · Fracture frequency · Reserves · Geostatistics · Oriented core

## Abbreviations

BM	Block model using the Kriging technique
CAD	Computer aided design
CIM	Canadian Institute of Mining, Metallurgy and Petroleum
CL	Confidence level
CVP	Cumulative volume proportion
DTM	Digital terrain model
FF	Fracture frequency (expressed in total number of joints per metre of rock core)
FF'	Fracture frequency of each joint set (expressed in $m^{-1}$ )
JORC	Joint Ore Reserves Committee
MF	Modifying factors in the process of transforming resources to reserves

NHPP	Non-homogeneous poisson process
NSD	Normalized standard deviation
pdf	Probability density function
PERC	Pan-European Reserves & Resources Reporting Committee
RR	Recovery ratio
SR	Stripping ratio
Wht	Whiteness (expressed in %)
$a$	Normalized standard deviation of the prediction at each block of the model
$\gamma(h)$	Geostatistical semi-variance function of the lag $h$
$f_{\lambda}$	Number of fractures per fixed length interval along scanline or rock core ( $m^{-1}$ )
$\lambda$	Basic linear frequency parameter of the NHPP
$\sigma_{OK}^2$	Kriging variance at the centroid of each block in the model

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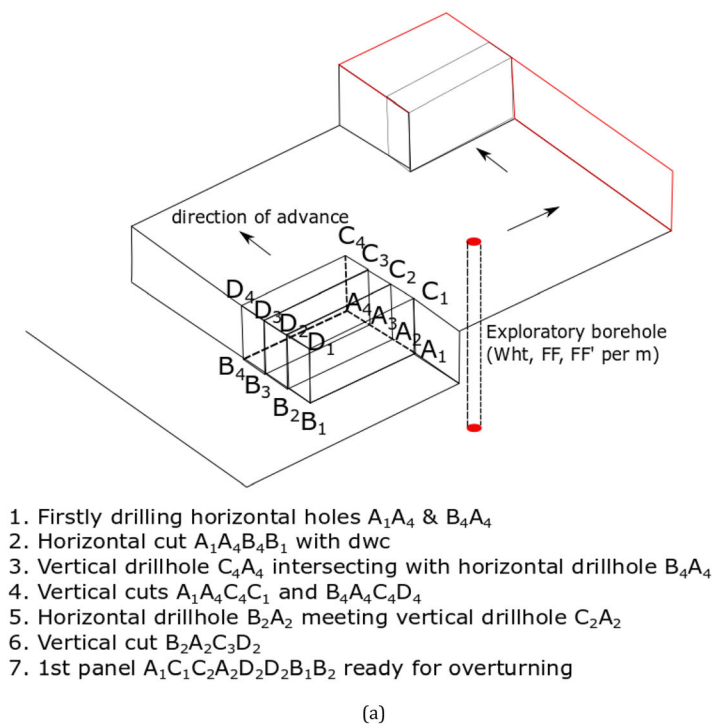
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## 1 Introduction

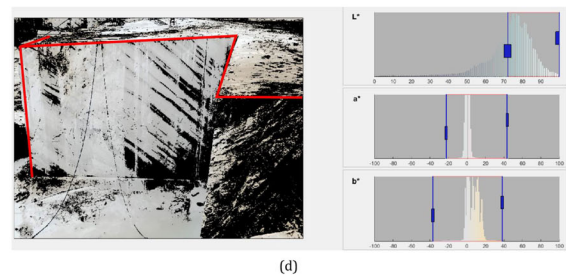
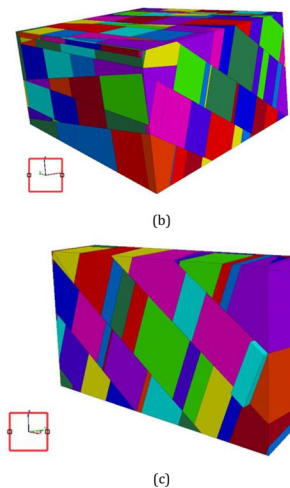
The decorative stone quarrying sector has some appreciable differences compared to other mining sectors. The former sector employs different extractive methods (i.e., combined drilling–

diamond wire cutting and chain saw cutting with limited use of smooth blasting) than those used in the metallic and industrial minerals extractive sectors. Further, in contrast to the current mining practice where a base-metal ore is characterized mainly by the grade of valuable mineral, the value of a decorative stone quarry comes from the tonnage of right-angled marble block volumes of commercial sizes (also called “squared” blocks). In marble quarrying, the basic ore extraction unit is the “bench block” that is first isolated from the bench by the hybrid method of drilling–diamond wire cutting, as shown in Fig. 1a. From this bench block, several “panels” (usually 2–4) are then sequentially extracted and overturned with loaders or excavators. The marble ore, similar to any other rock mass, is transected by natural joints. For example, a bench block with dimensions 10 m × 10 m × 6 m (length × width × height) in jointed marble simulated using the distinct element method [1] is shown in Fig. 1b. In this example, three joint sets transect the block, and the various blocks isolated by the joints are indicated with different colors. A single panel extracted from this block is shown in Fig. 1c. The jointed panel example demonstrates that only a certain number of marble blocks will be eventually selected from this panel based on criteria such as the possible dimensions of the squared blocks formed in a secondary stage from the natural marble blocks by additional cutting with diamond wire, dense drilling or another technique.

In this work, the terms joint and fracture are used as synonyms of any type of discontinuity in the rock mass. The value of the squared marble blocks is also affected by factors such as the coloration or aesthetical quality that suits the market and by the presence of a minimum number of structural defects (cracks, holes, hard or soft inclusions) or aesthetical defects (patches, inclusions or veins, alterations with different color from the background color, etc.). At the macro-scale, the color quality of white marbles is depicted mainly by the whiteness. The whiteness can be quantitatively described by the value of the L (vertical) Cartesian coordinate of the three dimensional CIE  $L^*a^*b^*$  color space [2]. In this space, the lightness coordinate L has values in the range 0% (black) to 100% (white). Built-in image analysis techniques in Matlab™ can be applied to photographs taken in the field under regular daylight conditions to infer the L histogram for the entire vertical face of a panel (Fig. 1d) or a block extracted from an overturned panel. In the image of the panel’s wall or face, we used a threshold  $L = 73\%$ ; the pixels with L lower than this value are shown in black in the processed digital image. According to market demands, only the blocks of this panel with  $L > 50\%$  (or more) and commercial size are marketable. The rest of the blocks that do not satisfy both the criteria of whiteness and size are considered as waste.

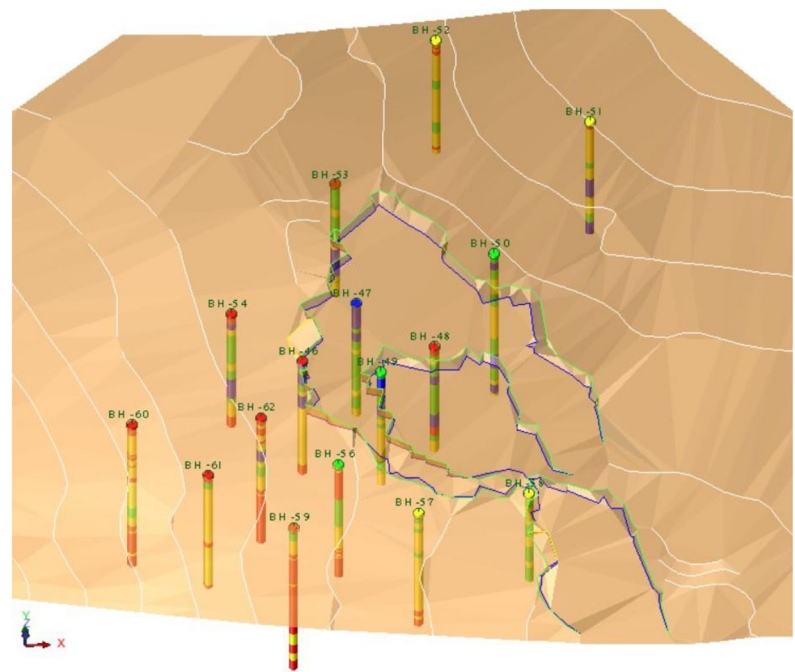


**Fig. 1** Cutting of bench blocks, block panels, and marble block extraction from each panel; **a** perspective view of two active benches and sequence of panel cutting from a bench block, **b** perspective view of a bench block transected by three principal joint sets (OY-axis points to the north), **c**



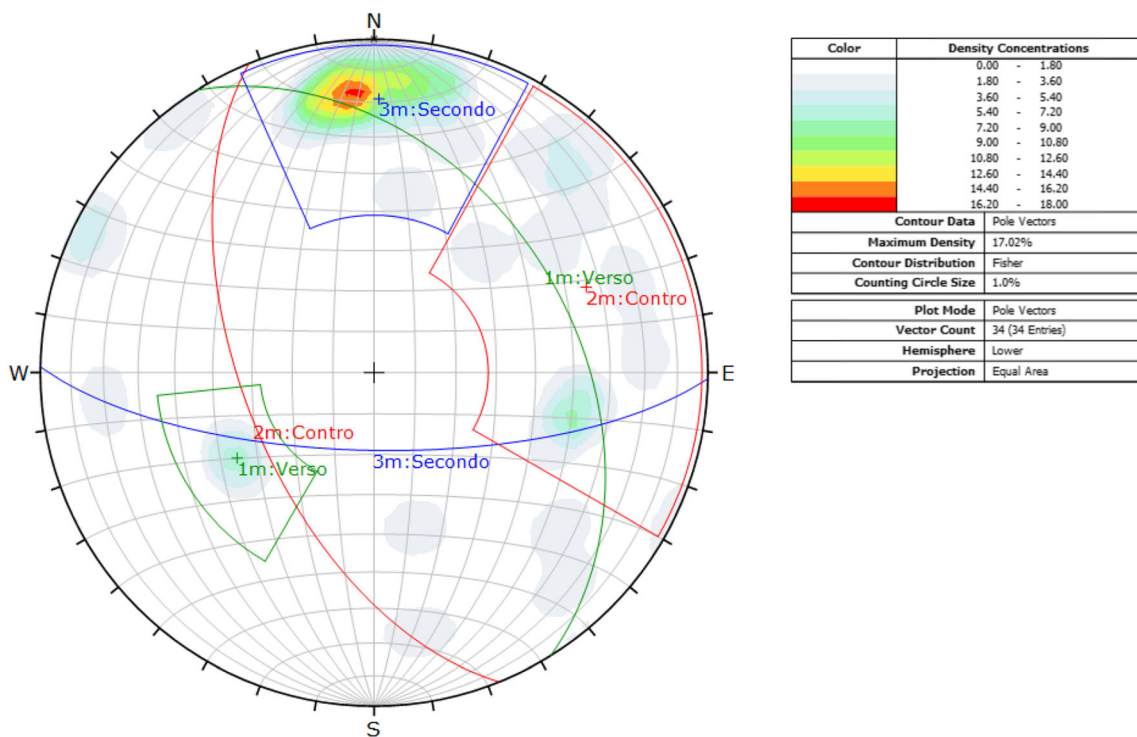
**d** view along the strike of the marble bedding planes that dip  $45^\circ$  of a panel isolated from a bench block that is ready for overturning, and **d** image analysis of a photograph of a panel with a lightness threshold below 73%

**Fig. 2** Digital terrain model of “current” quarry state with the vertical boreholes



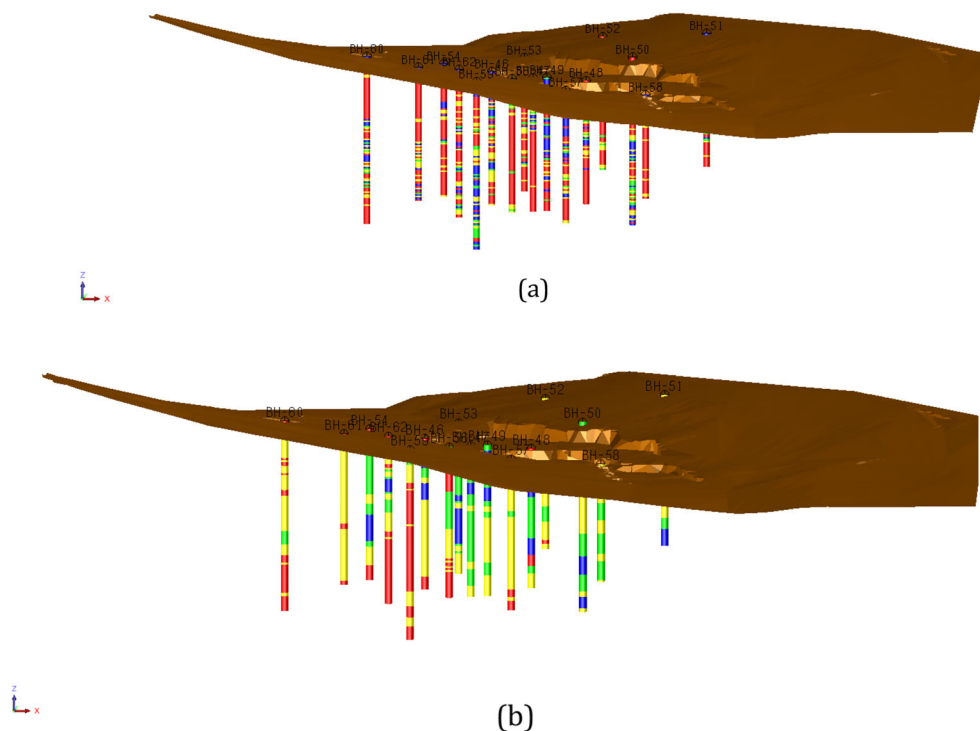
The decision to proceed with the exploitation of a marble orebody is based on factors such as market price and demand of the decorative stone at hand, estimated resources and the associated risk of the estimation, recovery ratio (RR) of squared marble blocks, stripping ratio (SR), location and topography, environmental and social criteria, and regulations (for example

on the final slope geometry of the quarry, pit slope accessibility, etc.). In this work, the term RR means the ratio of total volume of commercial blocks over either the simulated model block volume or the volume of the tilted panel (Fig. 1c). In the first case of the model, block volume RR is predicted, whereas in the case of the actual tilted panel at the bench floor, RR is measured.



**Fig. 3** Stereographic projection of mapped joints at the lower hemisphere. The three joint sets are the verso or grain, the contro or head-grain, and the secondo or secondary

**Fig. 4** Visualization of borehole data; **a** distribution of the four categories of fracture frequency (in  $\text{m}^{-1}$ ) along the boreholes and **b** distribution of the four categories of whiteness (in %) along the boreholes



Herein, we are concerned only with ores of white marble that represent a major portion of the decorative stones. Other types of decorative stones are colored marbles, travertines and onyx, granites, and sedimentary stones (such as limestones and sandstones among others). Our primary purpose is to compose a method to estimate white marble reserves. Therefore, the marble reserves are estimated using the Block Kriging technique [3, 4] and are classified according to international standards of public reporting, such as the PERC code [5], the Australasian Code JORC code [6], and the Canadian CIM code [7]. The proposed method is based on: (1) the appropriate color and joint intensity characterization of exploratory boreholes (coring or images of walls), (2) dedicated



**Fig. 5** Drill cores of HQ diameter in the wooden boxes. All the cracks were identified and colored according to the set they belong to. Some cores have been sawed in the processing plant along their axial plane for inspection of the color and other defects. Each of the three series of cores inside each box has as length of 2 m to facilitate core measurements

CAD software for quarry design (e.g., Autocad™) and geostatistical analysis (i.e., Geovia Surpac™ [8] or other similar software), (3) oriented diamond drill core data or oriented borehole wall images, and (4) statistical modeling of joint orientation, spacings and size, as well as Monte Carlo simulation techniques. The modeled orebody is discretized into orthogonal prismatic “model blocks” or panels of fixed volume of the order of 50–200  $\text{m}^3$ . This may be accomplished by a three dimensional spatial interpolation method at the resources evaluation phase [8–10]. Each model block may be then assigned with a market value and an extraction cost. The marble ore resource model is transformed into a reserve model by prescribing the final open pit layout and considering the primary *modifying factors* [6] of whiteness and block size. The optimum pit limits for the final evaluation of reserves are found by applying suitable criteria, such as the maximization of white marble ore reserves amenable to extraction with the minimum SR or other suitable multi-objective criteria already applied in open pit mining [11]. The latter topic is not considered in this work.

The structure of the paper is as follows: Section 2 is devoted to the methodology for marble characterization at the exploration phase and the resources evaluation phase. The steps of the methodology are illustrated via a case study of white marble ore deposit. One of the innovative aspects of the proposed method is the spatial interpolation of the whiteness, which is assessed from the drill cores or borehole walls. Whiteness is considered as a continuous random variable varying in the range 0–100%. Another new aspect of marble



**Fig. 6** HQ (63.5 mm) marble cores with the top line along the core-axis marked on them

resources estimation enables each model block to be quantitatively characterized by the fracture frequency or fracture intensity that is correlated with RR at the quarry site. The first quantity gives the market value of the block, while the second one the RR of commercial blocks that may be extracted from the “economical model block”. The orientation of the principal joint sets is considered for the alignment of the block model and for the orientation of the search ellipsoid in the Block Kriging process. The measured marble resources are those associated with a certain level of risk that is expressed with the Kriging variance of the model blocks. Section 3 addresses the marble reserves estimation and reporting according to the standard codes [5–7]. At this stage, the proven reserves obey the criteria of minimum risk of estimation, whiteness above a threshold value, and fracture frequency below an upper limit. Section 4 is devoted to the marble block volume distribution prediction from the fracture frequencies of each joint set measured along drill cores that presents the final volume of proven reserves according to the marketable volume size criterion. Finally, conclusions and recommendations are presented in Section 5.

## 2 Methodology for White Marble Reserves Estimation

### 2.1 Characterization of Drill Cores

The proposed method of classification of white marble ore resources and then the estimation of reserves of commercial

marble block volumes is demonstrated with the aid of a case study. The example case study pertains to a white dolomitic marble deposit lying under a relatively flat terrain with some already developed benches, as shown in the *digital terrain model* (DTM) in Fig. 2.

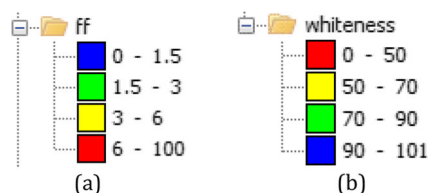
Initial joint orientation measurements obtained from a geological compass on the exposed marble surfaces indicated the presence of three main joint sets transecting the marble orebody. The oldest set is parallel to the layering or schistosity of the marble deposit, namely the grain system (or verso in Italian terminology). Another set is almost orthogonal and with the same strike to the former called the head-grain system (or contro). Finally, there is also a secondary system (or secondo) of deeply dipping joints with a strike orthogonal to the common strike of the other two families of joints. The poles of fracture planes processed with specialized software [12] for the identification of the principal sets are illustrated in the contoured *lower-hemisphere stereographic projection* in Fig. 3. In the same figure, the mean representative great circles of these sets are also displayed.

Sixteen vertical diamond boreholes spaced at 30–50 m apart have been drilled by a HQ core barrel to evaluate marble resources. The characterized drillholes with respect to fracture frequency (FF) and whiteness (Wht) are shown in Fig. 4a, b, respectively. These were created with the aid of commercial CAD software [8].

As mentioned previously, the market value of a model white marble block depends on the coloration (whiteness) and density of cracks that control the size of the extracted blocks. The degree of marble fracturing can be quantified in a fast manner via  $FF\ m^{-1}$  that is defined here as the total number of fractures per meter (m) of borehole. During drill core logging, the grain planes were marked with green, head-grain planes were marked with red, and secondary planes were marked with blue on the HQ cores inside the wooden boxes (Fig. 5). This procedure was followed to facilitate the easy and quick measurement of fracture counts per meter of core of each joint set separately and then calculate the sum of joints or fractures per meter denoted here as FF.  $FF\ m^{-1}$  of each joint set is used at a later stage (Sect. 5) to estimate the block size distribution. Some cores were sawed in half with a diamond cutting disc, and one of the flat surfaces of each sawed core was later polished to further observe the color and check for the presence of defects.

**Table 1** The four quality classes of marble cores according to their whiteness

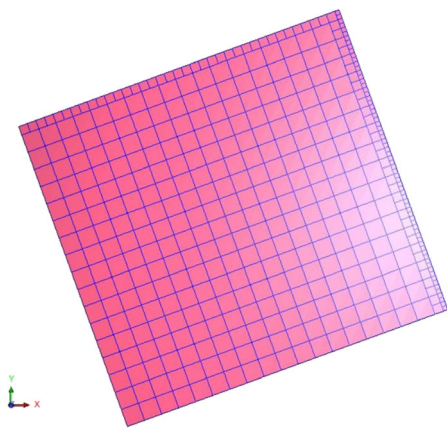
Coloration class	Whiteness [%]	Description
I (“blue color in the color scale”)	$\geq 90$	Snow white
II (“green color in the color scale”)	70–89	White-gray (snow white with some gray veins or “clouds”)
III (“yellow color in the color scale”)	50–69	Gray-white (more gray thin or wide veins than the white zones in between)
IV (“red color in the color scale”)	$< 50$	Gray to dark gray (no commercial value)



**Fig. 7** Color bars of FF  $m^{-1}$  and Wht % classes; **a** color bar indicating marble qualities according to their fracturation (FF in units of  $m^{-1}$ , i.e., total number of fractures per meter of drill core), and **b** color bar indicating marble qualities according to their whiteness and corresponding range of values in %

In the examined case, vertical quarry faces were exposed to measure a sufficient number of fracture orientations using a geological compass and the subsequent delineation of the principal joint sets. However, an alternative method to measure fracture orientations at the exploration stage is the oriented drill core technique that uses special instrumentation (extension tube of wireline core barrel, cylindrical cell with sensors attached at the tail of the barrel, and a twin cylindrical cell at the surface) and a wireline core barrel of appropriate diameter [13]. In this technique, which requires inclined boreholes, the extracted rock cores at the surface are first rotated to find and then draw the bottom (or top) line along the axis of the rock core at its in situ position (e.g., Fig. 6). The second step is the estimation of the *beta* and then of the *alpha* angles of the joints along the oriented core, and the final step is the transformation of these angles into dip direction and dip angles, respectively, using an appropriate method [13].

An alternative technique is the photographic imaging of borehole walls with a camera (optical televiewer logging technique) [14, 15]. In this technique a camera with known orientation is lowered or pushed–pulled with rods into the drillhole at a constant rate while taking wall colored pictures. The image obtained is unrolled into a strip to reveal an angle measure system from  $0^\circ$  to  $360^\circ$ . All joint planes appear as sinusoids on



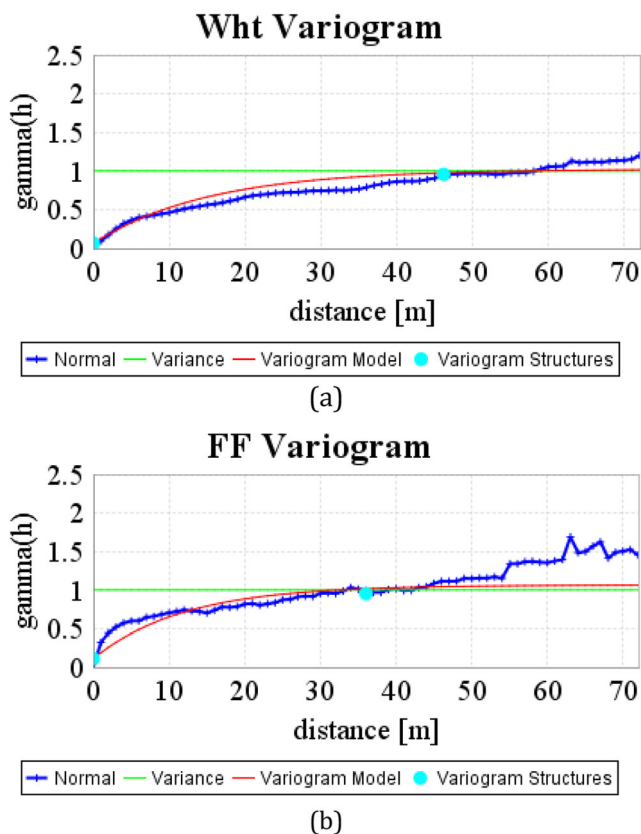
**Fig. 8** Plan view of the constructed block model of the research area. OY-axis points to the North

the unrolled image of the borehole. The dip angles of the joints are calculated by evaluating  $\tan^{-1}(h/d)$ , where  $h$  denotes the difference between the depth of the entry and depth of the exit of the joint into and out of the borehole walls on the points of maximum inclination of the joint plane, and  $d$  is the diameter of the borehole. The processing of the images is done with dedicated software. The depth from where the picture is taken is linked to the photograph of the rock core at the same depth. In wet and dirty boreholes, the alternative acoustic borehole imaging technique may be applied to obtain false colored images [16].

As mentioned previously, the Wht % is not treated as a categorical or discrete random variable. Specifically, whiteness is described by the variable  $L^*$  for the lightness from black (0%) to white (100%) according to CIE Lab notation. This procedure gives a universal, quantifiable character to Wht % that is independent from the specific qualitative characterization system adopted by each quarry, as is the usual approach in relevant publications [10]. The percent of whiteness is quantified by an experienced person of the quarry (geologist or mining engineer) every 1 m of the core, as is done for FF  $m^{-1}$ . This work is performed at open-air standard  $D_{65}$  illumination conditions.  $D_{65}$  corresponds roughly to the average midday light in Western Europe/Northern Europe (comprising both direct sunlight and the light diffused by a clear sky). The qualified person is a geologist or mining engineer who has significant past experience in marble quarrying and adequate knowledge of the marble deposit under investigation. This is a legitimate approach in everyday quarrying practice because the same qualified person from the quarry who has considerable experience with the specific marble deposit also performs the quality categorization of squared marble blocks for marketing purposes. Equipment such as portable spectrophotometers may also be used only for the color characterization of drill cores or small samples extracted from the benches in controlled conditions (i.e.,  $D_{65}$  illuminating source, dust-free, and other controlled conditions). A promising method relies on image processing of digital photographs of quarry blocks, panels, drillcores or borehole walls using specialized software (e.g., Matlab™ or other). For the classification of estimated marble reserves into quality classes, as well as for model visualization and hence communication

**Table 2** Block model geometrical parameters

	Description
Extent of BM along OX-axis ( $\Delta x$ )	450 m
Extent of BM along OY-axis ( $\Delta y$ )	420 m
Extent of BM along OZ-axis ( $\Delta z$ )	240 m
OZ-axis rotation	–20 degrees
Block size (X, Y, Z)	3 m × 3 m × 6 m



**Fig. 9** Best-fitted exponential models (continuous lines) on the experimental semivariograms of Wht and FF (lines with crosses); **a** down-the-hole semivariogram of whiteness (Wht %), and **b** down-the-hole semivariogram of fracture frequency (FF m<sup>-1</sup>)

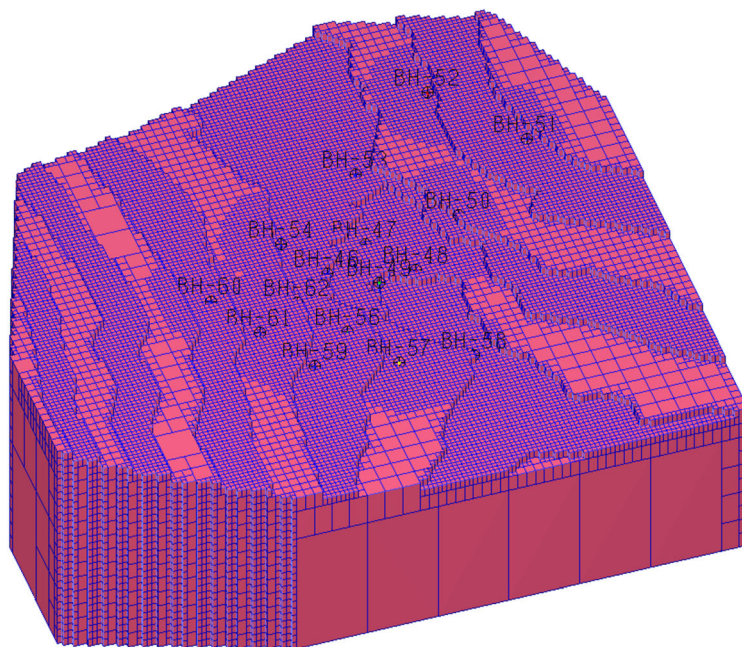
purposes, the Wht % quantitative visual assessments or measurements could be categorized into four classes, as shown in Table 1. In the block model, the four categories of defined marble resources according to their Wht % are shown with the chromatic scale provided in Fig. 7b.

Then the drill logs are transferred with a proper format into an electronic database (e.g., a worksheet in a workbook of Excel™). To visualize the model and schedule the excavation panels' sequence, four FF m<sup>-1</sup> classes are considered, as shown in Fig. 7a. The red color at the bottom of the scale refers to the faulted brecciated regions or zones that transect the deposit having FF > 6 m<sup>-1</sup>. The other three FF m<sup>-1</sup> categories, i.e., 0 ÷ 1.5 m<sup>-1</sup>, 1.5 ÷ 3 m<sup>-1</sup>, and 3 ÷ 6 m<sup>-1</sup> represent exploitable marble.

The spatial variation of the FF m<sup>-1</sup> and Wht % parameters of marble along the boreholes created with the CAD software after transferring data from Excel™ to Surpac™ is shown in Fig. 4a, b.

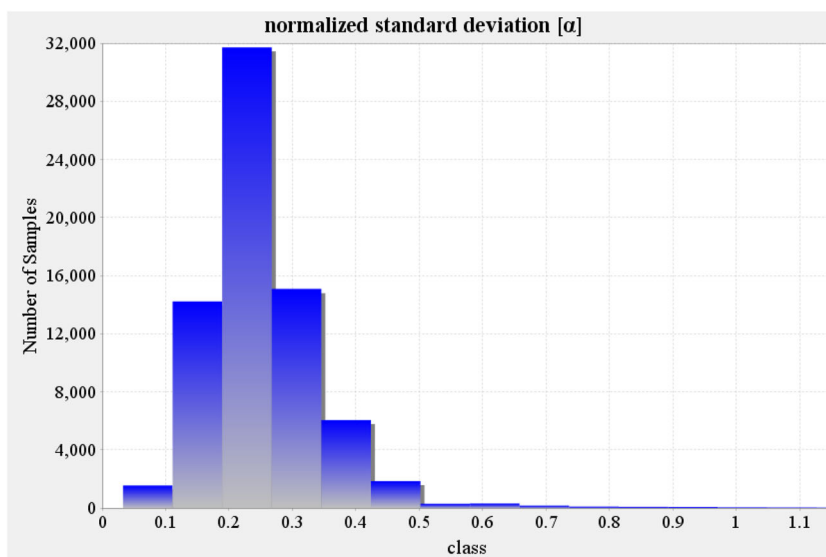
### 2.2 Block Kriging Model

Production planning is a key phase in the estimation of invested capital returns on every marble quarrying project. Therefore, the ore body is represented as a three-dimensional array composed of model blocks with orthogonal prismatic shape. In the proposed method, each block is assigned interpolated values of Wht % and FF m<sup>-1</sup> from neighboring boreholes with the Block Kriging technique [3, 4, 17]. Creation of such a block model (BM) requires the following steps:



**Fig. 10** Isometric view of the domain used for the creation of the block model with borehole collars

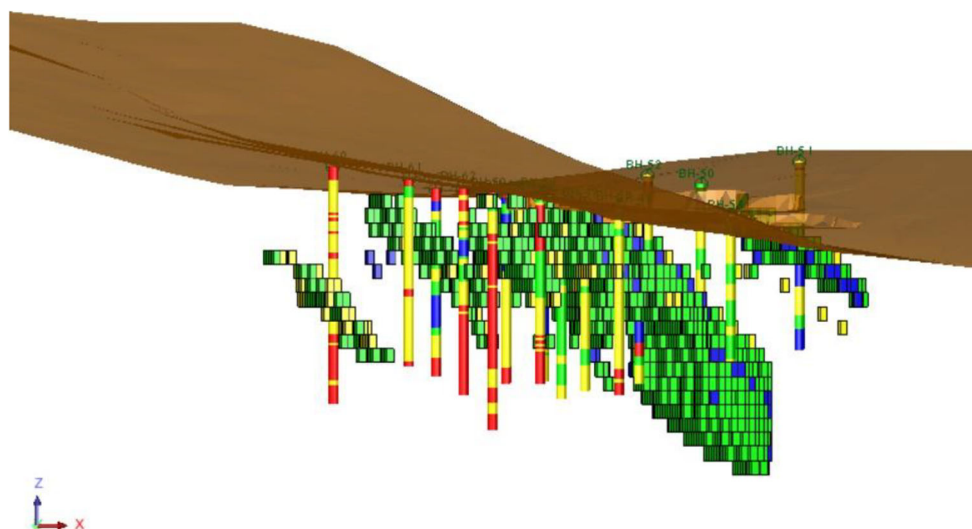
**Fig. 11** Frequency histogram of the normalized standard deviation (horizontal axis) of the blocks in the model



- Estimation of the minimum and maximum extents of the area that will be covered by the BM.
- Possible rotation of the BM around the vertical OZ-axis, which follows the extraction direction of the quarry (usually parallel to contro or secondo directions).
- Careful decision regarding the block dimensions that will be used by taking into account the bench height (Z value) and the cutting dimensions of chain saw or diamond-wire cutting machine on the horizontal plane OXY.
- Drillhole data compositing. In the proposed method case this is not needed since FF and Wht are evaluated per meter of core.
- Statistical analyses of raw and composited data.
- Construction of experimental variograms.
- Search for any possible anisotropy of experimental data by variography analysis.
- Best fitting of variogram models on the experimental data and estimation of the anisotropy ellipsoid axes.
- Attribute estimation, i.e., Wht % and FF m<sup>-1</sup> for each block using the Block Kriging technique [2, 3].

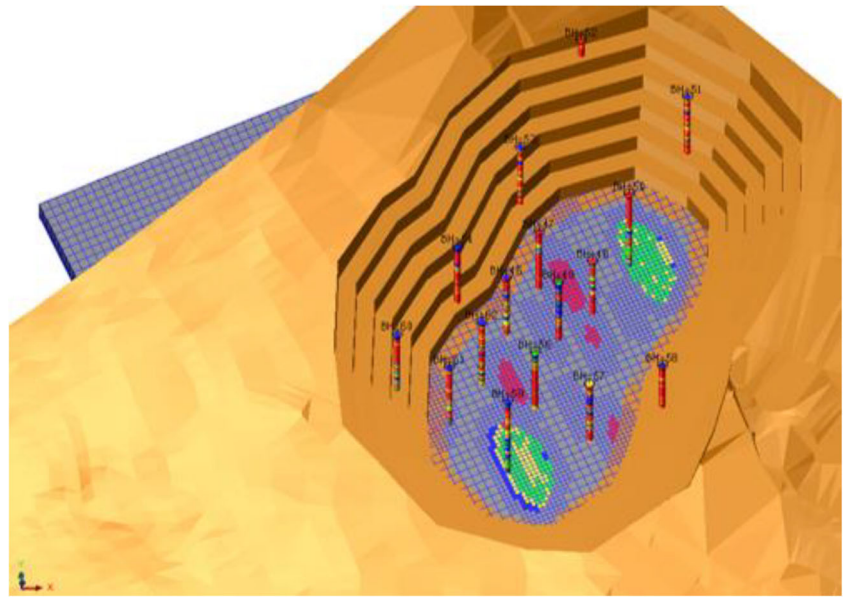
Taking into account all the factors presented above, a BM (Fig. 8) of the study area was constructed. The edges of the top and bottom faces of the blocks are oriented parallel to the planned directions of the quarry benches. For maximum marble block recovery, benches and associated block model edges are oriented along the strike of the head grains that exhibit the largest FF m<sup>-1</sup> values. The dimensions of the blocks in the model considered here were chosen to be 6 m in height × 3 m in depth × 3 m in width, which comply with current quarry panel sizes. In this case, the volume of each model block is 54 m<sup>3</sup>. All the geometrical parameters of this BM are presented in Table 2.

**Fig. 12** Isometric view of the measured and indicated marble resources with FF < 6 m<sup>-1</sup> and Wht > 70%





**Fig. 13** Top view of the final open pit quarry trough for marble reserves estimation



Drillhole data of  $FF\ m^{-1}$  and  $Wht\ \%$  per meter of borehole were used for the geostatistical analysis. Scatterplots for possible correlations among  $FF$  and  $Wht$  and frequency histograms of the  $Wht$  and  $FF$  were created. Geostatistical analysis was done by constructing:

- (i) One omnidirectional downhole semivariogram for  $Wht\ \%$  and  $FF\ m^{-1}$  (Fig. 8a and b) and
- (ii) Six directional semivariograms (for every 30 degrees), especially for the whiteness in layered marble deposits. It is expected that the range of the  $Wht\ \%$  semivariograms along the dip and dip-direction of grain planes will be much greater than that along the direction normal to the former two [17].

Due to the lack of data in this case study, the directional semivariograms could not produce reliable results. In this case, the  $Wht\ \%$  anisotropy is modeled by recourse from secondary information from geological evidence. The length of the three axes of the ellipsoid of anisotropy was chosen using a

combination of the orientation in space of the marble bedding planes and the omnidirectional (maximum) semivariogram range (Fig. 9a and b). The ratios of the three axes were major/minor = 4 and major/semi-major = 1. The rotation of the major axis of the ellipsoid had an azimuth of 330 degrees, while the semi-major axis dipped to  $-40$  degrees, that is, the dip angle of the bedding or verso joint planes. In this fashion the ellipsoid was aligned with the dip direction and dip of the schistosity or verso discontinuity planes, as shown in the stereographic projection in Fig. 3. The same anisotropy parameters were chosen for the  $FF\ m^{-1}$ . Finally, as listed above, spatial estimations were performed with the Block Kriging technique to assign the interpolated value of  $FF\ m^{-1}$  or  $Wht\ \%$  to the centroid of each block. The final model is enclosed according to the DTM on the top, lateral, and bottom faces of the model with the elementary six-sided orthogonal prismatic blocks, as shown in Fig. 10, for the present case study.

A quick method to classify marble ore resources as measured, indicated, and inferred may be based on the comparing the distance of the model blocks from the

**Table 3** Estimated white marble proved reserves with  $FF < 6\ m^{-1}$ ,  $Wht > 50\%$  and  $\alpha \leq 0.2$

Bench no.	Bench floor level (absolute elevation in m)	Proved marble reserves [ $m^3$ ]	Total excavation [ $m^3$ ]
8	856	864	407,268
7	874	21,762	547,776
6	892	34,128	705,294
5	910	51,570	878,580
4	928	80,298	1,069,146
3	946	54,972	1,276,722
2	964	54,972	1,051,218
1	982	540	163,404
Total		300,000	6,100,000

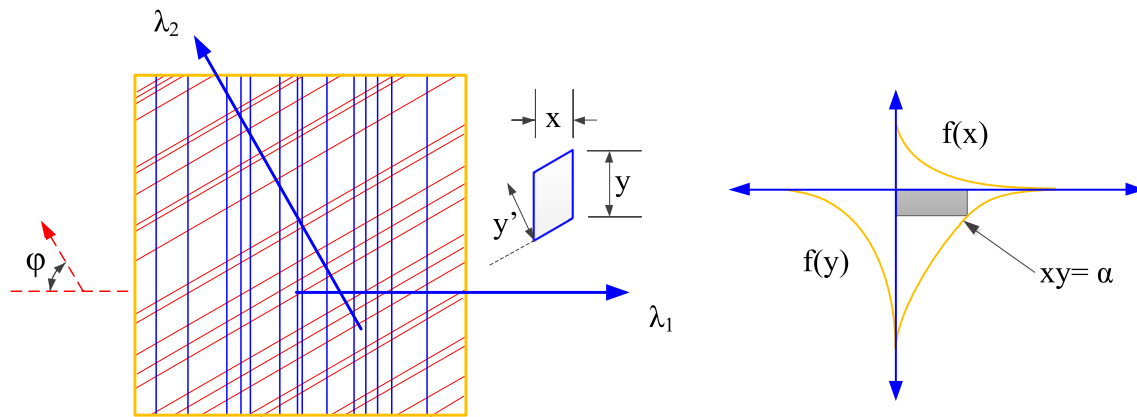


Fig. 14 Block area distribution in a plane with two intersecting discontinuity sets

neighboring boreholes and the range of the corresponding semivariogram. The motivation behind this classification is based on risk. For example, a common goal is to somehow classify parts of a resource model as measured, indicated, and inferred according to the risk or uncertainty associated with the estimated marble block sizes and coloration. Although motivated by risk, such a rule does not quantify it. Alternatively, the estimated value of  $FF\ m^{-1}$  or  $Wht\ \%$  with the Kriging technique, denoted with the symbol  $Z^*$ , and of the associated Kriging variance  $\sigma_{OK}^2$  at the centroid of each block in the model, may be used to classify resources. Considering a Gaussian-type confidence interval centered on the estimated value  $Z^*$ , then each block will be characterized by a *normalized standard deviation (NSD)*, i.e.,  $a = \sqrt{\sigma_{OK}^2}/Z^*$ . Blocks with a value of  $a$  below 0.2 are classified as Measured, because at a confidence level (CL) of 90%, we know that the true value lies in the open interval  $[Z^* - 1.6aZ^*, Z^* + 1.6aZ^*]$  or  $[Z^* - 0.3Z^*, Z^* + 0.3Z^*]$ . In other words, the estimated value of the attribute in the block will range plus/minus 30% with 90% confidence. If the value of parameter  $a$  is increased or equivalently the tolerance limit is increased, then the number of estimated blocks in the model is increased and consequently the resources, but the risk of not reaching the aimed yearly production of marketable marble blocks increases, too. According to our experience, a limit of  $\alpha \leq 0.2$  for both regional variables  $FF\ m^{-1}$  and  $Wht\ \%$  is adequate to estimate Measured white marble resources. Model blocks with a value of  $\alpha > 0.2$  and  $< 0.3$  are characterized as Indicated marble resources. In the case study under examination, most of the blocks fall outside the class of Measured resources, as seen in the histogram in Fig. 11. This is because the drilling campaign is at its initial stage. To increase the volume of measured resources, the grid of boreholes should be more dense in a second exploratory drilling campaign.

Figure 12 displays in an isometric view the most valuable or marketable model blocks of the measured and indicated

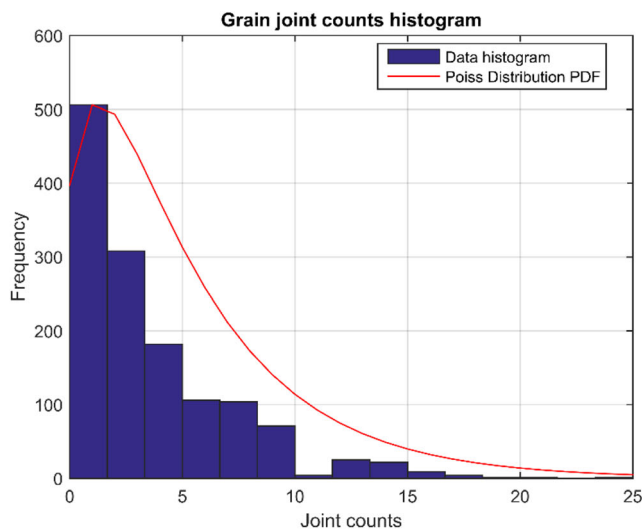
resources categories having  $FF\ m^{-1} < 6\ m^{-1}$  and  $Wht\ \% \geq 70\%$  (i.e. white-gray to white).

The basic steps described above for the creation of the marble ore resource model are indispensable for the best design of economical open pit excavation limits based on appropriate economical and other criteria.

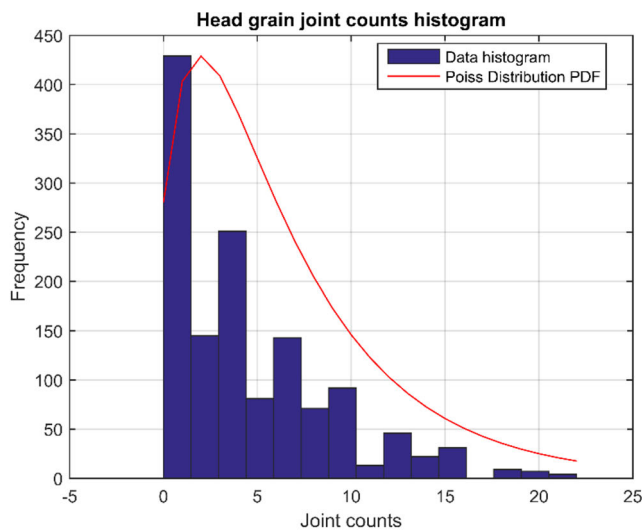
### 3 Estimation of the Marble Reserves

In order to proceed with the estimation of the Proved marble reserves from the Measured resources, we initially prescribe the final limits of the potential quarry excavation. This is the principal modifying factor (MF) of this transformation process. In the case of a deposit with flat surface topography, these limits refer to the area of the quarry at the surface (i.e., the DTM), the number of the final benches, and the elevation of the lower level that is approximately the elevation of the bottom of the boreholes. These limits circumscribe all the exploratory boreholes at this preliminary stage of studying the orebody. At a second stage, with more measured resources with acceptably low prediction risk, more sophisticated criteria may be employed.

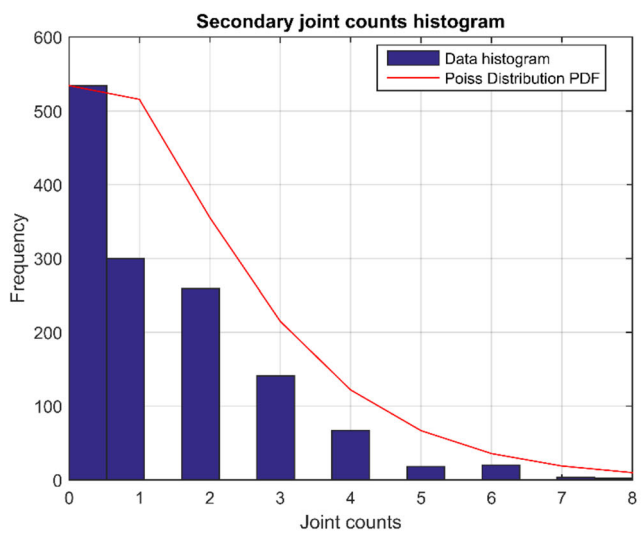
The DTM of the final marble quarry configuration is shown in Fig. 13. The final open pit quarry “trough” enclosing all the boreholes has eight benches of 18 m height (berm interval or bench height). This means that at the final quarry production stage, we assume that three benches of 6 m height each will merge into a single one with a height of 18 m. The final berm widths at each bench are equal to 10 m. The total volume enclosed by the open pit trough is calculated to be  $6,100,000\ m^3$ . The total volume of Proved reserves for an  $NSD\ \alpha \leq 0.2$  were estimated to be  $950,000\ m^3$  (round number). An additional marble volume of  $1,470,000\ m^3$  was classified as Probable reserves. However, considering the additional MFs of joints (FF) and color (Wht), the actual Proved reserves are much less than the estimation above.



(a)



(b)



(c)

◀ **Fig. 15** Best-fitted NHPP models (continuous curves) on  $FF' m^{-1}$  data (in the form of a histogram) per joint set; **a** grain or verso, **b** head-grain or contro, and **c** secondary or secondo

Table 3 shows the distribution per bench of these proved reserves characterized by  $FF < 6 m^{-1}$  (above that value the RR is almost zero) and by  $Wh > 50\%$  (below this value the marbles are not marketable). Thus, by excluding these classes of marble reserves the final proved reserves were estimated to be  $300,000 m^3$  (round number). Table 3 also shows the volume of total excavation per bench.

### 4 Estimation of Marble Block Volume Distribution

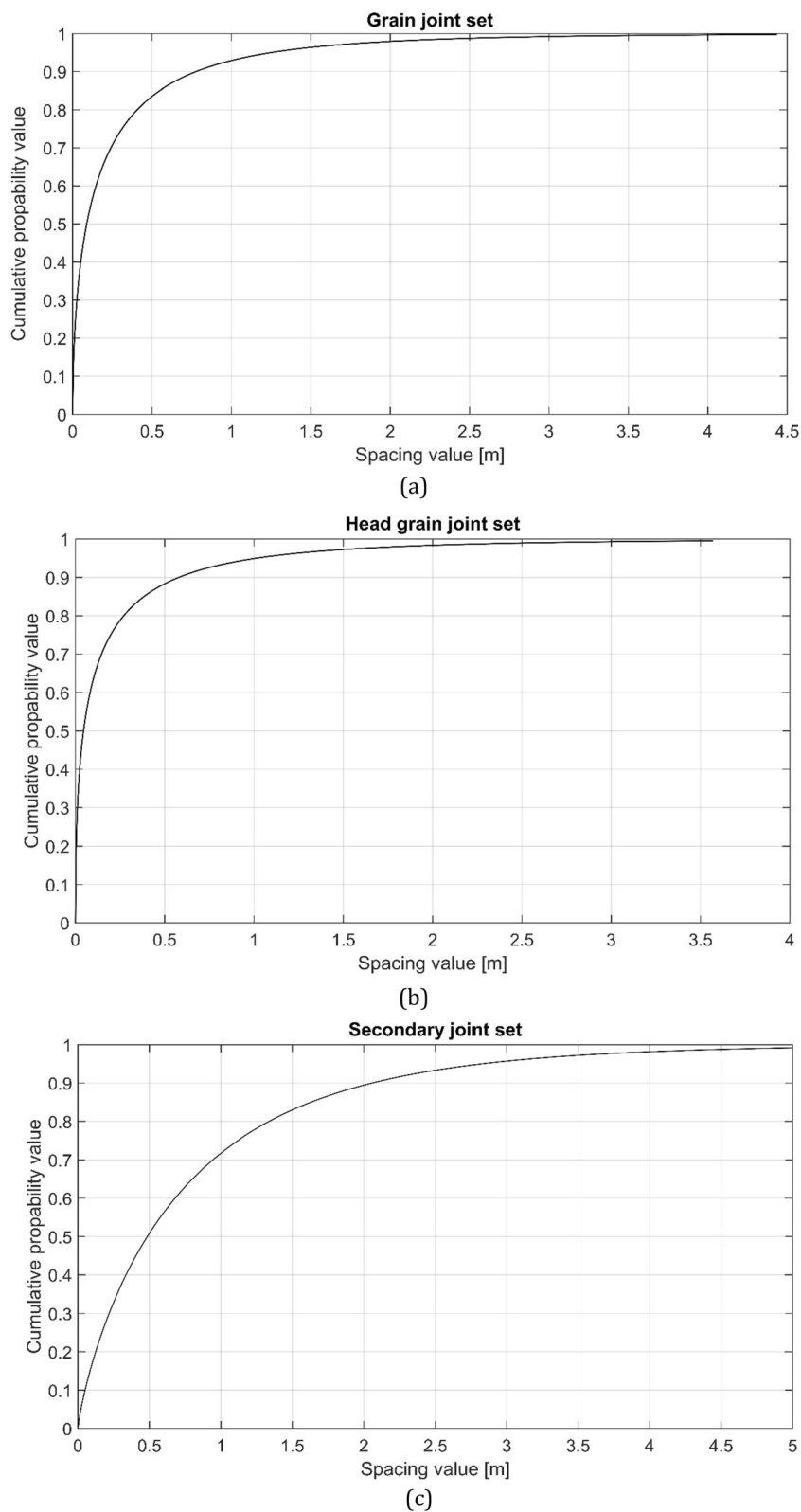
The  $FF m^{-1}$  factor was considered as an MF for the evaluation of proved marble reserves by eliminating those model blocks having  $FF > 6 m^{-1}$ . The next MF may be only obtained from the volume proportion of marble blocks separated by the joints. This MF pertains to the total volume of marble blocks that have a commercial or marketable size greater than  $2.5 m^3$ . Those marble blocks having a volume lower than this threshold volume are not marketable. The distribution of marble block volumes with commercial sizes from the classes of  $FF < 6 m^{-1}$  may be predicted from the statistical distribution of joint spacings, joint orientations, and joint sizes of each principal joint set. Joint orientations can be measured either using the compass and from oriented drill cores or from images of oriented borehole walls, as described in Section 2. Joint spacings distribution for each set can be accomplished from  $FF' m^{-1}$  measurements per joint set separately. In the first stage,  $FF' m^{-1}$  histograms for each set are constructed and then fitted with an appropriate pdf. In the second step, the joint spacing distribution for each joint set is derived analytically directly from the fracture counts  $FF' m^{-1}$  along the drill cores. The block area distribution is then constructed by a Monte Carlo simulation method from the pdf's  $f(x)$  and  $f(y)$  of spacing values on a plane, as shown in Fig. 14. The same procedure could be easily generalized in three dimensions ( $x y z$ ) to infer the block volume size distribution.

Specifically, it may be shown that  $FF' m^{-1}$  counts along drill cores follow the non-homogeneous Poisson process (NHPP) pdf, namely

$$P\left(\frac{f_\lambda \text{ events}}{\text{in interval } x}\right) = e^{-\lambda x} \frac{(\lambda x)^{f_\lambda}}{f_\lambda!} \tag{1}$$

where  $f_\lambda$  represents  $FF' m^{-1}$ ,  $x$  is the sampling length (or sample support), and  $\lambda$  denotes the linear frequency parameter of the negative exponential pdf with units of  $m^{-1}$ . In the frame of

**Fig. 16** Cumulative distributions of spacings per joint set; **a** grain or verso, **b** head-grain or contro, and **c** secondary or secondo

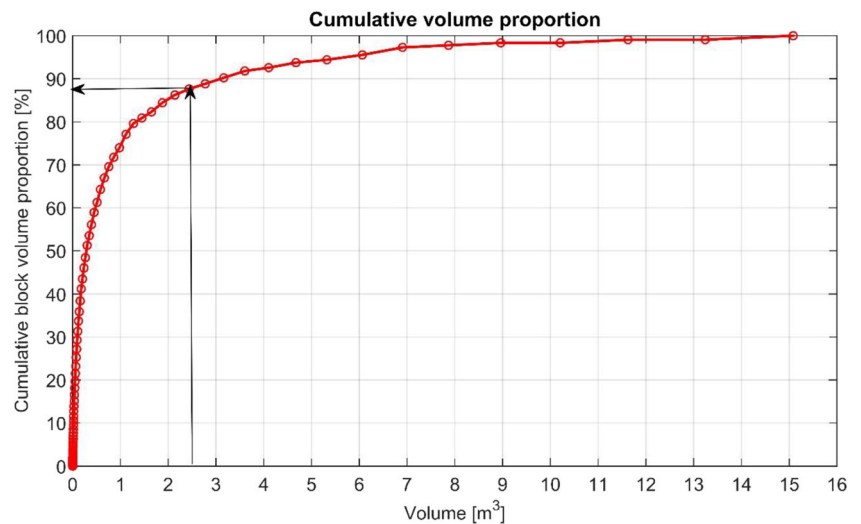


an NHPP, the *linear fracture frequency parameter*  $\lambda$  is not constant as occurs in the classical Poisson *pdf* given by Eq. (1), but also depends on the  $f_\lambda$  itself, as well as on the sampling interval  $x$ , in the following fashion [18].

$$\lambda = \lambda_0 + a_0 f_\lambda \tag{2}$$

The parameters  $\lambda_0$ ,  $a_0$  are not constant but depend on sampling length interval  $x$  with power relations [18]. Figure 15a–c

**Fig. 17** Cumulative block volume proportion curve for the case study at hand



displays the best-fitted NHPP pdf's on the statistical histograms of  $FF' m^{-1}$  per joint set separately.

The cumulative distributions of joint spacing of each joint set are then derived from the corresponding best-fitted pdf's of  $FF' m^{-1}$  by virtue of a semi-analytical method described in [18]. The fracture sizes have been also taken into account as described in [18]. The joint spacing cumulative distributions of the three joint sets are displayed in Fig. 16a–c.

In the final (third) step of the methodology, the cumulative volume proportion (CVP) distribution curve of the block volumes is produced by the Monte Carlo simulation technique. This distribution shows the volume percentage of marble block volumes of given size out of the total volume of marble contained within the volume space enclosed by the boreholes. It is derived from the cumulative distribution functions of the inferred joint spacing distributions of each of the three sets as previously mentioned. Figure 17 illustrates the marble block volume proportion curve produced from the simulation of 100,000 synthetic block volumes in the Monte Carlo simulation. It may be observed that 12% of six-sided blocks formed by the three mutually intersecting joint sets have marketable volumes above  $2.5 m^3$  or 7 t. In addition, the maximum natural block size that can be produced from the quarry is  $14 m^3$ . Finally, the Proved white marble reserves are those obeying the final criterion or MF of marketable size or equivalently 12% of  $300,000 m^3$  ( $36,000 m^3$ ).

Recapitulating as was shown above the proved reserves (PR) may be expressed with the following formula

$$PR = MR - MF_1 - MF_2 - MF_3 - MF_4 - MF_5 \tag{3}$$

where

- MR = Measured resources volume
- $MF_1$  = Modifying factor 1 accounting for the final open pit configuration.

- $MF_2$  = Modifying factor 2 representing the total volume of model blocks with  $FF > 6 m^{-1}$ .
- $MF_3$  = Modifying factor 3 corresponding to the total volume of model blocks with  $Wht < 50\%$ .
- $MF_4$  = Modifying factor 4 representing the total volume of marble blocks having volumes lower than a threshold volume of  $2.5 m^3$ .

### 5 Concluding Remarks

This paper outlined the basic steps of a methodology for the estimation of white marble resources and reserves in surface excavations. The two regional variables measured per meter of drill cores are the *total*  $FF m^{-1}$ , which is related to the volumes of intact marble blocks, and the *Wht %*, which defines the economic value of the commercial blocks. Similarly to the *FF* variable, the *Wht %* of marble was also regarded as a continuous random variable taking values in the range 0–100 (black to snow white) along the drill cores.

The mean orientations of the main joint sets transecting the marble deposit were found from stereographic analysis of joint orientation data. Orientation analysis is important for: (a) the subsequent exploration of possible anisotropy of the range of *Wht %* with directional variograms and (b) orientation of the edges of the top and bottom faces of the model blocks and of the quarry benches. This is the reason for the application of oriented drill coring or image analysis of borehole walls with televiewer techniques. The model blocks were then assigned the attribute values of *Wht %* and  $FF m^{-1}$  with the Block Kriging method taking into account the best-fitted theoretical semivariograms and geological evidence. The Measured resources are those with an NSD of block estimations of *Wht* or  $FF \leq 0.2$ . These resources are then transformed to the Proved

reserves by considering the following modifying factors, namely: (a) final open pit layout, (b) lower threshold of whiteness, (c) upper limit of fracture frequency, and (d) the natural block size distribution of the marble deposit. The volume distribution of the natural marble blocks isolated between mutually intersecting joints was found using  $FF' m^{-1}$  measurements of each joint set separately along the drill cores, a newly proposed generic NHPP pdf model of fracture counts along drill cores, and the Monte Carlo simulation technique. Additional MF's refer to (1) the "edge effect" of the panels caused by the horizontal and vertical diamond wire cuts on the lower horizontal face, as well as on the vertical side and back faces of each panel (Fig. 1c) that produces tetrahedral prismatic blocks by splitting originally hexahedral parallelepiped blocks, and (2) red or yellow "patches" due to ferroxides or other minerals in white marbles. The former effect or MF may be reduced at a certain degree by increasing the height and width of the panels (i.e. from 6 m to 8 m bench heights of the quarry and from 2 m to 3 m width of the overturned panels). This increase of panel dimensions requires more powerful equipment (excavators or loaders) in conjunction with powerful panel pushing devices for the overturning process of the panels.

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### Compliance with Ethical Standards

**Conflict of Interest** The authors declare that they have no conflict of interest.

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