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ORIGINAL PAPER

Determination of Some Elastic Constants of Materials Using Impact Analysis

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Abstract

Materials with increased stiffness and lesser weight are widely used in aerospace, automotive and other manufacturing industries. Nondestructive testing (NDT) is used to identify elastic characteristics of materials such as Young’s modulus and stiffness. NDT is popular due to its several advantages over destructive testing methods. This paper presents an algorithm for determination of elastic constants using Thomson multi-tapered periodogram. A Thomson multi-tapered periodogram is used to estimate the fundamental frequency of the material under consideration. The experimental set up comprises of mechanical assembly for free ball impact testing along with data acquisition hardware. The impact induced vibration signal obtained is preprocessed and power spectral density is obtained using Thomson multi-tapered periodogram to estimate the fundamental frequency of the material. Elastic constants as Young’s modulus and stiffness are determined using the estimated fundamental frequency of the test specimen. The experimental results are validated using finite element method technique (ANSYS). The experimentation is carried out for two different materials stainless steel SA 240 Gr 304 and copper with varying test conditions such as change in weight of the ball, change in release height of the ball and change area of plate. The Average percentage error in estimating the fundamental frequency for SS is observed to be 1.72% and 3.86% or copper. Average percentage error for computing Young's modulus of SS is observed to be 2.62% and 7.75% for copper. The experimental analysis shows that the proposed technique is robust to noise. The proposed method has been successfully used to obtain some elastic constants.

Keywords Elastic constants · Non-destructive testing · Vibration signal · Natural frequency · Thomson multi-taper periodogram

Introduction

Composite materials are widely used in manufacturing, automobile, civil industries. Analytical results of free vibration of rectangular plates are presented by A. W. Leiss in [1]. The Rayleigh–Ritz technique used for calculation of model of rectangular orthotropic plates is presented by L. R. Deobald and R. F. Gmson [2]. Shun-Fa Hwang Chao-Shui Chang proposed Finite element analysis method and optimum design to find the elastic constants [3]. Use of genetic algorithms (GAs) and finite element method to determine the elastic constants is presented in by C. Maletta and L. Pagnotta in [4]. The limitation of GAs is their higher computational complexity.

M. Alfano, L. Pagnotta present determination of Poisson’s ratio and dynamic Young’s modulus of the thin square plates of Carbon Steel specimens using first two frequencies of vibration test in [5].

Alain Giraudeau et al. propose deflectometry technique to find stiffness and damping parameters from thin vibrating plates [6]. These tests use high speed camera to tackle this problem. Determination of stiffness and damping properties of vibrating structures using Force Analysis Technique (FAT) is elaborated by Frederic et al. [7].

R. Rikardsa et al. present a technique to determine elastic properties of laminates plate based on experiment design and mathematical models using the finite element solutions

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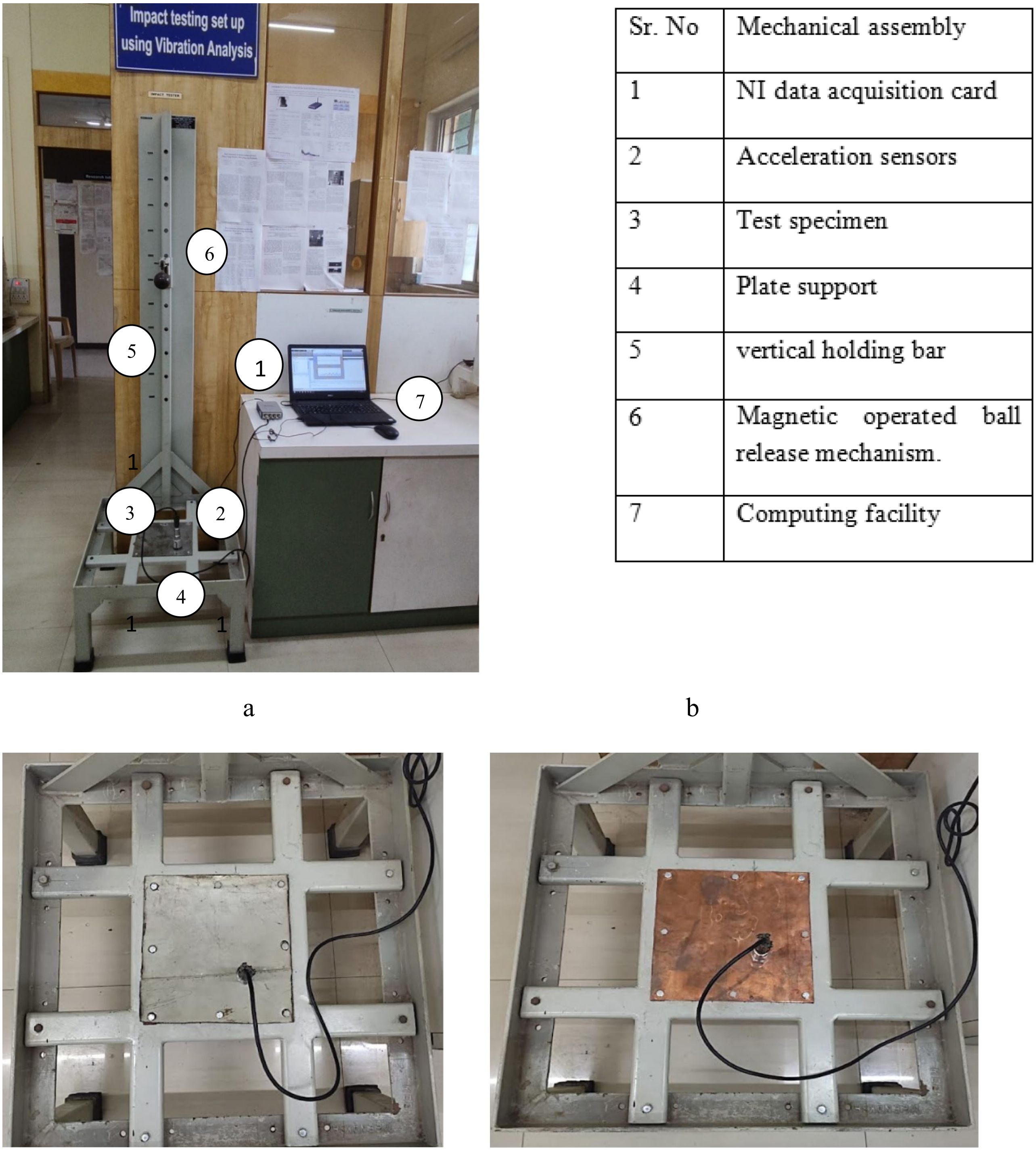


Sr. No	Mechanical assembly
1	NI data acquisition card
2	Acceleration sensors
3	Test specimen
4	Plate support
5	vertical holding bar
6	Magnetic operated ball release mechanism.
7	Computing facility

a

b

c

d

Fig. 1 **a** Free ball impact testing set up. **b** List of parts of Free ball impact testing set up. **c** Sample stainless steel SA 240 Gr 304 with 8 clamp at corners. **d** Figure copper plate with 8 clamp at corner

[8]. Mario Acosta-Floresa et al. present analytical model for the mechanical analysis of global stresses in symmetrical laminated composite materials subject to axial load in [9]. Tom Lauwagiea et al. compare the results of three different mixed numerical experimental methods for determining elastic properties of materials. Results obtained with typical

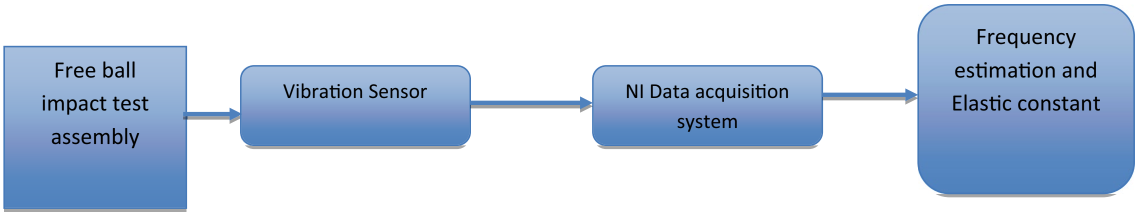


Fig. 2 Block diagram of the test set up

resonant beam and tensile tests are used for evaluation [10]. Arpita Mandal et al. propose a technique consisting of laminated constitutive model and a constitutive model. The elastic properties of core of a symmetric metal sandwich plate are computed using the model and its analysis [11].

Determination of fundamental frequency of the metallic plate using Fast Fourier Transform is presented by Vijaykumar et al.in [12]. The work analyzes the vibration signal obtained from free ball impact on the test metallic sheet. Use of ARX model for determining fundamental frequency of the test metal sheet is presented in [13]. The results are compared with FFT technique. Use of Welch's Periodogram for estimation of fundamental frequency of the composite materials is presented in [14].

A new technique to determine elastic constant of plate using loudspeaker and scanning laser interferometer for evaluating resonance response is presented by Marco Matter Thomas Gmür in [15]. Rayleigh–Ritz method to select the modal analysis of a plate is proposed by W. L. Li in [16]. ZijianWangab Jianxun Liu proposes a nondestructive method, utilizing longitudinal guided waves at low frequency and flexural guided waves at high frequency, to approximate bar waves and transverse bulk waves, respectively. According to the approximate velocities of bar waves and transverse bulk waves, the elastic constants can be determined inversely [17]. F. Bucciarelli & G. P. Malfense Fierro present use of sound wave to find the elastic constant [18]. Jing Yang and Jianchun Cheng propose wavelet transform and an artificial neural network for determination of elastic constants of fiber-reinforced plate in [19]. Yapeng Li; Tianran Liu propose elastic constants determination using ultrasound longitudinal and shear wave velocities [20].

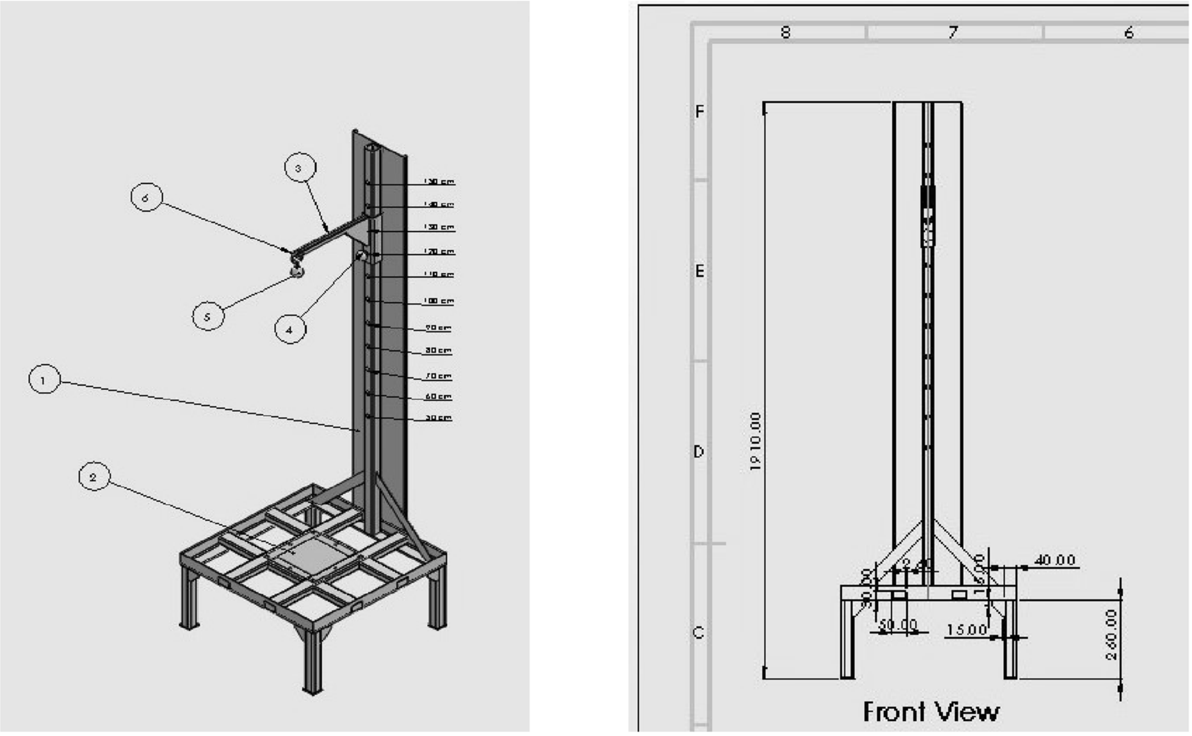
This paper presents a technique for estimating natural frequency of a material using Thomson Multi taper Periodogram method. The estimated frequency is further used to derive the elastic constants as Young's modulus and stiffness of the test material. The details of the technique are elaborated in Sect. 2.

Impact Testing Setup

A free ball testing setup is shown in Fig. 1a and b. It consists of data acquisition card, acceleration sensors and necessary mechanical assembly. The block diagram of the test set up is as shown Fig. 2.

Mechanical Assembly for Free Ball Impact Test Setup

The design details of free ball impact test set up is as shown in Fig. 3a. It consists of a fixed bottom frame to which the test specimen metal sheet is clamped. The acceleration sensors are placed at 37.24 mm from centre on the test specimen sheet as shown in Fig. 4. The vertical holding bar holds the ball used for impact testing. A vertical support panel holds the holding bar in upright position. The release mechanism of the impact ball is magnetically operated and can be used to release the ball at different heights ranging 50 cm to 150 cms with a step of 10 cms. The stand is manufactured using Wrought Stainless Steel with density 8000 kg / m^3. The stand lays on the rubber bushings on the plane ground. The weight of the stand is 32.05 kg which ensures stability of the assembly upon impact. The weight of the entire free ball impact assembly is approximately 37 kg. Two steel ball having weights 300 g and 200 g, respectively, are used for experimentation. Test specimen sheets of material stainless steel SA 240 Gr 304 and Copper of sizes 220 mm × 220 mm and with thickness 1.3 mm, 1 mm are used. The vibrations induced by falling ball on the test specimen sheet are sensed using MIL 521 piezoelectric acceleration sensor. The acceleration sensor location find out by ANSYS model which is 37.24 mm from center. The acceleration sensor is interfaced with MATLAB using data acquisition card NI cDAQ- 9234. The card is 24 bit with highest sampling frequency of 51.2 Ks/s. The vibration data is stored in Microsoft Excel format. Further vibration data acquired during free ball impact test is analyzed using Thomson



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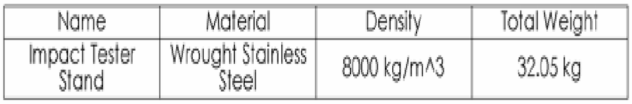
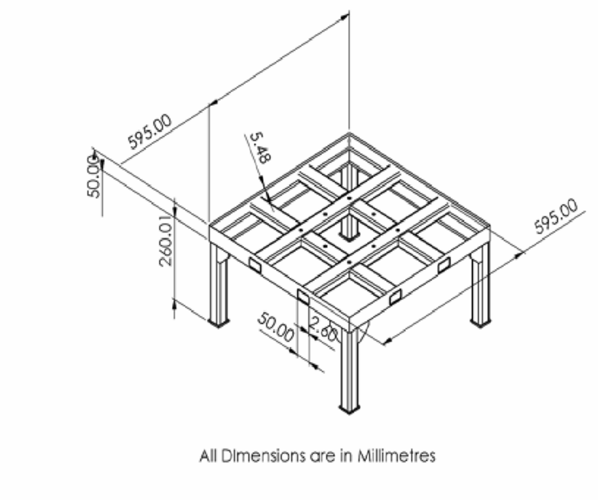


Name	Material	Density	Total Weight
Impact Tester Stand	Wrought Stainless Steel	8000 kg/m^3	32.05 kg



c

Fig. 3 **a** Isometric view of free ball impact test setup. **b** Design details of free ball impact testing assemble. **c** Dimensions and weight details of impact test set up stand

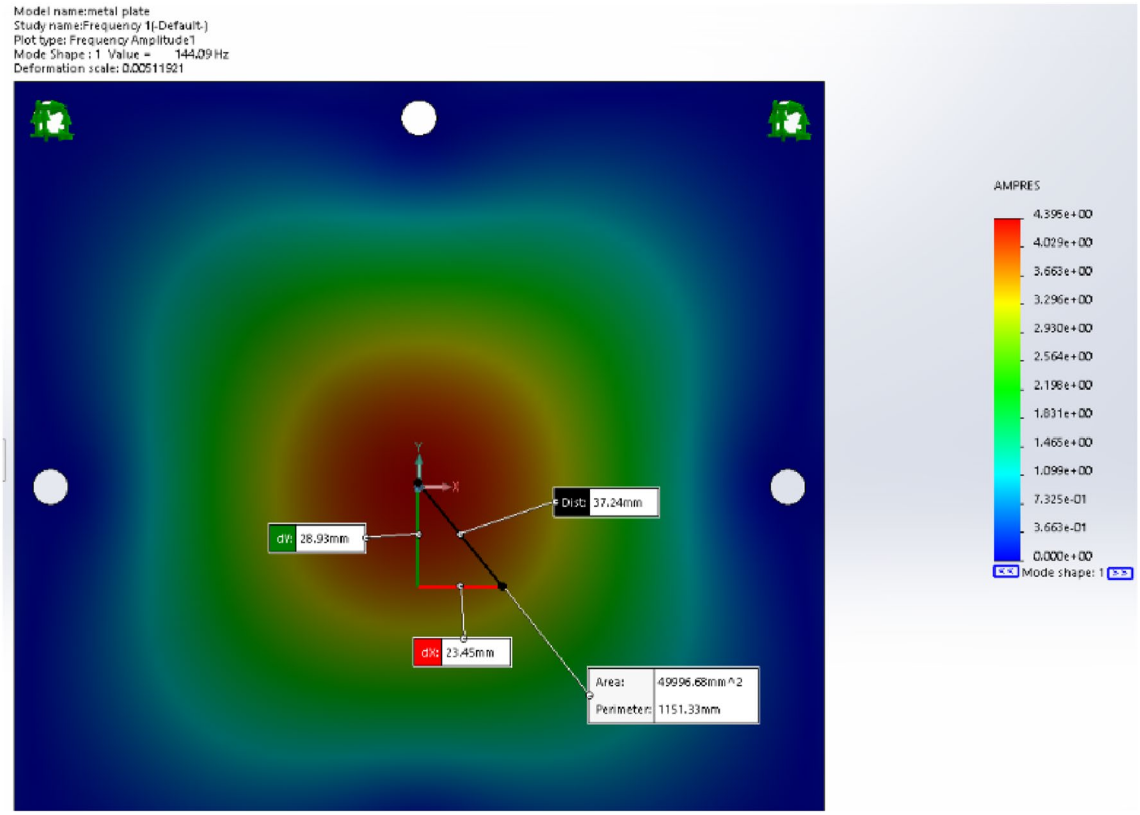


Fig. 4 Location of acceleration sensor on specimen sheets 37.24 mm from center

multi-taper periodogram. The algorithm for determining elastic constants is implemented using MATLAB 2018 platform.

Estimation of Fundamental Frequency and Determination of Elastic Constants

Elastic constants of the material such as Young’s Modulus and stiffness are dependent on the fundamental frequency of the material. Therefore, accurate estimation of the fundamental frequency of the material under consideration is of prime importance. In this paper, Thomson Multi-taper periodogram is used to estimate the fundamental frequency. This frequency is further used to compute mentioned elastic constants. The flowchart of the proposed technique for frequency estimation and determination of elastic constants is given in Fig. 5.

Filtering and Pre-processing

The vibration signal obtained from free ball impact on the test sheet is acquired in using accelerometer sensor MIL521 and NI cDAQ-9234 DAC card. The sampling frequency for data acquisition is set at 5.1 kHz. Initially the input signal is filtered using Hanning window. The advantage of using Hanning Window method is that it reduces the side lobes. The Hanning window is specified by Eq. (1) [21].

$$w(n) = 0.5\left(1 - \cos\left(2\pi\frac{n}{N}\right)\right), \quad 0 \le n \le N. \tag{1}$$

Window length, L = N + 1.

Computation of PSD Using Thomson Multi-taper Periodogram

The power spectral density of the filtered input signal is obtained using Thomson Multi taper periodogram.

Thomson's multitaper method estimate the power spectrum of a signal from N equally spaced samples by averaging K tapered periodograms as shown in Fig. 6. Brief overview of the standard nonparametric spectral estimation and the multitaper spectral estimation is presented in [22]. Discrete prolate spheroidal sequences (DPSS) are used as tapers since they provide excellent protection against spectral leakage

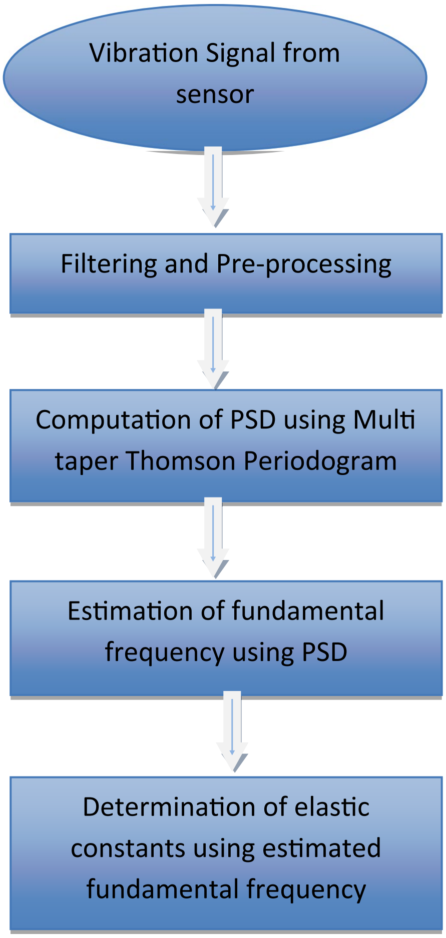


Fig. 5 Flowchart of the proposed technique

[23]. *K* modified periodograms obtained from Slepian sequence as window is used in Multi taper periodogram method. The multitaper periodogram using *k*th Slepian sequence, $g_k(n)$ is represented using Eq. (2) [24].

$$Sk(f) = \Delta t\left[\sum gk(n)x(n)e^{-j2\Pi fn\Delta t}\right]^2, \tag{2}$$

where
$sk(f)$ is discrete prolate spheroidal sequences (DPSS).
X(n) is input signal.
$g_k(n)$ is *k*th Slepian sequence.
K is averaging K tapered periodograms.
N is number of eigenvectors.
$e^{-j2\pi\pi fn\Delta t}$ is Fourier transform.

$\mathrm{e}^{-\mathrm{j}2\pi\mathrm{fn}\Delta\mathrm{t}}$ is the multitaper method averages the *K* modified periodograms to produce the multitaper PSD estimate as given by equation [3].

$$(s)^{(MT)}(f) = \frac{1}{k}\sum_{k=0}^{k-1} Sk(f), \tag{3}$$

$(s)^{(MT)}(f)$ is multitaper spectral estimate.
K is averaging K tapered periodograms.

The multitaper method averages modified periodograms obtained using a orthogonal windows or tapers therefore minimizers the variability in the estimate of PSD [24]. The tapers have optimal time and frequency resolution. Orthogonality and optimal time–frequency resolution of Thomson Multi taper periodogram makes it effective for estimating the spectral density. In this work, the number of tapers required for analyzing the vibration signal is elaborated in Sect. 4. Further the elastic constants of the test specimen are derived as given in the sub Sect. 3.3

Determination of Young's Modulus and Stiffness

The frequency estimated using the Thomson multi taper periodogram is further used to compute the elastic constants as Young's modulus and stiffness of the test specimen.

Young's Modulus

Young's modulus of the test specimen can be determined from the natural frequency using Eq. (4) as mentioned in [25, 26].

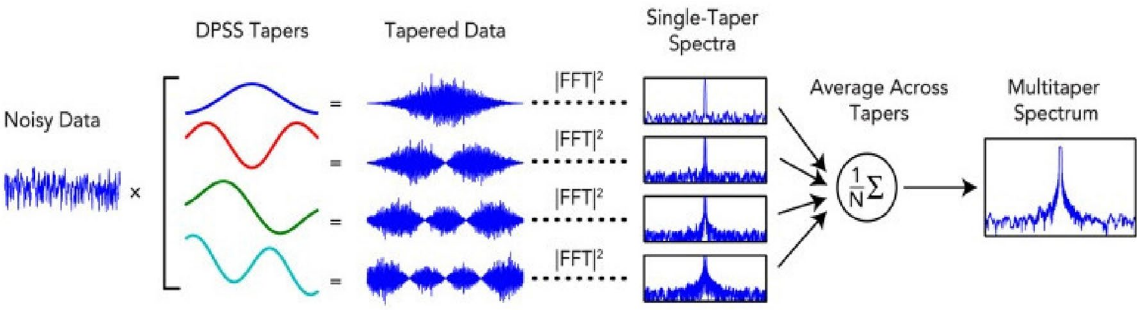
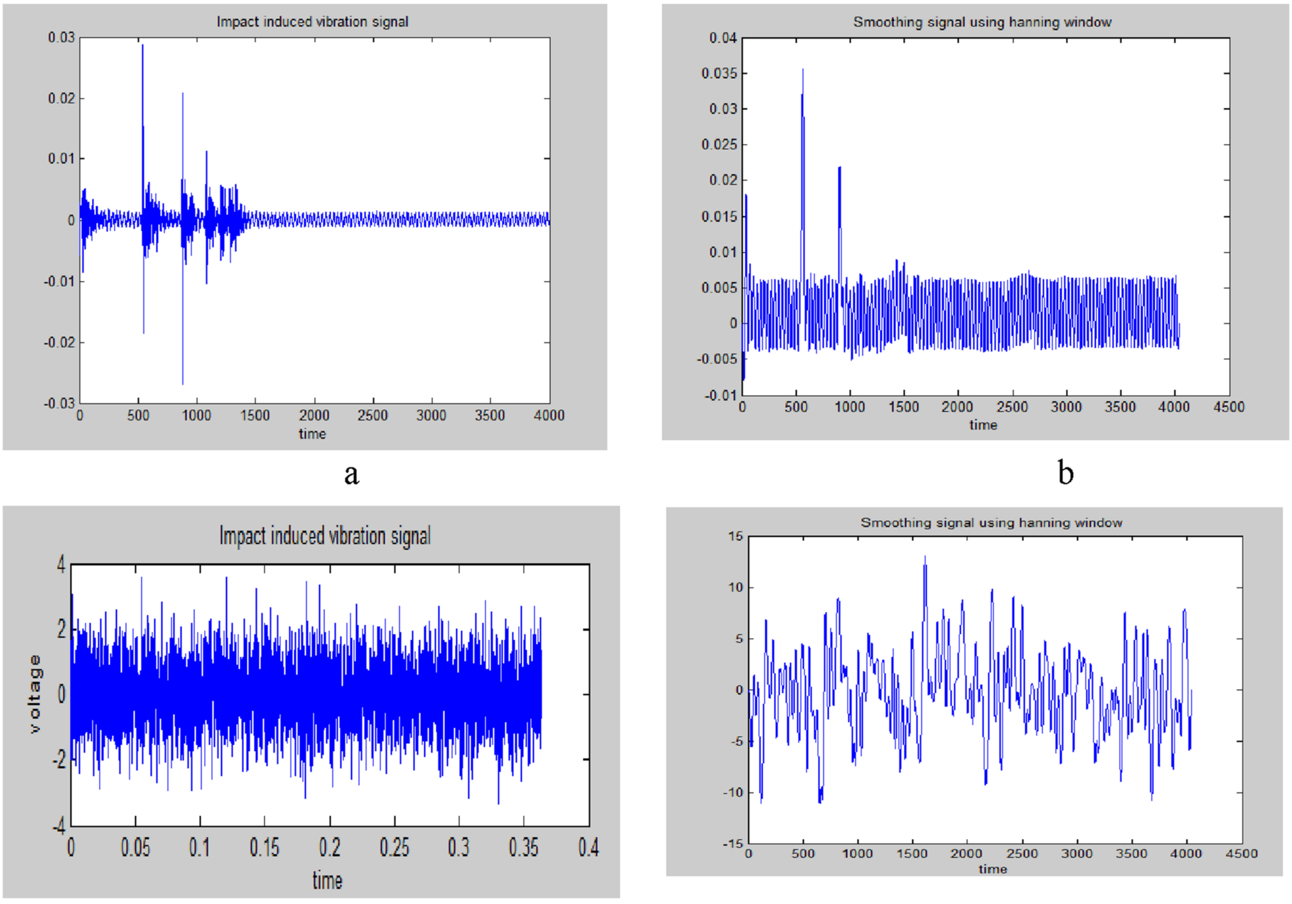


Fig. 6 Estimation of PSD using Thomson Multi taper periodogram [22]



a b c d

Fig. 7 **a** Impact induced vibration signal for stainless steel specimen sheet. **b** Preprocessing of vibration signal in ausing Hanning window **c** Impact induced vibration signal for copper specimen sheet. **d** Preprocessing of vibration signal in c using Hanning window

$$E = \frac{48}{\pi^2}\left(\frac{f}{\lambda(\upsilon)}\right)^2 \frac{ma^3\left(1-\upsilon^2\right)}{bt^3}, \tag{4}$$

where
λ = non-dimensional frequency factor.
ν = poisson's ratio.
m = mass of specimen.
f = natural frequency of specimen.
a = length of specimen.
b = width of specimen.
t = thickness of specimen.

Stiffness

Stiffness is the ability of a material to maintain external loads without changes of its deformations. It is given by Eq. (5). [27]

$$k = \frac{E * A}{L}, \tag{5}$$

where E = modulus of elasticity.
A = area.
L = length of material.

Experimental Results and Discussion

The proposed method for estimating fundamental frequency and determination of Young's modulus and stiffness is evaluated on test specimen sheets of materials stainless steel SA 240 Gr 304 and Copper of size 220 mm × 220 mm × 1.3 mm and 220 mm × 220 mm × 1 mm, respectively. The vibrations induced by the falling ball on the test specimen sheet are sensed using MIL 521piezoelectric acceleration sensor. The acceleration sensors are interfaced with MATLAB using data acquisition device NI cDAQ-9234. The card is 24 bit with highest sampling

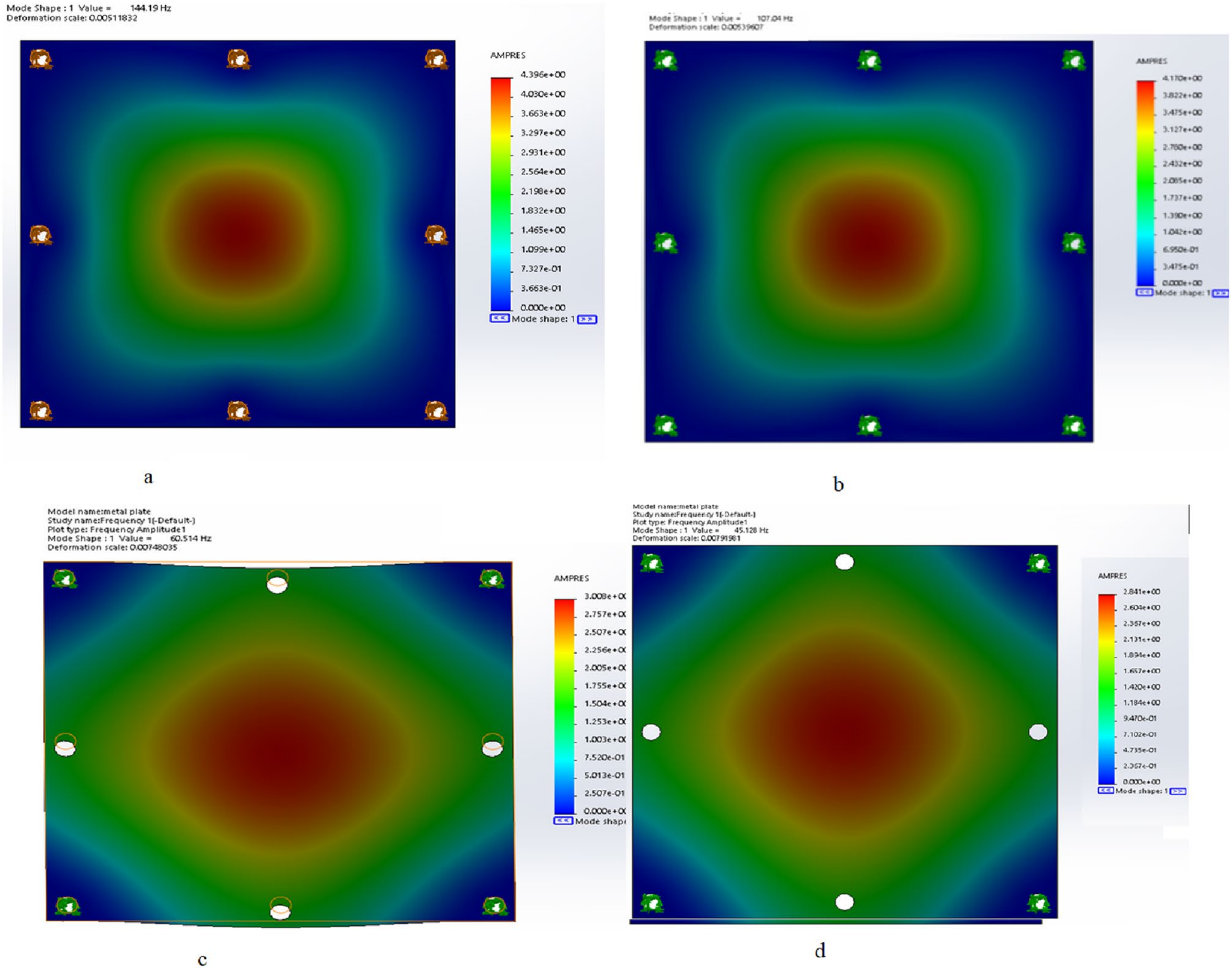


Fig. 8 **a** Stainless steel specimen sheet 220 × 220 × 1.3 mm ANSYS first modal natural frequency is 144.19 Hz for 8 clamp edge. **b** Copper specimen sheet 220 × 220 × 1 mm ANSYS first modal natural frequency is 107.04 Hz. for 8 clamp edges. **c** Stainless steel specimen sheet 220 × 220 × 1.3 mm ANSYS first modal natural frequency is 60.51 Hz for 4 clamp edge. **d** copper specimen sheet 220 × 220 × 1 mm ANSYS first modal natural frequency is 45.12 Hz. for 4 clamp edges

frequency of 51.2 Ks/s. The impact induced vibration signal is further smoothened using Hanning window.

The size of the Hanning window is given by [21]

$$w = f * SR, \tag{6}$$

where

W = window size.

SR = sampling Rate of the impact induced vibration signal.

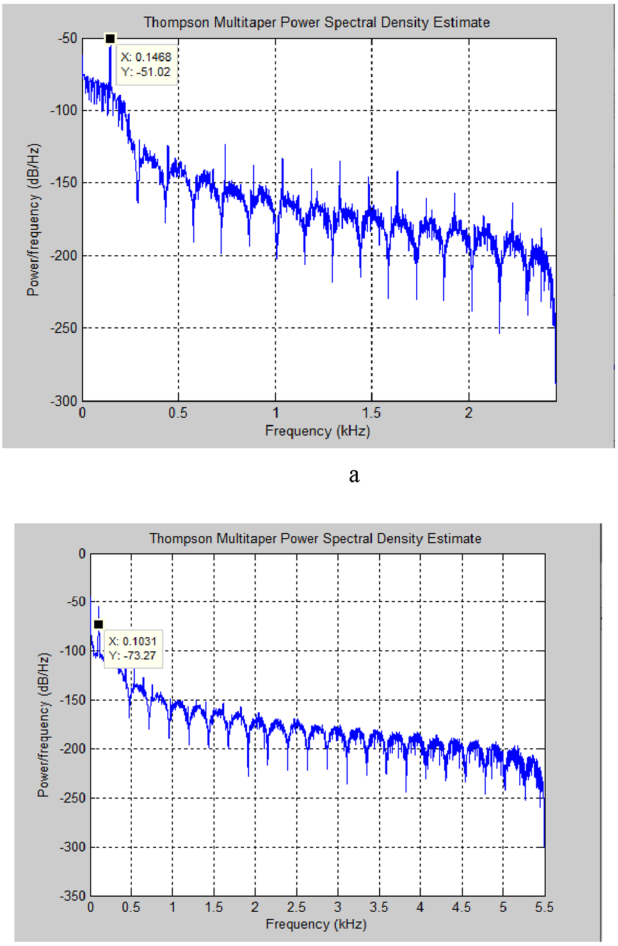
f = natural frequency of signal.

The Young’s modulus of stainless steel SA 240 Gr 304 material is observed to be in the range 200–215 [28]. Therefore, the natural frequency of stainless steel SA 240 Gr 304 material lies in the range 142 to 147 with mean of 144.5. To determine the window size of the Hanning window using Eq. (6) and considering sampling frequency of 5000 Hz, the size of the Hanning window is obtained as 35 samples for stainless steel SA 240 Gr 304 specimen sheet.

The Young’s modulus of Copper is observed to be in the range 125—135 [28]. Therefore, the natural frequency of Copper lies in the range 103 to 107 with mean 105. The size of the Hanning window is obtained as 48 samples for copper specimen sheet.

The vibration signal after impact without preprocessing for stainless steel SA 240 Gr 304 and Copper are as shown in Fig. 7a and c. Vibration signal after smoothing using Hanning window for SS and Copper test specimens are is shown in Fig. 7b and d, respectively.

Further the power spectral density of the smoothed vibration signal is computed using Thomson multi taper



a

b

Fig. 9 **a** Typical plot of power spectral density Stainless steel specimen sheet 220×220×1.3 mm. **b** shows typical plot of power spectral density copper specimen sheet 220×220×1 mm

periodogram. The number of tapers used to estimate the power spectral density of the vibration signal are determined using Eq. (7) given below [29].

$$L = 2TW - 1, \quad (7)$$

where

L—number of tapers of Thomson multi taper periodogram.

TW—time-bandwidth product.

$$\mathrm{TW} = N\,\Delta f/2, \quad (8)$$

where

N = size of window in second.

Δf = frequency resolution.

The vibration signals of length 16,400 samples and sampling frequency 5000 Hz;

N = 16,400/5000 = 3.28 s.

N = size of window in second.

We have considered n = 2500 samples for determining FFT of the vibration signal. Considering sampling frequency of 5000 Hz, the frequency resolution obtained as.

Fs = 5000, n = 2500, Fs/n = 5000/2500 = 2 Hz.

Hence, using Eq. (8)TW = N Δf/2 = (3.28 s *2 Hz)/2 = 3.28.

Therefore required number of tapers L = [2TW] − 1 = [2*3.28] − 1 = 5.56 tapers. We have used Thomson multi taper periodogram with 6 tapers to estimate the fundamental frequency of the input vibration signal.

Experimentation on proposed method is carried out by varying the release height of ball, weight of the ball and varying area of the test specimen sheet. The free ball impact set up has magnetic release mechanism to release the metallic ball to create impact from varying heights. The metal balls weighing 300 g and 200 g are used. Test specimen

Table 1 Estimated frequency and average error for Specimen sheets of SS and Copper with dimensions 220×220×1.3 mm and 220×220×1 mm using 300 gm ball for creating impact

	SS specimen plate					Copper specimen plate			
Sr no.	Height of Impact in cm	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average Error (%)	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average Error (%)
1	50	144.19	146.60	1.67	**1.85%**	107.04	101.30	5.36	**3.88%**
2	70		145.02	1.44			102.04	4.67	
3	90		146.80	1.81			103.10	3.68	
4	110		146.60	1.67			103.48	3.32	
5	130		147.32	2.17			103.60	3.21	
6	150		147.60	2.36			103.60	3.21	
	Average Frequency and error		**146.65**	**1.85%**			**102.85**	**3.88**	

Table 2 Estimated frequency and average error for specimen sheets of SS and Copper with dimensions 220 × 220 × 1.3 mm and 220 × 220 × 1 mm using 200gm ball

	SS specimen plate					Copper specimen plate			
Sr no.	Height of Impact in cm	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average Error (%)	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average Error (%)
1	50	144.19	146.10	1.32	**1.59%**	107.04	101.80	4.89	**3.84%**
2	70		146.12	1.33			102.02	4.68	
3	90		146.10	1.32			102.92	3.84	
4	110		146.50	1.60			102.92	3.84	
5	130		147.02	1.96			103.90	2.93	
6	150		147.10	2.01			103.94	2.89	
	Average Frequency and error		**146.49**	**1.59**			**103.07**	**3.84**	

sheets of SS and Copper of sizes 220 × 220 × 1.3 mm and 220 × 220 × 1 mm is used for experimentation. The release height of the ball is varied from 50 to 150 cm with the interval of 20 cm.

The estimated values of the Natural frequency of the test specimen sheets are compared with results obtained from finite element method (FEM) using ANSYS software version ANSYS 2021R1. The FEM is widely used and powerful numerical approximate method and has been used by many researchers for performance measurement [17, 23]. The ANSYS software solve complex structural engineering problems and make better, faster design decisions with the finite element analysis (FEA). It consists of a discretization of the element into a finite number of generally triangular or rectangular elements [23, 30].

Specimen sheets of stainless steel SA 240 Gr 304 and Copper of dimensions 220 × 220 × 1.3 mm and 220 × 220 × 1 mm clamped at 8 vertices are simulated in ANSYS for determining the natural frequency as shown in Fig. 8a, b.The FEM analysis ofstainless steel SA 240 Gr 304specimen sheet and Copper specimen sheet is illustrated in Fig. 8a and b. The natural frequencies for stainless steel SA 240 Gr 304 and Copper are obtained as 144.19 Hz and 107.04 Hz, respectively. The Specimen sheets of stainless steel SA 240 Gr-304 of dimension 300 × 300 × 1.3 mm clamped at 8 edges simulated in ANSYS for determining the natural frequency is 91.97 Hz [31].

Figure 9a, b shows plot of power spectral density obtained using Thomson multi-tapered periodogram induced of the vibration signal obtained by releasing the metallic ball of weight 200 gm at the height of 90 cm for SS and Copper specimen sheets, respectively.

Tables 1 and 2 present values of estimated natural frequency and average percentage error for SS and Copper specimen sheets for varying the release height of the ball from 50 to 150 cm with the interval of 20 cm using 300 g and 200 g metallic balls for creating impact, respectively. The average error is computed with respect to the natural frequency obtained by FEM (ANSYS).

Table 3 presents values of estimated natural frequency and average percentage error for SS sheets 300 × 300 × 1.3 mm for varying the release height of the ball from 50 to 150 cm with the interval of 20 cm using 200 gm metallic balls for creating impact, respectively. The average error is computed with respect to the natural frequency obtained by FEM (ANSYS).

Table 3 Estimated frequency and average error for specimen sheets of SS with dimensions 300 × 300 × 1.3 mm using 200 gm ball

	SS specimen plate with 300 × 300 × 1.3 mm				
Sr. no.	Height of Impact in cm	Frequency obtained by FEM (Ansys software)	Estimated frequency using proposed method	Error (%)	Average Error (%)
1	50	91.97	92.01	0.04	1.10%
2	70		92.30	0.3	
3	90		92.81	0.91	
4	110		93.35	1.50	
5	130		93.91	2.10	
6	150		93.60	1.77	
	Average Frequency and error		92.99	1.10	

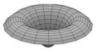


Table 4 Estimated frequency for specimen sheet of SS (220 × 220 × 1.3 mm) and Copper (220 × 220 × 1 mm) size in presence of additive noise

SS specimen plate				Copper specimen plate	
Sr. no.	Signal to Noise ratio(SNR)	Natural frequency	% error	Natural frequency	% error
1	80	146.60	2.31%	103.90	1.05%
2	70	146.60		103.90	
3	60	146.60		103.90	
4	50	146.60		103.90	
5	40	146.60		103.90	
6	30	146.60		103.90	
7	20	**150.0**		104.01	
8	10	**151.11**		104.02	

The proposed technique for estimating fundamental frequency is also tested by adding noise of varying power to

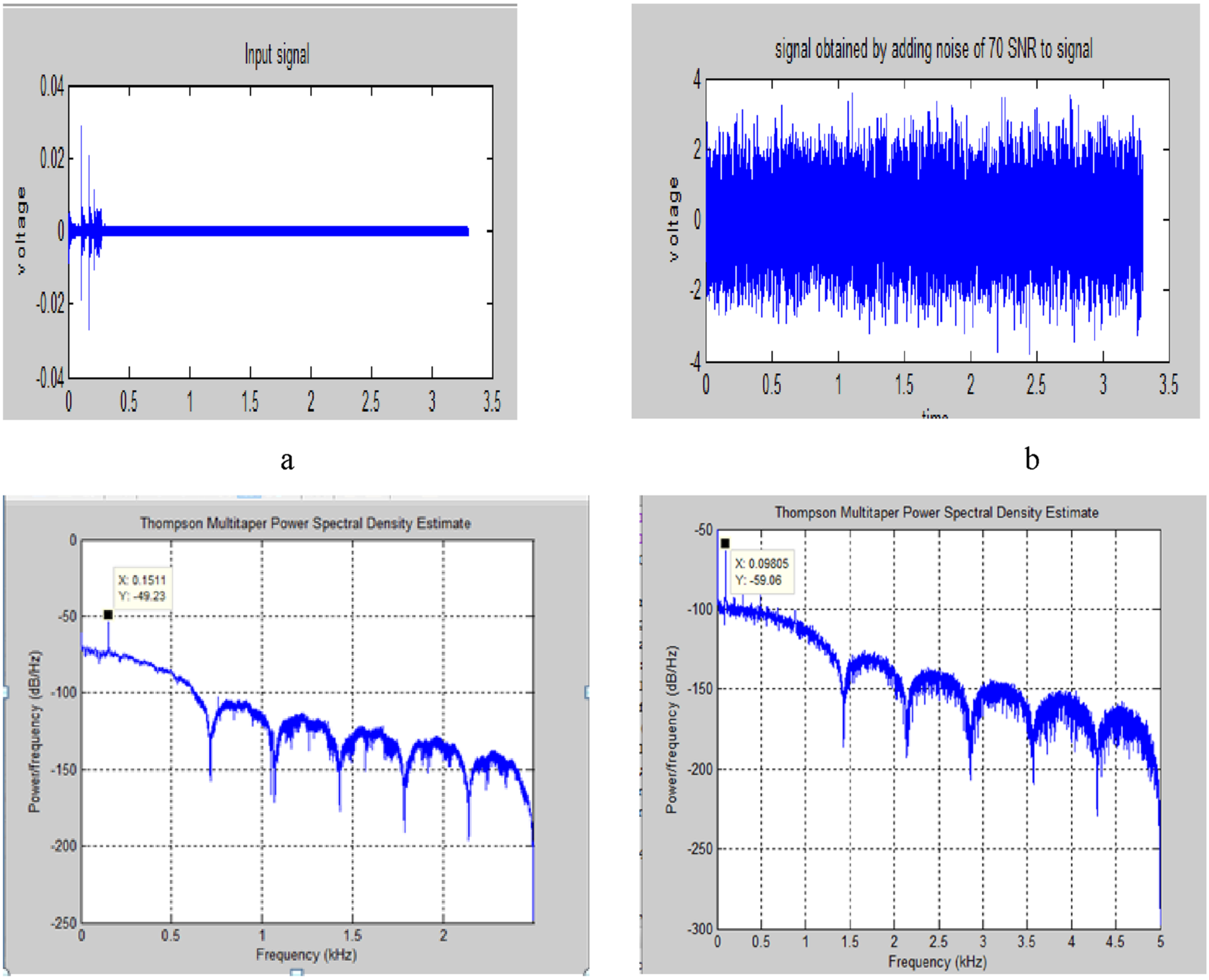


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Fig. 10 **a** Typical impact induced vibration signal. **b** Plot of signal obtained by adding noise of 70 SNR to signal **a**. **c** Plot of power density of corrupted vibration signal for SS. d Plot of power density of corrupted vibration signal for copper plate

Table 5 Elastic constants of SS and Copper plate

SS plate size of 220×220×1.3 mm				Copper plate of size of 220×220 x 1 mm			SS plate size of 300×300×1.3 mm		
Parameter	Finite element method by ANSYS	Proposed method	Average error (%)	Finite element method by ANSYS	Proposed method	Average error (%)	Finite element method by ANSYS	Proposed method	Average error (%)
Fundamental Frequency (Hz)	144.19	146.57	1.62%	107.04	102.85	3.86%	91.97	92.99	1.10%
Young's Modules (N/m^2)	187.69	192.62	2.62%	133.82	123.65	7.75%	195.227	199.62	2.25%
Stiffness (N/m^2)	17.06	17.58	0.3%	12.55	11.57	7.80%	17.88	18.28	2.23%

the vibration signal. Figure 10a shows the impact induced vibration signal SS plate and Fig. 10b illustrates the vibration signal by adding noise of 70 SNR. Plot of power spectral density of the corrupted vibration signal is shown in Fig. 10c. It is observed that the Thomson multi taper periodogram is robust for signal to noise ratio greater than 70% (Table 4).

The comparison of elastic constants estimated using proposed technique and obtained using FEM ANSYS has been carried out. The details of the comparison are presented in Table 5. The average error in estimating fundamental frequency, Youngs modulus and Stiffness for SS 304 specimen sheet with dimension 220×220×1.3 mm is observed to be 1.62%, 2.62% and 0.3%, respectively. The average error in estimating fundamental frequency, Youngs modulus and Stiffness for Copper specimen sheet with dimension 220×220×1 mm is observed to be 3.86%, 7.75% and 7.80%, respectively.

Conclusions

The proposed method effectively uses Thomson Multi taper periodogram for estimating the fundamental frequency for determining some of the elastic constants. Experimentation is carried out on stainless steel and copper plates by varying the weight of the ball used for impact, changing the release height of the ball and change in area of the test specimen sheet. The estimated frequency and elastic constants are compared with the results obtained by finite element method technique (ANSYS). The proposed method estimates natural frequency and elastic constants with absolute error of be 1.62% and 3.86% for SS and copper, respectively. The experimental results are observed to be invariant to size of specimen, weight of the ball used for impact and release height of the ball. The proposed method is also tested under presence of zero mean Gaussian additive noise. The method is observed to be robust to noise. The average percentage error in presence of noise is observed to be 2.31% for SS and 1.05% for copper. It proves that the Thomson multitaper periodogram correctly estimate the natural frequency in presence of noise. Hence, this technique can be effectively used to estimate natural frequency and some elastic constants under noisy environment.

Declarations

Conflict of interest The contents of this manuscript are not submitted to any other journal. There are no conflicts of interest.

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