



Gravitational Influence on a Nonlocal Thermoelastic Solid with a Heat Source via L–S Theory

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Abstract

Purpose The purpose is to obtain precise expressions of physical fields using the appropriate non-dimensional variables and normal mode analysis.

Methods Based on the Lord–Shulman (L–S) theory and taking into account gravitational influences as well as temperature-dependent features, the fundamental equations for a nonlocal thermoelastic solid are developed.

Results When a nonlocal thermoelastic media is swapped out for a thermoelastic one, this approach still holds true. Comparisons are done between the outcomes obtained and those expected for various nonlocal parameter values and for an empirical material constant. Additionally, comparisons are done between the outcomes for various gravity field values.

Conclusion The nonlocal parameter plays a big part in how the physical fields are distributed. The distributions of the physical fields are significantly influenced by the gravity field.

Keywords Gravity field · Lord–Shulman theory · Internal heat source · Nonlocal parameter

List of symbols

σ_{ij}	Component of stress tensor
e_{kk}	Dilation
e_{ij}	Components of strain tensor
δ_{ij}	Kronecker delta
ρ	Mass density
C_E	Specific heat at constant strain
λ, μ	Elastic constants
t	Time
u_i	Components of displacement vector
K	Thermal conductivity
Q	Internal heat source
τ_0	Thermal relaxation time
ε	$= a_0 e_0$ Is the elastic nonlocal parameter
T	Thermal temperature
T_0	Reference temperature, $\left (T - T_0) / T \right < 1$, $\theta = T - T_0$
α_t	Linear thermal expansion coefficient, $\gamma = (3\lambda + 2\mu)\alpha_t$
$\mu_0, \lambda_0, \gamma_0$	Constants of material

δ_0	Empirical material constant
$\bar{u}(z)$	Amplitude of the function $u(x, z, t)$
m	Complex time constant,
a	Wavenumber in the x – direction
V_0	Velocity of a moving internal heat source
Q_0	Magnitude of an internal heat source

Introduction

The equation of heat conduction is of the parabolic form, predicting infinite speeds of propagation for heat waves, and does not contain any elastic terms, according to the classical uncoupled theory of thermoelasticity. In order to solve the problem inherent in the traditional uncoupled hypothesis, Biot [1] defined the coupled thermoelasticity theory. To resolve this contradiction, a generalized theory with a finite speed of heat transfer in elastic solids (the hyperbolic heat transport equation) has been developed in recent years. Lord and Shulman (L–S) [2] provided this generalization theory, also referred to as the extended thermoelasticity theory, which only has one thermal relaxation time parameter. The first- and second-time derivatives of the strain are included in the Lord and Shulman energy equation. They considered a new law of heat conduction instead of the law of Fourier. Based on their theory, the linear correlation

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between temperature and heat flux includes temperature rate and thermal rates. According to this theory, the temperature propagation speed is finite due to the hyperbolic heat equation. Actually, as well known, the term “generalized” usually refers to thermodynamic theories based on hyperbolic (wave-type) heat equation, so that, a finite propagation speed for thermal signal is admitted. The generalized thermoelasticity has drawn extensive attention due to its applications in diverse fields, such as earthquake engineering, nuclear reactor design, and high-energy particle accelerators. Youssef [3] developed an isotropic elastic material with temperature-dependent mechanical and thermal properties that has a generalized thermoelasticity with the L–S theory. Bagri and Eslami [4] provide a one-dimensional generalized thermoelasticity model of a disk based on the L–S theory. L–S theory and linked theory were developed by Othman and Said [5, 6] to investigate the effects of rotation and a magnetic field on fiber-reinforced thermoelastic media. The L–S theory has lately been established in numerous studies covering a wide range of topics [7–17].

The nonlocal theory of elasticity was used to study applications in nano-mechanics including lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics, fracture mechanics, surface tension fluids, etc. Eringen [18] was interested in material bodies whose actions at any interior location depended on the conditions at all other sites within the body. The impact of harmonically fluctuating heat on FG nanobeams was demonstrated by Zenkour and Abouelregal [19] within the framework of a nonlocal two-temperature thermoelasticity theory. Yu et al. [20] examined size-dependent generalized thermoelasticity using Eringen's nonlocal model. In the context of the nonlocal theory, Ebrahimi et al. [21] examined the wave propagation properties in magneto-electro-elastic nanotubes while taking the shell model into consideration. The axisymmetric vibrational behavior of a size-dependent circular nano-plate with functionally graded material with different types of boundary conditions was investigated by Shariati et al. [22]. The authors Acharya and Mondal [23], Roy et al. [24], Zenkour [25], Bachher and Sarkar [26], Abouelregal and Mohammed [27], Zhou and Li [28], Kaur et al. [29], Gupta et al. [30], Said [31], Atta et al. [32], Yahya et al. [33], and Abouelregal et al. [34] have all published studies on nonlocal thermoelasticity.

Any study on the propagation waves in generalized thermoelasticity materials can be significant in structural engineering, geophysics, and seismology. Such a study becomes more realistic if the presence of gravity could be considered. In this study, a nonlocal thermoelastic solid with a moving internal heat source will be developed. Discussions covered location, temperature-dependent characteristics, and influences of the gravitational field. With the use of normal mode analysis, precise solutions to the physical field are obtained. The L–S theory was discussed in relation to the issue.

The Description of the Problem

A nonlocal thermoelastic solid that is affected by gravity field and has a moving internal heat source in the half-space ($x \geq 0$). The surface of the half-space is subjected to a thermal shock which is a function of x and z . Thus, all quantities are independent of y . The dynamic displacement $\underline{u} = (u, 0, w)$, $v = 0$, $\frac{\partial}{\partial y} = 0$.

The constitutive equations as presented by Hetnarski and Eslami [35] and Eringen [36]:

$$(1 - \varepsilon^2 \nabla^2) \sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma \theta \delta_{ij}, \quad (1)$$

The heat conduction equation as Lord and Shulman [2]

$$K \nabla^2 \theta = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \rho C_E \dot{\theta} + \gamma T_0 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \dot{e} - \left(1 + \tau_0 \frac{\partial}{\partial t}\right) Q. \quad (2)$$

The equation of motion

$$\rho \ddot{u}_i = \sigma_{ji,j} + F_i, \quad (3)$$

where $F_1 = \rho g \frac{\partial w}{\partial x}$, $F_2 = 0$, $F_3 = -\rho g \frac{\partial u}{\partial x}$.

According to Said [37]:

$$\mu = \mu_0 (1 - \delta_0 T_0), \lambda = \lambda_0 (1 - \delta_0 T_0), \gamma = \gamma_0 (1 - \delta_0 T_0). \quad (4)$$

The case of the temperature-independent modulus of elasticity if $\delta_0 = 0$.

When Eqs. (1) and (4) are introduced in Eq. (3), we obtain

$$\begin{aligned} \rho (1 - \varepsilon^2 \nabla^2) \ddot{u} = & (1 - \delta_0 T_0) \left((\lambda_0 + 2\mu_0) \frac{\partial^2 u}{\partial x^2} \right. \\ & \left. + (\lambda_0 + \mu_0) \frac{\partial^2 w}{\partial x \partial z} + \mu_0 \frac{\partial^2 u}{\partial z^2} - \gamma_0 \frac{\partial \theta}{\partial x} \right) \\ & + \rho g (1 - \varepsilon^2 \nabla^2) \frac{\partial w}{\partial x}, \end{aligned} \quad (5)$$

$$\begin{aligned} \rho (1 - \varepsilon^2 \nabla^2) \ddot{w} = & (1 - \delta_0 T_0) \left((\lambda_0 + 2\mu_0) \frac{\partial^2 w}{\partial z^2} \right. \\ & \left. + (\lambda_0 + \mu_0) \frac{\partial^2 u}{\partial x \partial z} + \mu_0 \frac{\partial^2 w}{\partial x^2} - \gamma_0 \frac{\partial \theta}{\partial z} \right) \\ & - \rho g (1 - \varepsilon^2 \nabla^2) \frac{\partial u}{\partial x}. \end{aligned} \quad (6)$$

We define the non-dimension variables as follows for convenience:

$$\begin{aligned} (x', z', \varepsilon', u', w') &= \frac{1}{l_0} (x, z, \varepsilon, u, w), g' = \frac{l_0}{d_0^2} g, (t', \tau_0 t) = \frac{d_0}{l_0} (t, \tau_0), \\ \theta' &= \frac{\gamma_0 \theta}{(\lambda_0 + 2\mu_0)}, Q' = \frac{\gamma_0 Q}{(\lambda_0 + 2\mu_0)}, \sigma'_{ij} = \frac{\sigma_{ij}}{\mu_0 (1 - \delta_0 T_0)}, \\ l_0 &= \sqrt{\frac{K}{\rho C_E T_0}}, d_0 = \sqrt{\frac{(\lambda_0 + 2\mu_0) (1 - \delta_0 T_0)}{\rho}}. \end{aligned} \quad (7)$$

When Eqs. (7) are introduced in Eqs. (2), (5), and (6), we obtain

$$(1 - \epsilon^2 \nabla^2) \ddot{u} = \frac{\partial^2 u}{\partial x^2} + (1 - \delta_0 T_0) \left(\frac{\lambda_0 + \mu_0}{\rho d_0^2} \frac{\partial^2 w}{\partial x \partial z} + \frac{\mu_0}{\rho d_0^2} \frac{\partial^2 u}{\partial z^2} \right) - \frac{\partial \theta}{\partial x} + g (1 - \epsilon^2 \nabla^2) \frac{\partial w}{\partial x}, \tag{8}$$

$$(1 - \epsilon^2 \nabla^2) \ddot{w} = (1 - \delta_0 T_0) \left(\frac{\mu_0}{\rho d_0^2} \frac{\partial^2 w}{\partial x^2} + \frac{\lambda_0 + \mu_0}{\rho d_0^2} \frac{\partial^2 u}{\partial x \partial z} \right) + \frac{\partial^2 w}{\partial z^2} - \frac{\partial \theta}{\partial z} - g (1 - \epsilon^2 \nabla^2) \frac{\partial u}{\partial x}, \tag{9}$$

$$\nabla^2 \theta = \frac{\rho C_E d_0 l_0}{K} (1 + \tau_0 \frac{\partial}{\partial t}) \dot{\theta} + \frac{\gamma^2 T_0 d_0 l_0}{K(\lambda + 2\mu)} (1 + \tau_0 \frac{\partial}{\partial t}) \dot{\epsilon} - \frac{l_0^2}{K} (1 + \tau_0 \frac{\partial}{\partial t}) Q. \tag{10}$$

Normal Mode Analysis

The following normal modes can be used to decompose the solution of the physical variables under consideration:

$$(u, w, \theta, \sigma_{ij})(x, z, t) = (u^*, w^*, \theta^*, \sigma_{ij}^*)(z) \exp(mt + iax), \tag{11}$$

$$Q = Q^* \exp(mt + iax), \quad Q^* = Q_0 V_0.$$

Introducing Eqs. (11) in Eqs. (8)–(10), we obtain

$$(N_1 D^2 - N_2) u^* + ia(-g \epsilon^2 D^2 + A_1 D + N_3) w^* - ia N_4 \theta^* = 0, \tag{12}$$

$$ia(g \epsilon^2 D^2 + A_1 D - N_3) u^* + (N_5 D^2 - N_6) w^* - N_4 D \theta^* = 0, \tag{13}$$

$$ia N_7 u^* + N_7 D w^* + (N_{10} - D^2) \theta^* = N_9, \tag{14}$$

where $D = \frac{d}{dz}$, $N_1 = \epsilon^2 m^2 + \frac{\mu_0(1-\delta_0 T_0)}{\rho d_0^2}$, $N_2 = a^2 + m^2(1 + a^2 \epsilon^2)$, $N_3 = g(1 + a^2 \epsilon^2)$, $N_4 = 1$, $N_5 = \epsilon^2 m^2 + 1$, $N_6 = \frac{\mu_0(1-\delta_0 T_0)}{\rho d_0^2} a^2 + m^2(1 + a^2 \epsilon^2)$,

$$N_7 = \frac{\gamma^2 T_0 d_0 l_0}{K(\lambda_0 + 2\mu_0)} m(1 + m\tau_0), \quad N_8 = \frac{\rho C_E d_0 l_0}{K} m(1 + m\tau_0),$$

$$N_9 = \frac{Q_0 V_0 l_0^2}{K} (1 + m\tau_0), \quad N_{10} = N_8 + a^2.$$

By solving Eqs. (12)–(14), we obtain

$$(D^6 - C_1 D^4 + C_2 D^2 - C_3) u^*(z) = -\frac{ia N_4 N_6 N_9}{C_0}. \tag{15}$$

The bound solution of Eq. (15) is:

$$u^*(z) = \sum_{j=1}^3 M_j \exp(-k_j z) + \frac{ia N_4 N_6 N_9}{C_0 C_3}. \tag{16}$$

Similarly,

$$w^*(z) = \sum_{j=1}^3 H_{1j} M_j \exp(-k_j z) + \frac{a^2 N_3 N_4 N_9}{C_0 C_3}, \tag{17}$$

$$\theta^*(z) = \sum_{j=1}^3 H_{2j} M_j \exp(-k_j z) + \frac{a^2 N_3^2 N_9 - N_2 N_6 N_9}{C_0 C_3}. \tag{18}$$

Using the above equations, we get

$$\sigma_{zz}^*(z) = \sum_{j=1}^3 H_{3j} M_j \exp(-k_j z) - C_4, \tag{19}$$

$$\sigma_{xz}^*(z) = \sum_{j=1}^3 H_{4j} M_j \exp(-k_j z) + C_5, \tag{20}$$

where k_j^2 ($j = 1, 2, 3$) are the roots of the characteristic equation: $(k^6 - C_1 k^4 + C_2 k^2 - C_3 = 0)$.

$$C_0 = g^2 a^2 \epsilon^4 - N_1 N_5, C_1$$

$$= \frac{1}{C_0} \left\{ \begin{array}{l} 2g N_3 a^2 \epsilon^2 + g^2 a^2 \epsilon^4 N_{10} + A_1^2 a^2 - N_1 N_4 N_7 - N_1 N_6 - N_2 N_5 - \\ N_1 N_5 N_{10} \end{array} \right\},$$

$$C_2 = \frac{1}{C_0} \left\{ \begin{array}{l} 2g N_{10} N_3 a^2 \epsilon^2 + 2A_1 a^2 N_4 N_7 + A_1^2 a^2 N_{10} + N_3^2 a^2 - N_2 N_4 N_7 - \\ N_2 N_6 - N_{10} N_1 N_6 - N_2 N_5 N_{10} - a^2 N_4 N_5 N_7 \end{array} \right\},$$

$$C_3 = \frac{1}{C_0} \{ N_{10} N_3^2 a^2 - N_2 N_6 N_{10} - a^2 N_4 N_6 N_7 \},$$

$$C_4 = \frac{N_4 N_9 (1 - \delta_0 T_0) (\lambda_0 a^2 N_6 + (\lambda_0 + 2\mu_0)(a^2 N_3^2 - N_2 N_6))}{\mu_0 C_0 C_3 (1 + \epsilon^2 a^2)},$$

$$C_5 = \frac{i a^3 N_3 N_4 N_9 (1 - \delta_0 T_0)}{C_0 C_3 (1 + \epsilon^2 a^2)},$$

$$H_{1j} = \frac{N_1 k_j^3 - g a^2 \epsilon^2 k_j^2 + (a^2 A_1 - N_2) k_j + N_3 a^2}{ia (g \epsilon^2 k_j^2 + (A_1 - N_5) k_j^2 - N_3 k_j + N_6)},$$

$$H_{2j} = \frac{N_1 k_j^2 - N_2 + ia (N_3 - A_1 k_j - g \epsilon^2 k_j^2) H_{1j}}{ia N_4},$$

$$H_{3j} = \frac{(1 - \delta_0 T_0) [ia \lambda_0 - (\lambda_0 + 2\mu_0) (k_j H_{1j} + N_4 H_{2j})]}{\mu_0 (1 + \epsilon^2 a^2 - \epsilon^2 k_j^2)},$$

$$H_{4j} = \frac{(1 - \delta_0 T_0) (ia H_{1j} - k_j)}{1 + \epsilon^2 a^2 - \epsilon^2 k_j^2}.$$

The Boundary Conditions

In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. To get the parameter M_n ($n = 1, 2, 3$), the initial and regularity conditions are used to solve the present problem at $z = 0$, are

a. Thermal boundary condition that the surface of the half-space is subjected to a thermal shock

$$\theta = f_1 f(x, t), \tag{21}$$

b. Mechanical boundary condition that the surface of the half-space is subjected to mechanical force

$$\sigma_{zz} = -f_2 G(x, t), \tag{22}$$

c. Mechanical boundary condition that the surface of the half-space is traction-free

$$\sigma_{xz} = 0, \tag{23}$$

where f_1, f_2 are constants and $f(x, t), G(x, t)$ are arbitrary functions.

We can find the following using Eqs. (18)–(20) in Eqs. (21)–(23):

$$\sum_{j=1}^3 H_{2j} M_j = f_1, \sum_{j=1}^3 H_{3j} M_j = -f_2 + C_4, \sum_{j=1}^3 H_{4j} M_j = -C_5. \tag{24}$$

By solving the aforementioned system in Eq. (24), we obtain

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{pmatrix}^{-1} \begin{pmatrix} f_1 \\ C_4 - f_2 \\ -C_5 \end{pmatrix}. \tag{25}$$

We determine the values of the coefficients $M_j (j = 1, 2, 3)$ by applying the inverse of the matrix approach of Eq. (25).

Validation and Applications

In the context of the L–S theory, we consider the numerical results for the physical constants as follows to compare the results for a nonlocal thermoelastic half-space solid with a moving internal heat source under the influence of the gravity field:

$$\lambda_0 = 7.76 \times 10^{10} \text{ N m}^{-2}, \quad \mu_0 = 7.78 \times 10^{10} \text{ N m}^{-2},$$

$$\rho = 8954 \text{ kg} \times \text{m}^{-3}, \quad a = 0.5, \quad T_0 = 293 \text{ K},$$

$$C_E = 383.3 \text{ J kg}^{-1} \text{ K}^{-1}, \quad \alpha_t = 2.78 \times 10^{-3} \text{ K}^{-1},$$

$$\tau_0 = 0.3 \text{ s}, \quad f_1 = 0.5, \quad f_2 = 1.5; K = 386 \text{ w m}^{-1} \text{ K}^{-1} \text{ s}^{-1},$$

$$m = m_0 + i\xi, m_0 = -0.3, \quad \xi = -0.3, \quad Q_0 = 3 \text{ K},$$

$$V_0 = 0.5 \text{ m s}^{-1}, \quad x = -0.5, \quad t = 0.5 \text{ s}.$$

Figures 1, 2, 3 and 4 display the vertical displacement distribution w , thermodynamic temperature distributions θ , and the stress components σ_{zz}, σ_{xz} for the nonlocal thermoelastic media with different values of the gravity field. Figure 1 depicts the variation of vertical displacement distribution w begins with positive values. Values of w decrease in the range $0 \leq z \leq 10$. With increasing the value of the gravity field g , values of w decrease. Figure 2 shows that the variation of thermodynamic temperature θ begins with a positive value and obeys the boundary conditions. Values of θ start with increasing attain their maximum values and then decrease. With increasing the value of g , values of θ decrease. Figure 3 introduces that the variation of stress

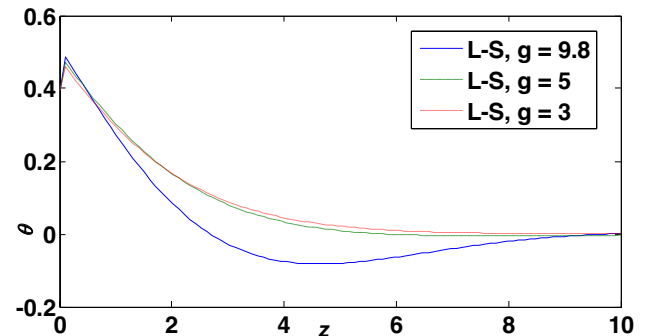


Fig. 2 Thermal temperature distribution θ for different values of the gravity field

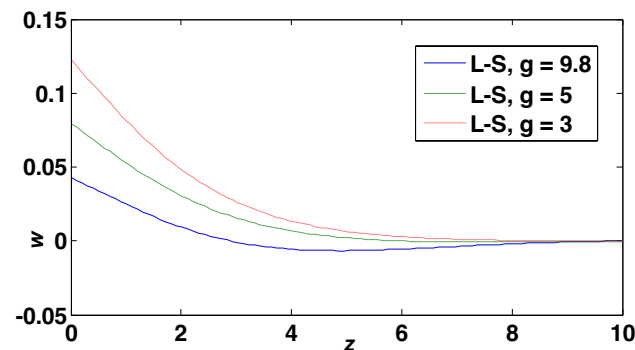


Fig. 1 Vertical displacement distribution w for different values of the gravity field

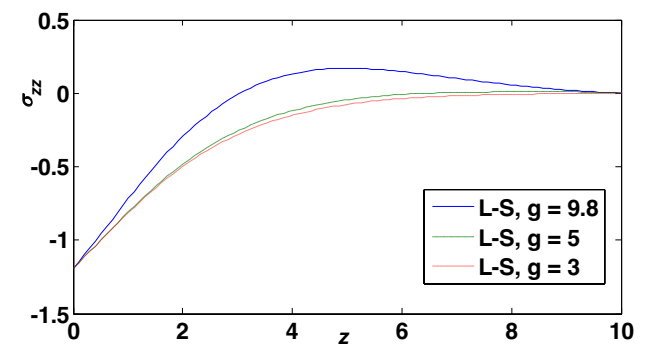


Fig. 3 Distribution of stress component σ_{zz} for different values of the gravity field

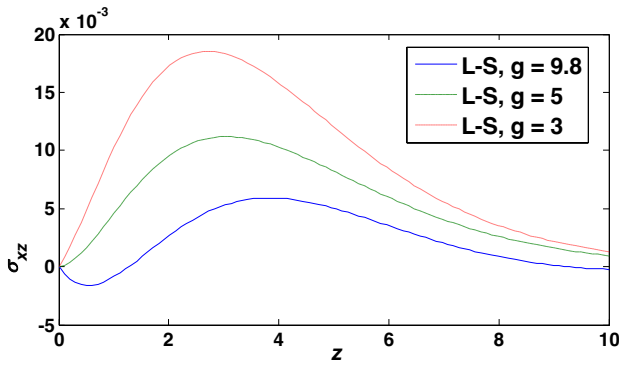


Fig. 4 Distribution of stress component σ_{xz} for different values of the gravity field

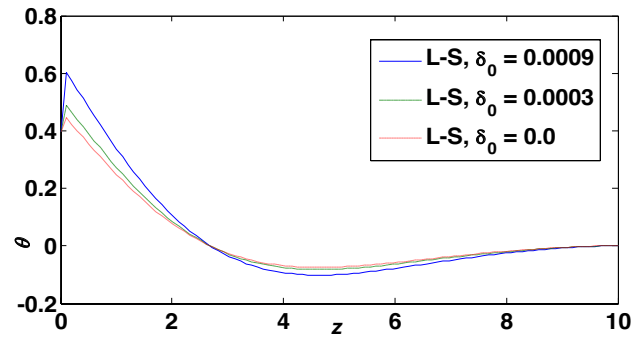


Fig. 6 Thermal temperature distribution θ for different values of an empirical material constant

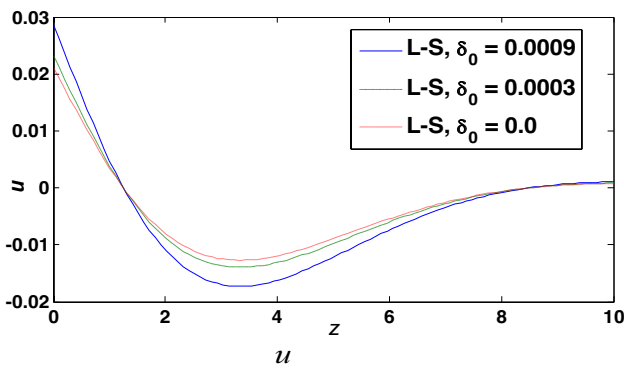


Fig. 5 Horizontal displacement distribution u for different values of an empirical material constant

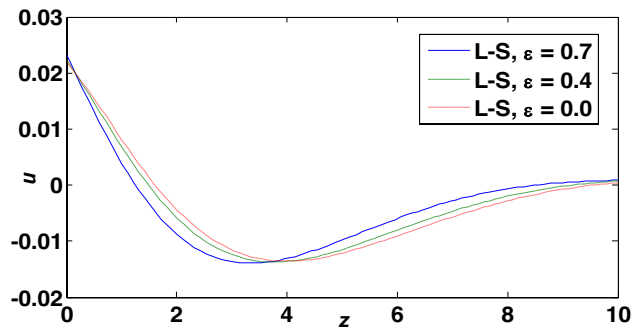


Fig. 7 Horizontal displacement distribution u for different values of a nonlocal parameter

component σ_{zz} begins with a negative value and satisfies the boundary conditions. Values of σ_{zz} increase in the range $0 \leq z \leq 10$. With increasing the value of g , values of σ_{zz} increase. Figure 4 depicts that the variations of stress component σ_{xz} begin from a zero value at $z = 0$ and satisfy the boundary conditions. With increasing the value of g , values of σ_{xz} decrease.

Figures 5 and 6 display the horizontal displacement components u and thermodynamic temperature distributions θ for the nonlocal thermoelastic media with different values of an empirical material constant δ_0 . Figure 5 depicts that the variation of horizontal displacement distribution u starting with positive values. Values of u decrease attain their minimum values in the range $0 \leq z \leq 3$ and then increase in the range $3 \leq z \leq 10$. With increasing the value of δ_0 , values of u increase and then decrease. Figure 6 depicts that the variation of thermodynamic temperature θ starting with a positive value. Values of θ decrease in the range $0 \leq z \leq 10$. With increasing the value of δ_0 , values of θ increase and then decrease.

Figures 7 and 8 display the horizontal displacement components u and stress component σ_{xz} distributions for the

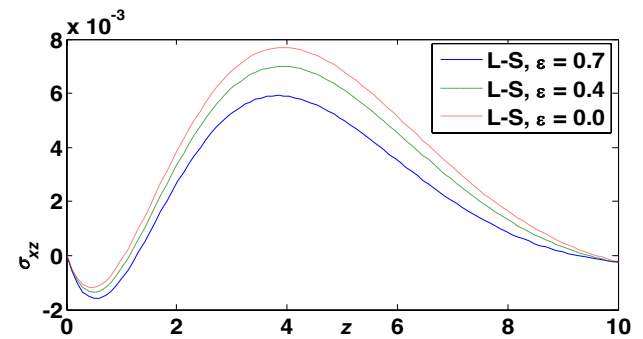


Fig. 8 Distribution of stress component σ_{xz} for different values of a nonlocal parameter

thermoelastic media with different values of a nonlocal parameter ϵ . Figure 7 depicts that the variation of horizontal displacement distribution u starting with decreasing to reach its maximum values and then increases. The increasing of the value of ϵ causes decreasing values of u and then increases. Figure 8 depicts that the variations of stress component σ_{xz} begin from a zero value and obey the boundary conditions. The values of

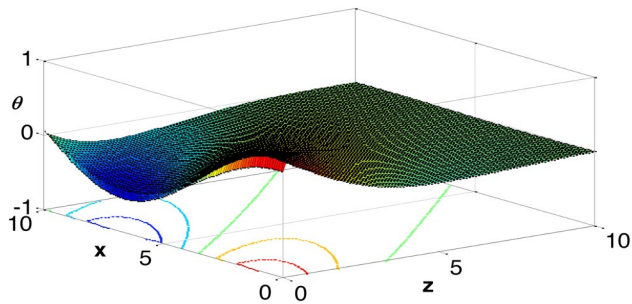


Fig. 9 Thermal temperature distribution θ in three-dimensional

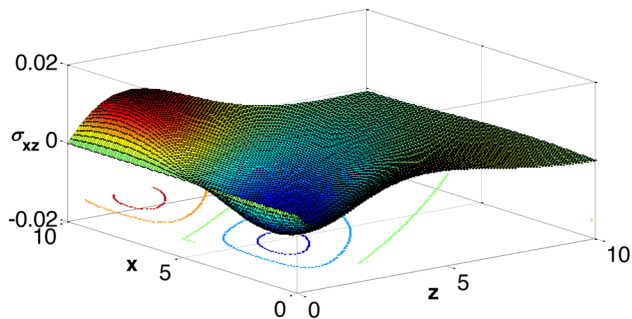


Fig. 10 Distribution of stress component σ_{xz} in three-dimensional

σ_{xz} attain their minimum values in the range $0 \leq z \leq 0.5$, then increase attain their maximum values in the range $0.5 \leq z \leq 4$, and again decrease. The increasing of the value of ε causes decreasing values of σ_{xz} . Figures 9 and 10 display 3D distributions of the non-dimensional thermal temperature θ and stress component σ_{xz} . These figures are very important to study the dependence of these physical quantities on the vertical component of distance.

Conclusion

The present theoretical results may provide interesting information and a mathematical foundation for working on the subject, because the increasing interest in the theory of thermoelasticity has many applications in such diverse fields as geophysics, acoustic wave damping in a magnetic field, machine element design of such equipment as heat exchangers, boiler tubes, nuclear devices emitting electromagnetic radiations, the development of magnetometers that are high in sensitivity and are super-conducting, the engineering of electrical power, plasma physics, etc. The new generalized nonlocal thermoelasticity model predicts novel characteristics for temperature, displacement, stresses, and strain. The conversations that have been held have led to the following conclusions:

- The nonlocal parameter plays a big part in how the physical fields are distributed.
- The distributions of the physical fields are significantly influenced by the gravity field.
- This impression supports the notion that the L–S theory is unquestionably a theory of generalized thermoelasticity.
- The physical fields are significantly impacted by the temperature-dependent characteristics.
- The physical fields are significantly impacted by the vertical distance.
- Even if a nonlocal thermoelastic medium is switched out for a thermoelastic one, the technique is still valid.

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Declarations

Conflict of interest The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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