



# Flight Test Design and Nonlinear Model Identification for Landing Gear Aerodynamics

Xiangqun Cai<sup>1</sup> · Xiaoyong Lei<sup>2</sup> · Shuling Dai<sup>3</sup> · Weiqi Li<sup>4</sup>

Received: 21 January 2024 / Revised: 19 June 2024 / Accepted: 30 June 2024  
© The Author(s), under exclusive licence to The Korean Society for Aeronautical & Space Sciences 2024

## Abstract

In this study, a nonlinear aerodynamic modeling method for landing gear extension and retraction process based on the least square method is proposed and corresponding flight tests are designed performed with a simulator. Firstly, the least square method is used to determine the baseline aerodynamic model from simulated flight test data. Then landing gear extension and retraction maneuver are conducted on the same simulator under quasi-steady conditions to acquire the increment of aerodynamic coefficients caused by the change of landing gear position. Thus, the nonlinear aerodynamic model considering the effect of landing gear can be obtained. Based on the above method, flight tests using a simulated Boeing 737-800 model is carried out, and nonlinear landing gear aerodynamic characteristics of the 737-800 model is acquired. Compared with the linear interpolation model, the fidelity of the nonlinear landing gear aerodynamic model proposed in this work is obviously improved.

## List of Symbols

alt	Altitude, ft	$\bar{c}$	Mean aerodynamic chord, inch
$a_x$	Acceleration along body $x$ -axis, ft/s <sup>2</sup>	$I_x$	Rotational inertia along body $x$ -axis, slug-ft <sup>2</sup>
$a_y$	Acceleration along body $y$ -axis, ft/s <sup>2</sup>	$I_{xz}$	Product of rotational inertia of the aircraft, slug-ft <sup>2</sup>
$a_z$	Acceleration along body $z$ -axis, ft/s <sup>2</sup>	$I_y$	Rotational inertia along body $y$ -axis, slug-ft <sup>2</sup>
$b$	Wing span, ft	$I_z$	Rotational inertia along body $z$ -axis, slug-ft <sup>2</sup>
$C_D$	Drag coefficient	IAS	Indicated air speed
$C_M$	Pitching moment coefficient	$m$	Mass of the aircraft, lbs
$C_L$	Lift coefficient	$p$	Body-axis roll rate, °/s
<hr/>			
Communicated by Jin Seok Park.			
<hr/>			
	Xiaoyong Lei leixy@buaa.edu.cn	$q$	Body-axis pitch rate, °/s
<sup>1</sup>	Ph.D. Candidate, School of Automation Science and Electrical Engineering, Beihang University, 37 Xueyuan Road, Haidian District, Beijing 100091, People's Republic of China	$\dot{q}$	Body-axis pitch acceleration, °/s <sup>2</sup>
<sup>2</sup>	Associate Professor, School of Automation Science and Electrical Engineering, Beihang University, 37 Xueyuan Road, Haidian District, Beijing 100091, People's Republic of China	$\bar{q}$	Dynamic pressure, psi
<sup>3</sup>	Professor, School of Automation Science and Electrical Engineering, Beihang University, 37 Xueyuan Road, Haidian District, Beijing 100091, People's Republic of China	QNH	Query normal height, inHg
<sup>4</sup>	Ph.D., School of Automation Science and Electrical Engineering, Beihang University, 37 Xueyuan Road, Haidian District, Beijing 100091, People's Republic of China	$r$	Body-axis yaw rate, °/s
		$S$	Wing area, ft <sup>2</sup>
		$T$	Thrust, lbs
		TAS	True airspeed, knot
		$v$	True air speed, knot
		$W$	Gross weight, lbs
		$x_{cg}$	Longitudinal center of gravity, mac
		$\alpha$	Angle of attack, °
		$\delta_E$	Deflection of elevator, °
		$\delta_H$	Deflection of horizontal stabilizer, °
		$\delta_{LG}$	Normalized landing gear position
		$\theta$	Pitch angle, °
		$\theta_{LG}$	The angle that the landing gear deflects from its retraction configuration, °

## Subscripts

$0$	Basic state
$q$	Pitch rate
$\alpha$	Angle of attack
$\delta_E$	Deflection of elevator
$\delta_H$	Deflection of horizontal stabilizer
$\delta_{LG}$	Normalized landing gear position

## 1 Introduction

THE landing gear system is one of the most important components of an aircraft [1]. During take-off and landing, the extension and retraction processes of the landing gear can lead to significant changes in the aerodynamic coefficients of the aircraft, especially the longitudinal coefficients [2], which in turn has a great impact on the whole aerodynamic characteristics of the aircraft. With the continuous growth of the size and weight of modern aircraft, the structure of the landing gear system has become more and more complex and the influence of the landing gear system on the aerodynamic characteristics during its extension and retraction process has become more obvious. In addition, because extending and retracting the landing gear usually takes seconds or even tens of seconds, the impact of this process on the aerodynamic characteristics of the aircraft is non-negligible. Therefore, in flight simulations, the landing gear aerodynamic characteristics of the landing gear extension and retraction process will greatly affect the accuracy of the whole model.

In the past landing gear system modeling, it was generally believed that the influence of the position of the landing gear on the aerodynamic characteristics of the aircraft during the landing gear extension and retraction process is linear, so linear interpolation using aerodynamic models of landing gear up and landing gear down configurations is usually used to generate a linear model to replace the real aerodynamic model of the landing gear extension and retraction process [3]. However, the actual flight test data show that due to the irregular shape of the landing gear and the interaction between the flow fields near the landing gear and other parts of the body, the change of landing gear position and aerodynamic coefficient is, in fact, nonlinear during the landing gear extension and retraction process. Therefore, it is difficult for the linear model to accurately reflect the aerodynamic characteristics of the aircraft with changing landing gear configuration. With the increasing requirements for the accuracy of aircraft aerodynamic characteristic models, the linear interpolation modeling method can no longer meet the requirements of modern simulators, which puts forward the need for a new modeling

method of the aerodynamic characteristics of the landing gear.

To improve the accuracy of the landing gear aerodynamic model, some researchers have approximated the nonlinear aerodynamic model by calculating the equivalent windward area of the landing gear at different positions, but such methods require repeated adjustment of the nonlinear model parameters, which is not only tedious but also showed limited effectiveness in the improvement of model accuracy. Some research institutions and aircraft manufacturers use wind tunnel to test the nonlinear aerodynamic characteristics of landing gear [4, 5]. However, it is difficult to obtain accurate aerodynamic characteristics using small wind tunnels due to the scale effect. Although full-scale wind tunnel can solve the problem caused by the scale effect, it has problems such as expensive test cost and serious wall interference. Several studies have discussed the application of system identification to the modeling of aircraft dynamics [6–8], including time and frequency domain methods, and several tools are available for the automatic identification process, and in this work, system identification using the least square method is used as a basis for landing gear aerodynamic model identification.

This paper proposes a model identification method and designs a series of flight tests to acquire the nonlinear aircraft aerodynamic model which can more accurately reflect the influence of landing gear configurations. Through the design of flight-test maneuvers, the increment of aerodynamic characteristics of aircraft caused by landing gear position can be obtained using system identification method, so as to obtain an accurate aircraft aerodynamic model for the landing gear extension and retraction stage. Experiments were conducted using flight simulators and results showed that a more accurate model can be obtained using the proposed model.

## 2 Mathematical Modeling of the Nonlinear Landing Gear Aerodynamic Model

### 2.1 Aerodynamic Equations

The aerodynamic model of an airplane can be used to describe the aircraft flying status. The aerodynamic model usually contains several aerodynamic forces and aerodynamic moments such as lift, drag, pitch moment, etc. Since the aerodynamic force and aerodynamic torque cannot be directly measured, it should be estimated according to the measured flight parameters such as the body axial angular rate and acceleration, dynamic pressure, aircraft mass, moment of inertia and aircraft profile data. According to these parameters, the rigid-body six-degree-of-freedom motion equations are used to calculate the lift, drag and pitching

moment coefficients of each flight-test status [9]. The derivation of the flight dynamics model is quite complex and goes beyond the scope of this paper’s discussion. To simplify the model, this paper primarily focuses on modeling three longitudinal aerodynamic coefficients, namely  $C_L$ ,  $C_D$ , and  $C_M$ , which are significantly affected by the landing gear retraction and extension. As a generally accepted form of the aerodynamic model, the calculation equations are as follows:

$$C_L = -\frac{ma_z}{\bar{q}s} \cos \alpha + \frac{ma_x - T}{\bar{q}s} \sin \alpha, \tag{1a}$$

$$C_D = -\frac{ma_x - T}{\bar{q}s} \cos \alpha - \frac{ma_z}{\bar{q}s} \sin \alpha, \tag{1b}$$

$$C_M = \frac{1}{\bar{q}s\bar{c}} \left[ I_y \dot{q} + (I_x - I_z)pr + I_{xz}(p^2 - r^2) \right]. \tag{1c}$$

In the above equations, parameters such as  $a_x$ ,  $a_y$ ,  $a_z$ ,  $\alpha$ ,  $p$ ,  $q$  and  $\bar{q}$  can be measured with a test flight (or in this work, with a simulator), which are used to reconstruct  $C_L$ ,  $C_D$  and  $C_M$  on the left of the equations.

### 2.2 Baseline Model

For the identification of landing gear aerodynamic models using both traditional linear interpolation and nonlinear modeling methods, a baseline model has to be constructed.

The simplified longitudinal baseline aerodynamic model can be decomposed into:

$$C_L = f(\alpha, \text{altitude, Mach, flap, } \delta_E, \delta_H, \delta_{LG}), \tag{2a}$$

$$C_D = f(\alpha, \text{altitude, Mach, flap, } \delta_E, \delta_H, \delta_{LG}), \tag{2b}$$

$$C_M = f(\alpha, \text{altitude, Mach, flap, } \delta_E, \delta_H, \delta_{LG}). \tag{2c}$$

It should be noted that the actual model of an aircraft is more complex, and other factors such as aircraft side slip and yaw rate should be taken into consideration. In this study, the focus is on the influence of landing gear on the longitudinal aerodynamic parameters of the aircraft, so the overall aerodynamic model has been simplified. The coefficients in the simplified aerodynamic model include aerodynamic coefficients caused by the changes in altitude, Mach, angle of attack, elevator and horizontal stabilizer.

To establish a baseline model, different statuses within the flight envelope are selected to create a flight test matrix. In each flight status, the  $C_L$ ,  $C_D$ , and  $C_M$  for each status can be represented using the following equations:

$$C_L = C_{L\alpha} \Delta\alpha + C_{Lq} \frac{\bar{q}\bar{c}}{2v} + C_{L\delta_E} \Delta\delta_E + C_{L\delta_H} \Delta\delta_H + C_{L0}, \tag{3a}$$

$$C_D = C_{D\alpha} \Delta\alpha + C_{Dq} \frac{\bar{q}\bar{c}}{2v} + C_{D\delta_E} \Delta\delta_E + C_{D\delta_H} \Delta\delta_H + C_{D0}, \tag{3b}$$

$$C_M = C_{M\alpha} \Delta\alpha + C_{Mq} \frac{\bar{q}\bar{c}}{2v} + C_{M\delta_E} \Delta\delta_E + C_{M\delta_H} \Delta\delta_H + C_{M0}. \tag{3c}$$

Using Eq. (1), the total aerodynamic force and aerodynamic moment on the left of Eq. (3) can be obtained from flight tests (in this work, from simulated flight tests) at the configurations of gear up/gear down. At this time, the least square method can be used to identify each aerodynamic derivative on the right of Eq. (3) and the baseline aerodynamics at a certain status can be obtained.

### 2.3 Linear Landing Gear Model

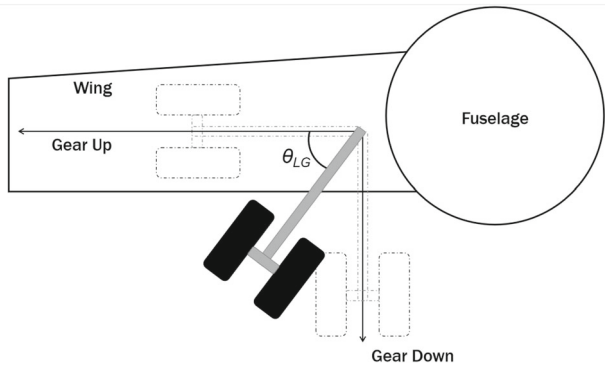
Traditionally, the linear interpolation method can be used to acquire a linear landing gear model. According to the classical aerodynamic theory, the lift, drag, and pitching moment caused by the angle of attack, velocity, elevator, and horizontal stabilizer are hardly affected by the landing gear position. The influence of landing gear position on the overall aerodynamic coefficients of the aircraft mainly lies in  $C_{L0}$ ,  $C_{D0}$ , and  $C_{M0}$ . Therefore, when the landing gear is retracted, the aforementioned coefficients in the Eqs. (1–3) are represented as  $C_{L0_{gearup}}$ ,  $C_{D0_{gearup}}$ , and  $C_{M0_{gearup}}$ ; when the landing gear is extended, the coefficients are represented as  $C_{L0_{gearDn}}$ ,  $C_{D0_{gearDn}}$ , and  $C_{M0_{gearDn}}$ . In traditional dynamic modeling processes, it is assumed that the coefficient increments caused by the landing gear are proportional to the projected area of the landing gear on the aircraft’s cross-section. Therefore, the model for this aerodynamic coefficient increment is simplified to a linear function of the normalized position of the landing gear, as follows:

$$C_L = C_{L\alpha} \Delta\alpha + C_{Lq} \frac{\bar{q}\bar{c}}{2v} + C_{L\delta_E} \Delta\delta_E + C_{L\delta_H} \Delta\delta_H + C_{L0_{gearup}} (1 - \delta_{LG}) + C_{L0_{gearDn}} \times \delta_{LG}, \tag{4a}$$

$$C_D = C_{D\alpha} \Delta\alpha + C_{Dq} \frac{\bar{q}\bar{c}}{2v} + C_{D\delta_E} \Delta\delta_E + C_{D\delta_H} \Delta\delta_H + C_{D0_{gearup}} (1 - \delta_{LG}) + C_{D0_{gearDn}} \times \delta_{LG}, \tag{4b}$$

$$C_M = C_{M\alpha} \Delta\alpha + C_{Mq} \frac{\bar{q}\bar{c}}{2v} + C_{M\delta_E} \Delta\delta_E + C_{M\delta_H} \Delta\delta_H + C_{M0_{gearup}} (1 - \delta_{LG}) + C_{M0_{gearDn}} \times \delta_{LG}. \tag{4c}$$

The above equations represent the linear landing gear aerodynamic model.



**Fig. 1** Diagram of landing gear position

The  $\delta_{LG}$  in Eq. (4), normalized landing gear position, refers to the ratio of the projected windward area at a certain status to the projected windward area at the gear down position. The mathematical equation is shown below (Fig. 1).

$$\delta_{LG} = \sin\theta_{LG}. \quad (5)$$

When the landing gear is retracted,  $\delta_{LG} = 0$ , When the landing gear is extended,  $\delta_{LG} = 1$ .

When the landing gear is retracted and extended respectively, performing the 2–3–1–1 maneuver (refer to Sect. 2.5) and utilizing the least square method, the coefficients in the above equation can be identified.

The linear model can accurately calculate the aerodynamic coefficients when the landing gear is fully retracted and extended, and by interpolation, it can compute the coefficients for intermediate statuses. However, during actual flight, extending the landing gear generates a significant amount of turbulence around it, which affects the aircraft's aerodynamic coefficients. Moreover, these effects are often non-linear with respect to the normalized landing gear position. Using only a linear landing gear model makes it difficult to accurately reflect the complex motion of the aircraft during landing gear extension and retraction.

## 2.4 Nonlinear Landing Gear Model

The nonlinear landing gear model is also derived from the baseline model but no longer calculates the aerodynamic coefficient increments  $C_{L\delta_{LG}}$ ,  $C_{D\delta_{LG}}$  and  $C_{M\delta_{LG}}$  by linear interpolation. Instead, it obtains the aerodynamic coefficient increments for different landing gear positions through specific (simulated) flight test results by executing specific flight maneuvers.

In the designed flight tests (details are described in Sect. 2.5) the landing gear is extended under constant altitude and Mach number at each status so that the angle of attack and Mach number remain basically unchanged during the extension process of the landing gear. During the extension

of the landing gear,  $C_{L\delta_{LG}}$ ,  $C_{D\delta_{LG}}$ ,  $C_{M\delta_{LG}}$  will vary with the normalized landing gear position, so they are the function of the normalized landing gear position and can be described as follows:

$$C_{L\delta_{LG}} = f_{C_{L\delta_{LG}}}(\delta_{LG}), \quad (6a)$$

$$C_{D\delta_{LG}} = f_{C_{D\delta_{LG}}}(\delta_{LG}), \quad (6b)$$

$$C_{M\delta_{LG}} = f_{C_{M\delta_{LG}}}(\delta_{LG}). \quad (6c)$$

At this time, The aerodynamic coefficients can be regarded as functions of the landing gear position and can be expressed as:

$$C_L = C_{L\alpha} \Delta\alpha + C_{Lq} \frac{\bar{q} \bar{c}}{2v} + C_{L\delta_E} \Delta\delta_E + C_{L\delta_H} \Delta\delta_H + f_{C_{L\delta_{LG}}}(\delta_{LG}) + C_{L_{0gearup}}, \quad (7a)$$

$$C_D = C_{D\alpha} \Delta\alpha + C_{Dq} \frac{\bar{q} \bar{c}}{2v} + C_{D\delta_E} \Delta\delta_E + C_{D\delta_H} \Delta\delta_H + f_{C_{D\delta_{LG}}}(\delta_{LG}) + C_{D_{0gearup}}, \quad (7b)$$

$$C_M = C_{M\alpha} \Delta\alpha + C_{Mq} \frac{\bar{q} \bar{c}}{2v} + C_{M\delta_E} \Delta\delta_E + C_{M\delta_H} \Delta\delta_H + f_{C_{M\delta_{LG}}}(\delta_{LG}) + C_{M_{0gearup}}. \quad (7c)$$

Since the aircraft test flight status point remains unchanged, that is, the angle of attack and Mach number remain basically unchanged during landing gear extension and retraction, it can be considered that only  $\Delta\delta_E$ ,  $\Delta\delta_H$  and  $\Delta\delta_{LG}$  are changed in each maneuver. Based on the aerodynamic derivatives identified in the baseline model and the data obtained from (simulated) flight tests, the additional aerodynamic coefficients  $C_{L\delta_{LG}}$ ,  $C_{D\delta_{LG}}$  and  $C_{M\delta_{LG}}$  are the function of the landing gear position can be described as:

$$f_{C_{L\delta_{LG}}}(\delta_{LG}) = C_L - C_{L\alpha} \Delta\alpha - C_{Lq} \frac{\bar{q} \bar{c}}{2v} - C_{L\delta_E} \Delta\delta_E - C_{L\delta_H} \Delta\delta_H - C_{L_{0gearup}}, \quad (8a)$$

$$f_{C_{D\delta_{LG}}}(\delta_{LG}) = C_D - C_{D\alpha} \Delta\alpha - C_{Dq} \frac{\bar{q} \bar{c}}{2v} - C_{D\delta_E} \Delta\delta_E - C_{D\delta_H} \Delta\delta_H - C_{D_{0gearup}}, \quad (8b)$$

$$f_{C_{M\delta_{LG}}}(\delta_{LG}) = C_M - C_{M\alpha} \Delta\alpha - C_{Mq} \frac{\bar{q} \bar{c}}{2v} - C_{M\delta_E} \Delta\delta_E - C_{M\delta_H} \Delta\delta_H - C_{M_{0gearup}}. \quad (8c)$$

Since it's a quasi-steady flight, the  $C_L$ ,  $C_D$ ,  $C_M$  at different  $\delta_{LG}$  on the left-hand side of the equations can be

calculated using formulas (1a, 1b, and 1c). The remaining aerodynamic coefficients on the right-hand side, except for  $C_{L_{\delta_{LG}}}$ ,  $C_{D_{\delta_{LG}}}$  and  $C_{M_{\delta_{LG}}}$ , have already been included in the obtained baseline model. Hence,  $f_{C_{L_{\delta_{LG}}}(\delta_{LG})}$ ,  $f_{C_{D_{\delta_{LG}}}(\delta_{LG})}$  and  $f_{C_{M_{\delta_{LG}}}(\delta_{LG})}$  can be calculated at this flight status.

During takeoff, cruise, and landing processes, selecting multiple statuses with different Mach numbers and flap configurations for flight testing will yield corresponding  $C_{L_{\delta_{LG}}}$ ,  $C_{D_{\delta_{LG}}}$ ,  $C_{M_{\delta_{LG}}}$  for each status. By utilizing aircraft angle of attack, Mach, flap configuration, and normalized landing gear position for different statuses, spline fitting can be performed to obtain a nonlinear landing gear model. It can be expressed as follows:

$$C_{L_{\delta_{LG}}} = f(\alpha, \text{Mach, flap, } \delta_{LG}), \tag{9a}$$

$$C_{D_{\delta_{LG}}} = f(\alpha, \text{Mach, flap, } \delta_{LG}), \tag{9b}$$

$$C_{M_{\delta_{LG}}} = f(\alpha, \text{Mach, flap, } \delta_{LG}). \tag{9c}$$

### 2.5 Least Square Method

As a widely used mathematical algorithm [10–12], the least square method can be used to identify the aerodynamic parameters during landing gear extension and retraction.

The identification process of the aerodynamic coefficient can be generalized as follows:

$$y = \theta_0 + \sum_{j=1}^n \theta_j \xi_j, \quad i = 1, 2, \dots, N, \tag{10}$$

where  $y$  is the dependent variable,  $\xi_j$  are functions of  $m$  independent variables  $x_1, x_2, \dots, x_m$  that are related to  $C_L, C_D, C_M$ , and  $\theta_0, \theta_1, \dots, \theta_n$  are model parameters to be identified.

The relationship between measured values and dependent variables in flight tests can be expressed as:

$$z = y + v(i), \tag{11}$$

where  $z$  is the measured output,  $v(i)$  is the random measurement noise.

The above model can be written with vector and matrix as:

$$y = X\theta, \tag{12}$$

$$z = X\theta + v, \tag{13}$$

where:

$z = [z(1)z(2) \dots z(N)]^T$ , which is  $N \times 1$  vector;  $\theta = [\theta_0 \theta_1 \dots \theta_n]^T$ , which is vector of  $n_p = n + 1$  unknown parameters;  $X = [1 \xi_1 \dots \xi_n]$ , which is a  $N \times n_p$  matrix;  $v = [v(1) v(2) \dots v(n)]^T$  is the  $N \times 1$  vector of the measurement noise.

Solving the values of aerodynamic parameters is the process of solving  $\theta$  in the above equations.

In the least square method, it is assumed that the mean error of process noise is 0 and uncorrelated, with constant variance, there is:

$$E(v) = 0, \quad E(vv^T) = \sigma^2 I. \tag{14}$$

Then the least squares solution can be obtained by finding the minimum difference between the measured value and the calculated value:

$$J(\theta) = \frac{1}{2}(z - X\theta)^T(z - X\theta). \tag{15}$$

The estimator of  $\theta$  must satisfy

$$\frac{\partial J}{\partial \theta} = -X^T z + X^T X \hat{\theta} = 0, \tag{16}$$

or

$$X^T(z - X\hat{\theta}) = 0. \tag{17}$$

The least-square estimator of  $\theta$  can be found:

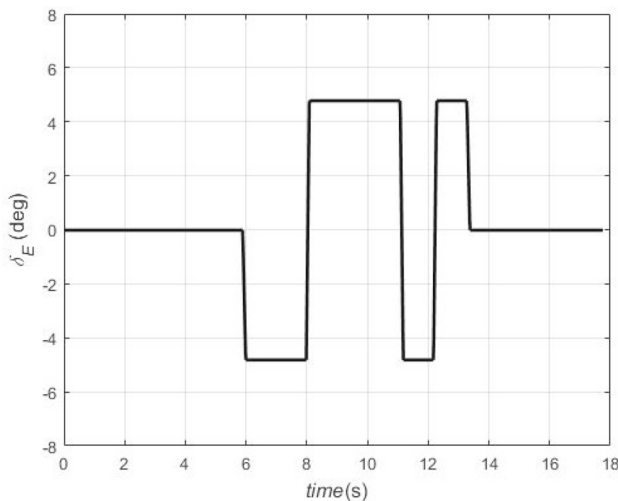
$$\hat{\theta} = (X^T X)^{-1} X^T z. \tag{18}$$

### 3 Flight-Test Design

In the flight-test maneuver design to identify the landing gear aerodynamic model, it is first required to determine the baseline aerodynamic characteristics of landing gear up/down configurations of the aircraft in the take-off and landing stages; On the basis of the acquired baseline model, the increment of aerodynamic characteristics of aircraft caused by the change of landing gear position during the extension and retraction of aircraft landing gear should be determined. To achieve the above objectives, three flight-test maneuvers were designed and should be performed at each flight configuration:

- (1) Longitudinal stick 2–3–1–1, landing gear up;
- (2) Longitudinal stick 2–3–1–1, landing gear down;





**Fig. 2** Input of longitudinal stick 2–3–1–1 maneuver, landing gear up/landing gear down

- (3) Landing gear extension during manually controlled or autopilot-controlled quasi-steady condition (Mach number, angle of attack, altitude unchanged).

### 3.1 Longitudinal Stick 2–3–1–1, Landing Gear Up

To identify the baseline characteristics of the aircraft in the landing gear up status, the longitudinal stick 2–3–1–1 maneuver is performed at a certain flight-test status point (pull the elevator back and hold it for 2 s; push forward and hold for 3 s; pull back and hold for 1 s; push forward and hold for 1 s) with the landing gear retracted. The input waveform is shown in Fig. 2.

### 3.2 Longitudinal Stick 2–3–1–1, Landing Gear Down

To identify the basic characteristics of the aircraft in the landing gear down status, the longitudinal stick 2–3–1–1 maneuver is performed at a certain flight-test status point (pull the elevator back and hold it for 2 s; Push forward and hold for 3 s; Pull back and hold for 1 s; Push forward and hold for 1 s) with landing gear extended. The input waveform is also shown in Fig. 2.

### 3.3 Landing Gear Extension at Quasi-Steady Condition

Keep trim condition with landing gear retracted for 5 s; then extend the landing gear while maintaining flight status through manual control or autopilot control until the landing gear is fully extended; keep trim condition at landing gear down configuration for at least 5 s.

Under trim conditions, it can be considered that Mach number, Angle of attack and altitude are almost constant, and the change of lift drag coefficient caused by these factors can be considered to be almost zero. Besides, generally, the aircraft engine installation angle is small, so the change in lift caused by the thrust component of the engine is very small, the change in lift/drag of the aircraft can be considered to be entirely caused by the change in landing gear position during landing gear extension or retraction under trim condition.

By conducting flight-tests at different flight statuses of angle of attack and Mach number, the relationship of lift/drag changes caused by landing gear position changes with angle of attack and Mach number can be obtained.

### 3.4 Flight-Test Matrix

To identify the increment of longitudinal aerodynamic characteristics caused by the retraction of the landing gear, a flight test matrix was constructed under different flight configurations (altitude during cruising, take-off and landing phases; Mach number and flap configuration).

The aircraft configurations included in the above flight test matrix include:

*Mach number* According to the speed envelope of the identified aircraft, from the minimum maneuvering speed of the identified aircraft to the maximum Mach number allowed to lower the landing gear. The smaller the test interval, the more accurate the model will be, and the higher the corresponding test flight cost will be.

*Altitude* From 500 feet above ground to typical landing gear lowering altitude, according to the envelope of the identified aircraft altitude. The smaller the test interval, the more accurate the model will be, and the higher the corresponding test flight cost will be.

*Flap configuration* Each flap configuration supported by the tested aircraft.

## 4 Case Study

In this part, we will use simulation data to evaluate the accuracy of the nonlinear model obtained from the above flight test design and identification methods.

## 5 Research Vehicle Model and Flight-Simulation Data

Using the Flightgear simulation model [13], flight-test and model identification of the B737-800 aircraft landing gear aerodynamic model were carried out. The B737-800 is a twin-engine turbofan short and medium-range aircraft jetliner produced by Boeing powered by two CFM56-7B

**Table 1** B737-800 mass and geometric configuration parameters

Parameters	Value	Unit
$W$	135,000	lbs
$I_x$	935,248.38	slug-ft <sup>2</sup>
$I_y$	2,482,936	slug-ft <sup>2</sup>
$I_z$	3,315,718	slug-ft <sup>2</sup>
$I_{xz}$	133,692.41	slug-ft <sup>2</sup>
$x_{cg}$	$0.23 \overline{c}$	in
$S$	1341	ft <sup>2</sup>
$c$	155.81	in
$b$	112.57	ft

turbofans [14]. The aircraft mass and geometric configuration parameters used in this study are the parameters used by Flightgear and are shown in Table 1.

### 5.1 Flight-Test Design

According to the flight envelope of B737-800, the flight-test matrix is designed according to the method described in Sect. 2.4. Different flight status points in the test flight matrix are shown in the Fig. 3:

The identification process of the model at each status is detailed in Sects. 2.1 to 2.4.

Taking the model identification of landing gear aerodynamic characteristics at the flight status point of 2000 ft, flap up and Mach 0.3 as an example, 2–3–1–1 maneuver was carried out at this flight status point by keeping the landing gear retracted/extended. Assume that all input and flight statuses are noise free. Figures 4 and 5 show the input and flight status of the longitudinal stick 2–3–1–1 maneuver in the test flight.

### 5.2 Identification of Baseline Model

The results from the flight simulation data using statuses and flight-test inputs described in Sect. 4.2 were used to identify the baseline model at this status (status: 2000 ft, flap up, speed 0.3 Mach).

The baseline aerodynamic model includes  $C_{L_0}$ ,  $C_{L_\alpha}$ ,  $C_{L_q}$ ,  $C_{L_{\delta_E}}$ ,  $C_{L_{\delta_H}}$ , and the corresponding drag coefficients  $C_{D_0}$ ,  $C_{D_\alpha}$ ,  $C_{D_q}$ ,  $C_{D_{\delta_E}}$ ,  $C_{D_{\delta_H}}$  and pitching moment coefficients  $C_{M_0}$ ,  $C_{M_\alpha}$ ,  $C_{M_q}$ ,  $C_{M_{\delta_E}}$ ,  $C_{M_{\delta_H}}$ . Through the simulator flight test data, the corresponding baseline aerodynamic model can be obtained by the least square method. The identification results for each parameter in the landing gear retracted and extended statuses are as follows (Table 2):

The aerodynamic model of this point is shown in Figs. 6 and 7, where the blue curve and the red dot line are the basic lift coefficient  $C_L$ , the basic drag coefficient  $C_D$  and the basic pitching moment coefficient  $C_M$  that are calculated

with the identified baseline model and simulator flight test data, respectively. The corresponding residuals are shown in Figs. 8 and 9.

The maximum residual errors of fitted  $C_L$  and the simulator flight test data is  $\pm 0.002$ , and the relative error is 0.18%. The maximum residual error between fitted  $C_D$  and the simulator flight test data is  $\pm 0.002$ , and the maximum relative error is about 2%. The maximum residual error between fitted  $C_M$  and the simulator flight test data is  $\pm 0.005$ , and the maximum relative error is about 1.85%. These results indicated that the identified baseline model is accurate.

At other statuses, the model also demonstrates good accuracy. Identification results at multiple other statuses are detailed in the supplementary information (figures S1–S12).

### 5.3 Linear Landing Gear Model

Based on the identification results in Sect. 4.3, it is evident that only  $C_{L_0}$ ,  $C_{D_0}$  and  $C_{M_0}$  exhibit significant differences between the landing gear retracted and extended statuses. The identification results for the remaining coefficients are very close, indicating that they are independent of the landing gear status. For ease of handling, we will take the average values of these coefficients as the coefficients for the linear model at this status (Table 3).

By substituting the above results into Eqs. (4a, 4b, and 4c), we can obtain the linear landing gear model.

### 5.4 Nonlinear Landing Gear Model

The input and statuses during simulator flight tests that are designed to acquire the incremental aerodynamic effects caused by landing gear position are shown in Fig. 10.

As can be seen from Fig. 10, the angle of attack, flight speed, Mach number and altitude remain basically unchanged during the flight test. The variation of the angle of attack is about 0.2 degree, the variation of Mach number is less than 0.005, the true airspeed varies by about 2 knots, and the change in altitude is about 20 feet. Short-term small fluctuations are observed in the longitudinal and normal acceleration of the aircraft and the body pitch angle rate, but the overall test process is close to the quasi-steady flight, so it can be considered that the aerodynamic coefficient changes caused by these fluctuations can be ignored.

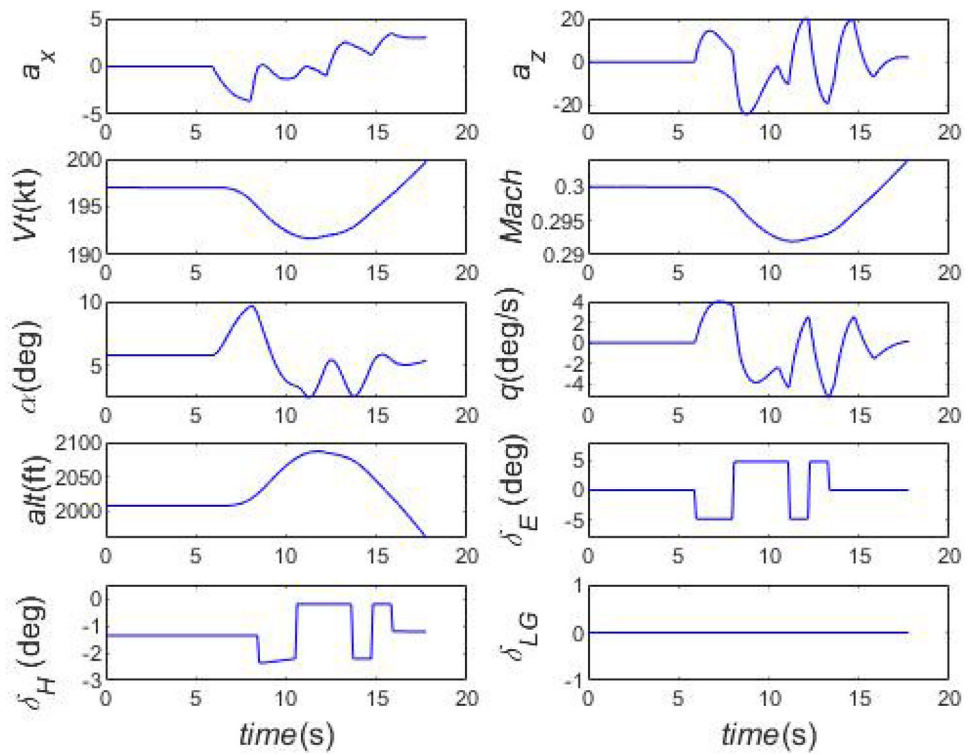
Using the baseline aerodynamic model and test flight data of each status, the aerodynamic coefficient increments  $C_{L_{\delta_{LG}}}$ ,  $C_{D_{\delta_{LG}}}$ , and  $C_{M_{\delta_{LG}}}$  caused by landing gear extension can be calculated by Eq. (8).

During this process, the change of angle of attack at other different status points along with the normalized landing gear position is shown in Fig. 11. It can be seen that the angle of attack remains almost unchanged with the position of the

Gear UP								Gear DOWN					
Flap UP								Flap UP					
Pressure ALT	2000	5000	10000	15000	20000	25000	30000	Pressure ALT	2000	5000	10000	15000	20000
MACH	0.3	0.3	0.3					MACH	0.3	0.3	0.3		
		0.4	0.4	0.4	0.4					0.4	0.4	0.4	0.4
		0.5	0.5	0.5	0.5	0.5				0.5	0.5	0.5	0.5
		0.6	0.6	0.6	0.6	0.6	0.6			0.6	0.6	0.6	0.6
Flap 1								Flap 1					
Pressure ALT	2000	5000	10000	15000	20000	25000		Pressure ALT	2000	5000	10000	15000	20000
MACH	0.3	0.3	0.3					MACH	0.3	0.3	0.3		
		0.4	0.4	0.4	0.4					0.4	0.4	0.4	0.4
		0.5	0.5	0.5	0.5	0.5							
Flap 5								Flap 5					
Pressure ALT	2000	5000	10000	15000	20000	25000		Pressure ALT	2000	5000	10000	15000	20000
MACH	0.3	0.3	0.3					MACH	0.3	0.3	0.3		
		0.4	0.4	0.4	0.4					0.4	0.4	0.4	0.4
		0.5	0.5	0.5	0.5	0.5							
Flap 15								Flap 15					
Pressure ALT	2000	5000	10000					Pressure ALT	2000	5000	10000		
MACH	0.2	0.2						MACH	0.2	0.2			
	0.3	0.3	0.3							0.3	0.3		
		0.4	0.4							0.4	0.4		
Flap 30								Flap 30					
Pressure ALT	2000	5000	10000					Pressure ALT	2000	5000	10000		
MACH	0.2	0.2						MACH	0.2	0.2			
	0.3	0.3	0.3							0.3	0.3		
		0.4	0.4							0.4	0.4		

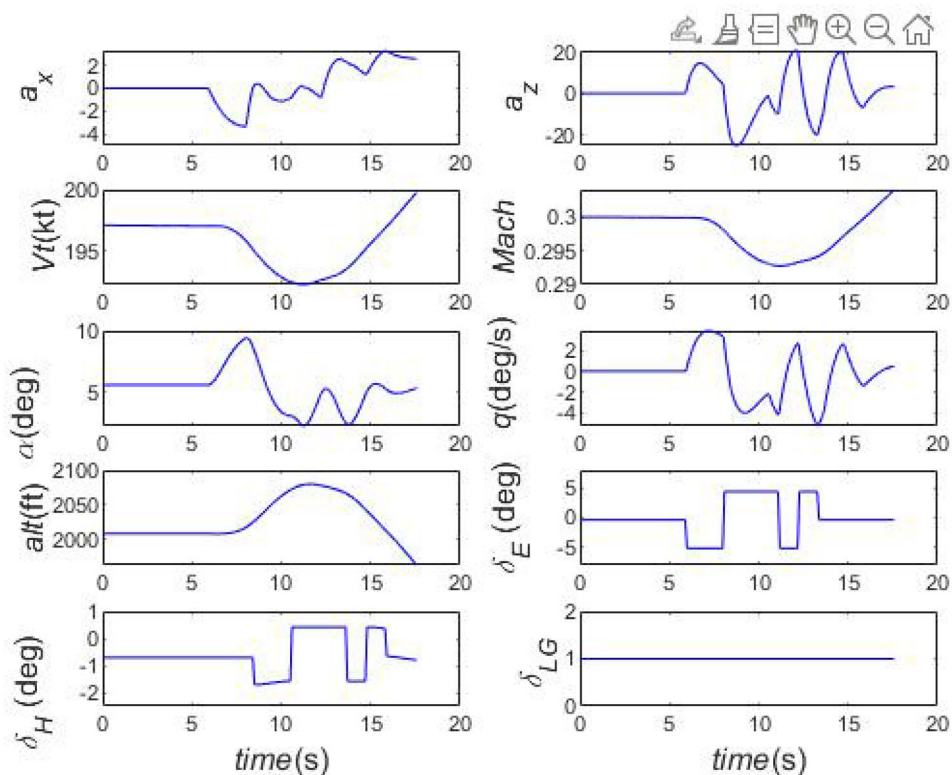
Fig. 3 Flight-test matrix for B737-800 landing gear model identification

Fig. 4 Statuses and flight-test inputs for baseline aerodynamic characteristics identification (status: 2000 ft, flap up, gear up, Mach 0.3)





**Fig. 5** Statuses and flight-test inputs for baseline aerodynamic characteristics identification (status: 2000 ft, flap up, gear down, Mach 0.3)



**Table 2** Parameters for baseline aerodynamic model (status: 2000 ft, flap up, speed 0.3 Mach)

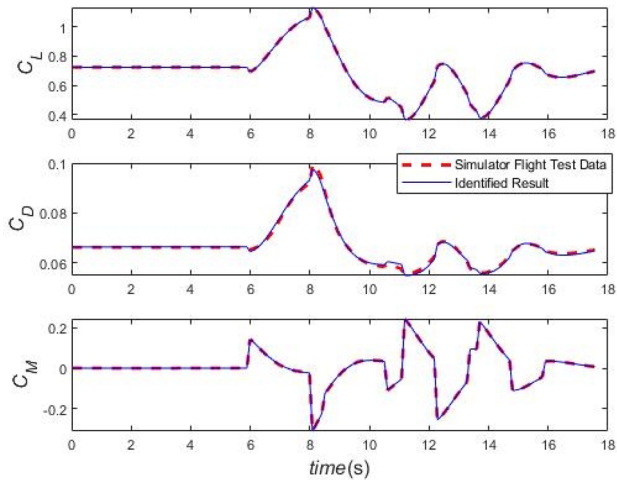
	$C_{L_0}$	$C_{L_\alpha}$	$C_{L_q}$	$C_{L_{\delta_E}}$	$C_{L_{\delta_H}}$
Gear down	0.7385	0.1068	0.00775	0.007054	0.01729
Gear up	0.7513	0.1017	0.00778	0.006983	0.01735
	$C_{D_0}$	$C_{D_\alpha}$	$C_{D_q}$	$C_{D_{\delta_E}}$	$C_{D_{\delta_H}}$
Gear down	0.06693	-0.00184	-0.08046	-0.00044	-0.00064
Gear up	0.03597	-0.00204	-0.08399	-0.00042	-0.00062
	$C_{M_0}$	$C_{M_\alpha}$	$C_{M_q}$	$C_{M_{\delta_E}}$	$C_{M_{\delta_H}}$
Gear down	0.13646	-0.03181	0.2725	-0.0318	-0.0733
Gear up	0.10609	-0.03175	0.2582	-0.0317	-0.0735

landing gear, which confirms that the flight test method proposed in this paper is reasonable.

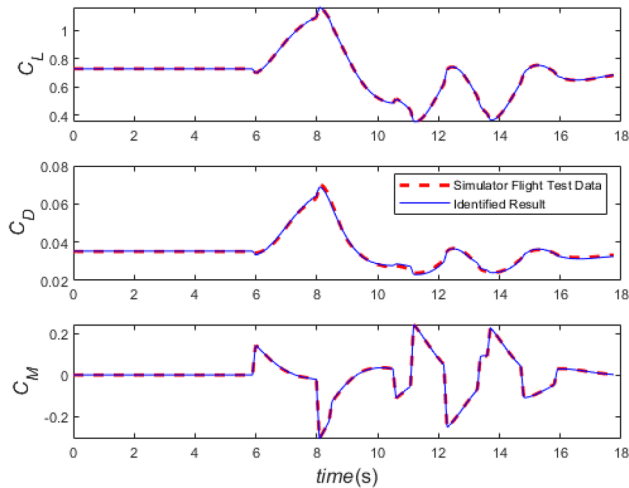
The above method was extended to other flight status points under different altitudes, Mach numbers and flap configurations within the flight envelope, and the incremental aerodynamic coefficients caused by the change of landing gear position are calculated. The identified aerodynamic model caused by the change of landing gear position is shown in Fig. 12.

As can be seen from the figure, the incremental changes of lift coefficients are relatively complex during the landing gear extension process, and the change trend is different at different angles of attack. For the drag coefficient increment,

when the landing gear position is less than 0.5, the drag coefficient increment increases with the increase of the landing gear position, but when the landing gear position is greater than 0.5, the drag coefficient increment remains relatively unchanged. For the pitching moment coefficient, when the landing gear position is less than 0.4, the increment of the pitching moment coefficient increases with the increase of the landing gear position. When the landing gear position is greater than 0.4, the increment of the pitching moment coefficient basically does not change. It can be seen that for the model used in this paper, aerodynamic coefficient increments caused by landing gear position show obvious nonlinear characteristics.



**Fig. 6** Identified longitudinal baseline aerodynamic coefficient model (landing gear down)

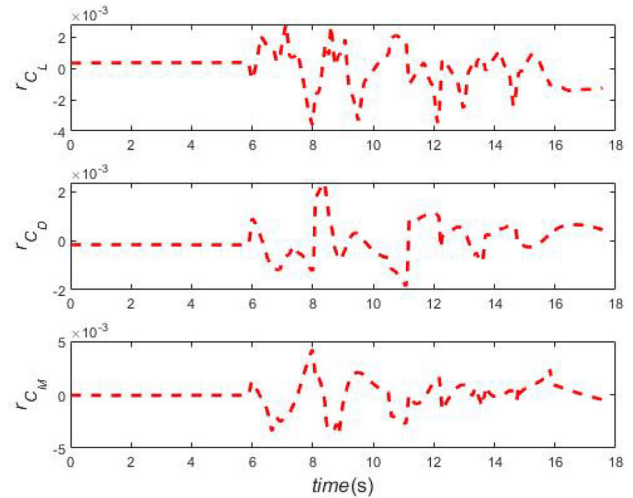


**Fig. 7** Identified longitudinal baseline aerodynamic coefficient model (landing gear up)

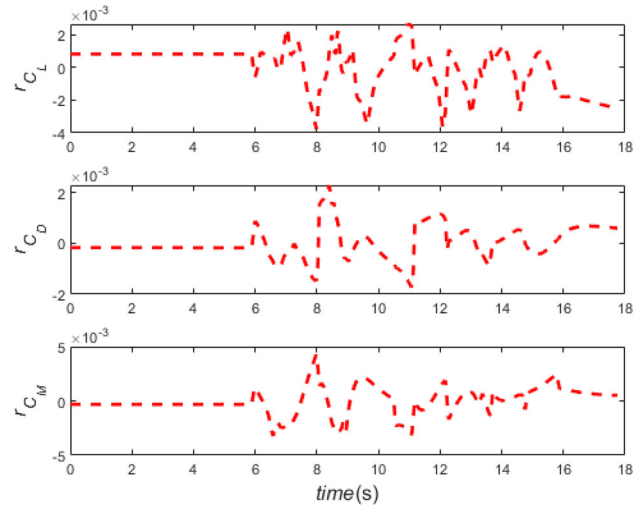
### 5.5 Comparison Between Linear Landing Gear Model and Nonlinear Landing Gear Model

For the purpose of comparing the accuracy between linear and nonlinear landing gear models, simulations are conducted using the previously obtained linear and nonlinear landing gear models, and the simulation results are compared with the simulated flight test data. Parameters such as pitch angle, angle of attack, pitch rate, true airspeed and altitude are observed to evaluate the accuracy of the acquired models.

Simulation settings and results for landing gear retraction with climbing speeds of 145 knots for a regular takeoff scenario and landing gear extension with landing speed of 135 knots for a regular landing scenario are presented below.



**Fig. 8** Residual error of identified longitudinal baseline aerodynamic coefficient model (landing gear down)



**Fig. 9** Residual error of identified longitudinal baseline aerodynamic coefficient model (landing gear up)

#### 5.5.1 Case 1: Takeoff at 145 Knots

In this case, we compared results from two landing gear models and the simulated flight results when retracting the landing gear at a climbing speed of 145 knots. The specific testing method is as follows:

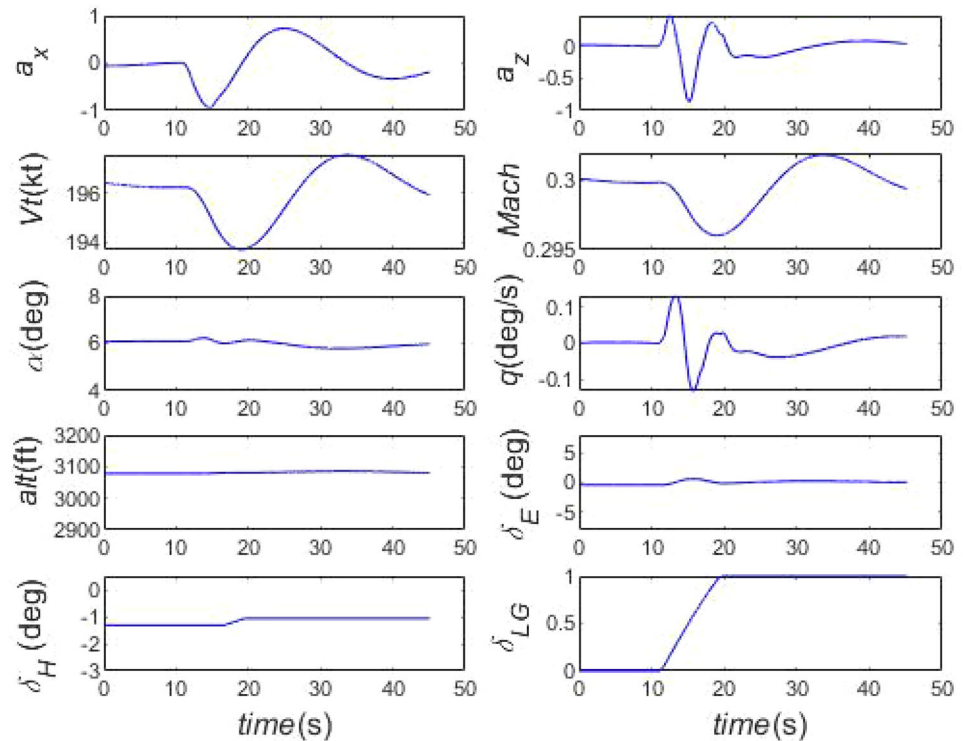
- The aircraft maintains a quasi-steady flight at an altitude of 2000 feet with the initial conditions listed in Table 4.
- Retract the landing gear while keeping other control inputs unchanged until the landing gear is fully retracted.

The setting of simulation parameters are shown in the table below.

**Table 3** Parameters for linear landing gear aerodynamic model

$C_{L_{0gearup}}$	$C_{L_{0gearDn}}$	$C_{Lq}$	$C_{L_{\delta_E}}$	$C_{L_{\delta_H}}$	$C_{L_{\alpha}}$
0.7513	0.7385	0.007765	0.007019	0.01732	0.1017
$C_{D_{0gearup}}$	$C_{D_{0gearDn}}$	$C_{Dq}$	$C_{D_{\delta_E}}$	$C_{D_{\delta_H}}$	$C_{D_{\alpha}}$
0.03597	0.06693	-0.08225	-0.00043	-0.00063	-0.00194
$C_{M_{0gearup}}$	$C_{M_{0gearDn}}$	$C_{Mq}$	$C_{M_{\delta_E}}$	$C_{M_{\delta_H}}$	$C_{M_{\alpha}}$
0.10609	0.13646	0.2653	-0.03175	-0.0734	-0.03175

**Fig. 10** Status and inputs in flight test for landing gear aerodynamic model identification



The simulation results are shown below (Figs. 13, 14).

**5.5.2 Case2: Landing at 135 Knots**

In this case, we compared results from two landing gear models and the simulated flight results when retracting the landing gear at a descending speed of 135 knots. The specific testing method is as follows:

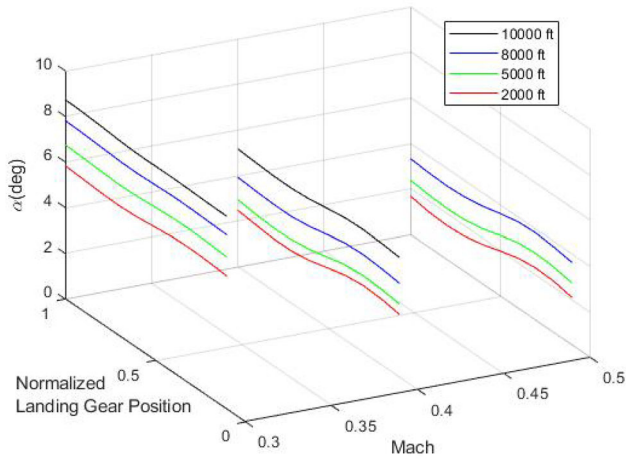
- a. The aircraft maintains quasi-steady flight at an altitude of 2000 feet with the initial conditions listed in Table 5.
- b. Extend the landing gear while keeping other control inputs unchanged until the landing gear is fully extended. The simulation parameters are shown in the table below.

The simulation results are shown below (Figs. 15, 16).

It should be noted that all simulation flights are conducted under standard atmospheric conditions, and at the beginning of each test, the aircraft is in quasi-steady flight.

As can be seen from these results, the comparative results indicate that the proposed nonlinear model demonstrates higher accuracy compared to the linear model within the normal operational envelope.

For the linear interpolation model, the simulated pitch angle, angle of attack, pitch rate and true airspeed all start to deviate when the landing gear is extended, and when the landing gear is fully extended, these errors begin to gradually decrease. This phenomenon is due to the fact that the linear interpolation model can only ensure the accuracy of quasi-steady aerodynamic data of landing gear fully up and down. Therefore, during the landing gear extension process, the model cannot accurately reflect the relationship between the landing gear position and the aerodynamic coefficient



**Fig. 11** Change of angle of attack at different status points along with the normalized landing gear position

increment, resulting in a deviation. When the landing gear is completely lowered, the linear interpolation model becomes accurate again and under the effect of the static stability of the aircraft, the simulation error of the model gradually decreases.

Comparisons are also made between the residuals of the proposed nonlinear model and the traditional linear model. Results showed that the nonlinear model obtained by using the method presented in this paper can accurately establish the nonlinear aerodynamic coefficient changes caused by different landing gear positions in the process of landing gear

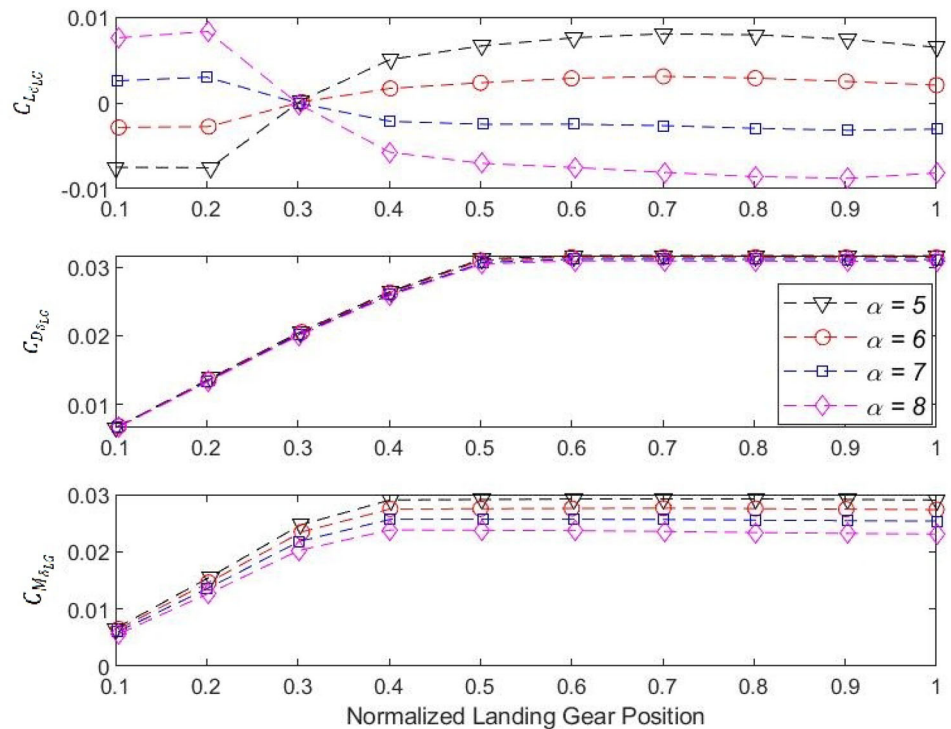
**Table 4** The settings of simulation parameters for takeoff at 145 knots

Initial conditions	Values	Units
$W$	135,000	lbs
$I_x$	935,248.38	slug-ft <sup>2</sup>
$I_y$	2,482,936	slug-ft <sup>2</sup>
$I_z$	3,315,718	slug-ft <sup>2</sup>
$x_{cg}$	133,692.41	slug-ft <sup>2</sup>
Altitude	2000	ft
IAS	145	Kt
TAS	149	Kt
$\delta_{LG}$	1	
Flap	5	
Sea level temperature	15	°C
QNH	29.92	inHg

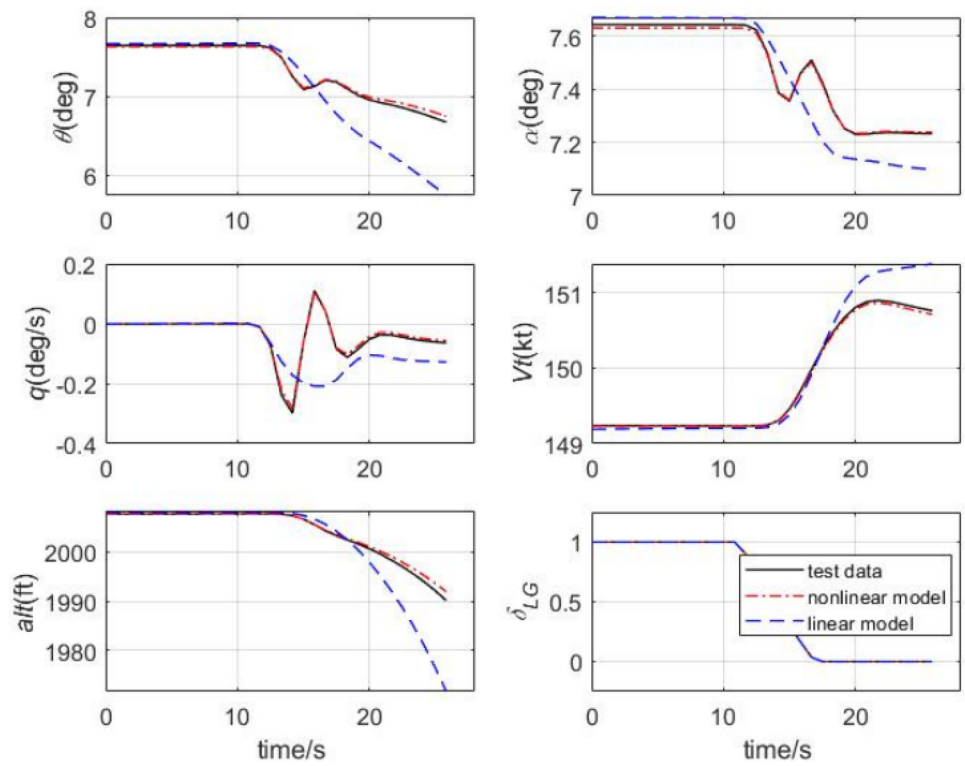
extension. The model has a higher accuracy in the whole process, and the flight status data always maintains a higher consistency with the simulated test flight data.

This paper also provides 3 other sets of simulation data presenting landing gear retraction during takeoff at 160 knots for heavy-weight takeoff, landing gear extension during landing at 160 knots for heavy-weight landing and one set representing landing gear extension during the cruise phase (with a speed of 250 knots). These results are included in the supporting information. These 5 sets of data essentially cover the whole envelope of landing gear operations under

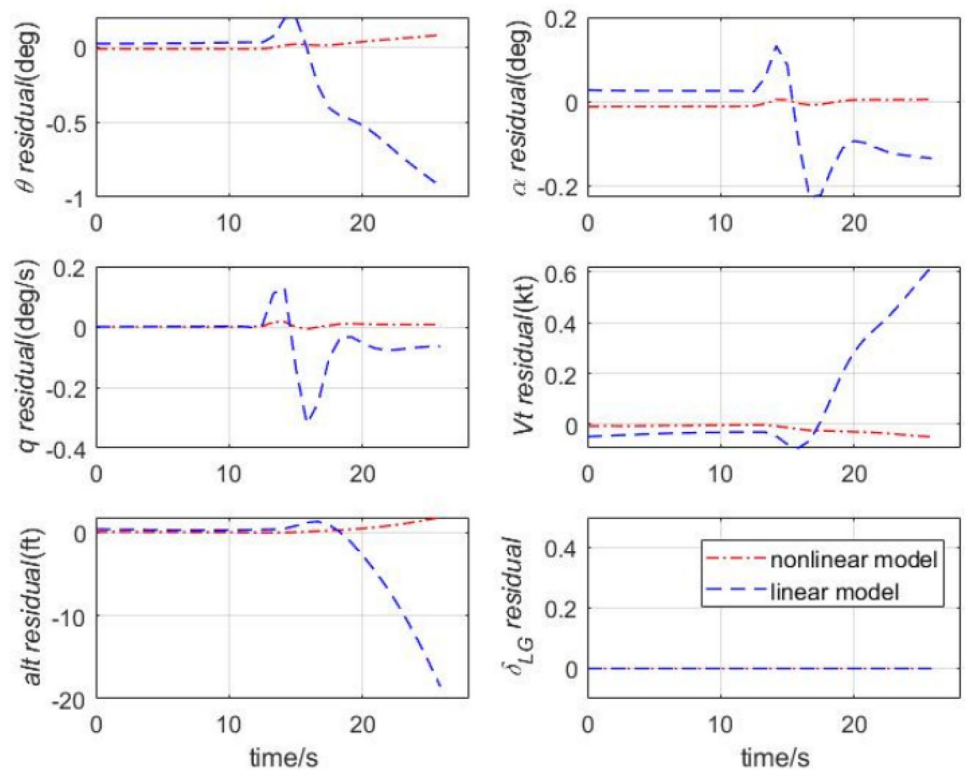
**Fig. 12** Incremental aerodynamic coefficient caused by landing gear position changes at different flight status points within the flight envelope



**Fig. 13** Comparison between the accuracy of the proposed nonlinear landing gear model and the linear interpolation model at takeoff at 145 knots



**Fig. 14** Comparison between the accuracy of the proposed nonlinear landing gear model and the linear interpolation model at takeoff at 145 knots





**Table 5** The settings of simulation parameters for landing at 135 knots

Initial conditions	Values	Units
$W$	135,000	lbs
$I_x$	935,248.38	slug-ft <sup>2</sup>
$I_y$	2,482,936	slug-ft <sup>2</sup>
$I_z$	3,315,718	slug-ft <sup>2</sup>
$x_{cg}$	133,692.41	slug-ft <sup>2</sup>
Altitude	2000	ft
IAS	135	Kt
TAS	139	Kt
$\delta_{LG}$	0	
Flap	30	
Sea level temperature	15	°C
QNH	29.92	inHg

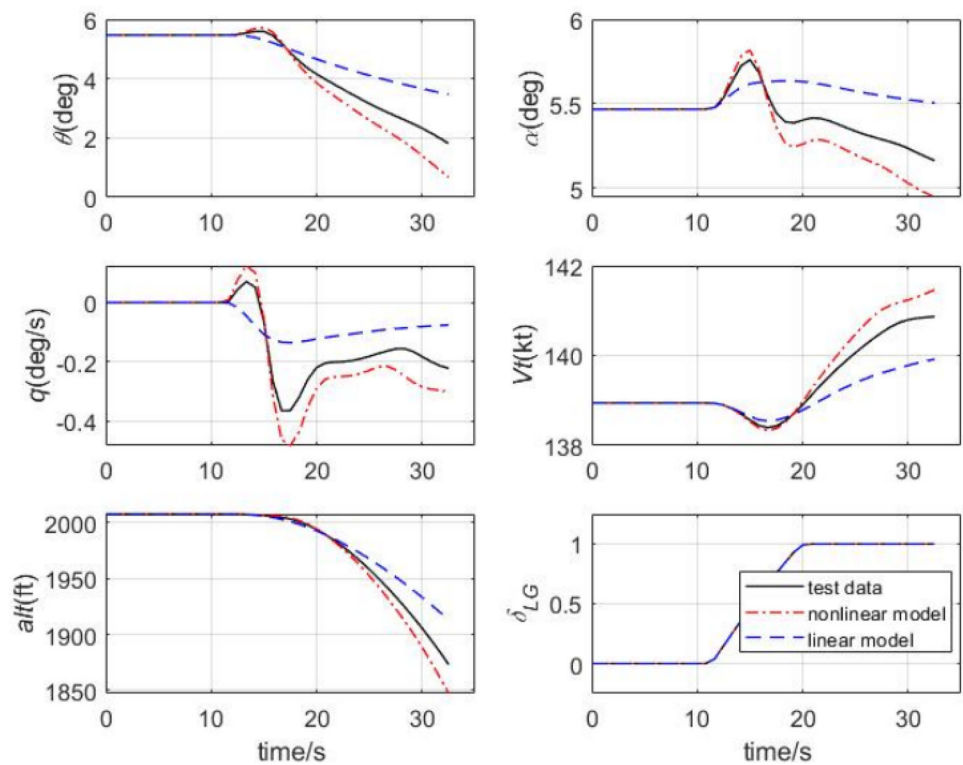
## 6 Conclusion

In this study, to obtain a more accurate nonlinear aerodynamic model for aircraft take-off and landing stage, flight test maneuvers were designed and performed with a simulated B737-800 aircraft. The nonlinear aerodynamic coefficient changes caused by the landing gear in the extension stage of the aircraft were obtained, and the aircraft aerodynamic model was optimized.

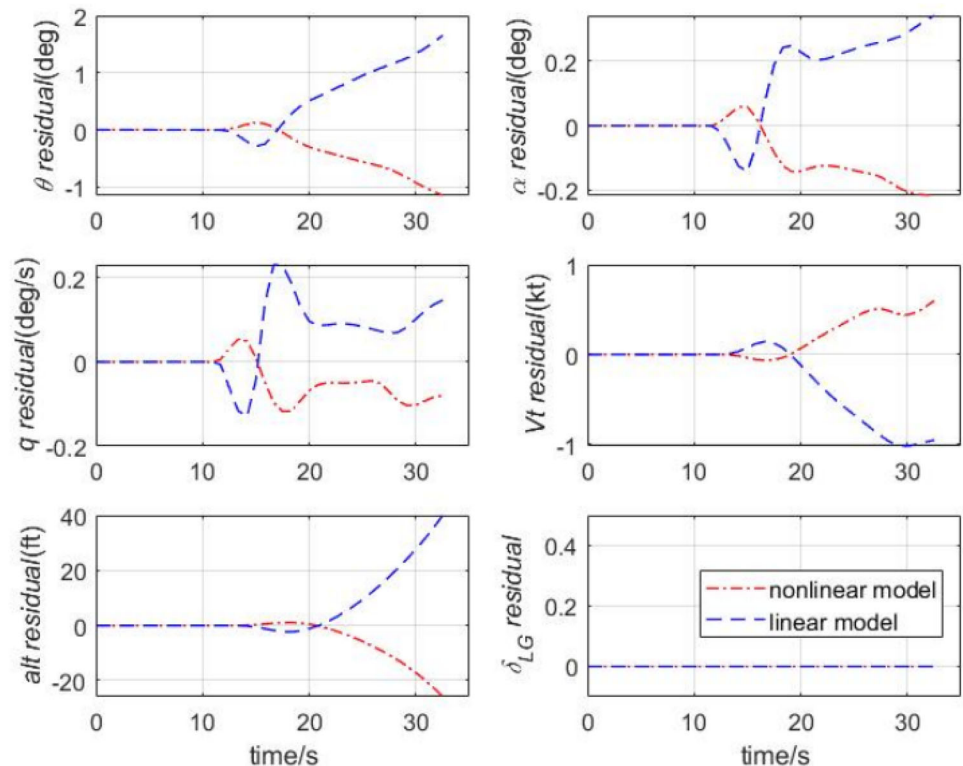
Compared with the linear interpolation model, the nonlinear aerodynamic model of the aircraft pitch channel obtained in this work shows higher accuracy, and can more accurately fit the aerodynamics caused by the change of landing gear position during landing gear extension and extraction processes, which makes the aerodynamic characteristics of aircraft at take-off and landing stage can be simulated better in flight simulators.

normal flight conditions. All results showed that the nonlinear landing gear model is more accurate within the envelope.

**Fig. 15** Comparison between the accuracy of the proposed nonlinear landing gear model and the linear interpolation model at landing at 135 knots



**Fig. 16** Comparison between the residuals of the proposed nonlinear landing gear model and the linear interpolation model at landing at 135 knots



**Supplementary Information** The online version contains supplementary material available at <https://doi.org/10.1007/s42405-024-00786-6>.

**Funding** The Funding was provided by State Key Laboratory of Virtual Reality Technology and Systems (Grant no. KG02-0012-16).

## References

- Schmidt R (2021) The design of aircraft landing gear. SAE International, Warrendale
- Huang Y-C (2020) Numerical simulation of the aerodynamic characteristics of a HTHL high-speed demonstrator aircraft. M.S. thesis, School of Aerospace Engineering, Xiamen University
- Paris AC, Alaverdi O (2005) Nonlinear aerodynamic model extraction from flight-test data for the S-3B viking. *J Aircr* 42(1):26–32. <https://doi.org/10.2514/1.3172>
- Götten F, Havermann M, Braun C, Marino M, Bil C (2020) Wind-tunnel and CFD investigations of UAV landing gears and turrets—improvements in empirical drag estimation. *Aerosp Sci Technol* 107:106306. <https://doi.org/10.1016/j.ast.2020.106306>
- Remillieux MC, Camargo HE, Ravetta PA, Burdisso RA, Ng WF (2008) Novel Kevlar-walled wind tunnel for aeroacoustic testing of a landing gear. *AIAA J* 46(7):1631–1639. <https://doi.org/10.2514/1.33082>
- Dorobantu A, Murch A, Mettler B, Balas G (2013) System identification for small, low-cost, fixed-wing unmanned aircraft. *J Aircr* 50(4):1117–1130. <https://doi.org/10.2514/1.C032065>
- Mehra RK, Prasanth RK (2004) Time-domain system identification methods for aeromechanical and aircraft structural modeling. *J Aircr* 41(4):721–729. <https://doi.org/10.2514/1.3596>
- Morelli EA, Grauer JA (2020) Practical aspects of frequency-domain approaches for aircraft system identification. *J Aircr* 57(2):268–291. <https://doi.org/10.2514/1.C035599>
- Eugene A, Morelli VK (2016) Aircraft system identification: theory and practice, 2nd edn. Sunflyte Enterprises, Williamsburg
- Jategaonkar RV (2015) Flight vehicle system identification: a time-domain methodology, 2nd edn. American Institute of Aeronautics and Astronautics Inc, Washington, DC
- Evans C, Fleming PJ, Hill DC, Norton JP, Pratt I, Rees D, Rodríguez-Vázquez K (2001) Application of system identification techniques to aircraft gas turbine engines. *Control Eng Pract* 9(2):135–148. [https://doi.org/10.1016/S0967-0661\(00\)00091-5](https://doi.org/10.1016/S0967-0661(00)00091-5)
- Melody JW, Başar T, Perkins WR, Voulgaris PG (2000) Parameter identification for inflight detection and characterization of aircraft icing. *Control Eng Pract* 8(9):985–1001. [https://doi.org/10.1016/S0967-0661\(00\)00046-0](https://doi.org/10.1016/S0967-0661(00)00046-0)
- Flightgear flight simulator. FlightGear 2020.3 for Windows 10, 2020. Available from: <https://www.flightgear.org/>
- Zhang J, Roumeliotis I, Zolotas A (2022) Model-based fully coupled propulsion-aerodynamics optimization for hybrid electric aircraft energy management strategy. *Energy* 245:123239. <https://doi.org/10.1016/j.energy.2022.123239>

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.