Journal of Vibration Engineering & Technologies (2021) 9:105–114
https://doi.org/10.1007/s42417-020-00213-6

ORIGINAL PAPER

Study of Thermoelastic Damping in Microstretch Thermoelastic Thin Circular Plate

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Received: 2 December 2019 / Revised: 1 May 2020 / Accepted: 6 May 2020 / Published online: 8 June 2020

Abstract
Purpose The purpose of this paper is to study detection, microstretch function, temperature distribution function and thermoelastic damping analysis due to thermal variations and stretch forces in homogeneous, isotropic microstretch, generalized thermoelastic thin circular plate.
Method This theory is based on the Kirchho-Love plate theory assumptions. The governing equations for the transverse vibrations of microstretch thermoelastic thin circular plate have been derived. The analytical expressions for detection, microstretch function, temperature distribution function and thermoelastic damping have been numerically analyzed for clamped and simply supported boundary conditions in case of both non-Fourier and Fourier microstretch thermoelastic circular plate with the help of MATLAB programming software.
Results Finally the analytical development for thermoelastic damping have been illustrated numerically for Silicon-like material. The computer simulated results have been presented graphically under different boundary conditions.
Conclusion It leads to the conclusion that thermal relaxation time and microstretch parameters contribute to an increase in the magnitude of the critical value of damping.

Keywords Microstretch · Circular plate · Thermoelastic damping · Bessel functions · Clamped · Simply supported

Introduction

The theory of micropolar elasticity for solids perhaps established by Eringen [5], which provides a model that can support body and surface couples by including the intrinsic rotation of microstructures. Eringen [6] extended his work to include the effect of axial stretch during rotation of molecules and developed theory of microstretch by considering microstructure expansions and contractions. Eringen [7] also extended the theory of microstretch elastic solids to include the effect of heat conduction. Eringen [8] gave an exposition of the development in the microcontinuum field theories for solids (micromorphic, microstretch, and micropolar) including electro magnetic and thermal effects.

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Micro-electro-mechanical resonators have high sensitivity along with rapid-response and are frequently used such as sensors, communicators and modulators. Many researchers have stated various dissipation mechanism in MEMS, such as support-related losses, thermoelastic damping, as well as the radiation of energy away from the resonators into its surroundings. Thermoelastic damping (TED) arises from thermal currents generated by contraction/extension in elastic media. Zener [31] first identified the existence of thermoelastic damping as a significant dissipation mechanism in flexural resonators. This work was experimentally described by Berry [1] for α-brass, in which the damping factor was measured as a function of frequency at room temperature. Lifshitz and Roukes [15] studied TED and derived quality factor for a beam with rectangular cross section. Nayfeh and Younis [17] developed analytical expressions for the quality factor of micro-plates of general shapes due to thermoelastic damping. Wong et al. [30] considered thermoelastic damping of the in-plane vibration of thin silicon rings. Sun and Tohmyoh [26] discussed thermoelastic damping analysis of the axisymmetric vibration of circular plate. Sun and Saka [27] studied the damping analysis of the vibrations in arbitrary direction in a coupled thermoelastic circular plate. Kumar and Ray [13] investigated the active constrained layer damping of smart laminated composite sandwich plates using the vertically reinforced 1–3 piezoelectric composite materials. Biswas and Ray [2] investigated the active constrained layer damping of geometrically nonlinear vibrations of rotating laminated composite beams using the vertically reinforced 1–3 piezoelectric composite materials. Sharma and Grover [25] discussed thermoelastic vibrations in micro- and nano-plate resonators with voids. Grover [9, 10] studied transverse vibrations in viscothermoelastic beam and in thin circular viscothermoelastic plate, respectively. Grover and Seth [11, 12] discussed analytical expressions for thermoelastic damping and frequency shift of dual-phase-lagging generalized viscothermoelastic thin beam and circular plates, respectively. Alghamdi [18] discussed analysis and numerical results for the thermoelastic of an isotropic homogeneous, thermally conducting, Kelvin–Voigt-type circular micro-plate in the context of Kirchhoff's Love plate theory.

The present paper is devoted to study deflection, microstretch function, temperature distribution function and thermoelastic damping due to thermal variations and stretch forces in homogeneous, isotropic thin circular plate. The obtained expressions for clamped and simply supported circular plate have also been computed numerically by using MATLAB programming software. The numerically illustrated results have been presented graphically for comparison and for explanation of analytical expressions of TED.

Model Description and Problem Formulation

In this section, the governing equations for microstretch thermoelastic coupling problem of thin circular plate are derived. To analysis the derivation for thin circular plate, the following basic hypotheses, involving the Kirchhoff–Love plate theory, are employed in this analysis:

(i) Normal stress σ_{zz} can be neglected relative to the principal stresses, i.e. $\sigma_{zz} = 0$.
(ii) The rectilinear element normal to the middle surface before deformation remains perpendicular to the strained surface after deformation and their elongation can be neglected, i.e. $e_{rz} = e_{\theta z} = 0$.
(iii) For small deformation vibration, the deformation along the middle surface can be neglected, i.e. $e_{zz} = 0$.

Equations of Transverse Motion for Thin Circular Plate

Now here, we consider a thin circular plate with uniform thickness h and radius a. A cylindrical coordinate system (r, θ, z) is employed with its origin at the center of the plate. The (r, θ) plane is kept on the neutral surface of the plate and the z axis is perpendicular to the neutral surface. In equilibrium, the plate is unstrained, unstressed and kept at the uniform temperature T_0 everywhere.

We define $u_r(r,\theta,z,t)$, $u_\theta(r,\theta,z,t)$ and $u_z(r,\theta,z,t)$ to be displacement components along with the radial, circumferential and axial directions, respectively, $\phi^*(r,\theta,z,t)$ and $T(r,\theta,z,t)$ are microstretch and temperature distribution functions of the field. By above hypothesis, the strain-displacement relations can be written as follows by [3, 14, 24, 28]

$$\begin{aligned} e_{rr} &= \frac{\partial u_r}{\partial r}, e_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, e_{zz} = \frac{\partial u_z}{\partial z} = 0, 2e_{r\theta} = \gamma_{r\theta} \\ &= \frac{\partial u_\theta}{\partial r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r}, \\ 2e_{rz} &= \gamma_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} = 0, 2e_{\theta z} = \gamma_{\theta z} = \frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta} = 0, \end{aligned} \tag{1}$$

where $\gamma_{ij} = \gamma_{ji}, (i,j = 1,2,3)$ represent the mechanical strain components.

By integration of $\frac{\partial u_z}{\partial z} = 0$ provides us

$$u_z(r,\theta,z,t) = w(r,\theta,t) \tag{2}$$

Also, the integration for $\gamma_{rz}, \gamma_{\theta z}$ as mentioned in equation(1), give us

$$u_r = -z\frac{\partial w}{\partial r} \text{ and } u_\theta = \frac{-z}{r}\frac{\partial w}{\partial \theta} \tag{3}$$

Therefore, strain components written in equation(1) are given by

$$e_{rr} = -z\frac{\partial^2 w}{\partial r^2}, e_{\theta\theta} = -z\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right), 2e_{r\theta} = -2z\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right) \tag{4}$$

Now, stress components for microstretch thermoelastic media are given by

$$\begin{aligned} \sigma_{rr} &= -\frac{2\lambda\mu^* + \mu^{*2}}{\lambda + \mu^*}\left(z\frac{\partial^2 w}{\partial r^2}\right) - \frac{\lambda\mu^*}{\lambda + \mu^*}\left(\frac{z}{r}\frac{\partial w}{\partial r} + \frac{z}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) \\ &\quad + \frac{\lambda_0\mu^*}{\lambda + \mu^*}\phi^* - \frac{\mu^*\beta_1}{\lambda + \mu^*}T \\ \sigma_{\theta\theta} &= -\frac{\lambda\mu^*}{\lambda + \mu^*}\left(z\frac{\partial^2 w}{\partial r^2}\right) - \frac{2\lambda\mu^* + \mu^{*2}}{\lambda + \mu^*}\left(\frac{z}{r}\frac{\partial w}{\partial r} + \frac{z}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) \\ &\quad + \frac{\lambda_0\mu^*}{\lambda + \mu^*}\phi^* - \frac{\mu^*\beta_1}{\lambda + \mu^*}T \\ \sigma_{r\theta} &= \mu^*\left[\frac{-z}{r}\frac{\partial^2 w}{\partial r\partial\theta} + \frac{z}{r^2}\frac{\partial w}{\partial \theta}\right], \end{aligned} \tag{5}$$

where λ, μ, K are material constants and $\mu^* = 2\mu + K$.

Thus, the expressions for radial moment M_{rr}, tangential moment $M_{\theta\theta}$ and twisting moment $M_{r\theta}$ of thin circular plates are evaluated as follows [14, 22]

$$\begin{aligned} M_{rr} &= \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_{rr} z dz \\ &= -\frac{h^3}{12}\left[\left(\frac{2\lambda\mu^* + \mu^{*^2}}{\lambda+\mu^*}\right)\frac{\partial^2 w}{\partial r^2} + \left(\frac{\lambda\mu^*}{\lambda+\mu^*}\right)\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right)\right] \\ &\quad + \frac{\lambda_0\mu^*}{\lambda+\mu^*}M_{\phi^*} - \frac{\beta_1\mu^*}{\lambda+\mu^*}M_T, \\ M_{\theta\theta} &= \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_{\theta\theta} z dz \\ &= -\frac{h^3}{12}\left[\frac{\lambda\mu^*}{\lambda+\mu^*}\frac{\partial^2 w}{\partial r^2} + \frac{2\lambda\mu^* + \mu^{*^2}}{\lambda+\mu^*}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right)\right] \\ &\quad + \frac{\lambda_0\mu^*}{\lambda+\mu^*}M_{\phi^*} - \frac{\beta_1\mu^*}{\lambda+\mu^*}M_T, \\ M_{r\theta} &= \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_{r\theta} z dz = \mu^*\frac{h^3}{12}\left[\frac{-1}{r}\frac{\partial^2 w}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial w}{\partial\theta}\right], \end{aligned} \tag{6}$$

where $M_{\phi^*} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \phi^* z dz$ and $M_T = \int_{\frac{-h}{2}}^{\frac{h}{2}} T z dz$, are denoted as the moments of thin circular plate due to the presence of microstretch and thermal effects, respectively.

The equation of transverse motion of thin circular plate can be derived in polar coordinates by considering the dynamic equilibrium of the element. Moment equilibrium about the tangential θ direction is given by

$$\frac{\partial M_{rr}}{\partial r} + \frac{1}{r}\frac{\partial M_{r\theta}}{\partial\theta} + \frac{M_{rr} - M_{\theta\theta}}{r} - Q_r = 0 \tag{7}$$

Moment equilibrium about the radial r direction

$$\frac{\partial M_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial M_{\theta\theta}}{\partial\theta} + \frac{2}{r}M_{r\theta} - Q_\theta = 0 \tag{8}$$

Force equilibrium in the z direction

$$\frac{\partial Q_r}{\partial r} + \frac{1}{r}\frac{\partial Q_\theta}{\partial\theta} + \frac{Q_r}{r} + f - \rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{9}$$

By solving system of Eqs. (7)–(9), the transverse motion equation for a thin circular plate can be obtained as

$$\begin{aligned} &\frac{\partial^2 M_{rr}}{\partial r^2} + \frac{2}{r}\frac{\partial^2 M_{r\theta}}{\partial r\partial\theta} + \frac{2}{r}\frac{\partial M_{rr}}{\partial r} - \frac{1}{r}\frac{\partial M_{\theta\theta}}{\partial r} \\ &\quad + \frac{2}{r^2}\frac{\partial M_{r\theta}}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2 M_{\theta\theta}}{\partial\theta^2} + f(r,\theta,t) = \rho h\frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{10}$$

Now, by substituting Eq. (6) into Eq. (10), the equation of motion for circular plate in the absence of body force ($f = 0$) is given by

$$\begin{aligned} &\left(\frac{2\lambda\mu^* + \mu^{*^2}}{\lambda+\mu^*}\right)\frac{h^3}{12}\nabla_1^2\nabla_1^2 w - \frac{\lambda_0\mu^*}{\lambda+\mu^*}\nabla_1^2 M_{\phi^*} \\ &\quad + \frac{\beta_1\mu^*}{\lambda+\mu^*}\nabla_1^2 M_T + \rho h\frac{\partial^2 w}{\partial t^2} = 0, \end{aligned} \tag{11}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$.

Equations of Microstretch Thermoelastic Media

Consider a homogeneous, isotropic, microstretch, thermally conductive media in the context of generalized thermoelasticity (LS model). The microstretch and heat conduction equations in the absence of stretch forces and heat sources are given by [19–21]:

$$(\alpha_0\nabla^2 - \lambda_1)\phi^* - \lambda_0\nabla\cdot\mathbf{u} + \beta_2 T = \frac{\rho j_0}{2}\frac{\partial^2\phi^*}{\partial t^2} \tag{12}$$

$$\begin{aligned} K^*\nabla^2 T &= \rho C_e\left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)T \\ &\quad + \beta_1 T_0\left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\mathbf{u} + \beta_2 T_0\left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)\phi^* \end{aligned} \tag{13}$$

Here, K^* is the thermal conductivity, C_e is the specific heat at constant strain, ρ is the density of medium, T is the temperature change and T_0 is initial uniform temperature, t_0 is thermal relaxation time, $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t_1}$ and $\beta_2 = (3\lambda + 2\mu + K)\alpha_{t_2}$, α_{t_1}, α_{t_2} are coefficients of linear thermal expansion, α_0, λ_0, λ_1 are microstretch constants, j_0 is the microinertia of microelement, t is time and $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$ is Laplacian operator.

Using Eq. (4) in Eqs. (12)–(13), we get

$$\begin{aligned} &\alpha_0\left(\frac{\partial^2\phi^*}{\partial r^2} + \frac{1}{r}\frac{\partial\phi^*}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi^*}{\partial\theta^2} + \frac{\partial^2\phi^*}{\partial z^2}\right) \\ &\quad - \lambda_1\phi^* + \lambda_0 z\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) + \beta_2 T \\ &= \frac{\rho j_0}{2}\frac{\partial^2\phi^*}{\partial t^2} \end{aligned} \tag{14}$$

$$\begin{aligned} &K^*\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho C_e\left(\frac{\partial T}{\partial t} + t_0\frac{\partial^2 T}{\partial t^2}\right) \\ &\quad - \beta_1 T_0 z\left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) \\ &\quad + \beta_2 T_0\left(\frac{\partial\phi^*}{\partial t} + t_0\frac{\partial^2\phi^*}{\partial t^2}\right) \end{aligned} \tag{15}$$

In summary, Eqs. (11) and (14)–(15) makes the governing equation of this problem.

Introducing non-dimensional quantities by [19, 20] as follows:

$$r' = \frac{r}{a}, z' = \frac{z}{h}, w' = \frac{w}{h}, t' = \frac{c_1}{a}t, t_0' = \frac{c_1}{a}t_0, T' = \frac{T}{T_0}, \phi^{*'} = \frac{\rho c_1^2}{\beta_1 T_0}\phi^*, \tag{16}$$
$$M'_{\phi^*} = \frac{\rho c_1^2}{h^2\beta_1 T_0}M_{\phi^*}, M'_T = \frac{1}{T_0 h^2}M_T$$

Upon utilizing these non-dimensional expressions in Eq. (11) and Eqs. (14)–(15), the following system of equations are given by

$$\left(\frac{\partial^2\phi^*}{\partial r^2} + \frac{1}{r}\frac{\partial\phi^*}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi^*}{\partial\theta^2} + A_R^2\frac{\partial^2\phi^*}{\partial z^2}\right) - p_1\delta_2^*\phi^* + \frac{p_0\delta_2^* z}{\bar{\beta}A_R^2}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) + \bar{v}\delta_2^* T = \frac{1}{\delta_2^2}\frac{\partial^2\phi^*}{\partial t^2} \tag{17}$$

$$\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2} + A_R^2\frac{\partial^2 T}{\partial z^2}\right) - \bar{c}\left(\frac{\partial T}{\partial t} + t_0\frac{\partial^2 T}{\partial t^2}\right) + \frac{\bar{\varepsilon}_T z}{\bar{\beta}A_R^2}\left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right) \times \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) - \bar{v}\bar{\varepsilon}_T\left(\frac{\partial\phi^*}{\partial t} + t_0\frac{\partial^2\phi^*}{\partial t^2}\right) = 0 \tag{18}$$

$$D\nabla_1^2\nabla_1^2 w - P\nabla_1^2 M_{\phi^*} + \bar{B}\nabla_1^2 M_T + \frac{\partial^2 w}{\partial t^2} = 0 \tag{19}$$

(After dropping the superscript for conveniences)

where

$$D = \frac{1}{12A_R^2}\left(\frac{2\lambda\mu^* + \mu^{*^2}}{(\rho c_1^2)^2}\right), P = \frac{p_0\mu^*\bar{\beta}}{\rho c_1^2}, \bar{B} = \frac{\bar{\beta}\mu^*}{\rho c_1^2}$$

and

$$A_R = \frac{a}{h}, c_1^2 = \frac{\lambda + 2\mu + K}{\rho}, c_4^2 = \frac{2\alpha_0}{\rho j_0}, \delta_2^* = \frac{\rho c_1^2 a^2}{\alpha_0},$$
$$\delta_2^2 = \frac{c_4^2}{c_1^2}, p_0 = \frac{\lambda_0}{\rho c_1^2}, p_1 = \frac{\lambda_1}{\rho c_1^2},$$
$$\bar{\beta} = \frac{\beta_1 T_0}{\rho c_1^2}, \bar{c} = \frac{\rho C_e c_1 a}{K^*}, \bar{\varepsilon}_T = \frac{\beta_1^2 T_0 a}{\rho c_1 K^*}, \tag{20}$$
$$\varepsilon_T = \frac{\bar{\varepsilon}_T}{\bar{c}}, \bar{v} = \frac{\beta_2}{\beta_1}$$

Boundary Conditions

The following two types of boundary conditions are considered by [22] as below:

Case I: The Plate is Clamped at the edge, i.e.

$$w(1,\theta,t) = \frac{\partial w}{\partial r}(1,\theta,t) = 0 \tag{21}$$

Case II: The Plate is Simply Supported at the edge, i.e.

$$w(1,\theta,t) = \nabla_1^2 w(1,\theta,t) = 0 \tag{22}$$

Analytical Solution

To analyze microstretch thermoelastic coupling effect on vibrations of thin circular plate, we solve the system of Eqs. (17)–(19). For this, consider harmonic vibration solution of the circular plate as follows

$$[w(r,\theta,t), \phi^*(r,\theta,z,t), T(r,\theta,z,t)] = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}[W_{nm}(r), \Phi^*_{nm}(r,z), \Theta_{nm}(r,z)]e^{i(n\theta+\omega_{nm}t)}, \tag{23}$$

where ω_{nm} is the frequency, $n = 0, 1, 2, 3, ...$ represents the number of nodal diameters and $m = 1, 2, 3, ...$ represents the number of nodal circles.

Substituting Eq. (23) in system of Eqs. (17)–(19), we obtained

$$\nabla_1^{*^2}\Phi^*_{nm} + A_R^2\frac{\partial^2\Phi^*_{nm}}{\partial z^2} - \left(p_1\delta_2^* - \frac{\omega_{nm}^2}{\delta_2^2}\right)\Phi^*_{nm} + \frac{p_0\delta_2^* z}{\bar{\beta}A_R^2}\nabla_1^{*^2}W_{nm} + \bar{v}\delta_2^*\Theta_{nm} = 0 \tag{24}$$

$$\nabla_1^{*^2}\Theta_{nm} + A_R^2\frac{\partial^2\Theta_{nm}}{\partial z^2} - \bar{c}i\omega_{nm}\tau_0\Theta_{nm} + \frac{\bar{\varepsilon}_T i\omega_{nm}\tau_0 z}{\bar{\beta}A_R^2}\nabla_1^{*^2}W_{nm} - \bar{v}\bar{\varepsilon}_T i\omega_{nm}\tau_0\Phi^*_{nm} = 0 \tag{25}$$

$$D\nabla_1^{*^2}\nabla_1^{*^2}W_{nm} - P\nabla_1^{*^2}M_{\Phi^*_{nm}} + \bar{B}\nabla_1^{*^2}M_{\Theta_{nm}} - \omega_{nm}^2 W_{nm} = 0, \tag{26}$$

where $\nabla_1^{*^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{n^2}{r^2}$ and

$$M_{\Phi^*_{nm}} = \int_{\frac{-1}{2}}^{\frac{1}{2}}\Phi^*_{nm}z\,dz, M_{\Theta_{nm}} = \int_{\frac{-1}{2}}^{\frac{1}{2}}\Theta_{nm}z\,dz, \tau_0 = 1 + i\omega_{nm}t_0 \tag{27}$$

To evaluate approximate solution under some assumptions and uncoupled system($\bar{\nu}=0$) of above Eqs. (24)–(25) and also there is no flow of heat and microstretch parameters on the upper and lower surfaces of the circular plate (i.e.$\frac{\partial \Phi^*_{nm}}{\partial z}=\frac{\partial \Theta_{nm}}{\partial z}=0$ at $z=\pm\frac{1}{2}$). Thus, the trial solution of resulting system of equations are given as follows:

$$\begin{aligned}\Theta_{nm}(r,z) &= \frac{\varepsilon_T}{\bar{\beta}A_R^2}\left(z-\frac{\sin pz}{p\cos(\frac{p}{2})}\right)\nabla_1^{*^2}W_{nm}\\ \Phi^*_{nm}(r,z) &= \frac{p_0\delta_2^*}{\bar{\beta}A_R^2(p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2})}\left(z-\frac{\sin qz}{q\cos(\frac{q}{2})}\right)\nabla_1^{*^2}W_{nm}\end{aligned} \tag{28}$$

where the values of p and q are still subjected to modifications.

Now by substituting expression (28) in Eqs. (24)–(25), we obtained

$$\begin{aligned}\nabla_1^{*^2}\Phi^*_{nm} &= -\left[\frac{p_0\delta_2^*q^*}{\bar{\beta}\left(p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2}\right)}\frac{\sin qz}{q\cos(\frac{q}{2})}+\frac{\bar{\nu}\delta_2^*\varepsilon_T}{\bar{\beta}A_R^2}\right.\\ &\quad\left.\times\left(z-\frac{\sin pz}{p\cos(\frac{p}{2})}\right)\right]\nabla_1^{*^2}W_{nm}\\ \nabla_1^{*^2}\Theta_{nm} &= -\left[\frac{\varepsilon_T p^*}{\bar{\beta}}\frac{\sin pz}{p\cos(\frac{p}{2})}-\frac{\bar{\nu}\bar{\varepsilon}_T i\omega_{nm}\tau_0}{\bar{\beta}A_R^2}\frac{p_0\delta_2^*}{\left(p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2}\right)}\right.\\ &\quad\left.\times\left(z-\frac{\sin qz}{q\cos(\frac{q}{2})}\right)\right]\nabla_1^{*^2}W_{nm},\end{aligned} \tag{29}$$

where

$$p^*=p^2+\frac{\bar{c}i\omega_{nm}\tau_0}{A_R^2},\ q^*=q^2+\frac{\left(p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2}\right)}{A_R^2}$$

Now from Eq. (27),

$$M_{\Phi^*_{nm}}=\int_{\frac{-1}{2}}^{\frac{1}{2}}\Phi^*_{nm}zdz \text{ and } M_{\Theta_{nm}}=\int_{\frac{-1}{2}}^{\frac{1}{2}}\Theta_{nm}zdz$$

with the help of Eq. (29), we get that

$$\begin{aligned}\nabla_1^{*^2}M_{\Phi^*_{nm}} &= \int_{\frac{-1}{2}}^{\frac{1}{2}}\nabla_1^{*^2}\Phi^*_{nm}zdz\\ &= \left[\frac{p_0\delta_2^*q^*}{12\bar{\beta}\left(p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2}\right)}g(\omega_{nm})-\frac{\bar{\nu}\delta_2^*\varepsilon_T}{12\bar{\beta}A_R^2}(1+f(\omega_{nm}))\right]\nabla_1^{*^2}W_{nm}\\ \nabla_1^{*^2}M_{\Theta_{nm}} &= \int_{\frac{-1}{2}}^{\frac{1}{2}}\nabla_1^{*^2}\Theta_{nm}zdz\\ &= \left[\frac{\varepsilon_T p^*}{12\bar{\beta}}f(\omega_{nm})+\frac{\bar{\nu}\bar{\varepsilon}_T i\omega_{nm}\tau_0}{12\bar{\beta}A_R^2}\left(\frac{p_0\delta_2^*}{p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2}}\right)(1+g(\omega_{nm}))\right]\nabla_1^{*^2}W_{nm},\end{aligned} \tag{30}$$

where

$$f(\omega_{nm})=\frac{24}{p^3}\left(\frac{p}{2}-\tan\frac{p}{2}\right),\ g(\omega_{nm})=\frac{24}{q^3}\left(\frac{q}{2}-\tan\frac{q}{2}\right)$$

Eliminating $M_{\Phi^*_{nm}}$ and $M_{\Theta_{nm}}$ from Eq. (26) by using Eq. (30), we get

$$D\nabla_1^{*^2}\nabla_1^{*^2}W_{nm}+\frac{\bar{B}}{12\bar{\beta}A_R^2}\left[F(\omega_{nm})+p_0G(\omega_{nm})\right]\nabla_1^{*^2}W_{nm}-\omega_{nm}^2W_{nm}=0 \tag{31}$$

where the value of coefficients $F(\omega_{nm})$ and $G(\omega_{nm})$ are defined by

$$\begin{aligned}F(\omega_{nm}) &= \varepsilon_T p^*A_R^2 f(\omega_{nm})+\bar{\nu}\bar{\varepsilon}_T i\omega_{nm}\tau_0\left(\frac{p_0\delta_2^*}{p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2}}\right)(1+g(\omega_{nm}))\\ G(\omega_{nm}) &= \bar{\nu}\delta_2^*\varepsilon_T(1+f(\omega_{nm}))-\left(\frac{p_0\delta_2^*q^*A_R^2}{p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2}}\right)g(\omega_{nm})\end{aligned} \tag{32}$$

On the other hand, by substituting Eq. (28) in Eq. (27), we obtained the following equation

$$\begin{aligned}\nabla_1^{*^2}M_{\Phi^*_{nm}} &= \left(\frac{p_0\delta_2^*}{12\bar{\beta}A_R^2(p_1\delta_2^*-\frac{\omega_{nm}^2}{\delta_2^2})}\right)[1+g(\omega_{nm})]\nabla_1^{*^2}\nabla_1^{*^2}W_{nm}\\ \nabla_1^{*^2}M_{\Theta_{nm}} &= \frac{\varepsilon_T}{12\bar{\beta}A_R^2}[1+f(\omega_{nm})]\nabla_1^{*^2}\nabla_1^{*^2}W_{nm}\end{aligned} \tag{33}$$

Now, on comparing both equations of (30) to both equations of (33), we obtained

$$F(\omega_{nm}) \simeq \varepsilon_T[1+f(\omega_{nm})]\nabla_1^{*^2}$$
$$G(\omega_{nm}) \simeq -\left(\frac{p_0\delta_2^*}{p_1\delta_2^* - \frac{\omega_{nm}^2}{\delta_2^2}}\right)[1+g(\omega_{nm})]\nabla_1^{*^2} \tag{34}$$

Therefore, by substituting Eq. (34) in Eq. (31), we obtained

$$D_{\omega_{nm}}\nabla_1^{*^2}\nabla_1^{*^2}W_{nm} - 12A_R^2\omega_{nm}^2W_{nm} = 0, \tag{35}$$

where

$$D_{\omega_{nm}} = \left\{\frac{2\lambda\mu^* + \mu^{*2}}{(\rho c_1^2)^2} + \frac{\mu^*}{\rho c_1^2}\left[\varepsilon_T(1+f(\omega_{nm})) - \varepsilon_\phi(1+g(\omega_{nm}))\right]\right\},$$
$$\varepsilon_T = \frac{\beta_1^2T_0}{\rho^2C_ec_1^2} \text{ and } \varepsilon_\phi = \left(\frac{p_0^2\delta_2^*}{p_1\delta_2^* - \frac{\omega_{nm}^2}{\delta_2^2}}\right) \tag{36}$$

Here ε_ϕ and ε_T describe the elasto-stretch and thermo-mechanical coupling constants of the circular plate respectively. As the gradient of microstretch element and thermal gradient along the axial length of the circular plate quite small as compared to that along its thickness direction. Therefore, under these assumptions, we have

$$p^2 = \frac{-i\omega_{nm}\tau_0\bar{c}}{A_R^2}\left[1 - \frac{\bar{\nu}p_0\delta_2^*}{p_1\delta_2^* - \frac{\omega_{nm}^2}{\delta_2^2}}\right]$$
$$q^2 = -\frac{(p_1\delta_2^* - \frac{\omega_{nm}^2}{\delta_2^2})}{A_R^2}\left[1 + \frac{\bar{\nu}\varepsilon_T}{p_0}\right] \tag{37}$$

Therefore the result mentioned in Eq. (28) represent the result of coupled Eqs. (24)–(25) with modified values of p and q given by Eq. (37).

Now Eq. (35) can be written as follows

$$\nabla_1^{*^2}\nabla_1^{*^2}W_{nm} - \eta^4W_{nm} = 0 \text{ where } \eta^4 = \frac{12A_R^2\omega_{nm}^2}{D_{\omega_{nm}}} \tag{38}$$

$$(\nabla_1^{*^2} - \eta^2)(\nabla_1^{*^2} + \eta^2)W_{nm} = 0 \tag{39}$$

Equation(39) can be rewritten as

$$\left\{\left[r^2\frac{d^2}{dr^2} + r\frac{d}{dr} - (\eta^2r^2 + n^2)\right]\left[r^2\frac{d^2}{dr^2} + r\frac{d}{dr} + (\eta^2r^2 - n^2)\right]\right\}W_{nm} = 0 \tag{40}$$

The solution of Eq. (40) can be obtained from [29] as

$$W_{nm} = c_1J_n(\eta r) + c_2Y_n(\eta r) + c_3I_n(\eta r) + c_4K_n(\eta r) \tag{41}$$

where J_n and Y_n are bessel functions of first and second kind, I_n and K_n are modified Bessel functions of first and second kind of order n respectively and c_i^s are arbitrary constants.

Now, because solution W_{nm} must be finite at all points within the plate. This makes constants c_2 and c_4 vanish since the Bessel functions of second kind Y_n and K_n become infinite at $r = 0$ ([23, 27]). Therefore,

$$W_{nm}(r) = c_1J_n(\eta r) + c_3I_n(\eta r) \tag{42}$$

Upon using Eq. (42) in Eq. (28), we obtained Microstretch function and Temperature distribution function of thin circular plate as follows:

$$\Phi_{nm}^*(r,z) = \frac{p_0\delta_2^*\eta^2}{\bar{\beta}A_R^2(p_1\delta_2^* - \frac{\omega_{nm}^2}{\delta_2^2})}\left(z - \frac{\sin qz}{q\cos(\frac{q}{2})}\right)[c_3I_n(\eta r) - c_1J_n(\eta r)]$$
$$\Theta_{nm}(r,z) = \frac{\varepsilon_T\eta^2}{\bar{\beta}A_R^2}\left(z - \frac{\sin pz}{p\cos(\frac{p}{2})}\right)[c_3I_n(\eta r) - c_1J_n(\eta r)] \tag{43}$$

Now by substituting expression (42) into boundary conditions (21) and (22), we obtained the following characteristics equations

$$CaseI: J_n(\eta)I_{n+1}(\eta) + I_n(\eta)J_{n+1}(\eta) = 0$$
$$CaseII: J_n(\eta)I_n(\eta) = 0 \tag{44}$$

The solution of characteristics Eq. (44) as $\eta = \sqrt{q_{nm}}$ where the value of $q_{nm}(n = 0,1,2; m = 1,2,3)$ are listed in Table 1 for clamped and simply supported plates which coincides with the available results [14] and [10].

Table 1 Values of $q_{nm}(n = 0,1,2; m = 1,2,3)$ for clamped and simply supported circular plates

	Case I: clamped			Case II: simply supported		
n	q_{n1}	q_{n2}	q_{n3}	q_{n1}	q_{n2}	q_{n3}
0	10.2158	39.7711	89.1041	5.7832	30.4713	74.8870
1	21.2604	60.8287	120.0792	14.6820	49.2185	103.4995
2	34.8770	84.5826	153.8151	26.3746	80.8500	135.0207

Thermoelastic Damping

Now, the vibration frequency of the circular plate considering microstretch, thermoelastic coupling effect can be obtained as

$$\omega_{nm} = \frac{\eta_{nm}^2}{2\sqrt{3}A_R}\sqrt{D_{\omega_{nm}}} = \omega_0\sqrt{1 + \frac{\rho c_1^2}{2\lambda + \mu^*}[\varepsilon_T(1 + f(\omega_{nm})) - \varepsilon_\phi(1 + g(\omega_{nm}))]} \quad (45)$$

where

$$\omega_0 = \frac{\eta_{nm}^2}{2\sqrt{3}A_R}\sqrt{\frac{2\lambda\mu^* + \mu^{*^2}}{(\rho c_1^2)^2}}$$

and the values of η_{nm}^2 are mentioned in Table 1 for $n = 0, 1, 2$ and $m = 1, 2, 3$.

Noting that $\varepsilon_\phi \ll 1, \varepsilon_T \ll 1$ for silicon material, therefore we can replace $f(\omega_{nm})$ with $f(\omega_0)$ and $g(\omega_{nm})$ with $g(\omega_0)$ and expand Eq. (45) into a series up to first order. Then Eq. (45) becomes

$$\omega_{nm} = \omega_0\left\{1 + \frac{\rho c_1^2}{2(2\lambda + \mu^*)}[\varepsilon_T(1 + f(\omega_0)) - \varepsilon_\phi(1 + g(\omega_0))]\right\} \quad (46)$$

Here, the values of parameters p and q given by Eq. (37) are complex, therefore, using Euler's theorem and transforming ω_{nm} with ω_0, we obtain

$$p = p_{oo}\left(\cos\left(\frac{\theta_1}{2}\right) - i\sin\left(\frac{\theta_1}{2}\right)\right), q = iq_{oo}, \quad (47)$$

where

$$p_{oo} = \frac{1}{A_R}\sqrt{\omega_0 r_1 \bar{c}\left(1 - \frac{\bar{v}p_0\delta_2^*}{p_1\delta_2^* - \frac{\omega_0^2}{\delta_2^2}}\right)},$$
$$q_{oo} = \frac{1}{A_R}\sqrt{\left(p_1\delta_2^* - \frac{\omega_0^2}{\delta_2^2}\right)\left(1 + \frac{\bar{v}\varepsilon_T}{p_0}\right)}, \quad (48)$$
$$\theta_1 = \tan^{-1}\left(\frac{1}{\omega_0 t_0}\right), r_1 = \sqrt{1 + \omega_0^2 t_0^2}$$

On using Eqs. (47)–(48) in Eq. (46) and simplifying Eq. (46), we obtain

$$\omega_{nm} = \omega_{nm}^R + i\omega_{nm}^I \quad (49)$$

where

$$\omega_{nm}^R = \omega_0\left(1 - \frac{\varepsilon_\phi\rho c_1^2}{2(2\lambda + \mu^*)}H_1 + \frac{\varepsilon_T\rho c_1^2}{2(2\lambda + \mu^*)}H_2\right)$$
$$\omega_{nm}^I = \omega_0\left(\frac{\varepsilon_T\rho c_1^2}{2(2\lambda + \mu^*)}H_3\right) \quad (50)$$

and

$$H_1 = 1 - \frac{12}{q_{oo}^2} + \frac{24}{q_{oo}^3}\tan h\left(\frac{q_{oo}}{2}\right)$$
$$H_2 = 1 + \frac{12}{p_{oo}^2}\cos\theta_1 - \frac{24}{p_{oo}^3}\cos\left(\frac{3\theta_1}{2}\right)\left[\frac{\sin(k') + \tan(\frac{3\theta_1}{2})\sin h(k'T^*)}{\cos(k') + \cos h(k'T^*)}\right]$$
$$H_3 = \frac{12}{p_{oo}^2}\sin\theta_1 + \frac{24}{p_{oo}^3}\cos\left(\frac{3\theta_1}{2}\right)\left[\frac{\sin h(k'T^*) - \tan(\frac{3\theta_1}{2})\sin(k')}{\cos(k') + \cos h(k'T^*)}\right]$$
$$k' = p_{oo}\cos\left(\frac{\theta_1}{2}\right), T^* = \tan\left(\frac{\theta_1}{2}\right)$$

The thermoelastic damping arises from thermal currents generated due to contraction and expansion of elastic structures. Thus, for thin choice of the value of characteristic time, we take relaxation times (due to stretch and thermal effects) as $t_R = t_0 = \omega^{-1}$. Therefore Eqs. (47)–(48) becomes

$$p = p_{oo}^*\left(\cos\left(\frac{\theta_1^*}{2}\right) - i\sin\left(\frac{\theta_1^*}{2}\right)\right), q = iq_{oo}^* \quad (51)$$

where

$$p_{oo}^* = \frac{1}{A_R}\sqrt{\omega_0 r_1^* \bar{c}\left(1 - \frac{\bar{v}p_0\delta_2^*}{p_1\delta_2^* - \frac{\omega_0^2}{\delta_2^2}}\right)},$$
$$q_{oo}^* = \frac{1}{A_R}\sqrt{\left(p_1\delta_2^* - \frac{\omega_0^2}{\delta_2^2}\right)\left(1 + \frac{\bar{v}\varepsilon_T}{p_0}\right)}, r_1^* = \sqrt{2}, \theta_1^* = \frac{\pi}{4} \quad (52)$$

Therefore we get the expression for thermoelastic damping in circular plate as given by

$$Q_{LS}^{-1} = 2\left|\frac{\omega_{nm}^I}{\omega_{nm}^R}\right| = \left|\frac{\varepsilon_T\rho c_1^2 H_3^{LS}}{(2\lambda + \mu^*)}\left(1 + \frac{\varepsilon_\phi\rho c_1^2}{2(2\lambda + \mu^*)}H_1^{LS}\right)\right| \quad (53)$$

where

$$H_1^{LS} = 1 - \frac{12}{(q_{oo}^*)^2} + \frac{24}{(q_{oo}^*)^3}\tan h\left(\frac{q_{oo}^*}{2}\right)$$
$$H_2^{LS} = 1 + \frac{6\sqrt{2}}{(p_{oo}^*)^2} - \frac{24}{(p_{oo}^*)^3}\cos\left(\frac{3\pi}{8}\right)\left[\frac{\sin(\hat{k}') + \tan(\frac{3\pi}{8})\sin h(\hat{k}'\hat{T}^*)}{\cos(\hat{k}') + \cos h(\hat{k}'\hat{T}^*)}\right]$$
$$H_3^{LS} = \frac{6\sqrt{2}}{(p_{oo}^*)^2} + \frac{24}{(p_{oo}^*)^3}\cos\left(\frac{3\pi}{8}\right)\left[\frac{\sin h(\hat{k}'\hat{T}^*) - \tan(\frac{3\pi}{8})\sin(\hat{k}')}{\cos(\hat{k}') + \cos h(\hat{k}'\hat{T}^*)}\right]$$
$$\hat{k}' = p_{oo}^*\cos\left(\frac{\pi}{8}\right), \hat{T}^* = \tan\left(\frac{\pi}{8}\right)$$

In the absence of thermal relaxation time, i.e when($t_0 \to 0$) for coupled thermoelasticity, we have

$$p = p_{oo}^{**}\left(\cos\left(\frac{\theta_1^{**}}{2}\right) - i\sin\left(\frac{\theta_1^{**}}{2}\right)\right), q = iq_{oo}^{**} \tag{54}$$

where

$$p_{oo}^{**} = \frac{1}{A_R}\sqrt{\omega_0 r_1^{**}\bar{c}\left(1 - \frac{\bar{\nu}p_0\delta_2^*}{p_1\delta_2^* - \frac{\omega_0^2}{\delta_2^2}}\right)},$$
$$q_{oo}^{**} = \frac{1}{A_R}\sqrt{\left(p_1\delta_2^* - \frac{\omega_0^2}{\delta_2^2}\right)\left(1 + \frac{\bar{\nu}\varepsilon_T}{p_0}\right)}, r_1^{**} = 1, \theta_1^{**} = \frac{\pi}{2} \tag{55}$$

Therefore, thermoelastic damping for coupled thermoelasticity plate is given by

$$Q_{CT}^{-1} = \left|\frac{\varepsilon_T\rho c_1^2 H_3^{CT}}{(2\lambda + \mu^*)}\left(1 + \frac{\varepsilon_\phi\rho c_1^2}{2(2\lambda + \mu^*)}H_1^{CT}\right)\right|, \tag{56}$$

where

$$H_1^{CT} = 1 - \frac{12}{(q_{oo}^{**})^2} + \frac{24}{(q_{oo}^{**})^3}\tan h\left(\frac{q_{oo}^{**}}{2}\right)$$
$$H_2^{CT} = 1 + \frac{12\sqrt{2}}{(p_{oo}^{**})^3}\left[\frac{\sin(\hat{\hat{k}}') - \sin h(\hat{\hat{k}}')}{\cos(\hat{\hat{k}}') + \cos h(\hat{\hat{k}}')}\right]$$
$$H_3^{CT} = \frac{12}{(p_{oo}^{**})^2} - \frac{12\sqrt{2}}{(p_{oo}^{**})^3}\left[\frac{\sin h(\hat{\hat{k}}') + \sin(\hat{\hat{k}}')}{\cos(\hat{\hat{k}}') + \cos h(\hat{\hat{k}}')}\right]$$
$$\hat{\hat{k}}' = p_{oo}^{**}\cos\left(\frac{\pi}{4}\right)$$

Validation of the Analytical Results

In this section, we will discuss one special case to investigate our results.

For Thermoelastic Plate

In absence of microstretch effects, elasto-stretch coupling constants are zero i.e.($\varepsilon_\phi = 0$) then

$$D_{\omega_{nm}} = \left\{\frac{4\mu(\lambda + \mu)}{(\rho c_1^2)^2} + \frac{2\mu}{\rho c_1^2}[\varepsilon_T(1 + f(\omega_{nm}))]\right\}$$

(for non dimensional parameters)

which completely agrees for dimensional case of generalized thermoelastic ($t_0 \neq 0$) circular plate as well as for coupled thermoelastic ($t_0 = 0$) circular plate [10]. Thus Thermoelastic damping for both generalized and coupled thermoelasticity is reduced to analytic expressions discussed below

$$Q_{LS}^{-1} = \left|\varepsilon_T\frac{\rho c_1^2}{2(\lambda + \mu)}\left\{\frac{6\sqrt{2}}{(p_{oo}^*)^2} + \frac{24}{(p_{oo}^*)^3}\cos\left(\frac{3\pi}{8}\right) \times \left(\frac{\sin h(\hat{k}'\hat{T}^*) - \tan(\frac{3\pi}{8})\sin(\hat{k}')}{\cos(\hat{k}') + \cos h(\hat{k}'\hat{T}^*)}\right)\right\}\right| \tag{57}$$

$$Q_{CT}^{-1} = \left|\varepsilon_T\frac{\rho c_1^2}{2(\lambda + \mu)}\left\{\frac{12}{(p_{oo}^{**})^2} - \frac{12\sqrt{2}}{(p_{oo}^{**})^3} \times \left(\frac{\sin h(\hat{\hat{k}}') + \sin(\hat{\hat{k}}')}{\cos(\hat{\hat{k}}') + \cos h(\hat{\hat{k}}')}\right)\right\}\right| \tag{58}$$

Numerical Simulations and Graphical Illustrations

With the aim to analyze the theoretical results obtained in the previous sections, we present some numerical simulations under this section. The material of the plate structure for this purpose has been taken as Silicon. Here, the physical parameters for Silicon material are as follows [4, 27] (under the temperature $T_0 = 293°$ K)

$\rho = 2.33\times10^3$ Kg/m^3, $\lambda = 38.354\times10^9$N/m^2, $\mu = 69.99\times10^9$N/m^2, $K = 1.0\times10^{10}$N/m^2,
$j_0 = 0.185\times10^{-19}$ m^2, $\beta_1 = 6.6056\times10^5$ N/m^2 deg, $\beta_2 = 2.0\times10^6 N/m^2$,
$K^* = 156$ J/m s deg, Ce $= 0.713\times10^3$ J/Kg deg
and the value of relevant stretch parameters are taken as
$\lambda_0 = 0.5\times10^{10}$ N/m^2, $\lambda_1 = 0.5\times10^{10}$ N/m^2, $\alpha_0 = 0.779\times10^{-9} N$

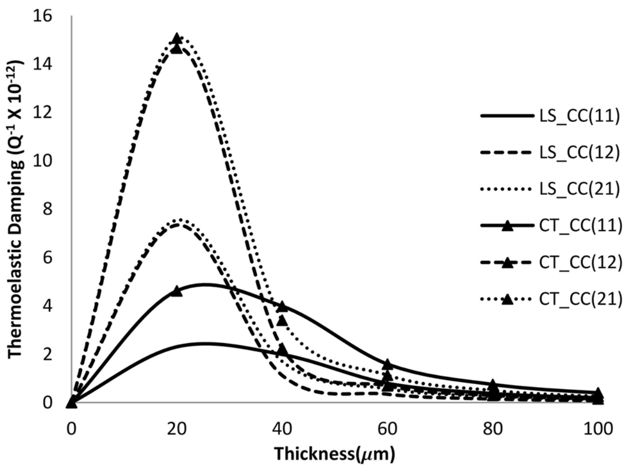

Fig. 1 Variation of TED of few modes in a clamped circular plates for a non-Fourier (generalized) and Fourier (coupled) microstretch thermoelastic plate with thickness(h) for fixed radius (a = 500 μm)

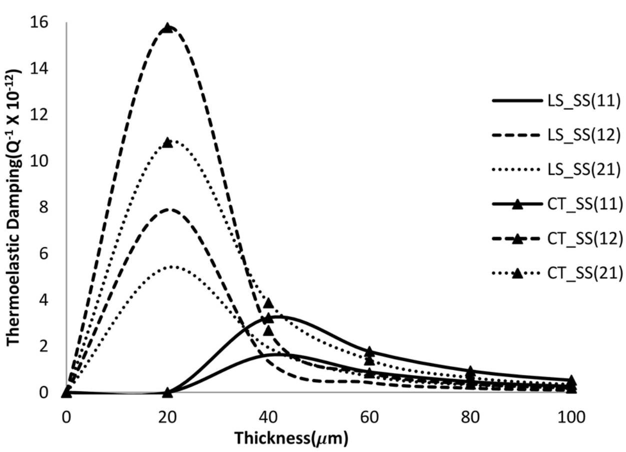

Fig. 2 Variation of TED of few modes in a simply supported circular plates for a non-Fourier (generalized) and Fourier (coupled) microstretch thermoelastic plate with thickness(h) for fixed radius (a = 500 μm)

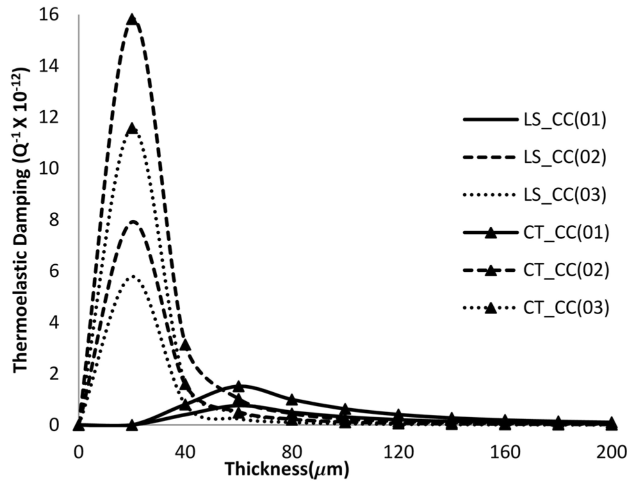

Fig. 3 Variation of TED of few modes in a clamped axisymmetric circular plates for a non-Fourier (generalized) and Fourier (coupled) microstretch thermoelastic plate with thickness(h) for fixed radius (a = 500 μm)

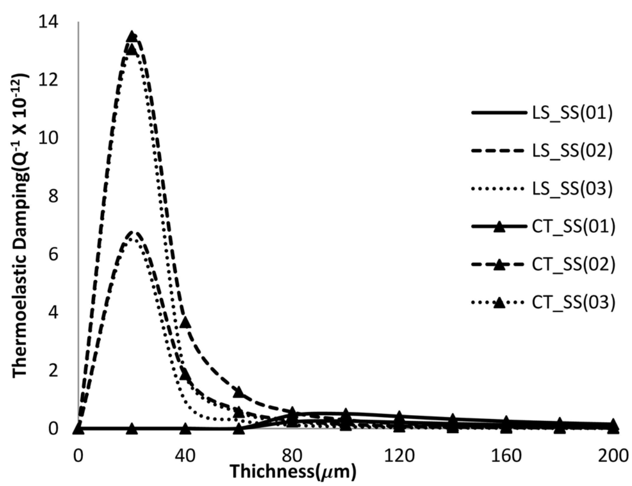

Fig. 4 Variation of TED of few modes in a simply supported axisymmetric circular plates for a non-Fourier (generalized) and Fourier (coupled) microstretch thermoelastic plate with thickness(h) for fixed radius (a = 500 μm)

As thermoelastic damping is an important design variable in plate resonators development. The dependency of thermoelastic damping factor on the plate dimensions, boundary conditions, microstretch effect, thermal relaxation time and vibration modes for silicon MEMS devices are discussed. The computer simulated results have been presented graphically in Figs. 1, 2, 3 and 4.

First, we consider the case of a circular plate with fixed radius $a = 500$ μm and varying thickness h for vibration modes(1, 1), (1, 2), (2, 1). Figure 1 depicts the behavior of TED in case of clamped plate although Fig. 2 depicts the behavior of TED in case of simply supported plate. According to these figures, the thermoelastic damping on vibration modes (1, 1), (1, 2), (2, 1) increases first and then decreases with increasing value of thickness. Thus, there exists a critical thickness for which the maximum value of the damping factor occurs. The value of damping factor of vibration modes is observed to have greater value in case of the Fourier (coupled) microstretch thermoelastic plate in comparison to non-Fourier (generalized) microstretch thermoelastic circular plate in both clamped and simply supported cases. It is also mentioned that for (1, 1) and (2, 1) modes, the value of thermoelastic damping is greater in case of clamped circular plate in comparison to simply supported circular plate where as for (1, 2) mode the value of thermoelastic damping is greater in case of simply supported circular plate in comparison to clamped circular plate.

In Figs. 3 and 4, the variation of thermoelastic damping in clamped and simply supported axisymmetric circular microstretch thermoelastic plates are described. Here, it is also observed that the damping factor firstly increases and then decreases in the considered range of thickness. The value of damping factor of vibration modes is observed to have greater magnitude in case of the Fourier (coupled) microstretch thermoelastic plate in comparison to non-Fourier (generalized) microstretch thermoelastic plate in both clamped and simply supported cases. It is also noticed that for (0, 1) and (0, 2) modes, the value of thermoelastic damping is higher in case of clamped circular plates in comparison to simply supported circular plate although for (0, 3) mode the value of thermoelastic damping is greater in case of simply supported circular plate in comparison to clamped circular plate.

Conclusion

It leads to the conclusion that thermal relaxation time and microstretch parameters contribute to increase in the magnitude of critical value of damping. For fixed radius and varying thickness, thermoelastic damping is higher in case of Fourier (coupled) microstretch thermoelastic in comparison to non-Fourier (generalized) microstretch thermoelastic case for both clamped and simply supported circular plate. It is also observed that the thermoelastic damping on vibration modes (1, 1), (1, 2), (2, 1) and for axisymmetric modes (0, 1), (0, 2), (0, 3) have symmetric behavior with different value of critical thickness.

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