Research Article

Impact of dummy variables in a probabilistic competitive environment

M. N. Srinivas[1] · C. Sreerag[2] · A. V. S. N. Murty[1]

Abstract
Most of the competitive environments are highly dynamic in nature and also fragile. The active and inactive participation of competitors may prompt an improper decision making. The article focuses on the behaviour and impact of dummy (inactive) competitors in the system. The study is to generate a maxima or minima in a time-probability distribution of all competitors in terms of inactive competitors. The inefficiency of the discrete ranking model without frequency to generate a maxima or minima in the corresponding position-probability distribution and also the capability of continuous infinite model with frequency for the same purpose conferred. The necessary conditions on behalf of with and without dummy variable to create a peak or dip in the position- probability distribution through one tail and two tail addition in the competitor frequency distribution curve is additionally contemplated. The cutting edge e-SCM scenario comprising of fake rating and reviewing is regularly critical in e-Marketing. The summed up model examined here along these lines causes us to anticipate the probabilities for selling and the related risk factor required by thinking about the number of fake rating as a spurious variable in the framework. A case study has been incorporated to study the impact of dummy variable in the system. The model will definitely find application in decision making and strategic formation process in the various fields of quality and inspection engineering, economics and social sciences etc.

Keywords Competition · Quality control · Fake rating · Dummy variable · e-SCM

1 Introduction

Competition exists between companies selling similar products and services with the goal of achieving profit and market share growth. Market competition stimulates companies to increase sales volume by developing the four components of the marketing mix. These Pillars stand for product, place, promotion, and price fixing. Proper and more accurate awareness and understanding of the competition is often crucial in designing a successful marketing strategy for stability and sustainability. Unless you know the strengths and weaknesses of the other competitors, it will promote other companies to come up in the forefront by taking an advantage. Therefore staying informed about products and services in the entire competition scenario is vital to the survival and sustainability of any business activity.

The present article while studying the competition environment as a whole in detail tries to explore the possibilities of a non-conventional probabilistic approach for strategic formation and decision making. The model described here is applicable not only in the industrial area but also other areas of research as well. The utilization of the inactive competitors which may influence the success of the other active competitors is made use of.

1.1 Economics background

Since the beginning of probability theory, there has been a refinement between probabilities that are given, as in

✉ M. N. Srinivas, mnsrinivaselr@gmail.com; C. Sreerag, sreeragc1347@gmail.com; A. V. S. N. Murty, avsnmurthy2005@gmail.com | [1]Department of Mathematics, School of Advanced Sciences, VIT, Vellore, Tamilnadu, India. [2]Department of Manufacturing, School of Mechanical Engineering, VIT, Vellore, Tamilnadu, India.

SN Applied Sciences (2019) 1:1115 | https://doi.org/10.1007/s42452-019-1121-0

Received: 10 May 2019 / Accepted: 19 August 2019 / Published online: 28 August 2019

a round of shot, and probabilities that are not given, however mirror an abstract level of conviction; Hacking [1] and Shafer [2] offer verifiable studies. The possibility of Bayesian approach or the most generally using techniques has its own limitation to practically impose in a complicated environment having some uncertainty. The result of a hurl of the main coin will be doled out a 50–50 dispersion because of limited proofs. The result of a hurl of the second coin will be allocated a similar dispersion as per Laplace's principle of indifference. However, Schmeidler [3] contends, the two conveyances feel unique what's more, subsequently; our eagerness to wager on them need not be the equivalent. The statement referred above unwrap the necessity and importance of other probabilistic approach in practical situation. The article "Probability and Uncertainty in Economic Modelling" by Gilboa et al. [4] while discussing the restriction and limitations of Bayesian approach put forward the possibility of other models in economics. There are some research articles that point out different modelling for a specific purpose but none of them approached the problem in a generalized way like "Incomes probabilistic model of the banking network" by Matalycki and Pankow [5]. This work is an investigation on the probabilistic model of wages change in the financial system. The Markov queuing system used as model in this article. Another article by Subbaiah et al. [6] proposed an inventory model for a demand rate depended on stock level, the model focussing the internal factors that can optimize and bring success to the firm without considering the external competitors. At the same time, the development of modern and managerial economics leads to the evolution of some inspired ideas and more number of technical terminologies in the dictionary of science. Scala's [7] explained the different terminologies and the fundamentals of probabilistic economics which are very relevant for developing a general novel model to the dilemma. The references are also extended to different authors [8–10] who studied economical oriented topics like random variable transformations, probabilistic distributions in order to categorize the different possibilities of probabilistic economics. One of the recent research studies of Lesmono and Limansyah [11] built up a scientific model for an inventory problem which comprises of numerous items. Requests for these items are following Gamma distribution. References [5, 6, 11] are independently efficient and reliable in their own field. However, there is nothing in common in these models even though they are fundamentally related in terms of sustainability and competition. The competition and struggle for holding a proper position is a reasonable starve for any industry and it is important to consider this competition probability in account. The proposed generalized model solves this problem as it is connected with the sense of competition and probability. Coming to the recent improvements in the risk free economics, "Time consistency for set-valued dynamic risk measures for bounded discrete-time processes" by Chen and Hu [12] presented two sorts of time predictable properties for set-esteemed dynamic hazard measures for discrete-time forms that are adjusted to a given filtration, named time consistency and multi-portfolio time consistency. Proportional portrayals of multi-portfolio time consistency are found for standardized unique hazard measures. The study can be considered as a different approach from the conventional approach of risk management in economics. However the complexity of applying the model as a generalized model with a fundamental background of competition is a reasonable question.

1.2 Game theory background

Developing a unique methodology in the field of probabilistic economics based on the science of competition is indirectly bounded to the recent development in game theory and its application. The conventional game theory has some major limitations. This limitation includes both the players should be from the same field, only two players can be considered at a time and the complexity of large pay off matrix. Harsanyi [13] put forward some of the fundamental limitations of classical game theory in the early 1980s. However, the development of game theory also gave birth to some categories in the apex of last century named as evolutionary game theory. Friedman's [14] is an introductory article about the evolution game theory in modern economics along with the purpose of getting benefits in the field of economics. After this, we can identify a series of research articles related to the application and improvements of evolutionary game theory in economics [15–17]. Samuelson [15] mentioned the possibilities of modelling of evolutionary game theory in economics with more detailed studies and also authors [16, 17] analysed the different welfare management strategies. The concept of co-evolutionary games was introduced by Perc and Szolnoki [18] and they put forward some modifications like introducing co-evolutionary rule in evolutionary games and studying the effect in different sections including population growth, teaching activity, mobility etc. The work highlighted the sensitive and highly dynamic issues in the social dilemma. The work can be blended with the concept of natural outcome probabilities of different skilled players on account of the active and inactive players. This may be helpful to develop a powerful strategy and decision making tool in the near future. In the last decade, many application oriented research works are evolved. Anastasopoulos et al. [19] created a feedback suppression system on a multicast-satellite model with the help of game theory. The modern game theory is able to consider two or more number of players in the same time. However, the complexity of such techniques makes the practical possibility more difficult. Ganzfried et al. [20] pointed out some difficulties in the field of artificial intelligence with multiple players (AI), also Li and Kendall [21] have the similar dispute about the practical possibility of handling multiple players while studying Nash Equilibrium and Evolutionarily Stable States. Thereby, all those literatures conclusively point out the complexity of handling more number of players using game theory. The recent scenarios indicate the need of new methodology, which is able to hold number of players or competitors at the same time with less complexity of science. The game theory works on the player's payoff contingent on the strategy implemented by the other player where as the model introduced here consider the system along with the probability of losing through fake activities (failure gates) and dummy (inactive) competitors as a whole. This provides valuable and at the same time less complicated data for proper decision making in terms of outcome probabilities, number of failure gates and the number of dummy variables introduced.

The reduction of complexity of science is more relevant in the recent economic environment. Helbing et al. [22] address the current social dilemma relating to traffic, epidemic controls etc. This likewise uncovers the further plausibility of using a probabilistic methodology in different parts of research, decreasing the complexity of science, which additionally spurs us to upgrade the present work. The research articles by Perc et al. [23, 24] were dealt with the human cooperation in a social environment. The works indirectly giving concepts of emergence (where individual serves the society for benefits of themselves and the society). The human cooperation and the participation of number of active and inactive individuals also contribute to the sustainability of diverse skilled players in the competition. The shifting of "Nash Equilibrium" can also be possible by adjusting the active and dummy competitors in the system. However, the current work mentioned in this article focuses only on the probabilistic superiority and inferiority of different skilled players.

The competitions in general can be classified into two types based on the way by which it is being conducted. First one is the single step competition, where there will be only two competitors and results in an ultimate winner and a looser. Second one is the compound step; multiple numbers of competitors compete in the same time resulting in a series of ranks. Sometimes the competitive environment itself forces us to do a sequence of single steps to arrive at the ultimate winner in the competition. The result obtained in the compound step may not be the absolute one for each and every rank and hence can be analysed only with the help of a probabilistic model alone.

The outcome probabilistic model mentioned above for discrete finite rank can very well be explained using the following equations,

$$P_d(n) = \frac{\lambda n_1 R_{M-2} + \left(\frac{M}{2} - \lambda\right) n_2 R_{M-2}}{R_M} \tag{1.1}$$

$$P_c(n) = \frac{\alpha + \beta}{\gamma} \tag{1.2}$$

where

$$\alpha = \frac{\lambda}{f(n)} R_{M-2} f(n) \left[\int_a^n f(x)dx - \left(\lim_{\Delta x \to 0} f(n) \cdot \Delta x \right) \right]$$

$$\beta = \frac{\left(\frac{A}{2} - \lambda\right)}{f(n)} R_{M-2} f(n) \left[\int_n^b f(x)dx - \left(\lim_{\Delta x \to 0} f(n) \cdot \Delta x \right) \right]$$

$$\gamma = \frac{1}{2} R_{M-2} \int_a^b \int_a^b f(z) \cdot \left(f(x) - \left(\frac{1}{b-a} \right) \right) dz \cdot dx$$

Equations (1.1) and (1.2) represent the outcome probabilities of discrete and continues frequency models respectively. Here $f(x)$ and $f(z)$ represent frequency distributions of ranks, in particular $f(n)$ is the frequency distribution of n^{th} rank. $f(n)\,\Delta x$ represents the number of competitors corresponding to n^{th} rank (Δx being strip width) and $\lim_{\Delta x \to 0} f(n) \cdot \Delta x$ is the area for elementary strip which represent number of competitors in the elementary strip. A represents total number of competitors $\left(A = \int_a^b f(x)dx\right)$ in the system in the continuous case and M represents total number of competitors in the discrete case. R_M represents the number of possible competition arrangements for M competitors, R_{M-2} represents the number of possible competition arrangement for M competitors by keeping one competitor fixed in a position, n_1 and n_2 represent number of competitors in front of n^{th} position and behind n^{th} position respectively. ${}^MC_2$ is the total number of combinations of M competitors and thus $M = n_1 + n_2 + 1, R_M = {}^MC_2 R_{M-2}$. λ represents total number of failure gates and both $P_d(n)$ and $P_c(n)$ represent the pass through probability of n^{th} rank in the discrete and continuous case respectively. a and b are lower and upper limits of ranks given. The above equations are used to study the effect of dummy competitors in the whole system and the scope of futuristic forecasting of the winner in a competition.

The estimation of ultimate n^{th} position in a rank list can be achieved through success-failure gateway. The purpose of the success-failure gateway method is to calculate the outcome probability for n^{th} position within a combination of success failure gateways (containing both success and failure gate). Success gates can be defined as control gates promoting the intermediate winner to the next level and failure gates are those which deny the winner from appearing to the next level or supports the intermediate looser to the next level. There are many possible competition criteria that can be followed. Knockout criteria, play off criteria are some of them. This work makes use of knockout criteria in particular.

2 Mathematical model

2.1 Effect in probability distribution with dummy competitors

Suppose the number of competitors in a competition M be a function of time. Therefore $M = h(t)$. In order to achieve particular probabilistic patterns like maxima or minima in the probabilistic distribution, let us introduce the variable $D(t)$ to represent the number of inactive competitors or dummy competitors in the system at time t. Usually dummy competitors are introduced in the least ranks so that these competitors have null effect in zero failure gate situations ($\lambda = 0$).

The grand total number of competitors $T(t)$ can be formulated as a linear model described below.

$$T(t) = h(t) + D(t) \tag{2.1}$$

2.2 Maxima or minima in time probability distribution of n^{th} position

If there is no maxima or minima for the $P(h(t))$, we have to design a $D(t)$ such that $P(T(t))$ gives a maxima or minima in the probability distribution of least rank for a failure gate $\lambda = g(t)$.

Thus,

$$P(n,t) = \frac{\left[g(t)(T(t)-1)\right]R_{T(t)-2}}{R_{T(t)}} \tag{2.2}$$

and $R_{T(t)} = T(t)C_2 R_{T(t)-2}$

where $T(t)C_2 = \frac{[T(t)(T(t)-1)]}{2}$

Therefore (2.2) becomes,

$$P(n,t) = \frac{2g(t)}{T(t)} \tag{2.3}$$

Differentiate (2.3) with respect to t and equate to zero

$$\frac{\partial}{\partial t}[P(n,t)] = \frac{T(t)g'(t) - g(t)T'(t)}{(T(t))^2} = 0$$

On solving,

$$g(t) = C \cdot T(t) \tag{2.4}$$

On Appling (2.1) in (2.4)

$$D(t) = (1/C)g(t) - h(t) \tag{2.5}$$

C is the constant that can be determined using the initial conditions at $t=0$.

Figure 1 represents the probability distribution of dummy, active and revised total number of competitors in the competition.

From the figure it is clear that the probability distribution of dummy and active competitors don't have any

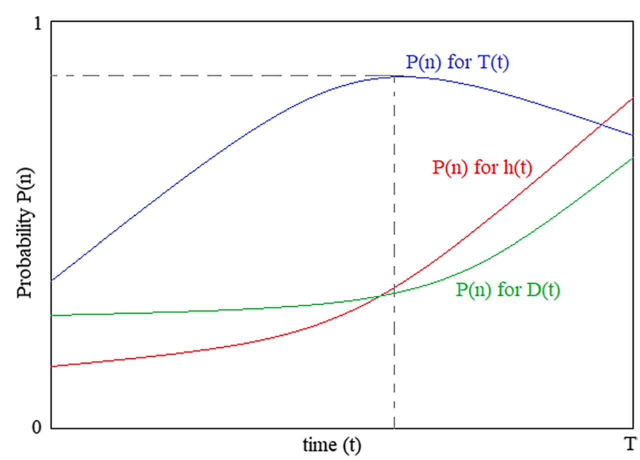

Fig. 1 Probability-time distribution of dummy, actual and total competitors. The arbitrary graphical representation of a n^{th} position which represents a situation where an actual competitor is incapable of finding a relative maximum anywhere in the time being (represented from 0 to T). The red line represents the probability-time distribution of actual competitor. We are then introducing a green line in the system which is nothing but representing the probability-time distribution for dummy competitor assuming that there will be a relative maximum or minimum in the probability-time distribution for the total competitors represented in blue line. The graphical representation provided is only for understanding the generalized physical meaning of the system to convey the practical application in general. The objective of the study is to identify the required number dummy competitor or dummy variable to create such a peak in the probability-time distribution (represented as blue line)

maxima or minima but the probability distribution of total number of competitors have a relative maxima. This indicates the impact and relevance of introducing dummy variable in the system to attain local maximum or minimum.

3 Analysis of maxima or minima in position probability distribution of n^{th} position without dummy variables

As we discussed in Sect. 2, the time probability distribution for the discrete rank model without frequency is insufficient to create a maxima or minima in the corresponding position probability distribution. In this connection, we are introducing continuous infinite rank model with frequency. However, continuous infinite rank model with frequency has its own speciality to create maxima or minima with and without dummy variables.

We know that the continuous infinite rank model as $\left(\lim_{\Delta x \to 0} f(n) \cdot \Delta x)\right)$

$$\frac{\lambda \int_a^n f(x)dx + \left(\frac{A}{2} - \lambda\right) \int_n^b f(x)dx}{AC_2} = P_c(n) \tag{3.1}$$

On differentiating (3.1) with respect to n and equating to zero to get maxima or minima in the position-probability distribution. i.e., $P_c'(n) = 0$.

This gives

$$P_c'(n) = \lambda \frac{d}{dn} \int_a^n f(x)dx + \frac{A}{2} \frac{d}{dn} \int_n^b f(x)dx - \lambda \frac{d}{dn} \int_n^b f(x)dx = 0 \tag{3.2}$$

The ratio form of (3.2) is

$$\frac{A_2 - A_1}{A_2} = \frac{A}{2\lambda} \tag{3.3}$$

where $\frac{d}{dn} \int_a^n f(x)dx = A_1$,

$$\frac{d}{dn} \int_b^n f(x)dx = A_2 \text{ and } A = A_1 + A_2 \tag{3.4}$$

Hence, by using (3.3) and (3.4)

$$A_1.A_2 + A_2^2 = 2\lambda(A_2 - A_1) \tag{3.5}$$

where A_1 and A_2 are the functions of n^{th} position. The rank (or) position of a competitor in a competition holding (3.5) will find a relative maxima or minima in the position- probability distribution. Figures 2 and 3 explain the comparison of position- probability distribution in discrete rank model without frequency and continuous infinite rank model with frequency.

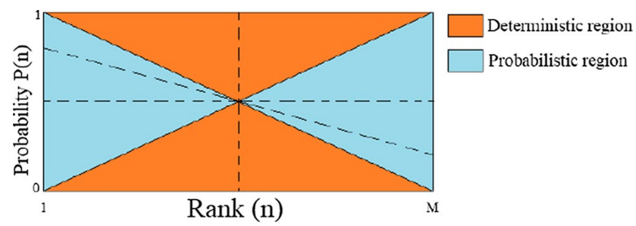

Fig. 2 Probabilistic and deterministic regions in a competition. The area of the figure classified into two different zones in the probability—number of competitor plot for an uniform frequency distribution of number of competitors. The blue region represents the probabilistic zone and orange represents the deterministic zone. Since the Eq. (1.1) is a straight line equation, the only possibility of variation in probability distribution is varying the slope of the line. The dotted lines passing through the blue coloured zone (probabilistic zone) represent the probability distribution with different slopes. However, the movement of the dotted line is restricted at the peak probabilities 0 and 1. Further movement is only through reducing the total number of competitors (M). Hence the rest zone is a non-probabilistic zone or deterministic zone

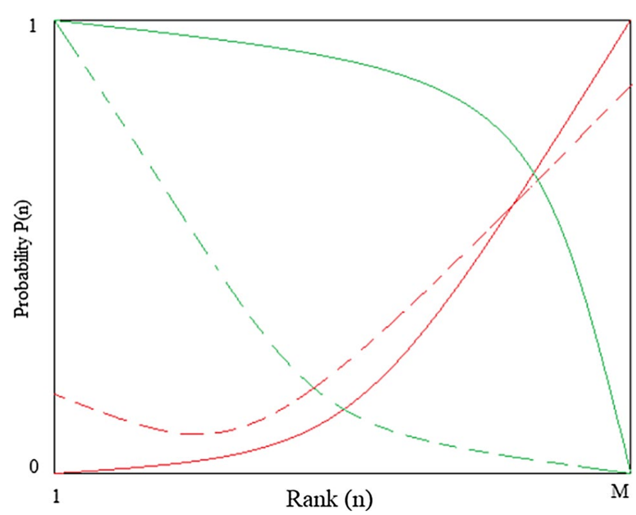

Fig. 3 Position probability distribution for M member competition with various failure gates. The red line indicates different practically possible scenarios where the outcome probabilities keep on increasing for lower ranks and this will usually happens on higher number of failure gates and green lines refers to the situation where higher ranks having having higher probabilities and lower ranks having the least, this kind of situation will arrays for lower number of failure gates. Figure 3 is provided for the understanding of this increasing and decreasing probability patterns

From Fig. 2, it is evident that that there is no possibility to attain local maxima or minima in the position- probability distribution of discrete rank model without frequency (since the basic model (1.1) is a linear function of n). Figure 3, represents the probability distribution function of a continuous infinite rank model with frequency. In Fig. 3, we find a possibility to attain local maxima or minima in the position-probability distribution since the basic model (1.2) is also a function of frequency distribution.

4 Analysis of maxima or minima in position probability distribution of n^{th} position with dummy variables

The position-probability distribution of the continuous infinite rank model with frequency having a dummy variable will find a local maxima or minima. This can be done mainly in two different ways. There are two essential methods to import dummy variable in the competition. These two methods include the addition of the dummy variable either on one tail or both the tails of the original frequency distribution (of the competitors).

4.1 One tail addition of the frequency distribution of the dummy variable

The dummy frequency is added to the right or left tail of original frequency to get total frequency distribution. The general equation of this analysis includes Eq. (3.1) along with some dummy variable as follows.

$$P_c(n) = \frac{1}{A_{C_2}}\left[\lambda \int_a^n \left[f(x) + D(x)\right]dx + \left(\frac{A}{2} - \lambda\right)\int_n^b f(x)dx\right] \tag{4.1}$$

where D represents the frequency of the dummy variable. On differentiating (4.1) with respect to n and equating to zero we get an extreme value in the position-probability distribution. i.e., $P_c'(n) = 0$. This gives

$$\lambda\frac{d}{dn}\int_a^n (f(x) + D(x))dx + \frac{A}{2}\frac{d}{dn}\int_n^b f(x)dx - \lambda\frac{d}{dn}\int_n^b f(x)dx = 0 \tag{4.2}$$

On simplifying (4.2) with the help of (3.4), we get

$$(A_1 + D_1)\cdot A_2 + A_2^2 = 2\lambda(A_2 - A_1 - D_1) \tag{4.3}$$

This implies that,

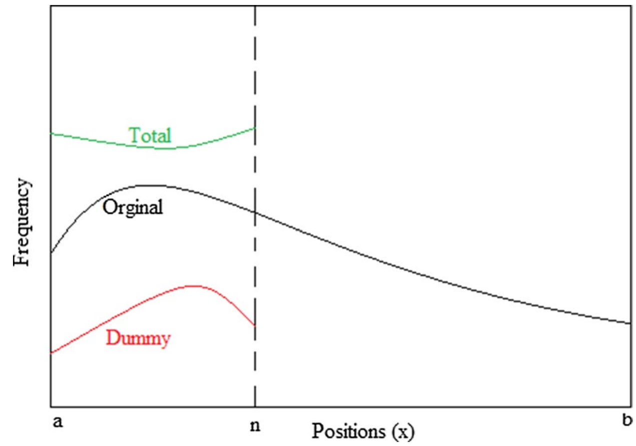

Fig. 4 One tail addition of dummy competitors on the original number of competitors resulting in the revised total number of competitors. The black line represents the frequency distribution of original number of competitors, red line represents the dummy competitor distribution and green line represents the distribution of total number of competitors. Since it is a one tail addition the dummy distribution is added only on any one of the sides (in the figure it is added in the left tail). Hence the total distribution is found to be varying only on the left tail and remains as original distribution on the right side. The left and right tail is pivoted with reference to the n^{th} position

$$D_1 = \frac{2\lambda(A_2 - A_1) - A_2(A_2 + A_1)}{(2\lambda + A_2)} \tag{4.4}$$

where A_1 and A_2 are functions of n^{th} position. Here the frequency of the dummy variable holding (4.4) permits a local maxima or minima in the position probability distribution of the n^{th} position. Figure 4 represents the one tail addition of such a dummy variable to the original frequency distribution of the n^{th} position.

4.2 Two tail addition of the frequency distribution of the dummy variable

The dummy frequency $D(x)$ is added to both right and left tail parts of the original frequency of the n^{th} position to get the frequency distribution of the revised total number of competitors. The general equation used for further analysis includes Eq. (3.1) along with some dummy variable as given below.

$$P_c(n) = \frac{1}{A_{C_2}}\left[\lambda \int_a^n \left[f(x) + D(x)\right]dx + \left(\frac{A}{2} - \lambda\right)\int_n^b \left[f(x) + D(x)\right]dx\right] \tag{4.5}$$

Also, let

$$\frac{d}{dn}\int_a^n D(x)dx = D_1, \quad \frac{d}{dn}\int_n^b D(x)dx = D_2 \tag{4.6}$$

where D_1 and D_2 are slope of the dummy variable-position distribution. On differentiating (4.5) with respect to n and equating to zero we get an extreme in the position-probability distribution. i.e., $P_c'(n) = 0$,

This gives $\lambda \frac{d}{dn} \int_a^n (f(x) + D(x))dx$

$$+ \frac{A}{2}\frac{d}{dn}\int_n^b (f(x) + D(x))dx - \lambda \frac{d}{dn}\int_n^b (f(x) + D(x))dx = 0 \tag{4.7}$$

On simplifying (4.6) with the help of (4.6), we get

$$(A_1 + D_1) \cdot (A_2 + D_2) + (A_2 + D_2)^2 = 2\lambda(A_2 + D_2 - A_1 - D_1) \tag{4.8}$$

Therefore whenever D_1 and D_2 satisfy (4.8) extreme values occur in the position probability distribution. The following Fig. 5 represents the addition of dummy frequency to the original frequency in the case of two tail addition.

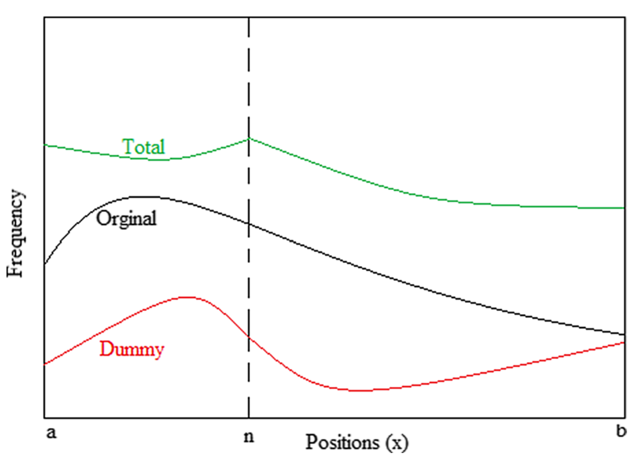

Fig. 5 Two tail addition of dummy competitors on the original number of competitors resulting in the revised total number of competitors. The black line represents the frequency distribution of original number of competitors, red line represents the dummy competitor distribution and green line represents the distribution of total number of competitors. Since it is a two tail addition the dummy distribution is added on both the sides (in the figure it is added in both left and right tails). Hence the total distribution is found to be varying only on both left and right tails. The left and right tail is pivoted with reference to the n^{th} position

5 Case study—analysis of number of fake rating in e-marketing

The contextual investigation directed here is to guarantee and clarify the promising conceivable outcomes of this model in present day e-marketing, e-Supply Chain Management (e-SCM). Fake rating and reviews are one of the serious issues in e-SCM, which legitimately impacting the item selling through online markets like amazon, Alibaba, flipcart and so on. The number of failure gates is thought to be a component of various outside variables like geography of market selling, demand balancing troubles, etc. The client's rating distribution for a set of products is given below.

The organization noticed an item selling difficulties in 6th level appraised items. The quantity of fake rating caused this unexpected depression in selling probability. The number of fake rating involved as well as the variations in the number of fake rating based on the number of failure gates is analysed using the necessary condition of one tail addition as given in Eq. (4.4).

6 Results

The number of customer ratings before the sixth level (A_1) is 2000 and after the sixth level (A_2) is 1000 (refer Fig. 6). The possible number of fake rating (D) that creates probabilistic sensitivity as mentioned in the Eq. (4.4) is then analysed and tabulated for different number of failure gates (λ).

This model is applicable in highly responsive e-supply chain network, where fake rating assumes a crucial job on the manageability of an online item in the e-marketing

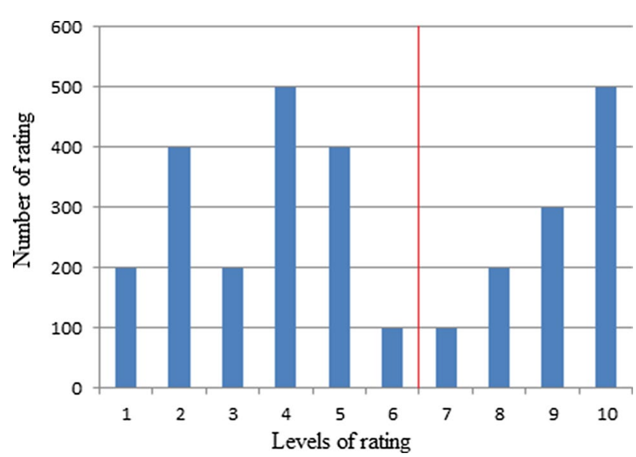

Fig. 6 The customer rating distribution of products with different rating levels

Table 1 Number of fake ratings for different number of failure gates in the sixth level rated products

Number of failure gates (λ)	Value of dummy variable or number of fake rating (D)
1500	600
1400	667
1300	740
1200	819
1100	905
1000	1000
900	1106
800	1223
700	1353
600	1500
500	1667

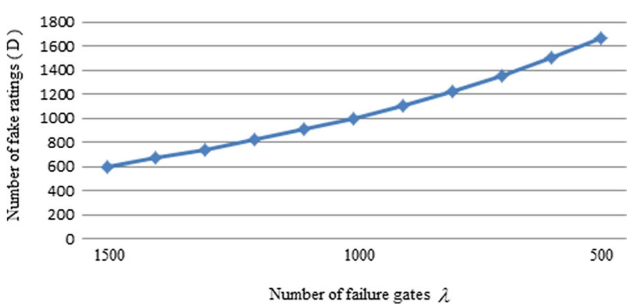

Fig. 7 Relation between number of failure gates and number of fake ratings in the case of sixth level rated products. It is clear that the number of fake rating is increasing with increasing number of failure gates this will allow the company to identify the appropriate number of failure fates that can be allowable to get some breathing time to tolerate number of fake rating thereby it improve the decision making (includes depth of advertising a product, required legal actions and so on). The selection of proper number of failure gates is the critical and based up on the situation of the company

condition. The number of fake rating is determined corresponding to number of failure gate extending from 1500 to 500 (Table 1).

Figure 7 given below is the graphical representation of the above case. This is to identify the relationship between the number of failure gates and corresponding number of fake ratings in the case of the sixth level rated products.

7 Concluding remarks

The article investigates the existence of the two local extremes of a probability distribution (n^{th} position) with and without introducing dummy variables for an instant of time 't'. The impact of dummy competitors on the discrete ranking model without frequency and continuous infinite model with frequency is studied. The necessary condition required for obtaining a maxima or minima in a position-probability distribution, without dummy variables also identified. The investigation further reached out by presenting dummy variable in both one tail and two tail expansion cases to arrive at required necessary conditions.

The case study discussed above discloses the fact that reduction in the number of failure gates will provide much breathing time to the competitor by increasing the limit of number of fake rating. The circumstance subsequently requests higher number of fake rating to make an unexpected probabilistic variety in the position-probability distribution. This is useful to take legitimate activities and techniques to guarantee the best-selling practice in e-marketing.

Additionally, the investigation can be stretched out for an optimal number of fake rating and comparing outcome probability for a specific rated product. This will indeed be more relevant in a highly responsive and dynamic current marketing scenario.

This study will definitely find its position in strategy analysis and decision making process in the field of economics, quality engineering. The study also expects to find some applications in particle science as well.

Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

References

1. Hacking I (1975) The emergence of probability. Cambridge University Press, Cambridge
2. Shafer G (1978) Non-additive probabilities in the work of Bernoulli and Lambert. Arch Hist Exact Sci 19(4):309–370
3. Gilboa I, Schmeidler D (1989) Maxmin expected utility with a non-unique prior. J Math Econ 18(2):141–153
4. Gilboa I, Postlewaite AW, Schmeidler D (2008) Probability and uncertainty in economic modeling. J Econ Perspect 22(3):173–188
5. Matalycki M, Pankow A (2003) Incomes probabilistic model of the banking network. Sci Res Inst Math Comput Sci 2(1):99–104
6. Subbaiah KV, Rao KS, Satyanarayana B (2004) Inventory models for perishable items having demand rate dependent on stock level. Opsearch 41(4):223–236
7. Scalas E (2008) Introduction to probability theory for economists. Universita del Piemonte Orientale (Amedeo Avogadro)
8. Hickman B (2009) Introduction to probability theory for graduate economics. http://home.uchicago.edu/~hickmanbr/uploads/PT_REVIEW_CH4.pdf

9. Menzel K (2009) Introduction to statistical methods in economics. MIT, New York
10. Floyd JE (2010) Statistics for economists a beginning. University of Toronto, Toronto
11. Lesmono D, Limansyah T (2017) A multi item probabilistic inventory model. J Phys Conf Ser 893(1):012024
12. Chen Y, Hu Y (2018) Time consistency for set-valued dynamic risk measures for bounded discrete-time processes. Math Financ Econ 12(3):305–333
13. Harsanyi JC (1982) A new general solution concept for both cooperative and noncooperative games. In: Papers in game theory. Springer, Dordrecht, pp. 211–232
14. Friedman D (1998) On economic applications of evolutionary game theory. J Evol Econ 8(1):15–43
15. Samuelson L (2002) Evolution and game theory. J Econ Perspect 16(2):47–66
16. Jormakka J, Mölsä JV (2005) Modelling information warfare as a game. J Inf Warf 4(2):12–25
17. Wooldridge M (2012) Does game theory work? IEEE Intell Syst (AI and Game Theory) 27:7680
18. Perc M, Szolnoki A (2010) Coevolutionary games—a mini review. BioSystems 99(2):109–125
19. Anastasopoulos MP, Taleb T, Cottis PG, Obaidat MS (2012) Feedback suppression in multicast satellite networks using game theory. IEEE Syst J 6(4):657–666
20. Ganzfried S, Sandholm T (2013) Improving performance in imperfect-information games with large state and action spaces by solving endgames. In: Workshops at the twenty-seventh AAAI conference on artificial intelligence
21. Li J, Kendall G (2015) On Nash equilibrium and evolutionarily stable states that are not characterised by the folk theorem. PLoS ONE 10(8):e0136032
22. Helbing D, Brockmann D, Chadefaux T, Donnay K, Blanke U, Woolley-Meza O, Perc M (2015) Saving human lives: what complexity science and information systems can contribute. J Stat Phys 158(3):735–781
23. Perc M (2016) Phase transitions in models of human cooperation. Phys Lett A 380(36):2803–2808
24. Perc M, Jordan JJ, Rand DG, Wang Z, Boccaletti S, Szolnoki A (2017) Statistical physics of human cooperation. Phys Rep 687:1–51