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ORIGINAL PAPER

Fault-Tolerant Control for Wing Flutter Under Actuator Faults and Time Delay

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Abstract
Purpose Investigating the design of the controller stabilizing the wing flutter system, which is robust against actuator faults, actuator saturation, time delay, parameter uncertainties and external disturbances.
Methods The model of the wing flutter system that considered the effects of actuator faults and saturation, time delay, parameter uncertainties and external disturbances is constructed by the Lagrange method. Then the finite-time fault-tolerant controller is derived, the stability of which is proved by the Lyapunov function.
Conclusions The simulation results elucidate that, the proposed fault-tolerant controller can handle the actuator faults effectively, meanwhile the wing flutter of the reentry vehicle can be suppressed instantly. The robustness of the actuator against actuator saturation, time delay, parameter uncertainties and external disturbances is also demonstrated.

Keywords Active flutter suppression · Fault-tolerant control · Time delay · Actuator fault · Observer · Actuator saturation

Introduction

With the technology in aeronautic field developing rapidly, performance of modern aircrafts improves a lot, and their characteristics of high air speed and flexibility caused by lighter weight and advanced composite materials make the aeroelasticity problems appear more frequently than before. Flutter is a considerable problem among those. Instability caused by flutter could lower control performance of aircrafts or even results in the disastrous structural failure [1]. The passive flutter suppression used traditionally is usually inefficient (because it will introduce adding structural weight), and they obtained not much effectiveness. To solve these inadequate problems, the active flutter suppression techniques have risen from early 1970 s, and it is realized by deflecting ailerons and flaps of the wing to change the aerodynamic distribution on the lifting surface.

Much attention has been paid on the technique of active flutter suppression over the past decade [2–6]. Although numerous controller design means for flutter suppression have been carried out [2–6], actuator fault or failure is not assumed to exist during the entire flutter suppression in most of the research. Practically, this assumption is not always satisfied because actuator malfunction may lead to some catastrophic failure. As a result, if no fault tolerance capability is considered during design process for the flutter suppression controller, the flutter suppression control could ultimately fail with an abrupt occurrence of actuator fault. Therefore, the faults of actuators and sensors should be prior thought over during the design process of flutter suppression controller, and introduce the fault-tolerant controller [7] for flutter suppression. Fault-tolerant control (FTC) methods generally have two classifications: passive fault-tolerant control (PFTC) and active fault-tolerant control (AFTC) schemes [8–11]. As designing based on limited faults and the controller parameters are fixed, passive fault-tolerant controller is unable to assure the control performance on the system, and an active fault-tolerant controller to suppress the flutter system including unexpected actuator faults or failures is hence investigated in this paper. So far as we have known, very little research on finite-time adaptive fault-tolerant control for wing flutter suppression can be seen in the published literature.

Time delay is also a key issue affecting the effectiveness of flutter suppression controllers. Non-synchronization in control forces may result from many control processes such as measuring system variables, controller calculating, and building up of the required control force in the actuator, which may

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degrade control efficiency and even result in the instability of the close-loop system for flutter control. Traveling in hypersonic air speed, the state variables of the reentry vehicle system change far more promptly, and a rather small time delay in the close-loop system for flutter suppression may result in the instability. The time-delay effect on the stability of aeroservoelastic systems is discussed in a few studies [12–14]. These studies only designed controllers to deal with time delay, but not consider the faults that may occur in actuators. When there are faults in the actuator, the above control schemes could no longer handle time delay in aeroservoelastic cases.

Revealing the disadvantages in traditional aeroservoelastic control methods, and dealing with the affection of time delay, actuator saturation and faults, parameter uncertainties and external disturbances to the controller, this paper mainly carries out the idea and derivation of the adaptive fault-tolerant controller for flutter suppression. The object model for flutter suppression is a two-dimensional airfoil with cubic nonlinearity. The actuator faults are taken into account during the derivation of the controller. This paper is organized as follows. Section 2 established the airfoil flutter dynamics equation. An observer-based finite-time adaptive fault-tolerant controller for flutter suppression is presented in "Design of Fault-Tolerant Flutter Controller Based on Observer". The next section depicts the result of "Numerical Simulations". "Conclusions" are briefed in final section.

7 Two-Dimensional Wing

Including the cubic hard spring nonlinearity, modeling of a nonlinearity two-dimensional airfoil flutter system is discussed in this section. A wing system model with two degree-of-freedom (2-DOF) is introduced and shown in Fig. 1. h is the plunge deflection, and downward donates the positive direction. p is the elastic axis, and the pitch angle about it is denoted by θ with nose up the positive direction. C is the mass center, and Q is the center of aerodynamics.c and S are the length of chord and wing span, respectively. x_C, x_Q, and x_p denote the distance between the leading edge, and C, Q and p, respectively. δ_{LEout} and δ_{LEin} (or δ_{REout} and δ_{REin}) are the control surface angles.

From Fig. 1, on the mass center of airfoil, the velocity is derived as

$$\dot{z} = \dot{h} + (x_C - x_p)\dot{\theta}. \tag{1}$$

The kinetic energy, potential energy and dissipation of the system can be expressed as

$$\begin{aligned} T &= \frac{1}{2}m_W\dot{z}^2 + \frac{1}{2}m_e\dot{h}^2 + \frac{1}{2}I_C\dot{\theta}^2, \\ U &= \frac{1}{2}K_h h^2 + \frac{1}{2}K_\theta\theta^2, \\ \zeta &= \frac{1}{2}C_h\dot{h}^2 + \frac{1}{2}C_\theta\dot{\theta}^2, \end{aligned} \tag{2}$$

where m_W and m_e are the mass and extra-mass of the wing, respectively, and I_C is the inertia moment about C. K_h and K_θ are the plunge and torsion stiffness coefficient, respectively. C_h and C_θ are the plunge and torsion damping coefficient, respectively.

The reentry vehicle is traveling in hypersonic flow, and the aerodynamic force and moment can be obtained by the widely known piston theory [15] as

$$\begin{aligned} L &= \frac{2\rho V\bar{\gamma}c}{M_\infty}\left[0.5c(1-x_0)\dot{\theta} + \dot{h} + V\theta + \frac{1}{12}V\bar{\gamma}^2(\kappa+1)M_\infty^2\theta^3\right] \\ T &= \frac{\rho V\bar{\gamma}c^2}{M_\infty}\left[\frac{1}{6}c(4-6x_0+3x_0^2)\dot{\theta} + (1-x_0)\dot{h} + V(1-x_0)\theta + \frac{1}{12}\bar{\gamma}^2(\kappa+1)M_\infty^2(1-x_0)V\theta^3\right], \end{aligned} \tag{3}$$

where $\bar{\gamma} = M_\infty/\sqrt{M_\infty^2 - 1}$ is the aerodynamic correction factor, with M_∞ denoting the Mach number. The ratio of specific heat is represented by κ. x_0 is the distance between the leading edge and p, which is non-dimensional.

The deflection of control surfaces will generate aerodynamic force and moment, and can be derived as

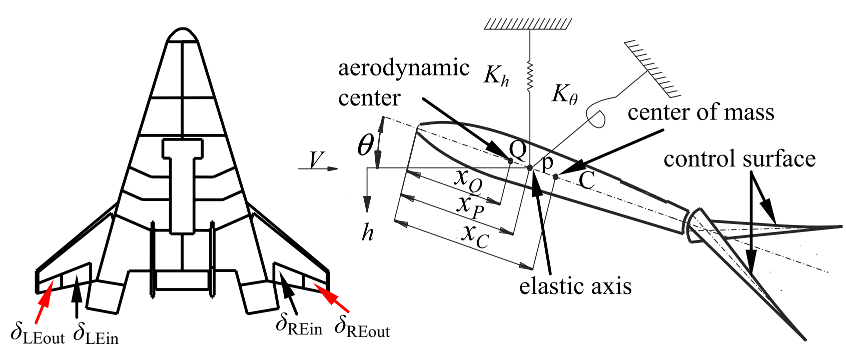

Fig. 1 Two-dimensional wing model with control surface

$$L_{\delta_{\mathrm{LEout}}} = L_{\delta_{\mathrm{LEin}}} = \frac{1}{2}\rho V^2 c a_{\mathrm{C}} s_\beta \delta_{\mathrm{LEout}},$$
$$M_{\delta_{\mathrm{LEout}}} = M_{L_{\delta_{\mathrm{LEin}}}} = \frac{1}{2}\rho V^2 c^2 b_{\mathrm{C}} s_\beta \delta_{\mathrm{LEout}}, \tag{4}$$

where a_{C} and b_{C} represent the coefficient relating control surface deflection to the lift force $L_{\delta_{\mathrm{LEout}}}$ and pitching moment $M_{\delta_{\mathrm{LEout}}}$, respectively, and s_β denotes the control surface span.

The cubic nonlinearity will cause a moment expressed as [16]

$$M(\theta) = K_\theta \theta + e_{n1}\theta^3. \tag{5}$$

When neglecting structural damping, we use the Lagrangian method to derive the dynamics equation of the 2-DOF wing aeroelastic system and rewrite it into matrix form as

$$\tilde{\mathbf{A}}\ddot{\mathbf{q}}(t) + \rho V\tilde{\mathbf{B}}\dot{\mathbf{q}}(t) + (\rho V^2\tilde{\mathbf{C}} + \tilde{\mathbf{D}})\mathbf{q}(t) + \tilde{\mathbf{f}}(t) = \tilde{\mathbf{b}}\mathbf{u}(t), \tag{6}$$

where $\tilde{\mathbf{A}}$ is the inertia matrix, and $\tilde{\mathbf{B}}$ is the aerodynamic damping matrix. $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ are, respectively, the aerodynamic and structural stiffness matrices. $\mathbf{q}(t)$ denotes the generalized displacement with the expression $\mathbf{q}(t) = [\ h(t)\ \ \theta(t)\ ]^{\mathrm{T}}$. $\mathbf{u}(t)$ represents the control input, expressed by $\mathbf{u}(t) = [\ \delta_{\mathrm{LEout}}\ \ \delta_{\mathrm{LEin}}\ ]^{\mathrm{T}}$. The coefficient matrices in Eq. (6) are detailedly given by

$$\tilde{\mathbf{A}} = \begin{bmatrix} m_{\mathrm{W}} + m_{\mathrm{e}} & m_{\mathrm{W}}(x_{\mathrm{C}} - x_{\mathrm{p}}) \\ m_{\mathrm{W}}(x_{\mathrm{C}} - x_{\mathrm{p}}) & m_{\mathrm{W}}(x_{\mathrm{C}} - x_{\mathrm{p}})^2 + I_{\mathrm{C}} \end{bmatrix},\quad \tilde{\mathbf{C}} = \begin{bmatrix} 0 & \frac{2\bar{\gamma}c}{M_\infty} \\ 0 & -\frac{\bar{\gamma}c^2}{M_\infty}(1 - x_0) \end{bmatrix},$$

$$\tilde{\mathbf{B}} = \begin{bmatrix} -\frac{2\bar{\gamma}c}{M_\infty} & -\frac{\bar{\gamma}c^2}{M_\infty}(1 - x_0) \\ \frac{\bar{\gamma}c^2}{M_\infty}(1 - x_0) & \frac{\bar{\gamma}c^3}{6M_\infty}(4 - 6x_0 + 3x_0^2) \end{bmatrix},\quad \tilde{\mathbf{D}} = \begin{bmatrix} K_h & 0 \\ 0 & K_\theta \end{bmatrix},$$

$$\tilde{\mathbf{b}} = \begin{bmatrix} -\frac{1}{2}\rho V^2 c a_{\mathrm{C}} s_\beta & -\frac{1}{2}\rho V^2 c a_{\mathrm{C}} s_\beta \\ \frac{1}{2}\rho V^2 c^2 b_{\mathrm{C}} s_\beta & \frac{1}{2}\rho V^2 c^2 b_{\mathrm{C}} s_\beta \end{bmatrix},\quad \tilde{\mathbf{f}} = \begin{bmatrix} -\frac{1}{6}\rho V^2\bar{\gamma}^3 c(\kappa + 1)M_\infty\theta^3 \\ \frac{1}{12}\rho V^2\bar{\gamma}^3 c^2(\kappa + 1)M_\infty(1 - x_0)\theta^3 + e_{n1}\theta^3 \end{bmatrix}.$$

Equation (6) can be changed in further as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{f}(t), \tag{7}$$

where $\mathbf{x}$ denotes the state variables expressed in form of vector as $\mathbf{x} = [h(t),\ \theta(t),\ \dot{h}(t),\ \dot{\theta}(t)]^{\mathrm{T}}$, and with $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\tilde{\mathbf{A}}^{-1}(\rho V^2\tilde{\mathbf{C}} + \tilde{\mathbf{D}}) & -\tilde{\mathbf{A}}^{-1}\rho V^2\tilde{\mathbf{B}} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{A}}^{-1}\tilde{\mathbf{b}} \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} \mathbf{0} \\ -\tilde{\mathbf{A}}^{-1}\tilde{\mathbf{f}}(t) \end{bmatrix}$.

Design of Fault-Tolerant Flutter Controller Based on Observer

Flutter dynamic Eq. (7) is used in this section, and all actuators are assumed to be fault free, and the system is regarded as nominal. Acting properly of the actuators for the active suppression of flutter is hard to be guaranteed. Some problems are likely to occur in them, such as saturation, float or loss of effectiveness. Therefore, during the flutter suppression controller design, it is necessary to consider fault tolerance capability.

On the other hand, there is an inevitable hysteresis $\tau(t)$ in the actuator instruction input $\mathbf{u}(t)$ denoted by $\mathbf{u}(t - \tau(t))$, and $\tau(t)$ involving the synthetic hysteretic effect from time for the actuator to response, D/A conversion, the measuring and filtering process of observer. Hence, it makes great sense to study on active flutter suppression system involving time delay.

Actuator faults of two types, which are the effectiveness loss and float fault, are synchronously taken into account in this section. Involving time delay, external disturbances, parameter uncertainties and saturation of actuator as well, we expand the flutter Eq. (7) as

$$\begin{cases} \dot{\mathbf{x}}(t) = [\mathbf{A} + \Delta\mathbf{A}(t)]\mathbf{x}(t) + \mathbf{B}\boldsymbol{\rho} sat(\mathbf{u}(t - \tau(t))) + \mathbf{B}\mathbf{u}_s(t) + \mathbf{f}(t,\ \mathbf{x}) + \mathbf{B}_1\mathbf{w}(t) \\ \mathbf{y}(t) = \mathbf{c}_1\mathbf{x}(t) \end{cases}, \tag{8}$$

where the uncertainties in state matrix $\mathbf{A}$ of control system of the wing flutter suppression is shown by $\Delta\mathbf{A}$. $\mathbf{u}(t - \tau(t))$ represents the desired input signal from controller, and $sat(\mathbf{u}(t))$ is the actual actuation vector of the actuators. The effectiveness coefficient of actuators of control surfaces is shown in the form of matrix by $\boldsymbol{\rho} = \mathrm{diag}(\rho_1,\ \ \rho_2)$ while $0 < \rho_i \le 1\ (i = 1,\ 2)$. $\rho_i = 1$ signifies the proper full actuation of the i-th actuator, while $0 < \rho_i < 1$ means the partial effectiveness loss in actuation of the i-th actuator. Satisfying $1 - \dot{\tau}(t) \ge \varpi^2$ with ϖ a constant, time-varying variables $\tau(t)$ represent the time delay. $\mathbf{u}_s(t) = (u_{s1},\ u_{s2}) \in \Re^{2\times 1}$ corresponds to the case representing that there is float fault in the i-th actuator on the i-th control surface. $\mathbf{w}(t)$ denotes the disturbances from external which is bounded. $\mathbf{y}(t)$ denotes the observed term from the filter, and $\mathbf{f}(t,\ \mathbf{x})$ represents a nonlinear appellation. Coefficient matrices $\mathbf{B}_1$ and $\mathbf{c}_1$ are obtained manually with suitable dimensions. Assuming that parameter uncertainty matrix $\Delta\mathbf{A}$ meets the matching condition as

$$\Delta\mathbf{A} = \mathbf{B}\mathbf{N}(t),\quad \mathbf{N}^{\mathrm{T}}(t)\mathbf{N}(t) \le \mathbf{I}, \tag{9}$$

where matrix $\mathbf{N}(t)$ is unknown but its ℓ^2 norm $\|\mathbf{N}\|$ is no larger than an unknown constant l^* as the expression of $\|\mathbf{N}\| \le l^*$.

Remark 1 For the reentry vehicle, atmospheric environment and aerodynamic characteristics are the main source of parameter uncertainties. When modeling the wing flutter system, there will be hardship in modeling the generalized aerodynamic forces because of these uncertainties, and uncertainties will be involved into the wing flutter dynamic equation. Control system of flutter suppression cannot remain invariant on account of the changing $\Delta\mathbf{A}(t)$. We think there would be specific relationship between them. Hence, it is reasonable to suppose $\Delta\mathbf{A} = \mathbf{BN}(t)$.

We define the actual actuation $sat(\mathbf{u}(t))$ as

$$\begin{cases} sat(\mathbf{u}(t)) = \mathbf{u}(t) + \boldsymbol{\delta}_{\mathbf{u}} \\ sat(\mathbf{u}(t)) = \mathbf{u}(t) = \begin{cases} \mathbf{u}_{\max}(t), & \mathbf{u}(t) > \mathbf{u}_{\max}(t) \\ \mathbf{u}(t), & |\mathbf{u}(t)| \le \mathbf{u}_{\max}(t) \\ -\mathbf{u}_{\max}(t), & \mathbf{u}(t) < -\mathbf{u}_{\max}(t) \end{cases} \end{cases} \tag{10}$$

where $\mathbf{u}_{\max}(t)$ and $\boldsymbol{\delta}_{\mathbf{u}}$ are the saturation level of input $sat(\mathbf{u}(t))$ and auxiliary variable. Through the RBF network [17], the unknown $\boldsymbol{\delta}_{\mathbf{u}}$ in this paper can be approximated as

$$h_j(x) = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{c}_i\|^2}{2b_j^2}\right),\ \boldsymbol{\delta}_{\mathbf{u}}(\mathbf{x}) = \mathbf{W}^{*\mathrm{T}}\mathbf{h}(\mathbf{x}) + \boldsymbol{\varepsilon}_{\mathbf{u}}(\mathbf{x}),\ \mathbf{x} \in \mathbf{D}_{\mathbf{x}}, \tag{11}$$

where $h_j(\mathbf{x})$ is the Gaussian function vector. $\mathbf{x}_i$ represents the network input, b_j is the width, and $\mathbf{c}_i$ denotes the center vector. The matrix of ideal weighting coefficients is given by $\mathbf{W}^*$. $\boldsymbol{\delta}_{\mathbf{u}}(\mathbf{x})$ is the network output. The estimation of $\boldsymbol{\delta}_{\mathbf{u}}(\mathbf{x})$ is given by $\hat{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x}) = \hat{\mathbf{W}}^{\mathrm{T}}\mathbf{h}(\mathbf{x})$ with $\bar{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x}) = \boldsymbol{\delta}_{\mathbf{u}}(\mathbf{x}) - \hat{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x})$. The estimation of $\mathbf{W}^*$ is represented by $\hat{\mathbf{W}}$. $\boldsymbol{\varepsilon}_{\mathbf{u}}(\mathbf{x})$ denotes the estimated error, and $\mathbf{D}_{\mathbf{x}} \in \mathfrak{R}^{4\times1}$ is a sufficiently large compact set.

The Design of Observer

In this section, an observer is developed to estimate the state variables of flutter control system which is needed when pursuing the control law. According to control system Eq. (8), the observer can be formed as follows:

$$\dot{\hat{\mathbf{x}}}(t) = [\mathbf{A} + \Delta\mathbf{A}(t)]\hat{\mathbf{x}}(t) + \mathbf{B}sat(\mathbf{u}(t-\tau(t))) + \mathbf{f}(t,\ \hat{\mathbf{x}}) + \mathbf{L}(\mathbf{y}(t) - \hat{\mathbf{y}}(t)). \tag{12}$$

The estimation error $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ denotes the differences between the true value $\mathbf{x}(t)$ and estimated value $\hat{\mathbf{x}}(t)$ of it. Derived from Eq. (8) and Eq. (12), the differential equation of the error can be expressed as

$$\begin{aligned}\dot{\mathbf{e}}(t) = {} & [\mathbf{A} + \Delta\mathbf{A}(t) - \mathbf{Lc}_1]\mathbf{e}(t) + \mathbf{B}(\boldsymbol{\rho} - \mathbf{I})\mathrm{sat}(\mathbf{u}(t-\tau(t))) \\ & + \mathbf{Bu}_{\mathrm{s}}(t) + \mathbf{f}(t,\ \mathbf{e}) + \mathbf{B}_1\mathbf{w}(t).\end{aligned} \tag{13}$$

Theorem 1 *Consider Eq.* (13), *and assume the observer gains to be obtained through the equations as*

$$\begin{aligned} & \widehat{\mathbf{P}}[\mathbf{A} - \mathbf{Lc}_1] + [\mathbf{A} - \mathbf{Lc}_1]^{\mathrm{T}} + \varepsilon_{\mathrm{f}}^{-1}\widehat{\mathbf{P}}\widehat{\mathbf{P}} + (\varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2 + \varepsilon_{\mathrm{A}})\mathbf{I} + \varepsilon_{\mathrm{A}}^{-1}\widehat{\mathbf{P}}\mathbf{BB}^{\mathrm{T}}\widehat{\mathbf{P}} < \mathbf{0} \\ & \left\|\widehat{\mathbf{P}}\mathbf{B}(\mathbf{I}-\boldsymbol{\rho})\right\|\mathrm{sat}(\mathbf{u}(t-\tau(t))) > \left\|\widehat{\mathbf{P}}\mathbf{B}\right\|\mathbf{u}_{\mathrm{s}}(t) + \left\|\widehat{\mathbf{P}}\mathbf{B}_1\right\|\mathbf{w}(t). \end{aligned} \tag{14}$$

Then system (13) *of* $\mathbf{e}(t)$ *is stable.*

Proof Select the Lyapunov function as $V(\mathbf{x}(t)) = \mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{e}(t)$. Take the derivative with respect to time t on both sides of the above function, expressed as

$$\begin{aligned}\dot{V}(\mathbf{x}(t)) = {} & \mathbf{e}^{\mathrm{T}}(t)\{\widehat{\mathbf{P}}[\mathbf{A} - \mathbf{Lc}_1] + [\mathbf{A} - \mathbf{Lc}_1]^{\mathrm{T}}\}\mathbf{e}(t) \\ & + 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{Bu}_s(t) + 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{B}_1\mathbf{w}(t) \\ & - 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{B}(\mathbf{I}-\boldsymbol{\rho})\mathrm{sat}(\mathbf{u}(t-\tau(t))) \\ & + 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\Delta\mathbf{A}(t)\mathbf{e}(t) + 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{f}(t,\ \mathbf{e}).\end{aligned} \tag{15}$$

Introduce Young's inequation for constants $\varepsilon_{\mathrm{f}} \ge 0$ and $\varepsilon_{\mathrm{A}} \ge 0$ with arbitrary positive value as $2x^{\mathrm{T}}y \le \varepsilon_{\mathrm{i}}x^{\mathrm{T}}x + \varepsilon_{\mathrm{i}}^{-1}y^{\mathrm{T}}y$, and $\mathbf{f}(t,\ \mathbf{e})$ meets $\|\mathbf{f}(t,\ \mathbf{e})\| \le L_{\mathrm{g}}\|\mathbf{e}(t)\|$ with $L_{\mathrm{g}} > 0$, known as the Lipschitz condition, we have

$$\begin{aligned} 2\left\|\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{f}(t,\ \mathbf{e})\right\| & \le \varepsilon_{\mathrm{f}}^{-1}\left\|\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\right\|^2 + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2\left\|\mathbf{e}^{\mathrm{T}}(t)\right\|^2 \\ 2\left\|\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\Delta\mathbf{A}(t)\mathbf{e}(t)\right\| & \le \varepsilon_{\mathrm{A}}^{-1}\left\|\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{B}\right\|^2 + \varepsilon_{\mathrm{A}}\left\|\mathbf{e}^{\mathrm{T}}(t)\right\|^2. \end{aligned} \tag{16}$$

According to Eq. (16), Eq. (15) can be changed into

$$\begin{aligned}\dot{V}(\mathbf{x}(t)) = {} & \mathbf{e}^{\mathrm{T}}(t)\{\widehat{\mathbf{P}}[\mathbf{A} - \mathbf{Lc}_1] + [\mathbf{A} - \mathbf{Lc}_1]^{\mathrm{T}} + \varepsilon_{\mathrm{f}}^{-1}\widehat{\mathbf{P}}\widehat{\mathbf{P}} + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2 \\ & + \varepsilon_{\mathrm{A}}^{-1}\widehat{\mathbf{P}}\mathbf{BB}^{\mathrm{T}}\widehat{\mathbf{P}} + \varepsilon_{\mathrm{A}}\}\mathbf{e}(t) + 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{B}_1\mathbf{w}(t) \\ \times {} & 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{B}(\boldsymbol{\rho} - \mathbf{I})\mathbf{u}(t) + 2\mathbf{e}^{\mathrm{T}}(t)\widehat{\mathbf{P}}\mathbf{Bu}_{\mathrm{s}}(t).\end{aligned} \tag{17}$$

Considering Theorem 1, it can be inferred from Eq. (17) that $\dot{V}(\mathbf{x}(t)) \le 0$, which proves the stability of system (13) of $\mathbf{e}(t)$, and the proof can be accomplished.

Fault-Tolerant Flutter Controller Design

In this section, a novel finite-time fault-tolerant flutter control algorithm is investigated to suppress the wing flutter. Considering Eqs. (8) and (11), the adaptive flutter control law is designed as

$$\mathbf{u}(t) = [\mathbf{K}_1 + \mathbf{K}_2(t) + \mathbf{K}_3(t)]\mathbf{x}(t) - \hat{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x}) \tag{18}$$

where control gain $\mathbf{K}_1$ is fixed and can be calculated through LMI algorithm (37) to guarantee flutter suppression system to be stable; $\mathbf{K}_2(t)$ and $\mathbf{K}_3(t)$ are adaptive gains to neutralize the affection from uncertainty in parameters, disturbances and float. The adaptive gain $\mathbf{K}_2(t)$ and $\mathbf{K}_3(t)$ are chosen as

$$\mathbf{K}_2(t) = \frac{\mathbf{B}^{\mathrm{T}}\bar{\tilde{\mathbf{P}}}^j\hat{k}_4(t)}{\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t)\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|}, \quad \mathbf{K}_3(t) = \frac{1}{2}\eta\mathbf{B}^{\mathrm{T}}\bar{\tilde{\mathbf{P}}}^j\hat{k}_5(t), \tag{19}$$

where $\tilde{\mathbf{P}}^j$ represents the corresponding matrix $\mathbf{P}$ of each faulty mode, and it is positively definite, with j denoting the j-th mode of faulty; $\bar{\tilde{\mathbf{P}}}^j$ is obtained from $\bar{\tilde{\mathbf{P}}}^j := \{\tilde{\mathbf{P}}^j : \max_j(\|\tilde{\mathbf{P}}^j\|)\}$ representing the maximum norm of $\tilde{\mathbf{P}}^j$. $\hat{k}_4$ as well as $\hat{k}_5$ are updated adaptively through the following equations:

$$\frac{\mathrm{d}\hat{k}_4(t)}{\mathrm{d}t} = -r_1\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t)\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|, \qquad \frac{\mathrm{d}\hat{k}_5(t)}{\mathrm{d}t} = -r_2\eta\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t)\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|^2. \tag{20}$$

$\hat{\mathbf{W}}$ is updated adaptively through the following equations:

$$\frac{\mathrm{d}\hat{\mathbf{W}}_i}{\mathrm{d}t} = r_3\mathbf{h}(\mathbf{x})\mathbf{x}^{\mathrm{T}}(t)\bar{\tilde{\mathbf{P}}}^j\mathbf{b}_i, \quad i = 1,\ 2, \tag{21}$$

where $\mathbf{b}_i$ is the i-th column of $\mathbf{B}$.

Denote

$$\tilde{k}_4(t) = k_4 - \hat{k}_4(t),\ \tilde{k}_5(t) = k_5 - \hat{k}_5(t),\ \tilde{\mathbf{W}}^{\mathrm{T}} = \mathbf{W}^{*\mathrm{T}} - \hat{\mathbf{W}}^{\mathrm{T}}. \tag{22}$$

Flutter control system Eq. (8) can be rewritten into the form of closed-loop, by substituting Eqs. (18) and (10), expressed as

$$\begin{cases} \dot{\mathbf{x}}(t) = [\mathbf{A} + \Delta\mathbf{A}(t)]\mathbf{x}(t) + \mathbf{B}\boldsymbol{\rho}[(\mathbf{K}_1 + \mathbf{K}_2(t-\tau(t)) + \mathbf{K}_3(t-\tau(t)))\hat{\mathbf{x}}(t-\tau(t)) - \\ \hat{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x}) + \boldsymbol{\delta}_{\mathbf{u}}(\mathbf{x})] + \mathbf{B}\mathbf{u}_s(t) + \mathbf{f}(t,\ \mathbf{x}) + \mathbf{B}_1\mathbf{w}(t) \\ \mathbf{y}(t) = \mathbf{c}_1\mathbf{x}(t) \end{cases} . \tag{23}$$

Assumption 1 For T_{f} of real working time, and when d_{s}, d_{w}, and $d_{\boldsymbol{\delta}}$ are given an arbitrary value, the nonlinear term $\mathbf{f}(t,\ \mathbf{x})$, fault of float $\mathbf{u}_s(t)$, estimation of auxiliary variable $\bar{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x})$, and disturbances from external $\mathbf{w}(t)$ are time variant and satisfying

$$\begin{aligned} &\int_0^{T_{\mathrm{f}}} \mathbf{u}_{\mathrm{s}}^{\mathrm{T}}(t)\mathbf{u}_{\mathrm{s}}(t)\mathrm{d}t \le d_{\mathrm{s}}, d_s \ge 0, \\ &\int_0^{T_{\mathrm{f}}} \mathbf{w}^{\mathrm{T}}(t)\mathbf{w}(t)\mathrm{d}t \le d_{\mathrm{w}}, d_{\mathrm{w}} \ge 0, \\ &\int_0^{T_{\mathrm{f}}} \bar{\boldsymbol{\delta}}_{\mathbf{u}}^{\mathrm{T}}(\mathbf{x})\bar{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x})\mathrm{d}t \le d_{\boldsymbol{\delta}}, d_{\boldsymbol{\delta}} \ge 0. \end{aligned} \tag{24}$$

Remark 2 The real output actuation applied by the actuators has its limitation considering the nature of the actuators in practice, and hence, the fault of float $\mathbf{u}_{\mathrm{s}}(t)$ is bounded as well as the auxiliary variable $\boldsymbol{\delta}_{\mathbf{u}}(\mathbf{x})$, with $\boldsymbol{\delta}_{\mathbf{u}}(\mathbf{x}) = \mathrm{sat}(\mathbf{u}(t)) - \mathbf{u}(t)$. Adding that in Eq (8), the disturbances $\mathbf{w}(t)$ from external, including shifting in atmospheric density, disturbance in gravity, offset of mass center, and error in inertia moment, are likewise bounded. Hence for flutter control system, Assumption 1 can be considered reasonable.

Definition 1 [18] When a positive definite matrix $\mathbf{R}$ which is symmetric and constants c_1, d_{w}, d_{f}, d_{s}, and T_{f} which are all positive are given, and if two constants c_1 and c_2 exist with $c_2 > c_1$, such that

$$\mathbf{x}_0^{\mathrm{T}}\mathbf{R}\mathbf{x}_0 \le c_1 \Rightarrow \mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) < c_2, \quad \forall t \in [0\ \ T_{\mathrm{f}}], \tag{25}$$

the derived feedback system (23) for flutter suppression can be said to be finite-time bounded (FTB) robustly in regard to $(\mathbf{R}, c_1, c_2, d_{\mathrm{w}}, d_{\mathrm{f}}, d_{\mathrm{s}}, T_{\mathrm{f}})$.

Definition 2 [19] If controller expressed as Eq. (18) exists, namely the derived feedback control system (23) for wing flutter is FTB as defined in Definition 1 in regard to $\mathbf{R}$, c_1, c_2, d_{w}, d_{f}, d_{s}, and T_{f}, and under the assumed zero initial condition, for $T_f > 0$ and for all acceptable $\mathbf{w}(t)$ that satisfies Assumption 1, the output of the flutter system will satisfy the following inequality:

$$\int_0^{T_{\mathrm{f}}} \mathbf{y}^{\mathrm{T}}(t)\mathbf{y}(t)\mathrm{d}t \le \gamma^2 \int_0^{T_{\mathrm{f}}} \mathbf{w}^{\mathrm{T}}(t)\mathbf{w}(t)\mathrm{d}t. \tag{26}$$

Then, the control law (18) for flutter suppression is known as the robust finite-time H_∞ controller for the nonlinear flutter control systems (23).

Lemma 1 [20] For arbitrary matrix $\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix}$ which is symmetric, the three expressions given below are isovalent

(1) $\mathbf{S} < \mathbf{0}$, (2) $\mathbf{S}_{11} < \mathbf{0}$, $\mathbf{S}_{22} - \mathbf{S}_{12}^{\mathrm{T}}\mathbf{S}_{11}^{-1}\mathbf{S}_{12} < \mathbf{0}$, (3) $\mathbf{S}_{22} < \mathbf{0}$, $\mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{12}^{\mathrm{T}} < \mathbf{0}$. (27)

Theorem 2 *When a positive definite matrix* $\mathbf{R}$ *which is symmetric and constants* c_1, $d_{\boldsymbol{\delta}}$, d_{w}, d_{s}, T_{f} *and* α_0 *which are all positive are given, the feedback control system (23) for flutter suppression is FTB in regard to* $\mathbf{R}$, c_1, c_2, $d_{\boldsymbol{\delta}}$, d_{w}, d_{s}, and T_{f}, *if a constant* $c_2 > 0$ *and a symmetric matrix* $\tilde{\mathbf{P}} > \mathbf{0}$ *exist, such that*

$$\begin{bmatrix} \boldsymbol{\Omega} & \tilde{\mathbf{P}}^j\mathbf{B} & \tilde{\mathbf{P}}^j\mathbf{B}_1 & \mathbf{0} & \mathbf{0} \\ * & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\mathbf{I} & \mathbf{0} \\ * & * & * & * & -\varpi^2 \end{bmatrix} < \mathbf{0}, \tag{28}$$

$$\mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) \le \frac{(\lambda_{\max}(\mathbf{P}^j)c_1 + 2\tau_2 c_1 + d_{\mathrm{s}} + d_{\mathrm{w}})e^{\alpha_0 t}}{\lambda_{\min}(\mathbf{P}^j)}, \tag{29}$$

where $\boldsymbol{\Omega} = \tilde{\mathbf{P}}^j\bar{\mathbf{A}} + \bar{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{P}}^j - \alpha_0\tilde{\mathbf{P}}^j + \varepsilon_{\mathrm{f}}^{-1}\tilde{\mathbf{P}}^j\tilde{\mathbf{P}}^j + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2 + 2\mathbf{I}$, $\tilde{\mathbf{P}}^j = \mathbf{R}^{\frac{1}{2}}\mathbf{P}^j\mathbf{R}^{\frac{1}{2}}$ *and* $\bar{\mathbf{A}} = \mathbf{A} + \Delta\mathbf{A}(t) + \mathbf{B}\boldsymbol{\rho}[\hat{\mathbf{K}}_1(t) + \mathbf{K}_2(t) + \mathbf{K}_3(t)]$.

Proof Form a candidate of the Lyapunov–Krasovskii functional for the closed-loop system (23) as $V(\mathbf{x}(t)) = \mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{x}(t) + \int_{t-\tau_1}^{t}\mathbf{x}^{\mathrm{T}}(s)\mathbf{x}(s)\mathrm{d}s + \int_{t-\tau_2}^{t-\tau_1}\mathbf{x}^{\mathrm{T}}(s)\mathbf{x}(s)\mathrm{d}s + \int_{t-\tau(t)}^{t}\mathbf{x}^{\mathrm{T}}(s)\mathbf{x}(s)\mathrm{d}s$. Then,

$$\begin{aligned}\dot{V}(\mathbf{x}(t)) = {}& \mathbf{x}^{\mathrm{T}}(t)\{\tilde{\mathbf{P}}^j[\mathbf{A} + \Delta\mathbf{A}(t) + \mathbf{B}\boldsymbol{\rho}(\hat{\mathbf{K}}_1(t) + \mathbf{K}_2(t) + \mathbf{K}_3(t))] \\ & + [\mathbf{A} + \Delta\mathbf{A}(t) + \mathbf{B}\boldsymbol{\rho}(\hat{\mathbf{K}}_1(t) + \mathbf{K}_2(t) + \mathbf{K}_3(t))]^{\mathrm{T}}\tilde{\mathbf{P}}^j + 2\mathbf{I}\}\mathbf{x}(t) \\ & + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\mathbf{u}_{\mathrm{s}}(t) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{f}(t,\ \mathbf{x}) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}_1\mathbf{w}(t) \\ & - \mathbf{x}^{\mathrm{T}}(t-\tau_2)\mathbf{x}(t-\tau_2) - \varpi^2\mathbf{x}^{\mathrm{T}}(t-\tau(t))\mathbf{x}(t-\tau(t)).\end{aligned} \tag{30}$$

Considering Eq. (30), and applying Eq. (16) into it, we have

$$\begin{aligned}\dot{V}(\mathbf{x}(t)) = {}& \mathbf{x}^{\mathrm{T}}(t)\{\tilde{\mathbf{P}}^j[\mathbf{A} + \Delta\mathbf{A}(t) + \mathbf{B}\boldsymbol{\rho}(\hat{\mathbf{K}}_1(t) + \mathbf{K}_2(t) + \mathbf{K}_3(t))] \\ & + [\mathbf{A} + \Delta\mathbf{A}(t) + \mathbf{B}\boldsymbol{\rho}(\hat{\mathbf{K}}_1(t) + \mathbf{K}_2(t) + \mathbf{K}_3(t))]^{\mathrm{T}}\tilde{\mathbf{P}}^j + 2\mathbf{I} \\ & + \varepsilon_{\mathrm{f}}^{-1}\tilde{\mathbf{P}}^j\tilde{\mathbf{P}}^j + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2\}\mathbf{x}(t) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\mathbf{u}_{\mathrm{s}}(t) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}_1\mathbf{w}(t) \\ & - \mathbf{x}^{\mathrm{T}}(t-\tau_2)\mathbf{x}(t-\tau_2) - \varpi^2\mathbf{x}^{\mathrm{T}}(t-\tau(t))\mathbf{x}(t-\tau(t)).\end{aligned} \tag{31}$$

Suppose a function is defined as follows:

$$J_1 = \dot{V}(\mathbf{x}(t)) - \alpha_0\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{x}(t) - \bar{\boldsymbol{\delta}}_{\mathbf{u}}^{\mathrm{T}}(\mathbf{x})\bar{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x}) - \mathbf{u}_{\mathrm{s}}^{\mathrm{T}}(t)\mathbf{u}_{\mathrm{s}}(t) - \mathbf{w}^{\mathrm{T}}(t)\mathbf{w}(t). \tag{32}$$

From condition inequation (28), we can show that $J_1 < 0$. Multiplying $e^{-\alpha_0 t}$ on both sides, above inequation can be derived into

$$\frac{\mathrm{d}}{\mathrm{d}t}(e^{-\alpha_0 t}V(\mathbf{x}(t))) < e^{-\alpha_0 t}(\bar{\boldsymbol{\delta}}_{\mathbf{u}}^{\mathrm{T}}(\mathbf{x})\bar{\boldsymbol{\delta}}_{\mathbf{u}}(\mathbf{x}) + \mathbf{u}_s^{\mathrm{T}}(t)\mathbf{u}_s(t) + \mathbf{w}^{\mathrm{T}}(t)\mathbf{w}(t)). \tag{33}$$

Consider $\tilde{\mathbf{P}}^j = \mathbf{R}^{\frac{1}{2}}\mathbf{P}^j\mathbf{R}^{\frac{1}{2}}$. Integrating from 0 to t, the inequation (33) can be derived into

$$\begin{aligned}V(\mathbf{x}(t)) < {}& e^{\alpha_0 t}[\mathbf{x}^{\mathrm{T}}(0)\tilde{\mathbf{P}}^j\mathbf{x}(0) + \int_{-\tau_1}^{0}\mathbf{x}^{\mathrm{T}}(s)\mathbf{x}(s)\mathrm{d}s \\ & + \int_{-\tau_2}^{-\tau_1}\mathbf{x}^{\mathrm{T}}(s)\mathbf{x}(s)\mathrm{d}s + \int_{-\tau(t)}^{0}\mathbf{x}^{\mathrm{T}}(s)\mathbf{x}(s)\mathrm{d}s] \\ & + \int_{-\tau(t)}^{0}\mathbf{x}^{\mathrm{T}}(s)\mathbf{x}(s)\mathrm{d}s] + e^{\alpha_0 t}[d_s + d_w] \\ \le {}& (\lambda_{\max}(\mathbf{P}^j)c_1 + 2\tau_2 c_1)e^{\alpha_0 t} + (\mathrm{d}_s + \mathrm{d}_w)e^{\alpha_0 t}.\end{aligned} \tag{34}$$

Meanwhile, the coming-up inequation is satisfied as

$$V(\mathbf{x}(t)) = \mathbf{x}^{\mathrm{T}}(t)\mathbf{R}^{\frac{1}{2}}\mathbf{P}^j\mathbf{R}^{\frac{1}{2}}\mathbf{x}(t) \ge \lambda_{\min}(\mathbf{P}^j)\mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) \tag{35}$$

It can be inferred from Eqs. (34) and (35) that

$$\mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) < \frac{(\lambda_{\max}(\mathbf{P}^j)c_1 + 2\tau_2 c_1 + d_{\mathrm{s}} + d_{\mathrm{w}})e^{\alpha_0 t}}{\lambda_{\min}(\mathbf{P}^j)}. \tag{36}$$

It can be inferred from Condition (29) that for $\forall t \in [0\ \ T_f]$, $\mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) < c_2$. Based on Definition 1, the proof comes to a completion.

Theorem 3 *When a positive definite matrix* $\mathbf{R}$ *which is symmetric and constants* c_1, $d_{\boldsymbol{\delta}}$, d_{w}, d_{s}, μ, T_{f} *and* α_0 *which are all positive are given, the feedback control system (23) for flutter suppression is FTB in regard to* ($\mathbf{R}$, c_1, c_2, $d_{\boldsymbol{\delta}}$, d_{w}, d_{s}, and T_{f} *and satisfies Eq. (26) for all acceptable* $\mathbf{w}(t)$, *if there exist positive constants* η, $\gamma_{\mathrm{f}} > \gamma_n$, ε_τ, and ε_{f}, *symmetric positive definite matrix* $\tilde{\mathbf{P}}^{-j}$ *for any* $\boldsymbol{\rho}$ *and any appropriately dimensioned matrices* Z, *which satisfy*

$$\begin{bmatrix} \boldsymbol{\Omega}_3 & \mathrm{Z}^{\mathrm{T}} & \mathbf{0} & \chi^{-1}\mathrm{Z}^{\mathrm{T}}\mathbf{c}_1^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & \boldsymbol{\Omega}_2 - \chi\tilde{\mathbf{P}}^{-j} & \mathbf{B}_1 & \mathbf{0} & \tilde{\mathbf{P}}^{-j} & \mathbf{B}\boldsymbol{\rho}\mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\ * & * & -\gamma_0^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & \hat{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & \bar{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -\varepsilon_\tau^2 & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -\mathbf{I} & \mathbf{0} \\ * & * & * & * & * & * & * & -\varpi^2 + \varepsilon_\tau^2 \end{bmatrix} < \mathbf{0}, \tag{37}$$

$$\mathbf{x}^{\mathrm{T}}(t)\mathbf{R}\mathbf{x}(t) \le \frac{(\lambda_{\max}(\mathbf{P}^j)c_1 + 2\tau_2 c_1 + d_{\mathrm{s}} + d_{\mathrm{w}})e^{\alpha_0 t}}{\lambda_{\min}(\mathbf{P}^j)}, \tag{38}$$

$$\begin{aligned}&\left\|\mathbf{x}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|k_4 + \left\|\mathbf{x}^{\mathrm{T}}(t-\tau(t))\tilde{\mathbf{P}}^j\mathbf{B}\right\|\mathbf{u}_s(t)\\ &+\left\|\mathbf{x}^{\mathrm{T}}(t-\tau(t))\tilde{\mathbf{P}}^j\mathbf{B}\right\|\boldsymbol{\varepsilon}_{\mathbf{u}\max} \le 0,\end{aligned} \tag{39}$$

where $\boldsymbol{\Omega}_3 = \chi^{-1}(\tilde{\mathbf{P}}^{-j} - \mathrm{Z} - \mathrm{Z}^{\mathrm{T}})$, $\boldsymbol{\Omega}_2 = \mathbf{A}\tilde{\mathbf{P}}^{-j} + \tilde{\mathbf{P}}^{-j}\mathbf{A}^{\mathrm{T}} - \alpha_0\tilde{\mathbf{P}}^{-j} + \varepsilon_{\mathrm{f}}^{-1}$, $\hat{\Omega} = -\mathbf{I} - \chi^{-1}\mathbf{c}_1\tilde{\mathbf{P}}^{-j}\mathbf{c}_1^{\mathrm{T}}$ *and* $\bar{\Omega} = -\left(\frac{1}{\eta} + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2 + 2\right)^{-1}$. *The* H_∞ *performance index of FTC system (23) for flutter suppression is expressed by* $\gamma_0 = \gamma_{\mathrm{f}}$ *for the fault conditions and* $\gamma_0 = \gamma_n$ *for the proper conditions. Then, an adaptive* H_∞ *FTC control law will exist for the system (23).*

Proof Choose the candidate of Lyapunov–Krasovskii functional same as in Theorem 2 and suppose the expression as below:

$$\begin{aligned}J_2 &= J_1 + r_1^{-1}\tilde{k}_4^2(t) + \frac{1}{2}\mu r_2^{-1}\tilde{k}_5^2(t) + \sum_{i=1}^{2}\rho_i r_3^{-1}\tilde{\mathbf{W}}_i^{\mathrm{T}}\tilde{\mathbf{W}}_i\\ J_3 &= \dot{J}_2 - \alpha_0\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{x}(t) + \mathbf{y}^{\mathrm{T}}(t)\mathbf{y}(t) - \gamma_0^2\mathbf{w}^{\mathrm{T}}(t)\mathbf{w}(t),\end{aligned} \tag{40}$$

Afterwards, considering Eqs. (9), (11), (19), (22) and (23), we can express J_3 as

Table 1 Structural parameters of two-dimensional wing

$S = 3.5$ m	$s_\beta = 1.6$ m	$V = 1406$ m/s
$c = 0.7$ m	$b_{\mathrm{C}} = -0.076$	$\rho = 0.0644$ kg/m^3
$m_{\mathrm{W}} = 1320$ kg	$a_{\mathrm{C}} = 3.82$	$e_{n1} = 10$
$m_{\mathrm{e}} = 490$ kg	$K_{\mathrm{h}} = 2\times10^6$ N/m	$K_\theta = 20000$ Nm/rad
$x_Q = 0.18$ m	$x_P = 0.28$ m	$x_{\mathrm{C}} = 0.525$ m
$I_{\mathrm{C}} = 13205$ kg m^2	$M_{\dot\theta} = -1.2$	$\kappa = 1.4$

Applying Eq. (16), for given positive number γ_0, ε_τ, ε_{f} and L_{g}, it can be derived that

$$\begin{cases}2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}_1\mathbf{w}(t) \le \gamma_0^{-2}\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}}\tilde{\mathbf{P}}^j\mathbf{x}(t) + \gamma_0^2\mathbf{w}^{\mathrm{T}}(t)\mathbf{w}(t)\\ 2l^*\left\|\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\right\|\|\mathbf{x}(t)\| \le l^*(\eta l^*\left\|\mathbf{x}^{\mathrm{T}}(t)\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|^2 + \frac{1}{\eta l^*}\|\mathbf{x}(t)\|^2)\\ 2\left\|\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{f}(t,\ \mathbf{x})\right\| \le \varepsilon_{\mathrm{f}}^{-1}\left\|\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\right\|^2 + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2\left\|\mathbf{x}^{\mathrm{T}}(t)\right\|^2\\ \mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\mathbf{K}_1\mathbf{x}(t-\tau(t)) \le \varepsilon_\tau^{-2}\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\mathbf{K}_1\mathbf{K}_1^{\mathrm{T}}\boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\tilde{\mathbf{P}}^j\mathbf{x}(t)\\ +\varepsilon_\tau^2\mathbf{x}^{\mathrm{T}}(t-\tau(t))\mathbf{x}(t-\tau(t))\end{cases}. \tag{42}$$

Using Eqs. (22), (39), (42), $l^*\eta l^*\left\|\mathbf{x}^{\mathrm{T}}(t)\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|^2 = \mu\eta\frac{l^{*2}}{\mu}\left\|\mathbf{x}^{\mathrm{T}}(t)\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|^2$ and let $k_5 = \frac{l^{*2}}{-\mu}$, we can change Eq. (41) into

$$\begin{aligned}J_3 \le\ &\mathbf{x}^{\mathrm{T}}(t)(\tilde{\mathbf{P}}^j\mathbf{A} + \mathbf{A}^{\mathrm{T}}\tilde{\mathbf{P}}^j - \alpha_0\tilde{\mathbf{P}}^j + 2\mathbf{I})\mathbf{x}(t) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\mathbf{K}_1\mathbf{x}(t-\tau(t))\\ &+2l^*\left\|\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\right\|\|\mathbf{x}(t)\| + \left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|\hat{k}_4(t-\tau(t)) - \mathbf{x}^{\mathrm{T}}(t-\tau_2)\mathbf{x}(t-\tau_2)\\ &-\varpi^2\mathbf{x}^{\mathrm{T}}(t-\tau(t))\mathbf{x}(t-\tau(t)) + \mu\eta\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|^2\hat{k}_5(t-\tau(t))\\ &+2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\tilde{\mathbf{W}}^{\mathrm{T}}\mathbf{h}(\mathbf{x}) + 2\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|\boldsymbol{\varepsilon}_{\mathbf{u}\max}(t) + 2\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\tilde{\mathbf{P}}^j\mathbf{B}\right\|\mathbf{u}_s(t)\\ &+2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{f}(t,\ \mathbf{x}) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}_1\mathbf{w}(t) + \mathbf{x}^{\mathrm{T}}(t)\mathbf{c}_1^{\mathrm{T}}\mathbf{c}_1\mathbf{x}(t) - \gamma_0^2\mathbf{w}^{\mathrm{T}}(t)\mathbf{w}(t)\\ &-2r_1^{-1}\tilde{k}_4(t-\tau(t))\dot{\hat{k}}_4(t-\tau(t)) - \mu r_2^{-1}\tilde{k}_5(t-\tau(t))\dot{\hat{k}}_5(t-\tau(t)) - 2r_3^{-1}\sum_{i=1}^{2}\rho_i\tilde{\mathbf{W}}_i^{\mathrm{T}}\dot{\hat{\mathbf{W}}}_i.\end{aligned} \tag{41}$$

$$
\begin{aligned}
J_3 \le\; & \mathbf{x}^{\mathrm{T}}(t)[\tilde{\mathbf{P}}^j\mathbf{A} + \mathbf{A}^{\mathrm{T}}\tilde{\mathbf{P}}^j - \alpha_0\tilde{\mathbf{P}}^j + \varepsilon_{\mathrm{f}}^{-1}\tilde{\mathbf{P}}^j\tilde{\mathbf{P}}^j + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2 + \varepsilon_\tau^{-2}\tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\mathbf{K}_1\mathbf{K}_1^{\mathrm{T}}\boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\tilde{\mathbf{P}}^j \\
& + \left(\frac{1}{\eta} + 2\right)\mathbf{I} + \gamma_0^{-2}\tilde{\mathbf{P}}^j\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}}\tilde{\mathbf{P}}^j + \mathbf{c}_1^{\mathrm{T}}\mathbf{c}_1]\mathbf{x}(t) - (\varpi^2 - \varepsilon_\tau^2)\mathbf{x}^{\mathrm{T}}(t-\tau(t))\mathbf{x}(t-\tau(t)) \\
& - \mathbf{x}^{\mathrm{T}}(t-\tau_2)\mathbf{x}(t-\tau_2) - 2\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|\tilde{k}_4(t-\tau(t)) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\tilde{\mathbf{W}}^{\mathrm{T}}\mathbf{h}(\mathbf{x}) \\
& - \mu\eta\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^j\mathbf{B}\right\|^2\tilde{k}_5(t-\tau(t)) - 2r_1^{-1}\tilde{k}_4(t-\tau(t))\dot{\tilde{k}}_4(t-\tau(t)) \\
& - \mu r_2^{-1}\tilde{k}_5(t-\tau(t))\dot{\tilde{k}}_5(t-\tau(t)) - 2r_3^{-1}\sum_{i=1}^{2}\rho_i\tilde{\mathbf{W}}_i^{\mathrm{T}}\dot{\tilde{\mathbf{W}}}_i.
\end{aligned}
\tag{43}
$$

Set

$$
\begin{aligned}
& \mathbf{x}^{\mathrm{T}}(t)[\tilde{\mathbf{P}}^j\mathbf{A} + \mathbf{A}^{\mathrm{T}}\tilde{\mathbf{P}}^j - \alpha_0\tilde{\mathbf{P}}^j + \varepsilon_{\mathrm{f}}^{-1}\tilde{\mathbf{P}}^j\tilde{\mathbf{P}}^j + \varepsilon_{\mathrm{f}}L_{\mathrm{g}}^2 + \left(\frac{1}{\eta}+2\right)\mathbf{I} \\
& + \varepsilon_\tau^{-2}\tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\mathbf{K}_1\mathbf{K}_1^{\mathrm{T}}\boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\tilde{\mathbf{P}}^j + \gamma_0^{-2}\tilde{\mathbf{P}}^j\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}}\tilde{\mathbf{P}}^j + \mathbf{c}_1^{\mathrm{T}}\mathbf{c}_1]\mathbf{x}(t) \\
& - (\varpi^2 - \varepsilon_\tau^2)\mathbf{x}^{\mathrm{T}}(t-\tau(t))\mathbf{x}(t-\tau(t)) - \mathbf{x}^{\mathrm{T}}(t-\tau_2)\mathbf{x}(t-\tau_2) < \mathbf{0}.
\end{aligned}
\tag{44}
$$

Considering Eq. (27), we can derive Eq. (44) as

$$
\begin{bmatrix}
\boldsymbol{\Omega}_1 & \tilde{\mathbf{P}}^j\mathbf{B}_1 & \mathbf{c}_1^{\mathrm{T}} & \mathbf{I} & \tilde{\mathbf{P}}^j\mathbf{B}\boldsymbol{\rho}\mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\
* & -\gamma^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & \bar{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & -\varepsilon_\tau^2 & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & -\mathbf{I} & \mathbf{0} \\
* & * & * & * & * & * & -\varpi^2 + \varepsilon_\tau^2
\end{bmatrix} < \mathbf{0},
\tag{45}
$$

where $\boldsymbol{\Omega}_1 = \tilde{\mathbf{P}}^j\mathbf{A} + \mathbf{A}^{\mathrm{T}}\tilde{\mathbf{P}}^j - \alpha_0\tilde{\mathbf{P}}^j + \varepsilon_f^{-1}\tilde{\mathbf{P}}^j\tilde{\mathbf{P}}^j$ and $\bar{\Omega} = -\left(\frac{1}{\eta} + \varepsilon_f L_g^2 + 2\right)^{-1}$.

Being post- and pre-multiplied by block-diagonal matrix $\mathrm{diag}(\tilde{\mathbf{P}}^{-j}, \mathbf{I}, \ldots, \mathbf{I})$ and for arbitrarily given constant $\chi > 0$, inequation (45) can be ulteriorly derived as

$$
\begin{bmatrix}
\boldsymbol{\Omega}_2 - \chi\tilde{\mathbf{P}}^{-j} & \mathbf{B}_1 & \mathbf{0} & \tilde{\mathbf{P}}^{-j} & \mathbf{B}\boldsymbol{\rho}\mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\
* & -\gamma_0^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & \hat{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & \bar{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & -\varepsilon_\tau^2 & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & -\mathbf{I} & \mathbf{0} \\
* & * & * & * & * & * & -\varpi^2 + \varepsilon_\tau^2
\end{bmatrix} + \boldsymbol{\Lambda}^{\mathrm{T}}\chi\tilde{\mathbf{P}}^{-j}\boldsymbol{\Lambda} < \mathbf{0},
\tag{46}
$$

where $\boldsymbol{\Omega}_2 = \mathbf{A}\tilde{\mathbf{P}}^{-j} + \tilde{\mathbf{P}}^{-j}\mathbf{A}^{\mathrm{T}} - \alpha_0\tilde{\mathbf{P}}^{-j} + \varepsilon_f^{-1}$, $\boldsymbol{\Lambda} = [\mathbf{I} \;\; \mathbf{0} \;\; \chi^{-1}\mathbf{c}_1^{\mathrm{T}} \;\; \mathbf{0} \;\; \mathbf{0} \;\; \mathbf{0} \;\; \mathbf{0}]$, and $\hat{\Omega} = -\mathbf{I} - \chi^{-1}\mathbf{c}_1\tilde{\mathbf{P}}^{-j}\mathbf{c}_1^{\mathrm{T}}$. Afterwards, according to Eq. (27), we can change Eq. (46) into

$$
\begin{bmatrix}
-\chi^{-1}\tilde{\mathbf{P}}^j & \mathbf{I} & \mathbf{0} & \chi^{-1}\mathbf{c}_1^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & \boldsymbol{\Omega}_2 - \chi\tilde{\mathbf{P}}^{-j} & \mathbf{B}_1 & \mathbf{0} & \tilde{\mathbf{P}}^{-j} & \mathbf{B}\boldsymbol{\rho}\mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\
* & * & -\gamma_0^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & \hat{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & \bar{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & -\varepsilon_\tau^2 & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & * & -\mathbf{I} & \mathbf{0} \\
* & * & * & * & * & * & * & -\varpi^2 + \varepsilon_\tau^2
\end{bmatrix} < \mathbf{0}.
\tag{47}
$$

Being post- and pre-multiplied by $\mathrm{diag}(\mathrm{Z}^{\mathrm{T}}, \mathbf{I}, \ldots, \mathbf{I})$ on both sides and considering $\mathrm{Z}^{\mathrm{T}}\tilde{\mathbf{P}}^j\mathrm{Z} \ge \mathrm{Z} + \mathrm{Z}^{\mathrm{T}} - \tilde{\mathbf{P}}^{-j}$, inequation (47) can be ulteriorly derived as

$$
\begin{bmatrix}
\boldsymbol{\Omega}_3 & \mathrm{Z}^{\mathrm{T}} & \mathbf{0} & \chi^{-1}\mathrm{Z}^{\mathrm{T}}\mathbf{c}_1^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & \boldsymbol{\Omega}_2 - \chi\tilde{\mathbf{P}}^{-j} & \mathbf{B}_1 & \mathbf{0} & \tilde{\mathbf{P}}^{-j} & \mathbf{B}\boldsymbol{\rho}\mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\
* & * & -\gamma_0^2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & \hat{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & \bar{\Omega} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & -\varepsilon_\tau^2 & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & * & -\mathbf{I} & \mathbf{0} \\
* & * & * & * & * & * & * & -\varpi^2 + \varepsilon_\tau^2
\end{bmatrix} < \mathbf{0},
\tag{48}
$$

where $\boldsymbol{\Omega}_3 = \chi^{-1}(\tilde{\mathbf{P}}^{-j} - \mathrm{Z} - \mathrm{Z}^{\mathrm{T}})$.

According to Eq. (48), we can change Eq. (43) into

$$J_3 < -2\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^{j}\mathbf{B}\right\|\tilde{k}_4(t-\tau(t)) - \mu\eta\left\|\hat{\mathbf{x}}^{\mathrm{T}}(t-\tau(t))\bar{\tilde{\mathbf{P}}}^{j}\mathbf{B}\right\|^2\tilde{k}_5(t-\tau(t)) + 2\mathbf{x}^{\mathrm{T}}(t)\tilde{\mathbf{P}}^{j}\mathbf{B}\boldsymbol{\rho}\tilde{\mathbf{W}}^{\mathrm{T}}\mathbf{h}(\mathbf{x}) - 2r_1^{-1}\tilde{k}_4(t-\tau(t))\dot{\hat{k}}_4(t-\tau(t)) - \mu r_2^{-1}\tilde{k}_5(t-\tau(t))\dot{\hat{k}}_5(t-\tau(t)) - 2r_3^{-1}\sum_{i=1}^{2}\rho_i\tilde{\mathbf{W}}_i^{\mathrm{T}}\dot{\hat{\mathbf{W}}}_i. \quad (49)$$

From Eqs. (20) and (21), Eq. (49) can be written as $J_3 < 0$, which implies that the flutter control system is ultimately uniformly bounded, and the state variables $\mathbf{x}(t)$ will converge to zero.

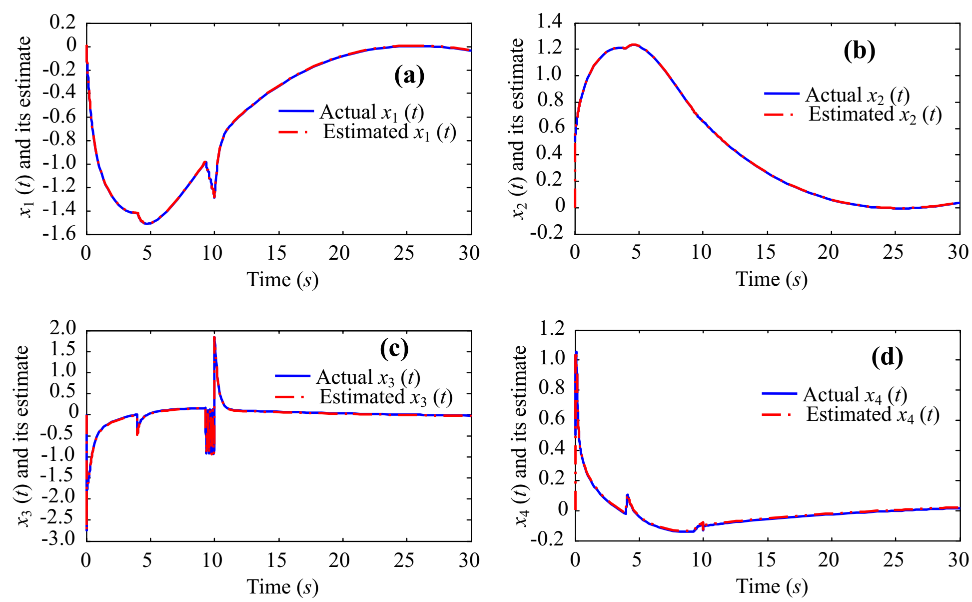

Fig. 2 The wing states $\mathbf{x}$ and its estimation $\hat{\mathbf{x}}$ in case of fault

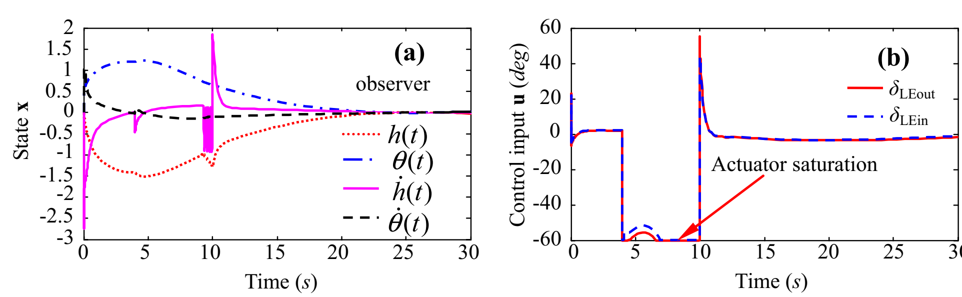

Fig. 3 Time response of the wing states $\mathbf{x}$ and control input $\mathbf{u}$ in case of fault

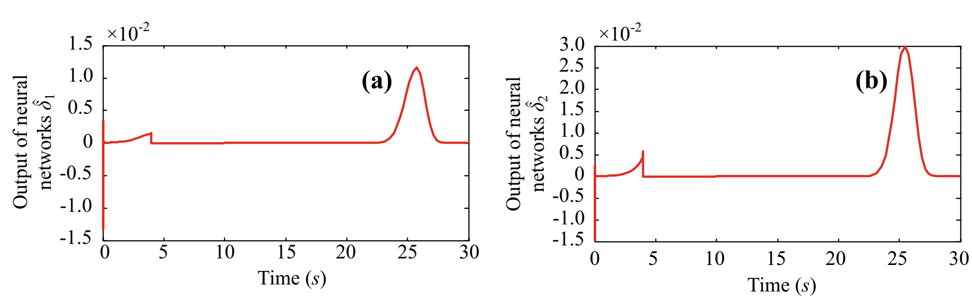

Fig. 4 The output of neural networks: **a** $\hat{\delta}_1$, **b** $\hat{\delta}_2$

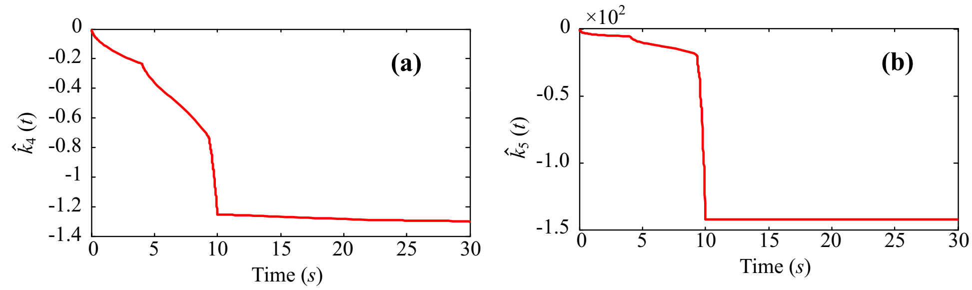

Fig. 5 The adaptive parameters $\hat{k}_4(t)$ and $\hat{k}_5(t)$ **a** $\hat{k}_4(t)$, **b** $\hat{k}_5(t)$

Remark 3 Assume that LMIs (37) and Eqs. (38–39) are satisfied, and control gain $\mathbf{K}_2(t)$, $\mathbf{K}_3(t)$, adaptive update laws $\hat{k}_4(t)$, $\hat{k}_5(t)$ and $\hat{\mathbf{W}}$ are given by (19), (20) and (21), then the closed-loop control system for flutter suppression (23) is stable, then γ_n and γ_f are minimized if the following optimization problem is solvable

$$\min \alpha_n\gamma_n^2 + \alpha_f\gamma_f^2, \ \text{ s.t. Eqs. } (37-39), \tag{50}$$

where α_f and α_n denote the weighting factors.

Numerical Simulations

In this section, the validity of our proposed approaches is illustrated by a providing numerical example. Parameters of the structural model of the 2-DOF wing are given in Table 1.

To depict the finer performance of the introduced control method, the succeeding faulty conditions are assumed to be experienced by the reentry vehicle: in the first 4 s which is the first case, the system is under proper condition, which means, both of the actuators works properly. Second condition starts from the 4th s and ends in the 10th s, and the first actuator is floating at $u_s(t) = 30 + 30\sin(0.1t) + 20\cos(0.5t)$ while the second one losing its capability, given by $\rho_2 = 1 - 0.05t$ until a loss of 50% capability. The third case is after the 10th s, the second actuator is floating and the first actuator is losing its capability, given by $\rho_1 = 1 - 0.01t$ until a loss of 70% capability. The system involves the disturbance $\mathbf{w}(t) = [-10\sin(0.1 \times t), \ 15]^{\mathrm{T}}$ from the beginning ($t \geq 0$).

Feedback and adapting gains of the controller are chosen through trial and error until a good performance is achieved. The parameters of controller b_j, η, r_1, r_2 and r_3 in Eqs. (11), (19), (20) and (21) are chosen as$b_j = 5$, $\eta = 100$, $r_1 = 0.25$, $r_2 = 0.25$, $r_3 = 4.5$

In Eqs. (7), (9) and (11), the initial values $\mathbf{x}(0)$ and the matrix $\mathbf{N}(t)$, and $\mathbf{c}_{ij}$ are chosen as$\mathbf{x}(0) = [0, \ 0.5, \ 0, \ 0.5]^{\mathrm{T}}$,

$$\mathbf{N}(t) = \begin{bmatrix} 0.5 \times \sin(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and } \mathbf{c}_i = \begin{bmatrix} -1 & -0.5 & 0 & 0.5 & 1 \\ -1 & -0.5 & 0 & 0.5 & 1 \end{bmatrix}.$$

Using Remark 3 with $\alpha_f = 1$, $\alpha_n = 5$, the H_∞ performances of feedback control system (23) for flutter suppression can be achieved as 1.4122 (faulty condition) and 0.4002 (proper condition) and. Figure 2 depicts that the wing states $\mathbf{x}$ and its estimation $\hat{\mathbf{x}}$ in case of fault. It is seen that the proposed observer (Eq. (12)) performed very well and provided the information of state variables despite those undesired effects in the control system for flutter suppression.

Figure 3 illustrates the flutter states $\mathbf{x}$ based on observer and control signals $\mathbf{u}(t)$ variations under different faults. In the simulation of this paper, two actuators of the vehicle are assumed to have some faults from 4th s to 10th s, while external disturbances existing from the beginning. From Fig. 3a, the actuator faults may result in variations in the state variables of wing flutter. When faults occur, parameters can be altered by controller $\mathbf{u}(t)$ to adaptively adjust the state variables, as shown in Fig. 3b, in order that effective compensation could be applied into the actuator and return the state variables to state. It can be seen from Fig. 3 that, under the control of the controller introduced in "Design of Fault-Tolerant Flutter Controller Based on Observer", the flutter can be suppressed within 1 s in the faulty conditions explained above, which proofs the reliability and the robustness of the control method for flutter suppression proposed in this article. Figure 4 depicts the output of neural networks $\hat{\delta}(t)$. From Fig. 4, it can be seen the parameters will be changed by the neural networks to adaptively adjust the controller for flutter suppression when the actuator encounters saturation situation.

For the purpose of counteracting the model uncertainties of wing flutter and fault of float in the actuators effectively and suppressing control system of wing flutter to be stable, control gain $\hat{k}_4$ and $\hat{k}_5$ are designed in "Design of Fault-Tolerant Flutter Controller Based on Observer". It can be seen from formula (20) that $\hat{k}_4$ and $\hat{k}_5$ will alter while the state variables of wing flutter vary. Under the conditions that uncertainties in parameters and faults in actuators exist, the state variables might have corresponding variations. It can be seen from Fig. 5 that the arguments to calculate $\hat{k}_4$ and $\hat{k}_5$ may adjust with the changing of the state variables of flutter system. Then, the control law (Fig. 3b) will be adjusted and will effectively counteract the uncertainties and float effectively (as shown in Fig. 3b). After the flutter system being stable, it can be seen from Fig. 5 that immutability is kept in $\hat{k}_4$ and $\hat{k}_5$.

The finite-time closed-loop flutter FTC system with actuator saturation, time delay, parameter uncertainties and external disturbances can be verified to be asymptotically stable under the condition that actuator faults exist. Good performance and effectiveness of suppression of the flutter by the proposed controller despite this undesirability in the control system are depicted by the simulation results.

Conclusions

In this paper, a novel finite-time H_∞ adaptive fault-tolerant control design scheme for wing flutter suppression is proposed, and it can deal with actuator saturation, time delay, external disturbances and parameter uncertainties in the wing flutter system. A radial basis function is used to approximate the actuator saturation. Loss of capability and float are considered as the actuator faults. The adaptively adjusting of the controller parameters to counteracting the actuator faults, time delay, disturbances and parameter uncertainties is proved. Numerical simulation results in advance verify the effectiveness of the proposed controller.

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