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ORIGINAL PAPER

Free Vibration Analysis of Stiffened Plates

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Abstract

Background Stiffened plates are one of the major structural elements in engineering structures that are subjected to vibration because of their use in a dynamic force environment. Hence, their analysis under free vibration condition is one of the requirements for their proper design consideration.

Purpose The stiffened plates depending on their use accommodate various stiffeners that are placed either concentrically or eccentrically. The orientation of the stiffeners with respect to the Cartesian coordinates is inevitable because of their strength contribution in a particular direction. The size and shape of the stiffeners play a vital role in strengthening the plates keeping the weight of the structure low. All these parameters contribute effectively to the vibration analysis of the stiffened plates significantly. Hence, in this paper, an attempt has been made to present a parametric study for the free vibration characteristics of stiffened plates having various boundary conditions and considering the above parameters.

Methods The free vibration analysis of the stiffened plates has been carried out using the finite element method. The stiffness and mass matrices of the plate and stiffener elements have been derived separately and assembled to form the global matrices for the entire structure. Some numerical examples have been solved using APDL and FEAST and compared with the results of the present method for the purpose of validation.

Conclusions Parametric study of the vibration characteristics of stiffened plates with different size, shape, orientation, and disposition (concentrically and eccentrically) have been presented and validated with the published results or FEAST/APDL software.

Keywords Free vibration analysis of plates · Concentric and eccentric stiffeners · Diagonally and orthogonally stiffened plates · Stiffeners with arbitrary orientation

Introduction

The free vibration analysis of an engineering structure is highly significant in view of its proper design when it is subjected to any external dynamic force. For this analysis, various analytical and numerical methods have been proposed in the past. Even, engineering structures comprising of different materials have also been addressed in these analyses. Some of the recently published literature for dynamic analyses using various methods have been reported for plates as well as shells which can be found in the references [1–4]. These studies, however, limit themselves to the structures without any stiffener reinforcement. As the dynamic analysis of a stiffened plate structure is too much involved when attempted by an analytical approach, it is preferable to go for a numerical method such as finite element.

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A stiffened plate is formed of a flat deck plate integrally attached with stiffeners in any direction. To make full use of the stiffness, stiffeners are often affixed to plates along with the main load-carrying directions. These are used in many engineering structures to acquire greater strength with a relatively small quantity of material use, which increases the strength–weight ratio and makes an economical structure.

The dynamic analyses of eccentrically stiffened plates have been reviewed by Srinivasan and Thiruvenkatachari [5] using the integral equation technique. Mukherjee and Mukhopadhyay [6] have investigated the free vibration characteristics of stiffened plates possessing symmetrical stiffeners by finite element method (FEM). Harik and Salamoun [7] have applied the analytical strip method to the analysis of rectangular stiffened plate where the plate and stiffeners have been modeled separately.

Palani et al. [8] have studied the vibration analysis of plates/shells with eccentric stiffeners by suggesting two isoparametric finite element models. The free vibration of rectangular stiffened plates has been analyzed by Koko and

Olson [9] using a super element. Also, they have applied the plate beam idealization technique to finite element method where the stiffeners should have been placed on the nodal lines. A compound finite element model has been developed by Harik and Guo [10] to investigate the free vibration of eccentrically stiffened plates in free vibration where the plate elements and beam elements have been treated as integral parts of a compound section.

A finite element model has been proposed by Holopainen [11] for the free vibration analysis of eccentrically stiffened plates with an arbitrary number and orientation of stiffeners within a plate element. Zeng and Bert [12] have investigated the free vibration characteristics of an orthogonally stiffened skew plate using both the Rayleigh–Ritz method and finite element method. A four-noded stiffened plate element has been developed by Barik and Mukhopadhyay [13] for the free vibration analysis, which has accommodated any shape of the plate and arbitrary orientation of the stiffener. Sheikh and Mukhopadhyay [14] have studied the linear vibration analysis of stiffened plates using the spline finite strip method, where spline functions and finite element shape functions have been used as the displacement interpolation functions in one direction and the other directions, respectively.

The free vibration analysis of stiffened plates with unidirectional and orthogonal stiffeners has been carried out by Siddiqui and Shivhare [15, 16], respectively, using ANSYS parametric design language code. The free vibration analysis of integrally stiffened plates with plate-strip stiffeners has been explored by Ahmad and Kapania [17] using the Rayleigh–Ritz method and compared with ABAQUS software results. The free vibration characteristics of stiffened plates have been presented by Nayak et al. [18] for various parameters using finite element method.

It may be observed from above that the free vibration study on plates with non-orthogonal stiffener placement is scanty. The present endeavor is to perform a parametric study on the free vibration characteristics of plates with varying stiffener orientation using the finite element method and compare the results with those obtained by the FEAST software, and the APDL code.

The FEAST is the Indian Space Research Organisation's structural analysis software based on Finite Element Method developed by structural engineering entity of Vikram Sarabhai Space Centre (VSSC) (http://www.vssc.gov.in). APDL code is an engineering structural solution software which is based on finite element analysis.

A continuous stiffened plated structure is assumed to be composed of a finite number of plate and stiffener elements interconnected at a limited number of nodes. The in-plane and transverse displacements within each element are uniquely defined with appropriate displacement functions. The effects of rotary inertia and torsional stiffness of the stiffeners are included in the formulation. The natural frequency results of stiffened plates are compared with the published ones. For the analysis of the stiffened plates, various types of stiffeners, both concentric as well as eccentric ones, have been used. The results for various parameters such as boundary conditions, aspect ratios, plate thickness–stiffener depth ratios, and stiffener width–depth ratios are analyzed. The effect of the orientation of the stiffeners placement on free vibration is also presented.

Theoretical Formulation

The stiffness and mass matrices are derived for the plate and the stiffener elements separately for the dynamic analysis of the stiffened plates.

Plate Element Matrices

The nodal displacements considered for the plate element shown in Fig. 1 are u_i, v_i, w_i, θ_{xi} and θ_{yi}, where u_i and v_i are the in-plane displacements, w_i is the lateral displacement and θ_{xi} and θ_{yi} are the rotations in the y and x directions, respectively, for the node i. The distributed nodal forces that act along the boundaries of an element are replaced by statically equivalent concentrated forces at the nodes of the element. These nodal forces are f_{ui}, f_{vi}, f_{wi}, m_{xi} and m_{yi}, where f_{ui} and f_{vi} are the in-plane nodal forces, f_{wi} is the nodal lateral force and m_{xi} and m_{yi} are the nodal moments in the y and x directions, respectively, at node i.

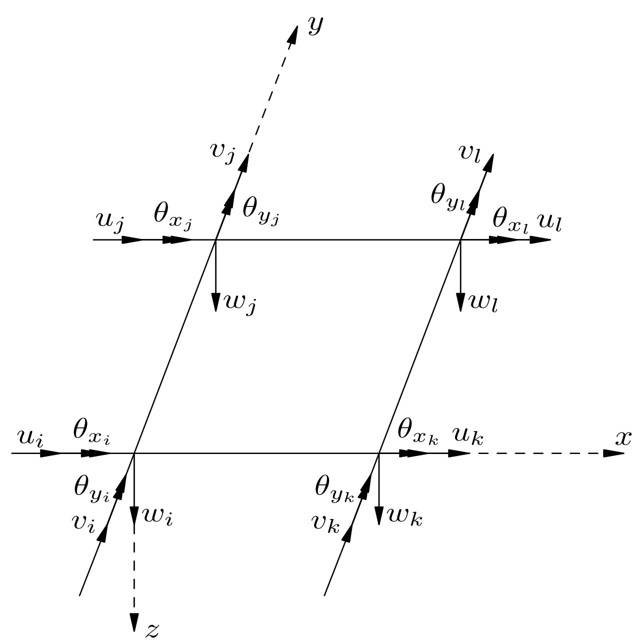


Fig. 1 Rectangular plate element

The displacement field is expressed as $\{u\}_e$

$$= \left\{ \begin{array}{c} u \\ v \\ -- \\ w \end{array} \right\}_e = \begin{bmatrix} N^m \\ -- \\ N^b \end{bmatrix} \{\delta\}_e, \tag{1}$$

where $[N]$ is the shape function, and m and b superscripts refer to the in-plane and bending effects, respectively, and expressed as

$$[N^m] = \begin{bmatrix} (1-s)(1-t) & 0 & (1-s)t & 0 & s(1-t) & 0 & st & 0 \\ 0 & (1-s)(1-t) & 0 & (1-s)t & 0 & s(1-t) & 0 & st \end{bmatrix}, \tag{2}$$

$$[N^b] = \begin{bmatrix} 1 - st - (3-2s)s^2(1-t) - (1-s)(3-2t)t^2 \\ -(1-s)t(1-t)^2 b \\ (1-s)^2 s(1-t)a \\ (3-2t)t^2(1-s) + s(1-s)(1-2s)t \\ (1-s)(1-t)t^2 b \\ s(1-s)^2 ta \\ (3-2s)s^2(1-t) + st(1-t)(1-2t) \\ -st(1-t)^2 b \\ -(1-s)s^2(1-t)a \\ (3-2s)s^2 t - st(1-t)(1-2t) \\ s(1-t)t^2 b \\ -(1-s)s^2 ta \end{bmatrix}^T, \tag{3}$$

where $s = \dfrac{x}{a}$ and $t = \dfrac{y}{b}$; a and b are the plate dimensions in x- and y-directions, respectively.

The plate strains produced from the assumed displacements are:

$$\{\epsilon\}_e = \left\{ \begin{array}{c} \epsilon^m \\ -- \\ \epsilon^b \end{array} \right\} = \left\{ \begin{array}{c} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ ---- \\ -\dfrac{\partial^2 w}{\partial x^2} \\ -\dfrac{\partial^2 w}{\partial y^2} \\ -2\dfrac{\partial^2 w}{\partial x \partial y} \end{array} \right\} = \begin{bmatrix} B^m \\ -- \\ B^b \end{bmatrix} \{\delta\}_e. \tag{4}$$

Plate stresses are expressed in terms of strains as:

$$\{\sigma\}_e = [D]\ \{\epsilon\}_e, \tag{5}$$

$$\text{where } \{\sigma\}_e = \left\{ \begin{array}{c} \sigma^m \\ -- \\ \sigma^b \end{array} \right\} = \left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \\ --- \\ M_x \\ M_y \\ M_{xy}, \end{array} \right\} \tag{6}$$

$$\text{Elasticity matrix } [D] = \begin{bmatrix} D^m \\ -- \\ D^b \end{bmatrix}$$

$$= \begin{bmatrix} D_1 & \nu D_1 & 0 & 0 & 0 & 0 \\ \nu D_1 & D_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{(1-\nu)D_1}{2} & 0 & 0 & 0 \\ -- & -- & -- & -- & -- & -- \\ 0 & 0 & 0 & D_2 & \nu D_2 & 0 \\ 0 & 0 & 0 & \nu D_2 & D_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{(1-\nu)D_2}{2}, \end{bmatrix} \tag{7}$$

where $D_1 = \dfrac{Eh}{(1-\nu^2)}$ and $D_2 = \dfrac{Eh^3}{12(1-\nu^2)}$.

Appropriate geometric transformation matrices are used to relate the displacements and forces from element coordinate system to the global coordinate system. Nodal displacements $\{q\}_e$ from the global coordinate system can be transformed into those of the element coordinate system $\{\delta\}_e$ by

$$\{\delta\}_e = [T]_e\ \{q\}_e, \tag{8}$$

where $[T]_e$ is a transformation matrix for element e. The finite element mass, stiffness and load matrices in the element coordinate system are

$$[m]_e = \int_{Ae} \rho\ h\ [N]^T\ [N]\ \mathrm{d}A, \tag{9}$$

$$[k]_e = \int_{Ae} [B]^T\ [D]\ [B]\ \mathrm{d}A, \tag{10}$$

$$[f]_e = \int_{Ae} [N]^T\ \{p\}_e\ \mathrm{d}A, \tag{11}$$

which can be expressed in global coordinate system as

$$[m]_{eg} = [T]_e^T\ [m]_e\ [T]_e, \tag{12}$$

$$[k]_{eg} = [T]_e^T \; [k]_e \; [T]_e, \tag{13}$$

$$[f]_{eg} = [T]_e^T \; \{f\}_e, \tag{14}$$

Stiffener Element Matrices

A stiffener element in the x-direction having an eccentricity e_c from the middle surface of the plate is shown in Fig. 2, where i and k are two ends of the stiffener. The torsional effect is considered after the evaluation of the in-plane and bending stiffness and mass matrices.

The nodal displacements and their corresponding nodal forces at node i, as shown in Fig. 2 are

$$\{\delta\}_{si} = \left\{u_{si} \; w_{si} \; \theta_{syi}\right\}, \tag{15}$$

$$\{f\}_{si} = \left\{f_{sui} \; f_{swi} \; m_{syi}\right\}. \tag{16}$$

The displacement functions of the centroidal line of the stiffener are,

$$\{u\}_s = \begin{Bmatrix} u_s \\ w_s \end{Bmatrix} = [N]_s \;\; \{\delta\}_s, \tag{17}$$

where

$$[N]_s = \begin{bmatrix} (1-s) & 0 \\ \frac{6e_c}{a}(s-s^2) & (1-3s^2+2s^3) \\ 3e_c(s-s^2) & a(s-2s^2+s^3) \\ s & 0 \\ \frac{6e_c}{a}(-s+s^2) & (3s^2-2s^3) \\ 3e_c(s-s^2) & a(-s^2+s^3) \end{bmatrix}^T. \tag{18}$$

and

$$\{\delta\}_s = \{u_{si} \;\; w_{si} \;\; \theta_{syi} \;\; u_{sk} \;\; w_{sk} \;\; \theta_{syk}\}^T. \tag{19}$$

This shape function is for stiffener in x-direction only. The strains for stiffener in terms of displacements are given by

$$\{\epsilon\}_s = \begin{Bmatrix} \frac{\partial u_s}{\partial x} \\ -\frac{\partial^2 w_s}{\partial x^2} \end{Bmatrix} = [B]_s \; \{\delta\}_s. \tag{20}$$

The elasticity matrix for the stiffener is expressed as

$$[D]_s = E_s \begin{bmatrix} A_x & 0 \\ 0 & I_x, \end{bmatrix} \tag{21}$$

where A_x is the cross-sectional area, I_x is the cross-sectional area moment of inertia and E_s is Young's modulus of the stiffener material. The stiffness and mass matrices can be found in a similar manner as that of the plate element and can be expressed as

$$[k]_s = \int_{Ae} [B]_s^T \; [D]_s \; [B]_s \; \mathrm{d}A \tag{22}$$

$$[m]_s = \int_{Ae} \rho_s \; h \; [N]_s^T \; [N]_s \; \mathrm{d}A. \tag{23}$$

The torsional stiffness matrix is given by:

$$\begin{Bmatrix} m_{sxi} \\ m_{sxk} \end{Bmatrix} = \frac{G_{sx}J_x}{a} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \theta_{sxi} \\ \theta_{sxk}, \end{Bmatrix} \tag{24}$$

where G_{sx} is the modulus of rigidity of the stiffener, J_x is the torsional constant and ρ_s is the density of the stiffener. This torsional stiffness matrix and stiffness matrix for the in-plane and bending effects can be combined to obtain the stiffness matrix for the eccentric stiffener element as

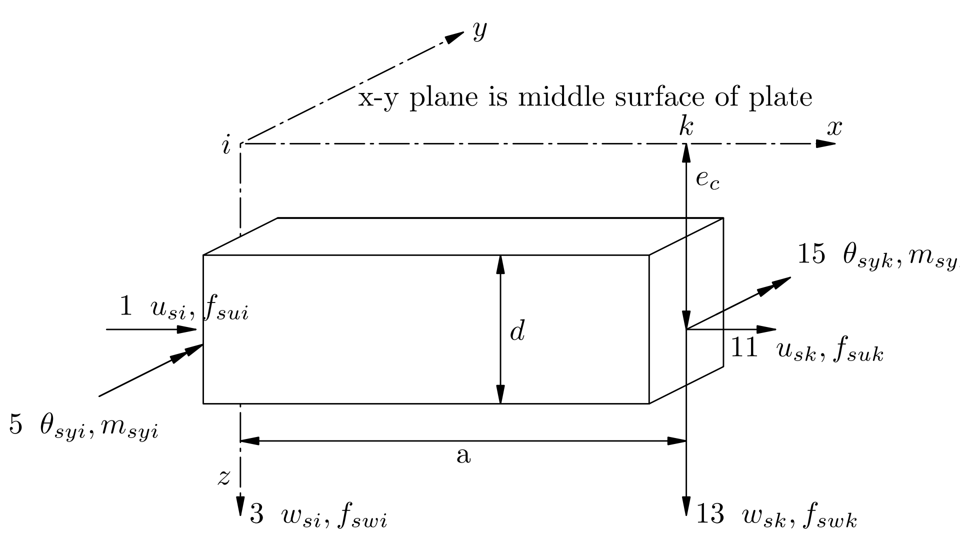


Fig. 2 Eccentric stiffener element

$$
[k]_s = \begin{bmatrix}
\frac{E_sA_x}{a} & & & & & & & & & \\
0 & 0 & & & & & \text{Sym.} & & & \\
0 & 0 & \frac{12E_sI_{ox}}{a^3} & & & & & & & \\
0 & 0 & 0 & K3 & & & & & & \\
0 & 0 & \frac{6E_sI_{ox}}{a^2} & 0 & K1 & & & & & \\
-\frac{E_sA_x}{a} & 0 & 0 & 0 & 0 & \frac{E_sA_x}{a} & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \\
0 & 0 & -\frac{12E_sI_{ox}}{a^3} & 0 & -\frac{6E_sI_{ox}}{a^2} & 0 & 0 & \frac{12E_sI_{ox}}{a^3} & & \\
0 & 0 & 0 & -K3 & 0 & 0 & 0 & 0 & K3 & \\
0 & 0 & \frac{6E_sI_{ox}}{a^2} & 0 & K2 & 0 & 0 & -\frac{6E_sI_{ox}}{a^2} & 0 & K1
\end{bmatrix} \tag{25}
$$

where $K1 = \frac{4E_sI_{ox}}{a} - \frac{e_c^2E_sA_x}{a}$ $\quad K2 = \frac{2E_sI_{ox}}{a} + \frac{e_c^2E_sA_x}{a}$

$K3 = \frac{G_{sx}J_x}{a}$ $\quad I_{ox} = I_x + A_xe_c^2.$

The mass matrix for the stiffener can be obtained as

$$
[m]_s = \rho_s\, A_x\, a \begin{bmatrix}
\frac{1}{J_x} & & & & & & & & & \\
0 & 0 & & & & & \text{Sym.} & & & \\
\frac{e_c}{2a} & 0 & S1 & & & & & & & \\
0 & 0 & 0 & \frac{J_x}{3A_x} & & & & & & \\
\frac{e_c}{4} & 0 & S2 & 0 & S3 & & & & & \\
\frac{1}{6} & 0 & \frac{e_c}{2a} & 0 & \frac{e_c}{4} & \frac{1}{J_x} & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \\
-\frac{e_c}{2a} & 0 & S5 & 0 & S4 & 0 & 0 & S1 & & \\
0 & 0 & 0 & \frac{J_x}{6A_x} & 0 & 0 & 0 & 0 & \frac{J_x}{3A_x} & \\
\frac{e_c}{4} & 0 & -S4 & 0 & S6 & \frac{e_c}{4} & 0 & -S2 & 0 & S3
\end{bmatrix}
$$

where $S1 = \frac{13}{35} + \frac{6}{5}\frac{I_{ox}}{A_xa^2}$ $\quad S2 = \frac{11a}{210} + \frac{I_{ox}}{10A_xa} + \frac{e_c^2}{2a}$

$S3 = \frac{a^2}{105} + \frac{2}{15}\frac{I_{ox}}{A_x} + \frac{e_c^2}{6}$ $\quad S4 = \frac{13a}{420} - \frac{I_{ox}}{10A_xa} - \frac{e_c^2}{2a}$

$S5 = \frac{9}{70} - \frac{6}{5}\frac{I_{ox}}{A_xa^2}$ $\quad S6 = -\frac{a^2}{140} - \frac{I_{ox}}{30A_x} + \frac{e_c^2}{3}.$ (26)

Similarly, the element matrices for the y-direction stiffener can be obtained.

Stiffener with Arbitrary Orientation

The plate stiffened with arbitrarily oriented stiffener are shown in Fig. 3, where the stiffener is along x'-axis and makes an angle β (anticlockwise) with the x-axis of the plate.

Similar to Eq. (19), the displacement of the stiffener in $x' - y'$ axis is given by

$$\{\delta'\}_{s\beta} = \{u'_{si}\ \ v'_{si}\ \ w'_{si}\ \ \theta'_{sxi}\ \ \theta'_{syi}\ \ u'_{sk}\ \ v'_{sk}\ \ w'_{sk}\ \ \theta'_{sxk}\ \ \theta'_{syk}\}^T, \tag{27}$$

which can be expressed in terms of the displacements in $x - y$ axis as

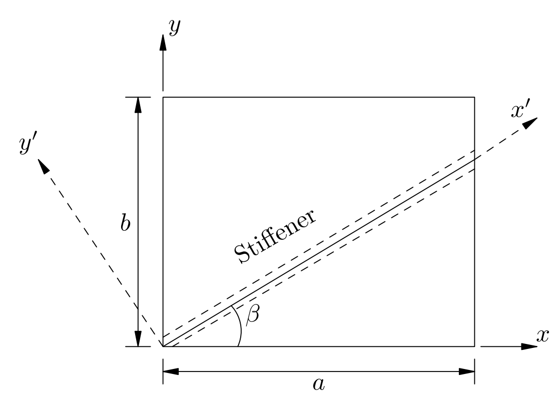


Fig. 3 Plate with arbitrarily oriented stiffener

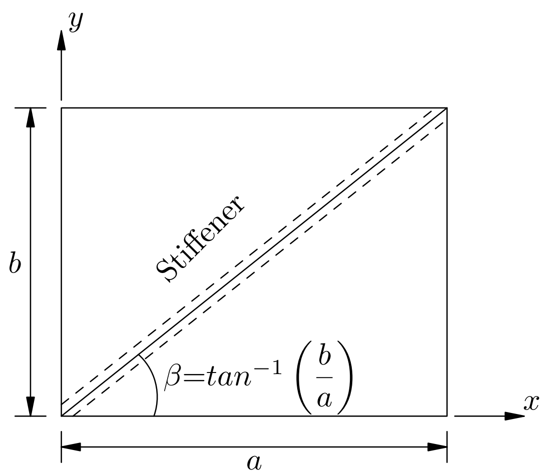


Fig. 4 Plate with diagonal stiffener

Putting Eq. (29) in (30)

$$\{u\}_{s\beta} = [N]_{s\beta}\ \ [T]_{s\beta}\ \{\delta\}_{s\beta}. \tag{31}$$

The strains for stiffener are given by

$$\{\epsilon\}_{s\beta} = [B]_{s\beta}\ \{\delta'\}_{s\beta}. \tag{32}$$

Putting Eq. (29) in (32)

$$\{\epsilon\}_{s\beta} = [B]_{s\beta}\ [T]_{s\beta}\ \{\delta\}_{s\beta}. \tag{33}$$

The stiffness and mass matrices for the stiffener are given by

$$\begin{aligned}[k]_{s\beta} &= \int_{Ae} [T]^T_{s\beta}\ [B]^T_{s\beta}\ [D]_s\ [B]_{s\beta}\ [T]_{s\beta}\ \mathrm{d}A \\ &= [T]^T_{s\beta}\ [k]_s\ [T]_{s\beta},\end{aligned} \tag{34}$$

$$\{\delta'\}_{s\beta} = \begin{bmatrix} \cos\beta & \sin\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sin\beta & \cos\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\beta & \sin\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sin\beta & \cos\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cos\beta & \sin\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sin\beta & \cos\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\beta & \sin\beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{Bmatrix} u_{si} \\ v_{si} \\ w_{si} \\ \theta_{sxi} \\ \theta_{syi} \\ u_{sk} \\ v_{sk} \\ w_{sk} \\ \theta_{sxk} \\ \theta_{syk}, \end{Bmatrix} \tag{28}$$

$$\{\delta'\}_{s\beta} = [T]_{s\beta}\ \{\delta\}_{s\beta}. \tag{29}$$

Similar to Eq. (17), the displacement functions of the arbitrarily oriented stiffener are,

$$\{u\}_{s\beta} = [N]_{s\beta}\ \ \{\delta'\}_{s\beta}. \tag{30}$$

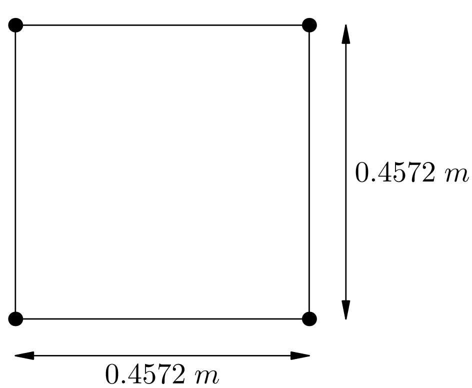


Fig. 5 Square bare plate point supported at corners

$$\begin{aligned}[m]_{s\beta} &= \int_{Ae} \rho_s\ h\ [T]^T_{s\beta}\ [N]^T_{s\beta}\ [N]_s\ [T]_{s\beta}\ \mathrm{d}A \\ &= [T]^T_{s\beta}\ [m]_s\ [T]_{s\beta}.\end{aligned} \tag{35}$$

Table 1 Non-dimensional frequencies $\left(\Omega = \omega\sqrt{\frac{\rho_p h a^4}{D}}\right)$ for a square plate point-supported at corners

Mode	Non-dimensinal frequency			
Number	FEM	APDL	FEAST	[19]
1	7.121	7.104	7.116	7.117
2	15.773	15.728	15.747	15.730
3	19.590	19.595	19.529	19.130
4	38.518	38.339	38.428	38.420
5	44.2727	44.258	44.196	43.550

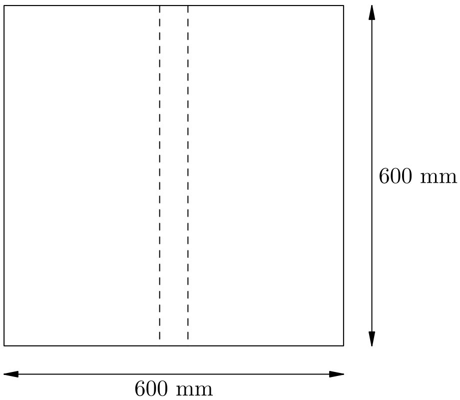


Fig. 6 Clamped square plate with a central concentric stiffener

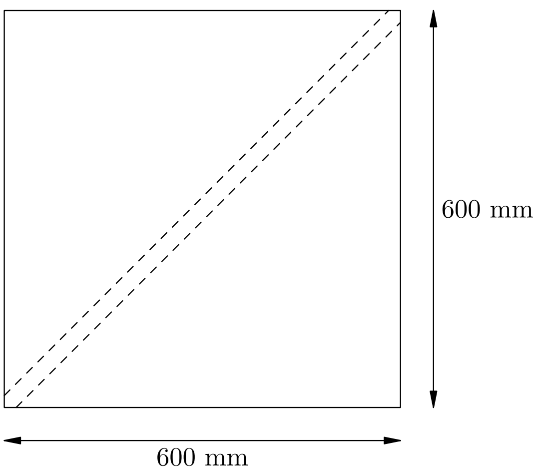


Fig. 7 Clamped square plate with a diagonal stiffener

Diagonal Stiffener

The plate with diagonal stiffener is where the stiffener is attached along the two diagonal corners of the plate element as shown in Fig. 4. The stiffness and mass matrices are same as Eqs. (34) and (35).

Boundary Conditions

The essential boundary conditions considered are

(a) for a clamped support, the known displacement conditions are

$$u = v = w = \theta_x = \theta_y = 0$$

(b) for a simply support ($x =$ constant),

$$v = w = \theta_x = 0$$

(c) for a simply support ($y =$ constant),

$$u = w = \theta_y = 0.$$

The nomenclatures C, S and F represent clamped, simply supported and free edges, respectively.

Solution Procedure

Free Vibration Analysis

The equation for free vibration of the system is

$$[M]\ \{\ddot{q}\} + [K]\ \{q\} = \{0\}, \tag{36}$$

where $[M]$ is the global mass matrix, $[K]$ is the global stiffness matrix, $\{\ddot{q}\}$ is the acceleration vector and $\{q\}$ is the displacement vector. Since the free vibrations are periodic, by assuming

$$\{q\} = \{q_0\}\ e^{i\omega t}$$

the time variable is separated from Eq. (36) and is obtained as

$$[K]\ \{q_0\} = \omega^2\ [M]\ \{q_0\}, \tag{37}$$

where $\{q_0\}$ is amplitude of $\{q\}$, ω is angular frequency of vibrations, and t is the time. Eq. (36) is a standard form of the general eigenvalue problem. If the two square matrices

Table 2 Frequency (Hz) of a clamped square plate with a central concentric stiffener

Mode Number	Frequency (Hz)						
	Central Stiffener				Diagonal Stiffener		
	FEM	APDL	FEAST	[20]	FEM	APDL	FEAST
1	49.87	49.65	49.73	50.45	52.97	52.82	52.69
2	63.03	62.84	62.91	63.71	67.49	67.01	66.56
3	74.07	73.89	73.96	75.16	83.97	82.92	82.15
4	84.30	84.07	84.18	85.50	92.31	91.63	90.89
5	112.17	111.98	112.05	113.69	121.34	120.08	118.06

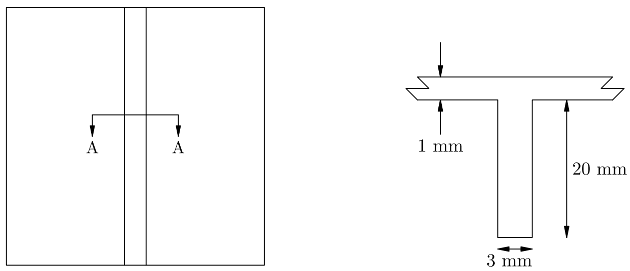


Fig. 8 Clamped square plate with a central eccentric stiffener and the section A–A

Table 3 Natural frequency (Hz) of square plate with an eccentric stiffener

Mode number	Natural frequency (Hz)		
	Central stiffener		Diagonal stiffener
	FEM	[14]	FEM
1	54.29	54.759	57.21
2	65.33	65.435	70.63
3	79.61	80.805	87.77
4	84.57	85.745	95.31
5	116.15	118.521	129.67
6	118.49	120.966	132.83

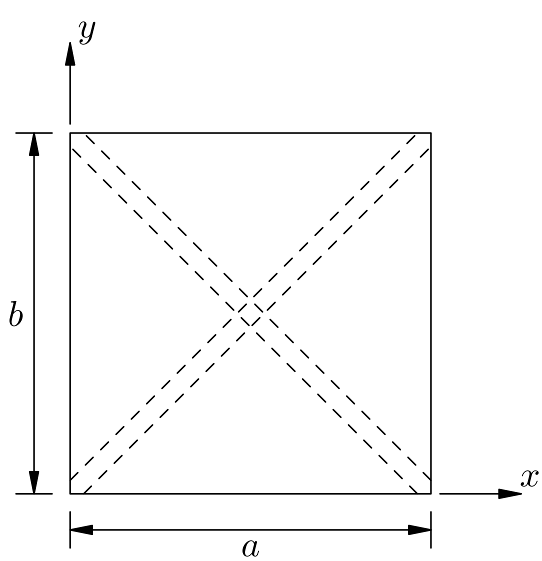
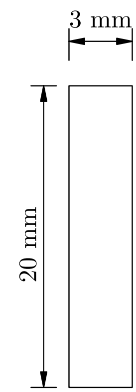


Fig. 9 Plate with diagonal R-beam stiffeners

$[K]$ and $[M]$ are of size $(f \times f)$, then there will be, in general, f different values of the natural frequency ω.

Fig. 10 Rectangular beam



Numerical Examples

Free Vibration Analysis of Square Plate Point Supported at Corners

A square plate shown in Fig. 5 of thickness 0.0016 m point-supported at corners is considered for free vibration analysis. The Young's modulus $E = 206$ GPa, density $\rho = 7929$ kg/m^3 and Poisson's ratio $\nu = 0.3$. The results obtained by the present FEM, FEAST and APDL software for a grid size of (24 $\times$ 24) are compared with finite difference results of Cox and Boxer [19] in Table 1. The results are in excellent agreement.

Clamped Square Plate with a Central Concentric Stiffener

A square plate clamped in all edges (C–C–C–C), having a centrally placed stiffener as presented by Nair and Rao [20] in Fig. 6 using the package STIFPTI has been analyzed by the present method. The dimensions of the plate are 600 mm $\times$ 600 mm $\times$ 1.0 mm and the material properties are: Young's modulus $E = 6.87 \times 10^7$ N/mm^2, Poisson's ratio $\nu = 0.34$ and density $\rho = 2.78 \times 10^{-6}$ kg/mm^3. A stiffener of $A_s = 67.0\,\text{mm}^2$, $I_s = 2290\,\text{mm}^4$ and $J_s = 22.33\,\text{mm}^4$ is taken. The frequency results obtained from FEM, APDL and FEAST software are compared with Nair and Rao [20] and presented in Table 2 showing excellent agreement. Also, the result is compared with the diagonally stiffened plate as shown in Fig. 7 with eccentric stiffener in Table 2.

Clamped Square Plate with Eccentric Stiffener

A clamped square plate (C–C–C–C) results having a centrally placed eccentric stiffener shown in Fig. 8 is of dimension 600 mm $\times$ 600 mm $\times$ 1.0 mm has been compared with those of Sheikh and Mukhopadhyay [14]. The stiffener is of width $w = 3\,\text{mm}$ and depth $d = 20\,\text{mm}$. The material

Table 4 Frequency (Hz) of square plate with diagonal concentric stiffeners

Boundary condition	Method	Mode sequence number				
		1	2	3	4	5
C–C–C–C	FEM	34.97	72.82	73.21	97.44	107.23
	APDL	34.41	71.13	71.13	95.84	104.86
C–S–C–S	FEM	29.31	69.39	69.66	82.95	97.42
	APDL	28.78	68.51	68.75	80.83	95.47
S–S–S–S	FEM	24.74	56.40	67.98	75.13	77.43
	APDL	24.25	55.73	67.02	73.47	75.63
C–F–F–F	FEM	4.31	16.24	23.16	33.10	33.58
	APDL	4.02	15.91	22.73	32.47	32.97
S–F–S–F	FEM	10.87	28.57	37.94	40.56	54.83
	APDL	10.53	28.01	37.35	39.72	53.97

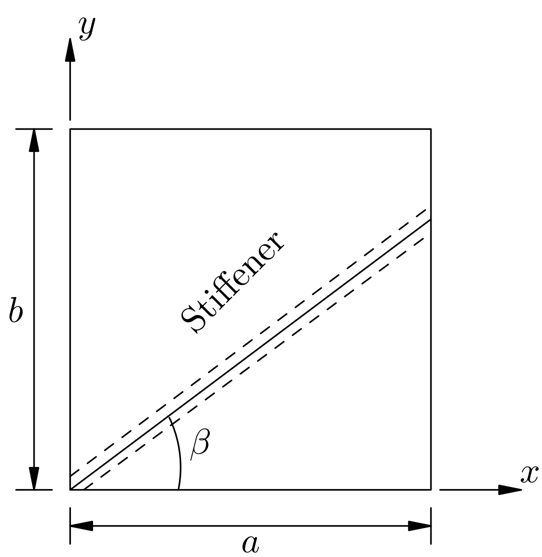


Fig. 11 Plate with arbitrary stiffener

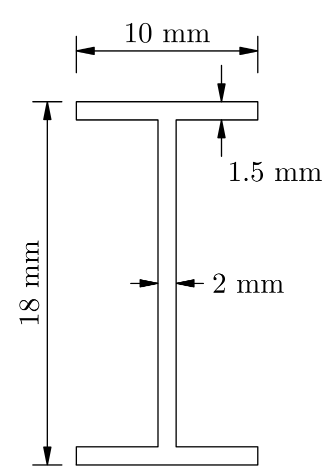


Fig. 12 I-beam

properties for both the plate and stiffener are: Young's modulus $E = 6.87 \times 10^7$ N/mm^2, Poisson's ratio $\nu = 0.34$ and density $\rho = 2.78 \times 10^{-6}$ kg/mm^3. The natural frequencies are shown in Table 3 with results of spline finite strip method which compare well. Also, the result is compared with the diagonally stiffened plate as shown in Fig. 7 with eccentric stiffener in Table 3.

Plate with Diagonal Concentric Stiffener

A plate with diagonal concentric stiffeners shown in Fig. 9 is analyzed using FEM. The plate dimensions are: $a = 1$ m, $b = 1$ m and thickness $h = 0.005$ m. The material properties of the plate are: Young's modulus $E = 20$ GPa, Poisson's ratio $\nu = 0.2$ and unit weight $\rho = 2400$ kg/m^3. The rectangular beam (R-beam) stiffeners are attached diagonally for the free vibration analysis as shown in Fig. 10 and having the material properties, Young's modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.3$ and unit weight $\rho = 7850$ kg/m^3. The results obtained for free vibration with different boundary conditions are compared with those by APDL software using SHELL181 element and presented in Table 4.

It may be observed from Table 4 that the results from present formulation compare well with those obtained using APDL software package. For C–C–C–C boundary condition, the frequency is greater than that of other boundary conditions for the plate with concentric rectangular stiffeners.

Plate with Arbitrary Stiffener Orientation

Plates with arbitrary concentric stiffener orientation as shown in Fig. 11 are considered for free vibration analysis. The plate dimensions are: $a = 1000$ mm, $b = 1000$ mm and thickness $h = 5$ mm. The material properties of the plate are: Young's modulus $E = 20$ GPa, Poisson's ratio $\nu = 0.2$

Table 5 Natural frequency (Hz) of plate with arbitrary stiffeners

Mode no.	Frequency (Hz)					
	15°		30°		45°	
	FEM	APDL	FEM	APDL	FEM	APDL
1	24.65	23.81	26.16	25.49	27.85	26.94
2	49.86	49.05	52.45	51.82	55.34	54.58
3	50.84	49.92	54.92	53.97	57.68	56.82
4	75.29	74.11	79.83	78.85	85.07	83.97
5	88.49	86.53	95.92	93.76	101.26	99.03

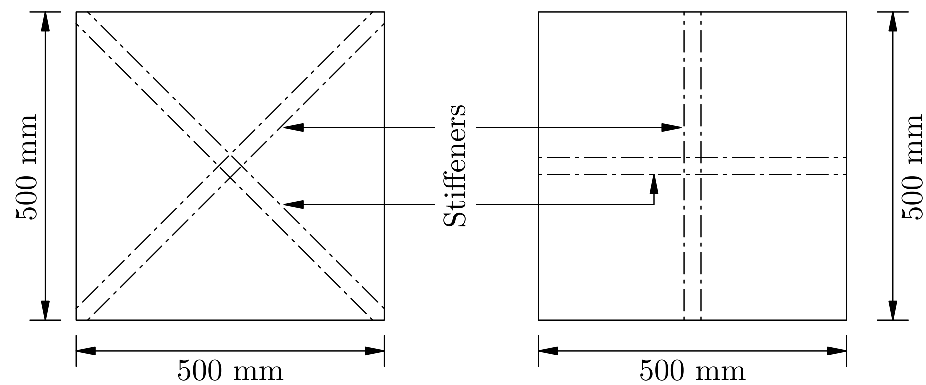


Fig. 13 Plate with diagonal and orthogonal eccentric stiffeners

Table 6 Non-dimensional frequency $\left(\omega b^2\sqrt{\frac{12\rho(1-\nu^2)}{Eh^2}}\right)$ for plate with diagonal and orthogonal stiffeners

Mode no.	Stiffened plate	
	Diagonal stiffeners	Orthogonal stiffeners
1	12.46	10.34
2	21.73	18.02
3	22.09	18.48
4	23.35	19.95
5	31.93	24.81

and unit weight $\rho = 2400\,\mathrm{kg/m^3}$. The I-stiffeners are attached diagonally for the free vibration analysis shown in Fig. 12 and having the material properties, Young's modulus $E = 210\,\mathrm{GPa}$, Poisson's ratio $\nu = 0.3$ and unit weight $\rho = 7850\,\mathrm{kg/m^3}$. The frequency results for β values 15°, 30° and 45° with all edges clamped are compared with APDL software package in Table 5, where the angle 45° is taken for the diagonal stiffener. It is observed that the frequency value increases with the increase of β values and the rate of increament is more in higher mode in comparison to the lower ones.

Table 7 Natural frequency(Hz) of square plate with diagonal eccentric stiffeners

Boundary condition	Stiffener type	Mode sequence number				
		1	2	3	4	5
C–C–C–C	I-beam	55.52	95.56	97.13	102.81	138.72
	R-beam	53.01	92.48	94.02	99.35	135.89
C–S–C–S	I-beam	49.95	80.13	85.75	95.39	121.74
	R-beam	47.44	79.80	84.38	92.83	118.23
S–S–S–S	I-beam	46.41	75.04	75.62	80.02	109.91
	R-beam	43.83	73.26	73.83	77.84	107.15
C–F–F–F	I-beam	7.15	27.49	28.02	37.59	46.51
	R-beam	6.95	26.22	27.41	36.17	45.17
S–F–S–F	I-beam	18.63	32.58	40.65	62.59	69.65
	R-beam	17.91	31.86	40.03	60.56	67.34

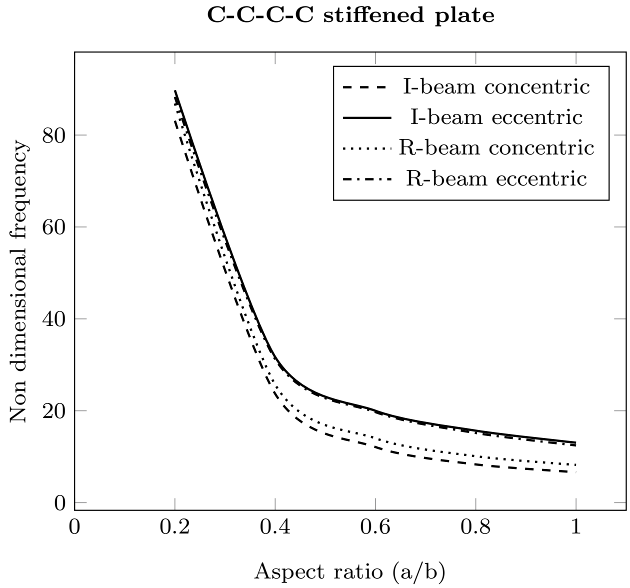


Fig. 14 Non-dimensional frequency of the C–C–C–C stiffened plate for different aspect ratios

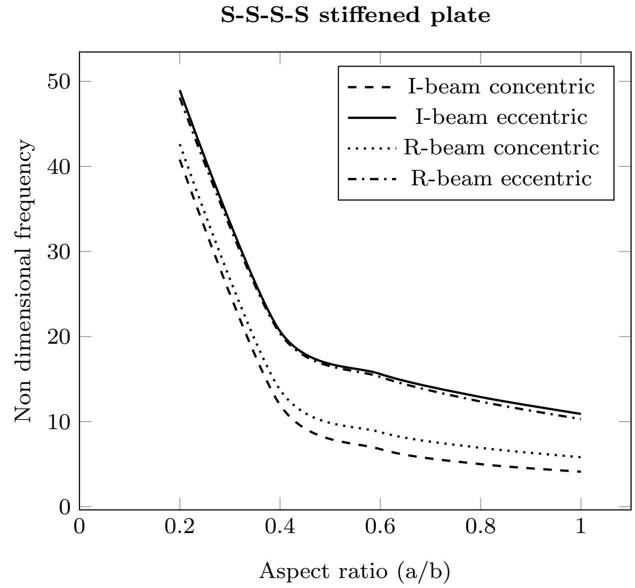


Fig. 16 Non-dimensional frequency of the S–S–S–S stiffened plate for different aspect ratios

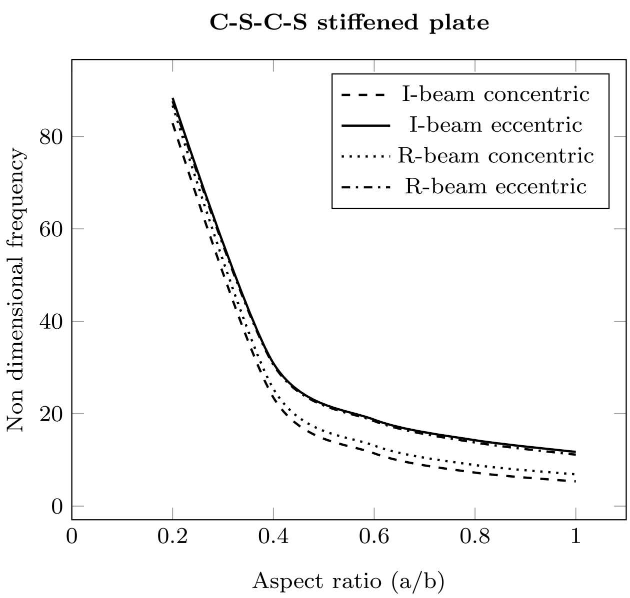


Fig. 15 Non-dimensional frequency of the C–S–C–S stiffened plate for different aspect ratios

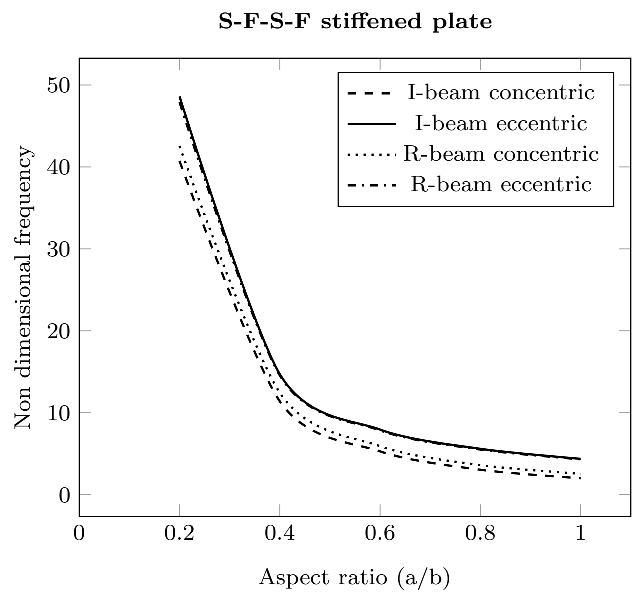


Fig. 17 Non-dimensional frequency of the S–F–S–F stiffened plate for different aspect ratios

Plate with Diagonal Stiffeners and Orthogonal Stiffeners

The plates with diagonal and orthogonal eccentric stiffeners as shown in Fig. 13 are considered for free vibration analysis separately. The plate dimensions are: $a = 1$ m, $b = 1$ m and thickness $h = 0.005$ m. The material properties of the plate are: Young's modulus $E = 20$ GPa, Poisson's ratio $\nu = 0.2$ and unit weight $\rho = 2400\,\mathrm{kg/m^3}$. The rectangular beam (R-beam) stiffeners are attached diagonally for the free vibration analysis shown in Fig. 10 and having the material properties, Young's modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.3$ and unit weight $\rho = 7850\,\mathrm{kg/m^3}$. The non-dimensional frequency $\left(\omega b^2\sqrt{12\rho(1-\nu^2)/Eh^2}\right)$ results for both plates with all edges clamped are compared in Table 6. The higher

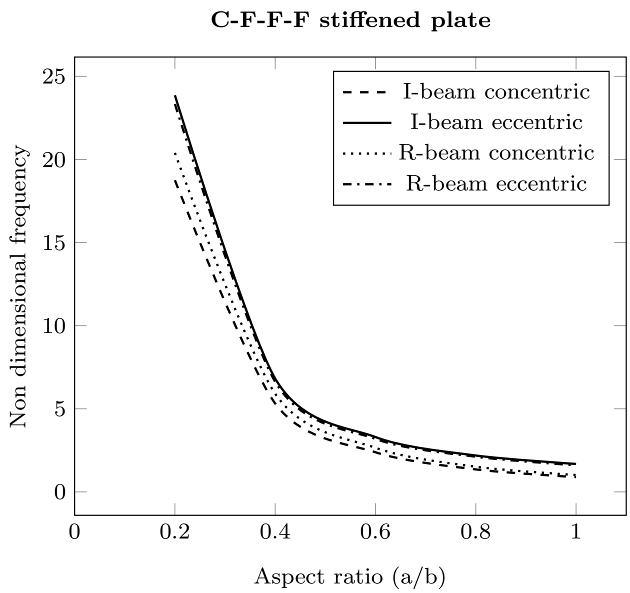


Fig. 18 Non-dimensional frequency of the C–F–F–F stiffened plate for different aspect ratios

frequency results of diagonally stiffened plate shows that it becomes stiffer in comparison to the plate with orthogonal stiffeners.

Plate with Diagonal Eccentric Stiffener

The free vibration response of a plate with diagonal eccentric stiffeners shown in Fig. 9 is analyzed. The plate dimensions and material properties are same as in section 4.4. Two types of stiffeners (I-beam and R-beam) are considered for the analysis which are shown in Figs. 10 and 12, respectively. The stiffeners are of Young's modulus $E = 210\,\text{GPa}$, Poisson's ratio $\nu = 0.3$ and unit weight $\rho = 7850\,\text{kg/m}^3$. The results for natural frequency with various boundary conditions are obtained for both the beams and presented in Table 7.

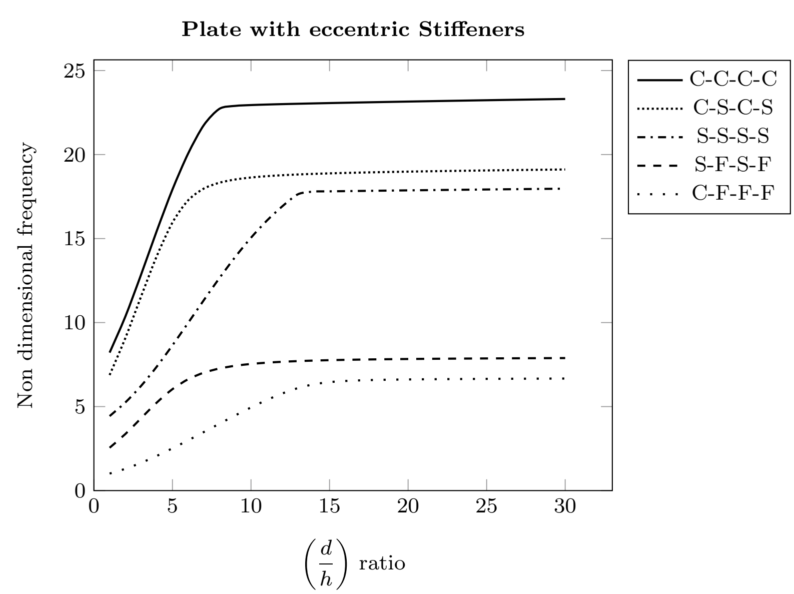
From Table 7, it is observed that the plate with I-eccentric stiffener's natural frequencies are higher than that of plate with R-eccentric stiffener for all boundary conditions. All edges clamped stiffened plate's frequencies are greater compared to those of the other boundary conditions because of the increased stiffness of clamped plates.

Stiffened Plate with Different Aspect Ratios

Stiffened plate with the same thickness and material properties as considered in section 4.7 is analyzed for both the I- and R-stiffeners. The non-dimensional fundamental frequency $\left(\omega b^2 \sqrt{12\rho(1-\nu^2)/Eh^2}\right)$ results for different aspect ratios $\left(\frac{a}{b}\right)$ such as 0.2, 0.4, 0.6, 0.8 and 1.0 are investigated with different boundary conditions and presented in Figs. 14, 15, 16, 17 and 18.

It is observed from Figs. 14, 15, 16, 17 and 18 that the non-dimensional fundamental frequency for I-eccentric stiffened plate is marginally higher than that of plate with eccentric R-stiffener. But for concentric stiffeners, R-beam stiffened plate has higher non-dimensional frequency than that of I-beam stiffened plate. It is also noted that the non-dimensional frequency is decreasing with the increase of aspect ratios for all type of stiffeners and for all boundary conditions. It is also observed that the non-dimensional frequency for all edges clamped stiffened plate is greater compared to that of other boundary conditions.

Fig. 19 Non-dimensional frequency of the eccentric stiffened plate for different $\left(\frac{d}{h}\right)$ ratios



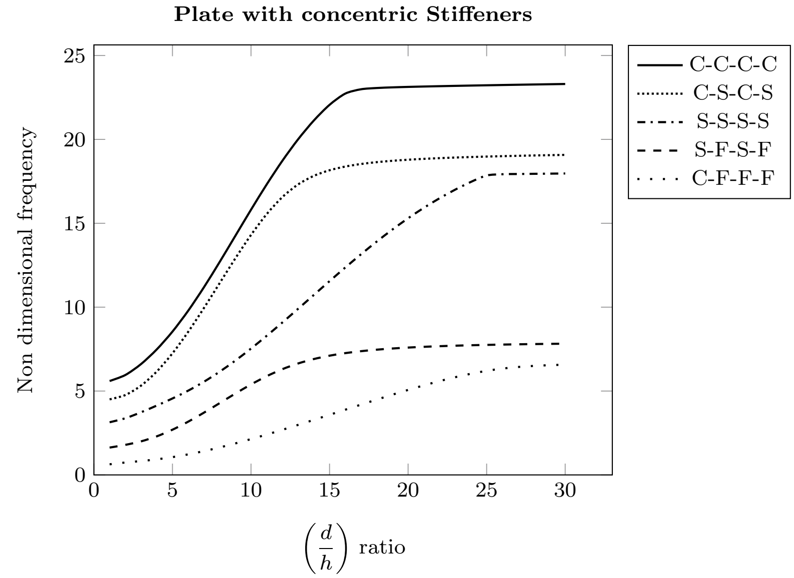


Fig. 20 Non-dimensional frequency of the concentric stiffened plate for different $\left(\frac{d}{h}\right)$ ratios

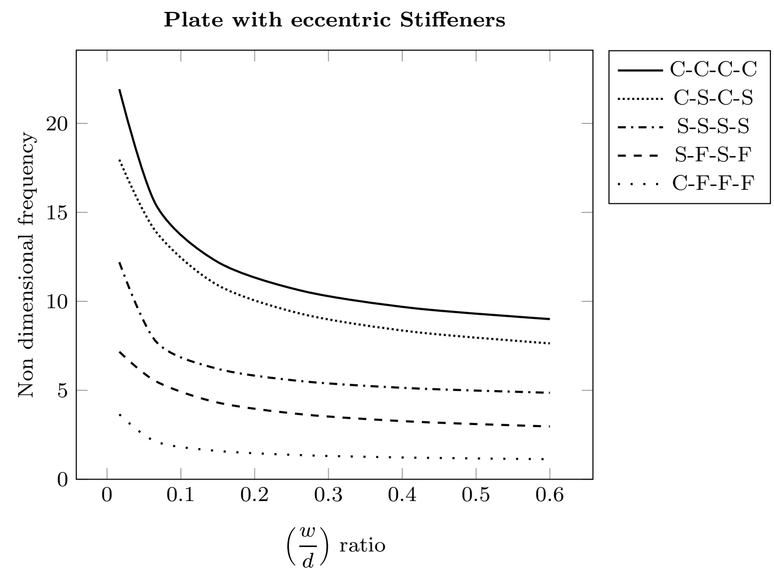


Fig. 21 Non-dimensional frequency $\left(\omega b^2\sqrt{\frac{12\rho(1-\nu^2)}{Eh^2}}\right)$ of the stiffened plate for different $\left(\frac{w}{d}\right)$ ratios

Stiffened Plate with Various Stiffener Depth-to-Thickness of Plate $\left(\frac{d}{h}\right)$ Ratio

The plate investigated in section 4.7 is considered for free vibration analysis, but now with the I-stiffener shown in Fig 12. The non-dimensional frequency $\left(\omega b^2\sqrt{12\rho(1-\nu^2)/Eh^2}\right)$ results for different stiffener depth–thickness of plate ratios are explored with various boundary conditions for both eccentric as well as concentric stiffeners and presented in Fig. 19 and 20.

From Fig. 19 and 20, it is inferred that the non-dimensional frequency is increasing with the increase of $\left(\frac{d}{h}\right)$ ratio and with the further increase of ratio, non-dimensional frequency values remain unchanged. It is also observed that the non-dimensional frequency value is higher for eccentric stiffeners than that of concentric stiffeners for the same boundary condition. The non-dimensional frequency is higher for C–C–C–C plate compared to other boundary conditions.

Stiffened plate with various stiffener width–depth $\left(\frac{w}{d}\right)$ ratio

The plate with diagonal stiffeners considered in section 4.7 is investigated for the free vibration analysis to study the effect of various stiffener width–depth $\left(\frac{w}{d}\right)$ ratios by keeping

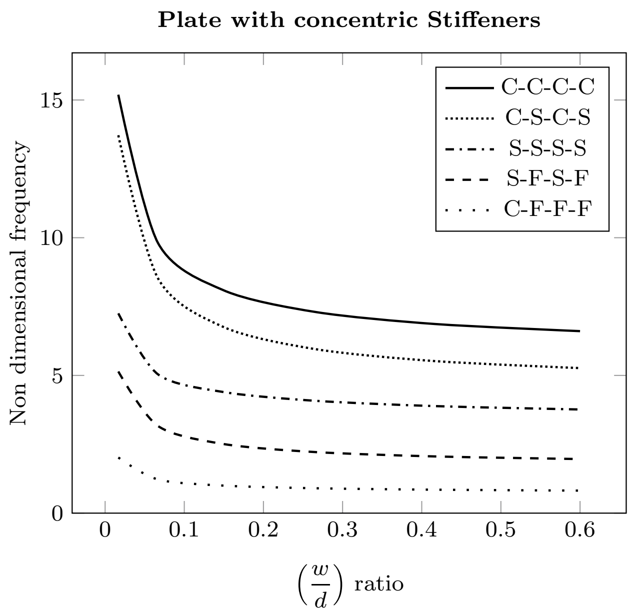


Fig. 22 Non-dimensional frequency $\left(\omega b^2\sqrt{\frac{12\rho(1-\nu^2)}{Eh^2}}\right)$ of the stiffened plate for different $\left(\frac{w}{d}\right)$ ratios

the area of rectangular beam stiffener constant. The non-dimensional frequency $\left(\omega b^2\sqrt{\frac{12\rho(1-\nu^2)}{Eh^2}}\right)$ results for different boundary conditions of the plate with eccentric and concentric stiffeners are presented in Figs. 21 and 22.

Figures 21 and 22 shows that the non-dimensional frequency is decreasing with the increase of $\left(\frac{w}{d}\right)$ ratio, and with the further increase of ratio, the change in frequency values is minimal. It is also observed that the eccentric stiffened plate achieves greater frequency values than that of concentric stiffened plate with the increase of $\left(\frac{w}{d}\right)$ ratio.

Conclusions

A finite element method is presented for free vibration analysis of stiffened plates, where the plate and stiffener element matrices are derived separately. The stiffener disposition in the plate is considered for both the concentric as well as eccentric ones. The orientation of the stiffener position has been considered in a general manner by which its effect on the free vibration has been observed for different orientation. The difference in results between the orthogonally and diagonally stiffened plates has been discussed. A detailed parametric study is carried out for the dynamic response of the stiffened plates by varying the plate aspect ratio, stiffener depth–plate thickness ratio, stiffener width–depth ratio. Results have been compared either with the published ones or with the ANSYS/FEAST results wherever possible.

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