Short Communication

A space-fractional Pennes bioheat conduction model for skin tissue

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Abstract
This study attempts to develop a modified Pennes bioheat conduction model for skin tissue, by using space-fractional derivative. The developed fractional model is applied to capture the thermal behaviour of the skin tissue subjected to heating and cooling procedures. It is found that with the increase of the fractional order, the predicted temperature of the skin tissue at the heating stage increases, while it decreases at the cooling stage. Irrespective of the value of the fractional order, the simulated temperature along the skin tissue at both heating and cooling stages became stable after several seconds thermal conduction. The fractional model reduces to the original Pennes model when the fractional order equals to two.

Keywords Fractional derivative · Bioheat conduction · Finite difference method

1 Introduction

Bioheat exchange between the human body and the surrounding environment has drawn attention from researchers for a long time. Skin tissue, as the largest living organ covering human body, can be damaged when subjected to rapid increase or decrease of environmental temperature, for example, the accidental exposure to boiling hot water. According to the report of the World Health Organisation, millions of people got injured by thermal flux every year. Therefore, there is an urgent need to understand and evaluate the thermal behaviour occurred within human tissues, for example, the skin tissue in this study. To quantify the temperature variation of skin tissue, Pennes [1] firstly developed a bioheat transfer equation, by assuming a Fourier law for heat conduction. However, It was reported that non-Fourier heat conduction behaviour was observed [2] during experiment, which indicated that a bioheat conduction model with non-Fourier kernel could be better for representing the bioheat conduction of skin tissue. To capture such non-Fourier thermal behaviour, a number of different models have been proposed, e.g., the dual-phase-lag model [3, 4], hyperbolic single-phase-lag model [5] and thermal wave model [6]. Even though a good model performance was usually reported, the development of a more concise and alternative method is still of interest for researchers.

Recently, the application of fractional calculus in solving real-world problems, for example, the creep and relaxation of solid materials [7], anomalous diffusion [8] and heat transfer [9] of fluid, have drawn increasing attention from worldwide researchers [10, 11]. By considering the delayed responses of the heat flux vector, different time-fractional bioheat conduction equations have been proposed [12–14], for example, the space–time-nonlocal generalization of the Fourier law and the space–time-fractional heat conduction equation [15, 16], and the space–time fractional bioheat equation governing the process of heat transfer in tissues during thermal therapy [17, 18]. In these models, the effect of fractional order on simulating the thermal behaviour of skin tissues have been investigated, it was found that the time-fractional bioheat model could be more robust and flexible than the classical bioheat equation. However, due to the porous characteristics of

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the skin tissue, the temperature gradient within the skin tissue could be more efficiently described via fractional order derivatives instead of the integer-order derivatives [19].

Therefore, an attempt is made in this study to evaluate the thermal behaviour of the skin tissue, based on a space-fractional Pennes bioheat conduction equation. The study is structured as follows: Sect. 2 generalises the original Pennes bioheat equation by incorporating space-fractional non-Fourier law; Sect. 3 provides a finite difference computational method for solving the fractional Pennes bioheat equation, where a numerical analysis of the skin tissue subjected to boiling water will be carried out. Section 4 summarises the study.

2 Space-fractional bioheat conduction model

Pennes [1] assumed that the bioheat transfer rate within skin tissue (q_t) was determined by three influencing factors, i.e., heat conduction rate (q_c), blood perfusion (q_b), and metabolic heat generation (q_m), such that

$$q_t = q_c + q_b + q_m \tag{1}$$

where q_t and q_b can be expressed as:

$$q_t = \rho c \frac{\partial T(x,t)}{\partial t} \tag{2}$$

$$q_b = \rho_b c_b w_b [T_b(x,t) - T(x,t)] \tag{3}$$

where ρ, c, T, x and t are the density, specific heat, temperature, distance and time of the skin tissue, respectively. ρ_b, c_b, w_b and T_b are the density, specific heat, blood perfusion rate and artillery temperature of the blood. Instead of using the Fourier law to represent q_c, the space-fractional non-Fourier law is employed in this study as:

$$q_c = -\frac{\partial q}{\partial x} = -\frac{\partial}{\partial x}\left[-k\frac{\partial^{\alpha-1}T(x,t)}{\partial x^{\alpha-1}}\right] \tag{4}$$

where $q = -k\frac{\partial^{\alpha-1}T(x,t)}{\partial x^{\alpha-1}}$, is the heat flux; k is the thermal conductivity of the skin tissue, which has a physical dimension of $\frac{W}{m^{3-\alpha}\,^{\circ}C}$; $\alpha \in (1,2]$, is the fractional-derivative order. Therefore, combining Eqs. (1)–(4), one has the following space-fractional Pennes bioheat equation:

$$\rho c \frac{\partial T(x,t)}{\partial t} = k\frac{\partial^{\alpha} T(x,t)}{\partial x^{\alpha}} + \rho_b c_b w_b \left[T_b(x,t) - T(x,t)\right] + q_m \tag{5}$$

It can be found that Eq. (1) reduces to the original Pennes bioheat equation when $\alpha = 2$. To facilitate numerical analysis, the Gruwald-Letnikov fractional derivative is suggested, such that

$${}_aD_x^{\alpha}T(x) = \frac{d^{\alpha}T(x)}{dx^{\alpha}} = \lim_{\substack{h\to 0 \\ x-a=nh}} h^{-\alpha}\sum_{r=0}^{n}(-1)^r\begin{bmatrix}\alpha \\ r\end{bmatrix}T(x-rh) \tag{6}$$

where a indicates the initial location, and

$$\begin{bmatrix}\alpha \\ r\end{bmatrix} = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-r+1)}{r!} \tag{7}$$

3 Computational analysis

The space-fractional derivative can be computed by using modified forward difference method [20], such that:

$$\frac{\partial^{\alpha}T(x,t)}{\partial x^{\alpha}} = \frac{1}{\Gamma(-\alpha)}\lim_{i\to\infty}\frac{1}{(\Delta x)^{\alpha}}\sum_{n=0}^{i+1}\frac{\Gamma(i-\alpha)}{\Gamma(i+1)}T(x-(n-1)\Delta x, t) \tag{8}$$

where Δx is the size of the discretized space step. $i = 1, 2, 3, \ldots, n-1$; $n = L/\Delta x$. L indicates the thickness of the skin tissue. Then, Eq. (1) can be re-written as:

$$\rho c\frac{T_i^{j+1} - T_i^j}{\Delta t} = \frac{k}{\Gamma(-\alpha)(\Delta x)^{\alpha}}\sum_{\xi=0}^{i+1}\frac{\Gamma(\xi-\alpha)}{\Gamma(\xi+1)}T_{i-\xi+1}^{j+1} + \rho_b c_b w_b[T_b - T_i^j] + q_m \tag{9}$$

where Δt is the size of the discretized time step. $j = 1, 2, 3, \ldots, m-1$; $m = h/\Delta t$. h indicates the total time for bioheat transfer. Furthermore, we define the following functions:

$$g_{\alpha,\xi} = \frac{\Gamma(\xi-\alpha)}{\Gamma(-\alpha)\Gamma(\xi+1)} \tag{10}$$

$$g_{\alpha,\xi} = g_{\alpha,\xi-1}\frac{\xi-1-\alpha}{\xi}, (\xi = 2, 3, ..., n) \tag{11}$$

$$\delta_{\alpha,x}T_i^j = \frac{1}{(\Delta x)^{\alpha}}\sum_{\xi=0}^{i+1}g_{\alpha,\xi}T_{i-\xi+1}^j \tag{12}$$

Then, substituting Eqs. (5)–(7) into Eq. (4), one has:

$$(1 - \vartheta\delta_{\alpha,x})T_i^{j+1} = \xi T_i^j + \omega \tag{13}$$

where $\vartheta = \frac{\Delta t k}{\rho c}$, $\xi = 1 - \frac{\rho_b c_b w_b \Delta t}{\rho c}$, $\omega = \frac{\Delta t(\rho_b c_b w_b T_b + q_m)}{\rho c}$.

In this study, following boundary condition is considered:

Table 1 Model parameters

ρ (10^3 kg/m^3)	c (10^3 J/kg/°C)	ρ_b (10^3 kg/m^3)	c_b (10^3 J/kg/°C)	w_b (/s)	k (W/m$^{3-\alpha}$/°C)	q_m (W/m^2)	L (10^{-3} m)
1.06	2.846	1.0	3.86	0.1	0.21	368.1	0.3

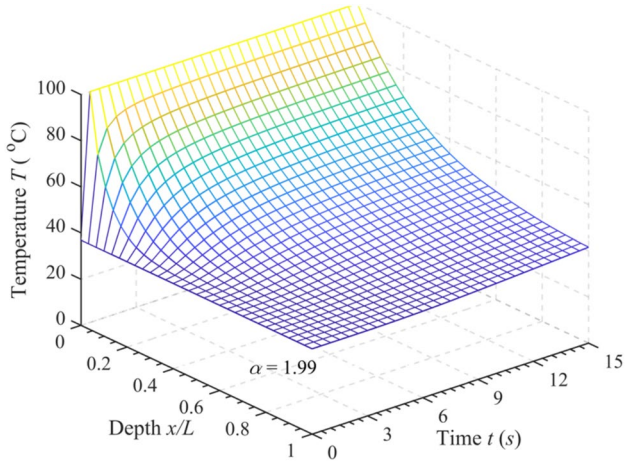

Fig. 1 Temperature variation in skin tissue

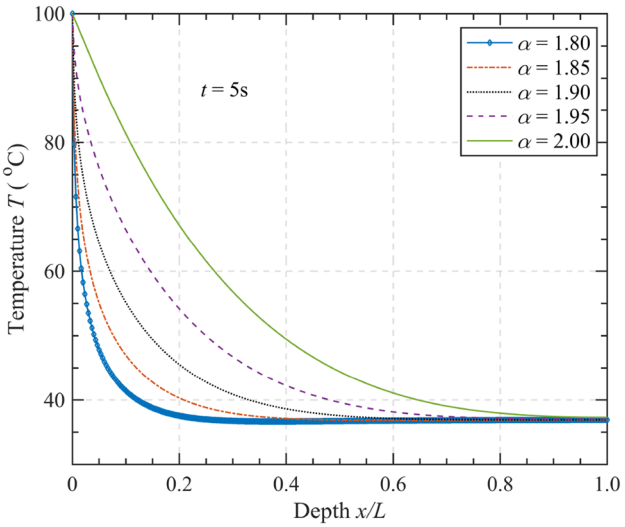

Fig. 2 Effect of fractional order on the temperature variation in skin tissue at $t = 5$ s

$$T(x,0) = T_{37}, \quad T(0,t) = T_{100}, \quad T_b = T_{37} \tag{14}$$

where $T_{37} = 37$ °C, $T_{100} = 100$ °C. This boundary condition is intended to analyse the thermal behaviour of skin tissue (37 °C) that is the accidentally exposed to boiling water (100 °C) for 5 s. Then, a cooling procedure is also carried out by immersion into 0 °C water, where $T(0,t) = T_0$ and $T_0 = 0$ °C. The values of each model parameter are listed in Table 1. Note that much more generalized boundary conditions on the bioheat conduction of skin tissue, as reported in [21], can be encountered, which needs further investigation.

4 Results and discussion

The numerical results of the developed space-fractional bioheat conduction model are provided and discussed in this section.

Figure 1 provides the simulated variation of the temperature of skin tissue with time. It can be found that at the initial time ($t = 0$–1 s), the temperature of the skin surface increases drastically to 100 °C; however, the temperature of the internal skin tissue exhibits a general increase with time, indicating the time effect for bioheat conduction. For the skin tissue far away from the skin surface, the temperature remains the same as the body temperature, due to efficient blood perfusion.

Figure 2 presents the effect of fractional order on the temperature variation along the skin tissue at $t = 5$ s. It can be found that with the increase of fractional order, the predicted temperature of the skin tissue at the same depth increases. $\alpha = 2$, indicating the model prediction by the original Pennes bioheat condition equation, which means that the original Pennes bioheat conduction model predicts the highest temperature at the same boundary condition. Furthermore, to show the effect of fractional order on simulating the temperature variation with time, Fig. 3 provides the typical results of the skin tissue at $x = L/6$. It can be found that the simulated temperature increases with time. As the fractional order increases, the predicted temperature at the same time increases.

Figure 4 shows the simulated cooling procedure of the skin tissue by immersing into 0 °C water immediately after exposure to boiling water ($t = 5$ s). It can be found that the temperature of the skin surface drops down to 0 °C, while the rest internal skin tissue still experiences relatively high temperature before sufficient bioheat transfer. After 5 s cooling, the temperature along the skin tissue becomes stable.

To investigate the effect of fractional order on the simulated heating and cooling procedures, Fig. 5 shows the typical results of the skin tissue at $x = L/10$. It can be found that, the temperature increases drastically at the heating stage, with the majority value of the temperature increase

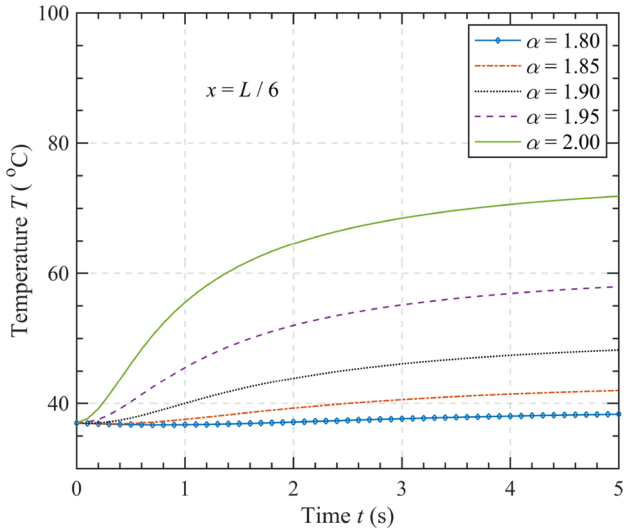

Fig. 3 Variation of the temperature of skin tissue with time at $x=L/6$

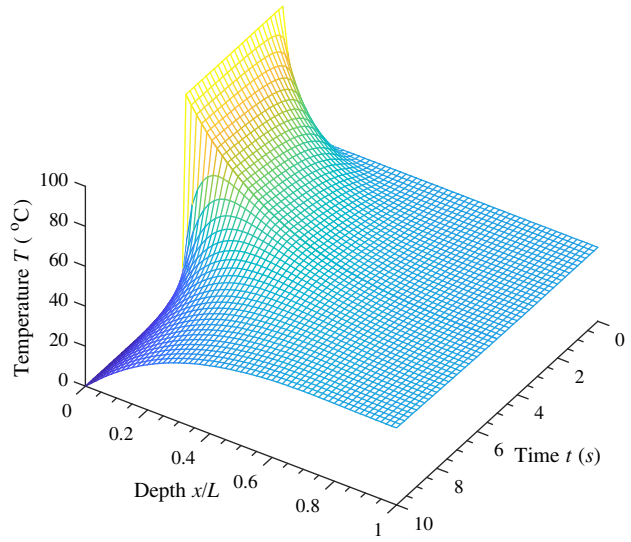

Fig. 4 Cooling of skin tissue after exposure to boiling water

completing at $t<1$ s. Then, the temperature near the skin surface approaches stable. With the increase of fractional order, the simulated temperature of the skin tissue drops faster, with most of the temperature reduction completing within one second. It indicates that the original Pennes bioheat conduction model predicts the most efficient heat transfer within the skin tissue. The space-fractional Pennes bioheat conduction model may be used, when a tissue requires longer thermal relaxation.

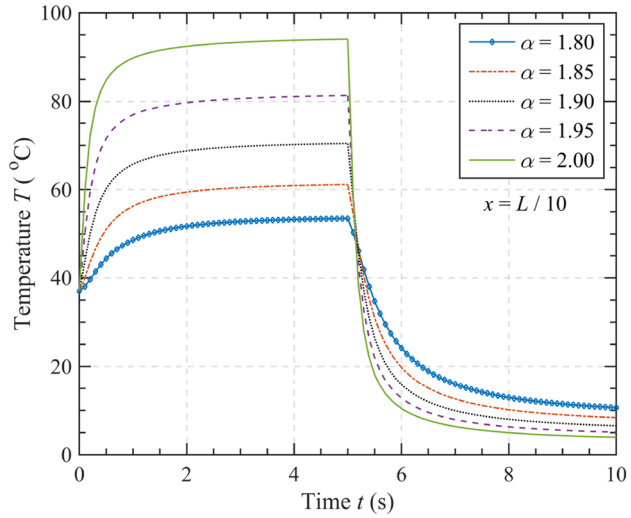

Fig. 5 Heating and cooling of skin tissue at $x=L/10$

5 Conclusions

Bioheat conduction within skin tissue is an important research topic, especially in the field of hyperthermia. Based on the hypothesis that heat conduction within skin tissue obeyed the space-fractional non-Fourier law, this study developed a space-fractional bioheat conduction model for the skin tissue. Then, a numerical application of the fractional model for simulating heating and cooling of the skin tissue was carried out. It was found that with the increase of fractional order, the predicted temperature of the skin tissue at the heating stage increased; however, it decreased at the cooling stage. When the fractional order increased to 2, the fractional model reduced to the original Pennes model. Irrespective of the value of fractional order, the simulated temperature along the skin tissue at both heating and cooling stages became stable after five seconds thermal conduction. The original Pennes model predicted the most efficient heat transfer process. However, the space-fractional Pennes bioheat conduction model could be used, when a tissue experienced longer thermal relaxation. This study only evaluated the developed bioheat conduction model at one boundary condition, a more generalised boundary condition should be investigated for future application.

Authors' contribution Jiangong Yang: software, validation, formal analysis, writing—original draft preparation. Yifei Sun: conceptualization, methodology, writing—review and editing, supervision. All authors have read and agreed to the published version of the manuscript.

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Data Availability All the data used can be provided by the corresponding author upon proper request.

Compliance with ethical standards

Conflicts of interest The authors declare that there is no conflict of interest.

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