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# Kalman Filter-Based Fusion Estimation Method of Steering Feedback Torque for Steer-by-Wire Systems

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## Abstract
Universal challenge lies in torque feedback accuracy for steer-by-wire systems, especially on uneven and low-friction road. Therefore, this paper proposes a fusion method based on Kalman filter that combines a dynamics-reconstruction method and disturbance observer-based method. The dynamics- reconstruction method is designed according to the vehicle dynamics and used as the prediction model of the Kalman filter. While the disturbance observer-based method is performed as an observer model of the Kalman filter. The performance of all three methods is comprehensively evaluated in a hardware-in-the-loop system. Experimental results show that the proposed fusion method outperforms dynamics reconstruction method and disturbance observer-based method. Specifically, compared with the dynamics-reconstruction method, the root mean square error is reduced by 36.58% at the maximum on the flat road. Compared with the disturbance observer-based method, the root mean square error is reduced by 39.11% at the maximum on different-friction and uneven road.

**Keywords** Kalman filter · Steering feedback torque estimation · Steer-by-wire system

### Abbreviations

| | |
|---|---|
| ADAS | Advanced driver-assistance systems |
| HIL | Hardware-in-the-loop |
| SBW | Steer-by-wire |
| DR | Dynamics-reconstruction |
| DO | Disturbance observer |
| RMSE | Root mean square error |

## 1 Introduction

Advanced driver-assistance systems (ADAS) are one of the fast growing technologies because of its potential to reduce the driving load or road accidents. Steer-by-wire (SBW) systems are able to assist drivers by eliminating the mechanical link between the steering wheel and the wheels. Without a proper design of steering feedback torque, it is difficult for drivers to feel the road conditions [1–4], which affects the judgment of vehicle response. Therefore, it is essential to design a method to estimate the feedback torque [5–7]. At present, the methods to acquire the feedback torque mainly include the dynamics-reconstruction (DR) [8, 9] and disturbance observer (DO)-based method [10].

The steering feedback torque mainly consists of the self-aligning torque caused by the caster and camber of the king-pin and the returning torque caused by lateral forces. In Ref. [11], a suspension model with wheel positioning information was developed to calculate the self-aligning torque. A two-degree-of-freedom vehicle model was used to calculate the front wheel sideslip angle in Refs. [12, 13], and a linear tire model was applied to estimate the steering feedback torque. Nevertheless, this method does not work under extreme conditions due to nonlinear tire characteristics. Then, Ref. [14] applied the Magic Formula with tire nonlinearity to estimate the tire lateral force under all operating conditions. In addition, the change in the front wheel load is another essential factor that affects the steering feedback torque. The front wheel load was obtained by calculating the vertical stiffness and was used as the input of the tire model to estimate the steering feedback torque [15, 16].

A high-fidelity vehicle model can take care of the above factors. However, parameters in the accurate vehicle model, especially the tire model, should be determined

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before extensive experiments. Further, the coefficient of friction is difficult to directly measure by the existing onboard sensors. In addition, it is impossible to describe the extra steering feedback torque generated from the interaction between the uneven road and tire. As a result, all the aforementioned issues will lead to poor estimation performance when only the vehicle dynamics model is used.

A position sensor or a tension sensor was installed on the steering input shaft or the knuckle arm to measure the steering feedback torque, but the method requires accurate and stable sensors, especially in harsh environments [17, 18]. An observer was designed [19] to estimate the steering torque thanks to the linear correlation between the motor current and motor torque within a certain range. However, due to neglecting the torque needed to overcome friction and inertia, the load estimation has a great error. As a result, friction and the inertia compensation of the steering system were used as inputs of the state equation to reduce such an error [20]. To filter out the nonlinear noise in the current measurement value, in Refs. [20, 21], a median filter was applied, and a smooth change in the estimation result of the steering feedback torque was achieved. However, when the steering torque change frequency was more than 2.5 Hz, this estimation method produces a lag of over 130 ms, and the peak error is more than 20%. The Kalman filter was used to mitigate the effect of measurement noise on the estimation accuracy [22]. The authors assumed that the current state was equal to the previous state and used the variation rate of the current state to describe the measurement noise. However, when the steering wheel was commutated, the frequency of the current motor changed rapidly, and the highest frequency was approximately 4 Hz. Therefore, the measurement noise characteristics studied in this method are not realistic. Above all, since the measured value of the motor current contains nonlinear noise, it is impossible to avoid the phase lag and amplitude error by using the filter to process the noise.

In summary, to improve the estimation accuracy of the steering feedback torque on low-friction and uneven roads, a fusion method is developed. The proposed method contributes in the following three aspects:

(1) The fusion method improves the torque estimation accuracy on low-friction roads by considering the nonlinear cornering force acquired from the observation model.
(2) The fusion method improves the torque estimation accuracy on uneven roads by reducing the influence of the current noise by using the cornering force acquired from the linear tire model in the observation model.
(3) Compared with the DR method and the DO-based method, the superiority of the fusion method is verified on the hardware-in-the-loop (HIL) platform, which is established based on a SBW system with dual motors.

The remainder of this paper is organized as follows: Sect. 2 presents an estimation method based on a vehicle model. In Sect. 3, an observation method based on a DO is developed. In Sect. 4, the fusion method is designed. Section 5 comprehensively evaluates the performance of the three methods, followed by conclusions in Sect. 6.

## 2 Estimation Method Based on the Vehicle Dynamics Model

As mentioned above, the fusion method reduces the requirements for the accuracy of complex vehicle dynamics models. Therefore, a bicycle model with basic vehicle dynamics is utilized. This model uses the front wheel angle $\delta$ and longitudinal velocity $u$ as inputs and the lateral velocity $v$ and equivalent cornering stiffness $k$ as outputs. Meanwhile, it is assumed that the equivalent cornering stiffnesses of the front and rear wheels are equal. Thus, the state equation can be described as

$$\begin{cases} 2k\dfrac{v}{u} + \dfrac{1}{u}(a-b)k\omega_{\mathrm{r}} - k\delta = ma_{\mathrm{y}} \\ (a-b)\dfrac{v}{u}k + \dfrac{1}{u}\left(a^2+b^2\right)\omega_{\mathrm{r}}k - ak\delta = I_z\dot{\omega}_{\mathrm{r}} \end{cases} \tag{1}$$

where $a$ and $b$ represent the distances from the center of mass to the front and back axes, respectively; $m$ is the mass of the vehicle; $I_z$ is the inertia of the vehicle; $a_{\mathrm{y}}$ is lateral acceleration; and $w_{\mathrm{r}}$ denotes the yaw rate.

The lateral force of the front wheel can be calculated through a linear tire model as shown in Eq. (2). During the steering process, the driver overcomes the aligning torque caused by the returning torque and self-aligning torque (see Fig. 1). Since the self-aligning torque only takes a small proportion of the steering feedback torque, its influence is ignored.

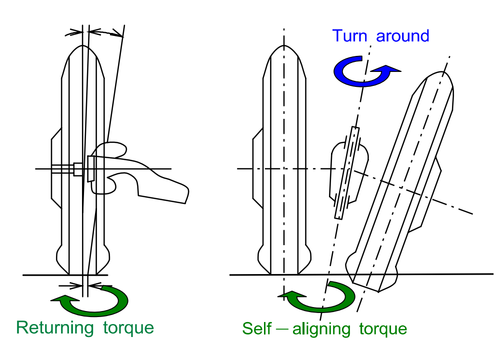



**Fig. 1** Steering feedback torque

$$F_y = k\alpha \tag{2}$$

where $\alpha$ denotes the front wheel slip angle and $F_y$ denotes the lateral forces.

The slip angle of the front wheel can be calculated by the bicycle model as shown in Eq. (3). Assuming the transmission ratio from the steering wheel to the front wheel is $i$ and combining Eqs. (2) and (3), the steering feedback torque $T_{\mathrm{stld}}$ can be calculated by Eq. (4).

$$\alpha = \frac{v}{u} + \frac{a\omega_{\mathrm{r}}}{u} - \delta \tag{3}$$

$$T_{\mathrm{stld}} = \frac{k}{i}\left(\frac{v}{u} + \frac{a\omega_{\mathrm{r}}}{u} - \delta\right) \tag{4}$$

The above model is developed in CarSim and Simulink co-simulation environment. The simulation results of a double lane change on high-friction surfaces are shown in Fig. 2. When the steering wheel angle is near zero, the estimated value of the steering feedback torque changes in the speediest way. Equation (5) shows the equivalent cornering stiffness obtained from Eq. (1), which is related to the steering wheel angle, yaw rate, and lateral acceleration. It can be seen from Fig. 3 that there is a phase difference among the steering wheel angle, yaw rate, and lateral acceleration. Thus, the change rate of the numerator and denominator is different in Eq. (5) such that it leads to the high change rate of the steering feedback torque.

$$k = \frac{I_z\dot{\omega}_{\mathrm{r}} - \frac{1}{2}(a-b)ma_y}{\frac{1}{2}(a-b)\delta - a\delta - \frac{1}{2u}(a-b)^2\omega_{\mathrm{r}} + \frac{1}{u}\left(a^2 - b^2\right)\omega_{\mathrm{r}}} \tag{5}$$

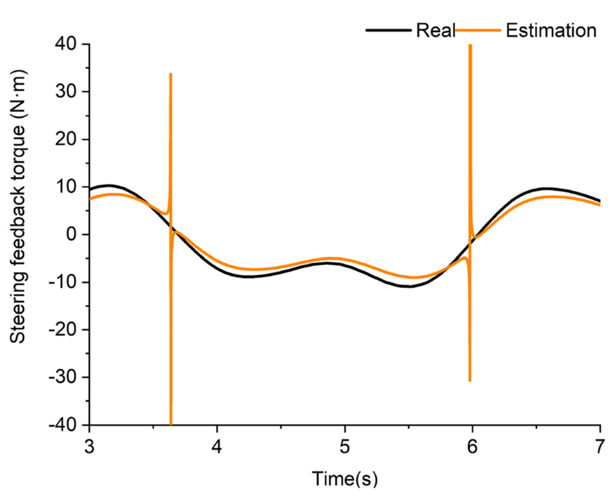



**Fig. 2** Steering torque estimation vs. test results under double lane change tests on the high-friction road

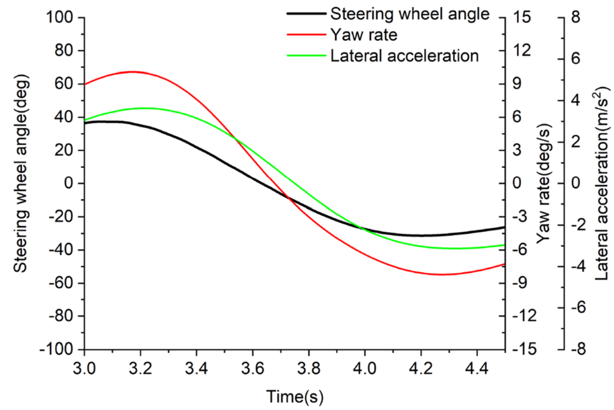



**Fig. 3** Partial enlarged view of the steering wheel angle, yaw rate and lateral acceleration

To prevent this issue that the estimated value of the steering feedback torque changes in the speediest way with near zero of steering wheel angle, the correlation between the lateral stiffness and lateral acceleration is obtained through a steady-state circular test. In this test, the steering wheel angle, yaw rate and lateral acceleration change slowly, and their changing phases are consistent. Therefore, the calculated value of the equivalent cornering stiffness is smooth.

When the vehicle is driving on a low-friction road, the tire force is easily saturated. Equation (4) is used to calculate the steering feedback torque, and the estimated value of the steering feedback torque will increase with increasing steering angle. If the steering angle increases continuously and the lateral tire force becomes saturated, the estimation method based on the linear vehicle model would perform poorly. To improve the estimation accuracy, the steering angle needs to be obtained when the tire lateral force reaches saturation, which is represented by $\delta_{\mathrm{emax}}$. Therefore, a logic rule is built to obtain $\delta_{\mathrm{emax}}$, which consists of the following two parts. The flow chart of the state machine is shown in Fig. 4.

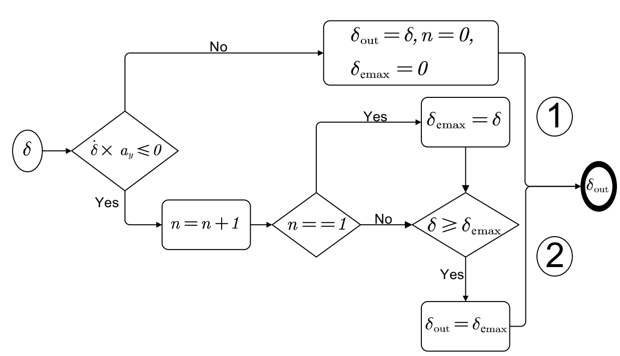



**Fig. 4** Logic flow chart to obtain $\delta_{\mathrm{emax}}$

Whether or not the signs of the steering angle rate and the lateral acceleration rate are different is used as the judgment basis. The tire lateral force has saturated and the value of the steering angle is $\delta_{\mathrm{emax}}$ when the product of the two varying rates is negative at the first time. Otherwise, the steering wheel angle is identical with the output of the state machine.

(1) When the tire force is located in the linear region, which means $\dot{\delta} \times \dot{a}_y > 0$, $\delta$ is the output (see Channel 1 in Fig. 4);
(2) When the tire force enters the nonlinear region for the first time, which means $\dot{\delta} \times \dot{a}_y \leq 0$, n = 1. Then, $\delta_{\mathrm{emax}}$ is set as $\delta$ and will be used for the next time steps until the tire force exists in the nonlinear region. Thus, $\delta_{\mathrm{emax}}$ is the output (see Channel 2 in Fig. 4). At the next time step, if the tire force is still located in the nonlinear region, $\delta_{\mathrm{emax}}$ is always the output (see Channel 2).
(3) Once the tire force exits the nonlinear region at a time step, as in step 1, the output will be set to $\delta$ (see Channel 1). Meanwhile, $n$ and $\delta_{\mathrm{emax}}$ will be reset to zero.

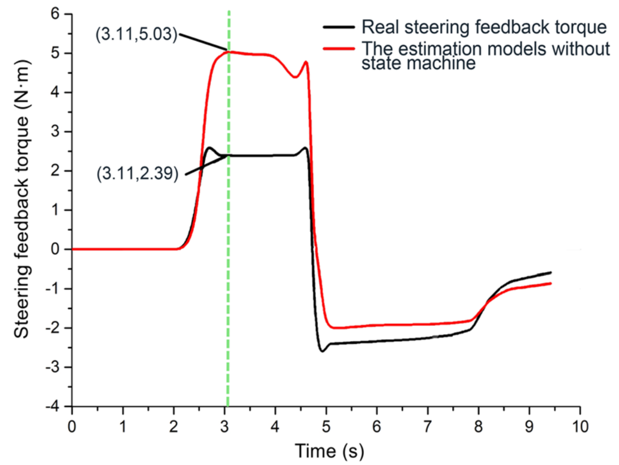
The experimental results with and without the proposed rule on low-friction roads are compared, as shown in Fig. 5. It can be seen from the figure that the maximum errors of the estimation models of the steering feedback torque with and without state machines are 2 and 2.64 N·m, respectively, and the error is reduced by 28.47%. It can be seen from Fig. 5(b) that without using a complex tire model, the state machine improves the estimation accuracy of the steering feedback torque on low- friction surfaces. The DR method can describe the saturation of the tire lateral force, but it cannot reflect the nonlinear region of the tire force.

## 3 Steering Feedback Torque Observation Method Based on a Disturbance Observer

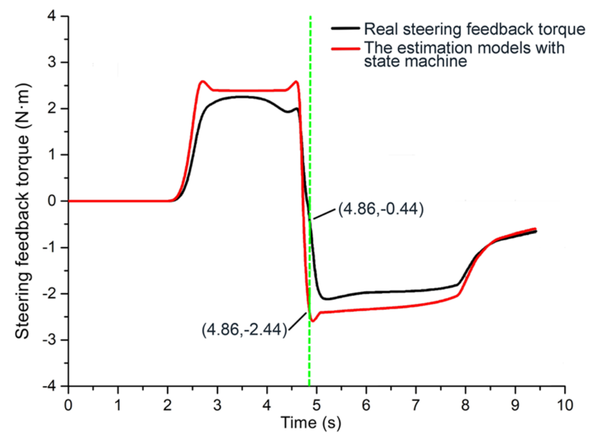
Figure 6 shows the force analysis of the rack, and the torque conservative equation is presented in Eq. (6), which is used to measure steering feedback torque for the proposed fusion method discussed in the next section.

$$M\ddot{X} = GK(I + \gamma) - F_{\mathrm{f}} - F_{\mathrm{stld}} \tag{6}$$

where $M$ is the equivalent mass of the steering system, which derives from the mass of the rack and the rotational inertia of the motor; $X$ denotes the displacement of the rack; $G$ denotes the transmission ratio from the motor to the rack; $K$ represents the motor torque coefficient converting the current to the torque; $F_{\mathrm{f}}$ indicates the equivalent friction of the steering machine, which includes the kinetic friction of the turbo-worm and the pinion; $F_{\mathrm{stld}}$ denotes the equivalent load force of the steering feedback torque; $I$ denotes the motor current; and $\gamma$ denotes the noise of the motor current.




**(a)** DR method without Logic flow chart




**(b)** DR method with Logic flow chart

**Fig. 5** Comparison of the simulation results of low friction coefficient surfaces

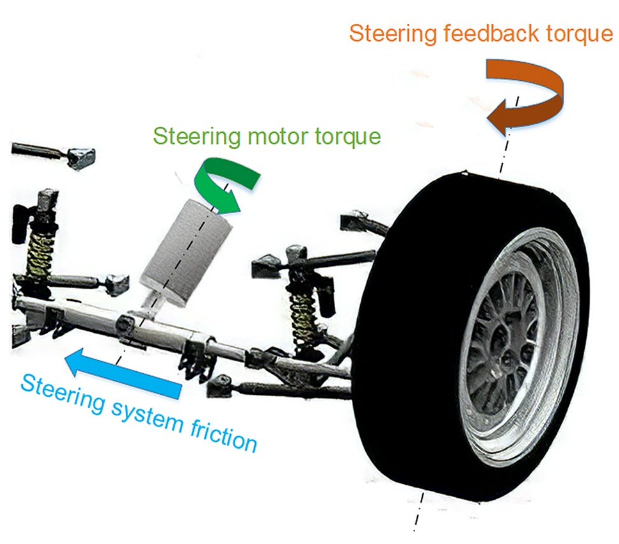



**Fig. 6** Force analysis of the rack

Taking $F_f$ and $F_{stld}$ as the system disturbance in Eq. (7), the motor current $I$ as the input, and the rack displacement $x$ as the output, the DO-based method is built as shown in Fig. 7.

$$d_r = F_f + F_{stld} \tag{7}$$

This paper analyzes the performance of the observation method by tests with sine steering inputs, where the frequencies are 1 Hz, 1.5 Hz and 2 Hz. Such frequencies are related to the frequencies of excitations from different roads. The results are shown in Table 1, which shows that the lagging phase increases from 6.7% to 16% and the maximum error increases from 2.07 to 2.74 N·m as the command frequency increases from 1 to 2 Hz. Thus, this observation method performs poorly when the frequency of the steering feedback torque or the frequency of the road excitation is high.

## 4 Fusion Estimation Method of the Steering Feedback Torque Based on the Kalman Filter

In view of the drawbacks of each method discussed in the previous two sections and to accurately estimate the steering feedback torque on low-friction and uneven roads, this paper proposes a fusion method to estimate the torque based on the Kalman filter, which fuses two parts: a DR method and a DO-based method. Figure 8 is a schematic diagram of the structure of the proposed fusion estimation method.

The Kalman filter combines the prediction value and the measurement value, which are derived from the prediction model and the measurement model, respectively. According to the Kalman filter principal, the fusion estimation method for the steering feedback torque is divided into three parts: the prediction model, the optimization process, and the covariance matrix update.

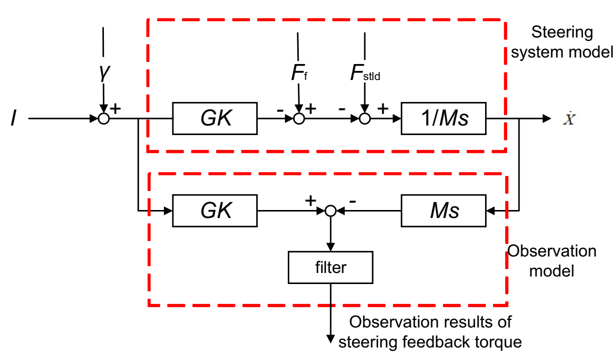



**Fig. 7** The DO-based method

**Table 1** Sine test results of the steering feedback torque

| Steering angle command(deg, Hz) | Lag time/ms | Lag phase | Maximum error/ (N·m) |
|---|---|---|---|
| 40, 1 | 134 | 6.7% | 2.07 |
| 40, 1.5 | 93.7 | 14.1% | 2.25 |
| 40, 2.0 | 80 | 16% | 2.74 |

### 4.1 Prediction model

Taking the front wheel angle and longitudinal velocity as inputs and the lateral velocity and the yaw rate as states, the state transition matrix is described as

$$\begin{bmatrix} \dot{v}_{(k|k-1)} \\ \dot{w}_{r(k|k-1)} \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} v_{(k-1|k-1)} \\ w_{r(k-1|k-1)} \end{bmatrix} \cdot \Delta t + \boldsymbol{B} \cdot \begin{bmatrix} \delta_{(k-1|k-1)} \\ u_{(k-1|k-1)} \end{bmatrix} \cdot \Delta t \tag{8}$$

where $\left[v_{(k-1|k-1)}\ w_{r(k-1|k-1)}\right]^T$ and $\left[\delta_{(k-1|k-1)}\ u_{(k-1|k-1)}\right]^T$ denote the state vector and the input vector at the last time step, respectively;$\left[\dot{v}_{(k|k-1)}\dot{w}_{r(k|k-1)}\right]^T$ denotes the estimation of the state vector at the current time step; $\Delta t$ denotes the discrete time step of the system model; $\boldsymbol{A}$ and $\boldsymbol{B}$ denote the system state coefficient matrix and system control coefficient matrix, respectively.

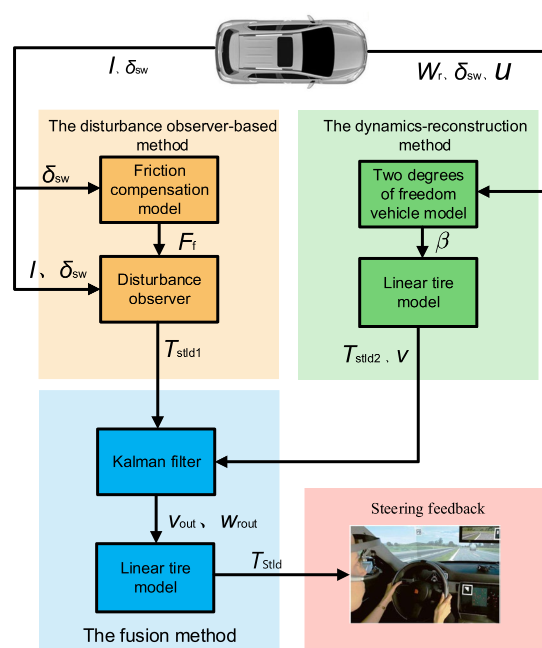



**Fig. 8** Schematic diagram of the fusion estimation method of the steering feedback torque

$$A=\begin{bmatrix}\frac{2k}{um} & \frac{(a-b)k}{um}-u\\ \frac{(a-b)k}{uI_z} & \frac{(a^2-b^2)k}{uI_z}\end{bmatrix},\ B=\begin{bmatrix}-\frac{k}{m}\\ -\frac{ak}{I_z}\end{bmatrix}$$

The state covariance matrix of the Kalman filter can be calculated through

$$\boldsymbol{P}=\begin{bmatrix}\mathrm{cov}(v,v) & \mathrm{cov}(v,w_r)\\ \mathrm{cov}(w_r,v) & \mathrm{cov}(w_r,w_r)\end{bmatrix} \tag{9}$$

where $\boldsymbol{P}$ denotes the state covariance matrix.

Assuming the noise of each state is white noise, the covariance matrix prediction equation is obtained as follows:

$$\boldsymbol{P}_{(k|k-1)}=A\boldsymbol{P}_{(k-1|k-1)}\boldsymbol{A}^{\mathrm{T}}+\begin{bmatrix}10\\01\end{bmatrix}\boldsymbol{Q} \tag{10}$$

where $\boldsymbol{P}_{(k|k-1)}$ and $\boldsymbol{P}_{(k-1|k-1)}$ denote the state covariance matrix at the current and the last moment, respectively, and $\boldsymbol{Q}$ denotes the noise of the states.

## 4.2 Optimization process

Assuming the measurement noise is white noise, according to the Kalman gain calculation equation, i.e. Equation (11), the optimal state estimation at the current time step can be calculated through Eq. (12).

$$\boldsymbol{K}_{(k|k)}=\boldsymbol{P}_{(k|k-1)}\boldsymbol{C}^{\mathrm{T}}\left(C\boldsymbol{P}_{(k|k-1)}\boldsymbol{C}^{\mathrm{T}}\right)^{-1}+\boldsymbol{R} \tag{11}$$

$$\begin{bmatrix}v_{(k|k)}\\ w_{r(k|k)}\end{bmatrix}=\begin{bmatrix}v_{(k|k-1)}\\ w_{r(k|k-1)}\end{bmatrix}+\boldsymbol{K}_{(k|k)}\times\left[\boldsymbol{Y}_{(k|k)}-\boldsymbol{C}\begin{bmatrix}v_{(k|k-1)}\\ w_{r(k|k-1)}\end{bmatrix}\right] \tag{12}$$

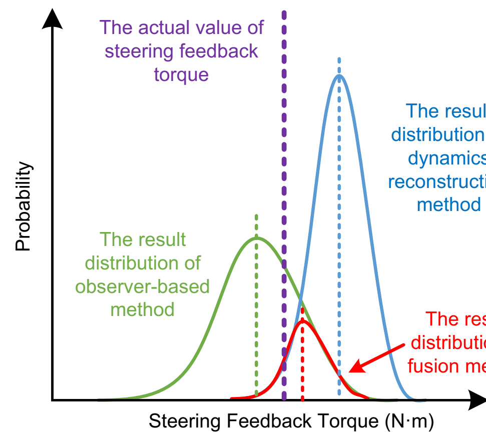
where $Y_{(k|k)}$ denotes the measurement value at the current step and $R$ denotes the noise of the measurement signals.

## 4.3 Covariance matrix update

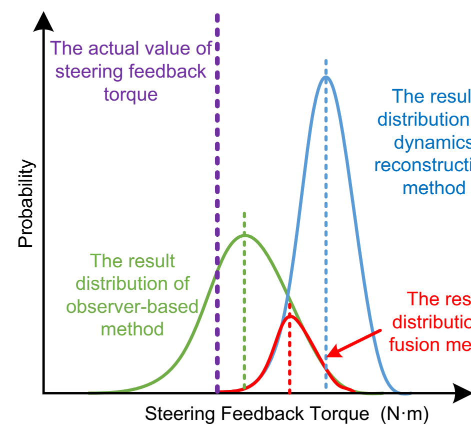
The covariance matrix can be updated through Eq. (13). Considering the initial static state of the vehicle, the initial state and covariance matrix are assumed to be 0.

$$\boldsymbol{P}_{(k|k)}=\left[\begin{bmatrix}1&0\\0&1\end{bmatrix}-\boldsymbol{K}_{(k|k)}\boldsymbol{C}\right]\boldsymbol{P}_{(k|k-1)} \tag{13}$$

The DR method will cause a large mean error regarding the probability distribution of the torque estimation due to the linearization of the tire model. However, the DO-based method will cause a very small error. It will cause a large bandwidth regarding the probability distribution of torque estimation due to the extremely large noise in the current. However, the DR method will not cause such large bandwidth because the input signals contain only slight noise. Therefore, the two methods can remedy the drawbacks of each other. To improve the estimation accuracy, this paper combines the two methods and proposes the fusion model to achieve a probability distribution of the torque estimation with a smaller mean error and bandwidth (see Fig. 9), improving the estimation accuracy of the steering feedback torque.




**(a)** The real steering feedback torque is located between the two estimated results of the DR method and the DO-based method




**(b)** The real steering feedback torque is located on one side of the two estimated results of the DR method and the DO-based method

**Fig. 9** Two fusion results of the steering feedback torque

## 5 Model Verification and Analysis

An HIL test bench based on dual steering machines is designed to verify the feasibility and accuracy of the proposed fusion method. One of the steering systems is used as test hardware to simulate the steering process of the vehicle; the other acts as load simulation hardware to apply a force from 0 to 1000 N to the rack.

### 5.1 Hardware-in-the-loop Bench for Steer-by-wire Systems Based on Dual Motors

With the help of the LabVIEW real-time system, an HIL bench for the SBW system is built. One steering motor is tested, and the other steering motor simulates the steering torque. The communication architecture based on the dual steering motor HIL platform is shown in Fig. 10.

The HIL mainly consists of five parts:

- Wire-controlled steering system
- Steering torque estimation system
- Vehicle real-time dynamic simulation model
- Real-world steering feedback torque acquisition system
- Steering feedback torque simulation system

The SBW system provides a real driving environment. This system also transmits the steering angle signal to the vehicle real-time dynamic simulation model, which is then used to provide the attitude information of the vehicle during the driving process and transmit the actual steering feedback torque information to the steering feedback torque simulation system.

The simulation based on CarSim can present the real-world test environment to a large extent. Therefore, this paper believes that the tire cornering force output by the software is almost equal to the actual steering load torque. In addition, the gear ratio is known from the wheel to the rack, so the aim is to make the measured force of the rack equal to the calculated value when the driver controls the virtual vehicle in CarSim through the real steering wheel. This software can provide various simulation environments including

**Fig. 11** The HIL bench

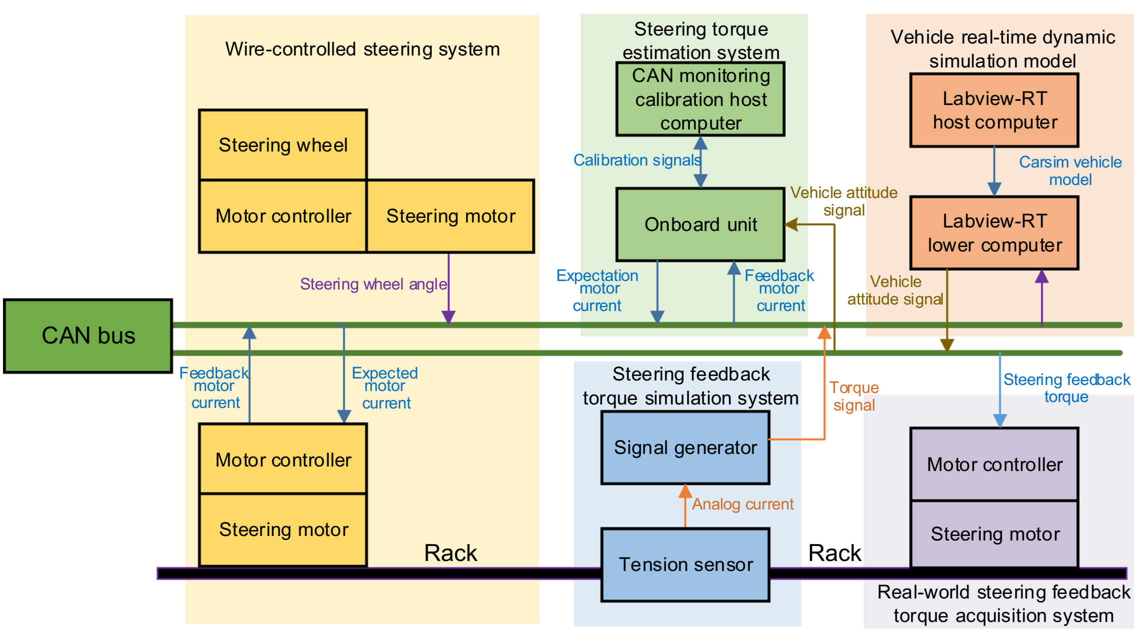



**Fig. 10** Communication architecture diagram of the HIL platform

various coefficients of friction and rough road conditions, and the steering feedback torque motor simulates the steering feedback torque by applying a force of 0 to 1000 N to the rack. Therefore, the test bench can complete the task of reproducing the transfer load under different adhesion coefficients and rough road conditions. The real-world steering feedback torque collection system can collect the transformation load torque applied to the rear rack, which is used to verify the accuracy of the model. The HIL test bench based on dual steering gears is shown in Fig. 11.

## 5.2 Verification and Analysis

In this section, the proposed methods are verified in two typical conditions: low friction and uneven road. Then, its universality is studied on medium friction and high friction road.

### 5.2.1 Verification Tests on a Low-Friction Road

The tire force easily reaches the nonlinear zone on roads with low friction coefficients, when the estimation accuracy of the DR method based on the linear tire model is low. To verify the accuracy of the steering system estimation framework by combining the observed current and the linear tire model, the double lane change test on icy and snowy roads is chosen. It can be seen from Fig. 12 that the proposed fusion model can accurately estimate the steering feedback torque when the tire lateral force is saturated. The fusion method avoids utilizing complex nonlinear tire models and suspension models and improves the accuracy of the steering feedback torque estimation on low-friction surfaces. The maximum error is reduced by 45.5% compared with the vehicle dynamic method.

### 5.2.2 Verification Tests on Uneven Road

As mentioned in the introduction, the estimation accuracy of the steering feedback torque based on the DR method on uneven roads is low. The DO-based method causes a significant phase lag and amplitude errors. The proposed fusion method is compared with the other two methods under uneven road conditions to verify if the fusion estimation model is effective.

It can be seen from Fig. 13 that the error of the proposed fusion estimation model is smaller than that of the DR model. Further, the maximum error of the fusion estimation model is 12.15 N·m less than that of the DR model. It can be seen from Fig. 14 that the fusion estimation model has a phase lag reduction of 40 ms and an amplitude error of 6.05 N·m compared to the DO-based method. Therefore, the steering feedback torque estimation accuracy of the fusion

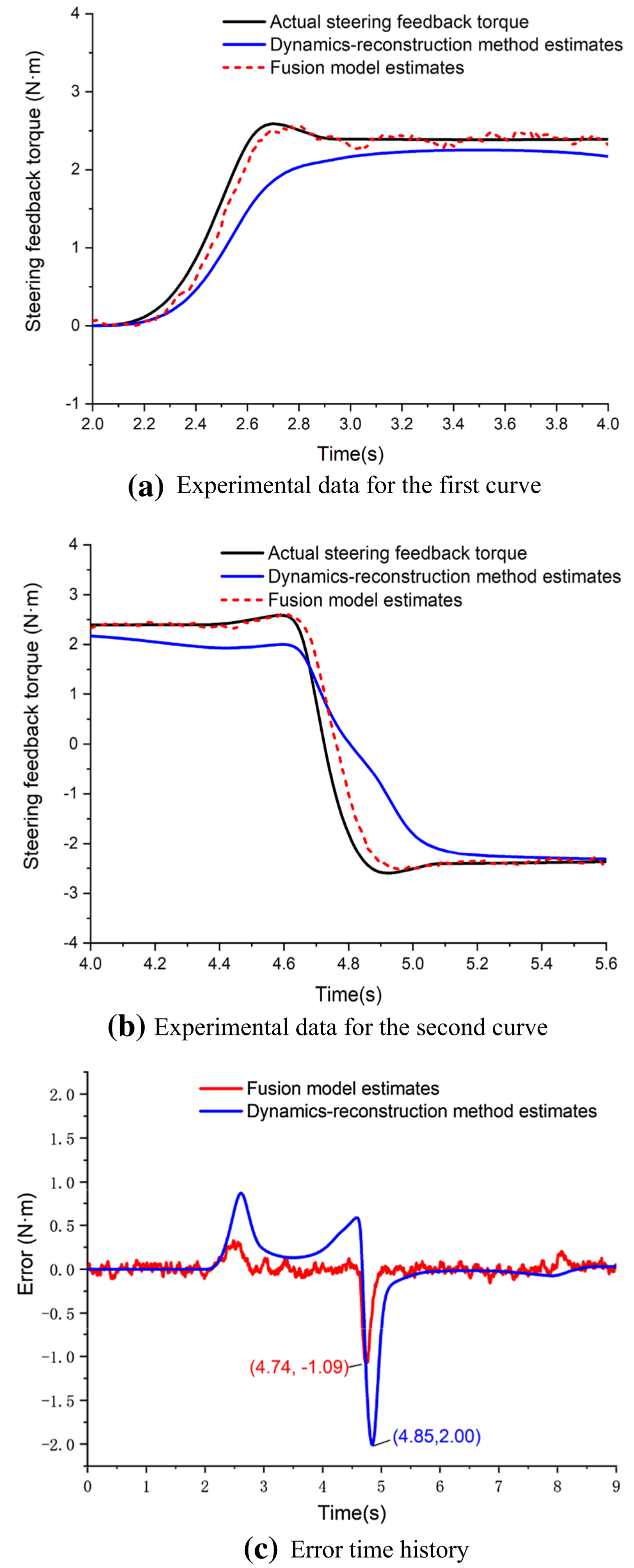



**Fig. 12** Partial enlarged view and error time history of the estimated results of the low-friction road test

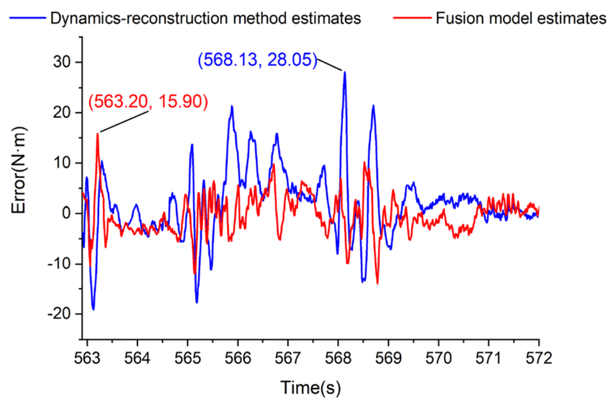



**Fig. 13** Error time history of the estimated results of the uneven road test

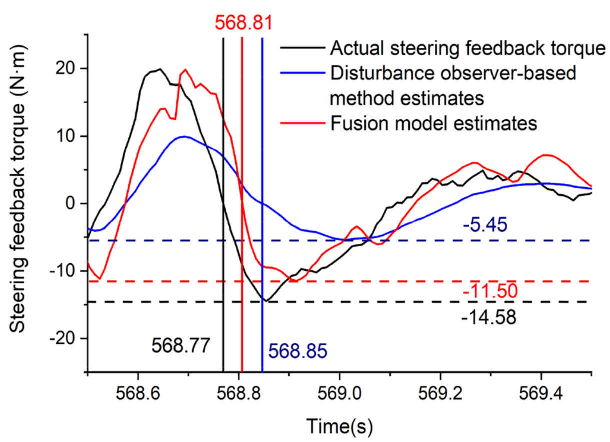



**Fig. 14** Partial enlarged view of the estimated results of the uneven road test

estimation model on uneven roads is higher than that of the DO-based method and DR model.

#### 5.2.3 Verification Tests on Road with Different Friction Coefficients

Double lane change tests were completed on two road surfaces with different adhesion coefficients ($\mu$ = 0.5 and $\mu$ = 0.8) to verify the adaptability of the fusion load estimation model, where $\mu$ is the adhesion coefficient of the road surface. The estimation results of the three estimation models and the comparison results of estimation errors are shown in Table 2, and following two conclusions are obtained:

(1) On a flat road with different friction coefficients, the maximum error of the proposed fusion model can be reduced by 52.20% and 40.77%, and the root mean square error (RMSE) can be reduced by 36.58% and 32.94% compared to DR model and DO-based method, respectively.
(2) On uneven roads, the RMSE of the proposed fusion model can be reduced by 78.24% and 39.11% compared to DR model and DO-based method, respectively.

## 6 Conclusion

This paper proposes a fusion method based on Kalman filter that combines vehicle dynamics and observation methods. Compared with the DR model, the proposed method

**Table 2** The test results on HIL bench

| Method name | index | Double lane change tests on dry concrete pavement ($\mu$ = 0.8) | Double lane change tests on wet soil pavement ($\mu$ = 0.5) | Double lane change tests on icy road ($\mu$ = 0.1) | Uneven road test |
|---|---|---|---|---|---|
| DR model (1) | Maximum error/(N·m) | 9.71 | 10.58 | 7.49 | 28.05 |
| | RMSE/(N·m) | 3.847 | 2.6946 | 1.5628 | 24.9863 |
| | Phase error/ms | 210 | 466 | 393 | 103 |
| | Amplitude error/(N·m) | – | – | – | 3.767 |
| DO-based method (2) | Maximum error/(N·m) | 11.16 | 6.34 | 5.41 | 16.29 |
| | RMSE/(N·m) | 4.238 | 2.0089 | 1.3470 | 8.9280 |
| | Phase error/ms | 114 | 221 | 136 | 77 |
| | Amplitude error/(N·m) | – | – | – | 9.10 |
| Fusion model (3) | Maximum error/(N·m) | 6.61 | 5.54 | 3.58 | 15.90 |
| | RMSE/(N·m) | 2.842 | 1.7088 | 1.0084 | 5.4359 |
| | Phase error/ms | 42 | 98 | 42 | 37 |
| | Amplitude error/(N·m) | – | – | – | 3.082 |

takes into account the nonlinear characteristics of the tire. Compared with the DO-based method, the proposed method reduces the influence of the motor current noise on the estimated value and shortens the lag caused by filtering. Experimental results show that the proposed fusion method has the highest accuracy among the three mentioned methods. Compared with the DR method, the RMSE is reduced by 36.58% at the maximum on the flat road. Compared with the DO-based method, the RMSE is reduced by 39.11% at the maximum on different-friction and uneven road.

It is hardly guaranteed in real-life situations to make the equivalent cornering stiffnesses of the front equal to rear wheels like in the prediction model. Therefore, future work will focus on the estimation method of equivalent cornering stiffness for the front and rear wheels.

## Declarations

**Conflict of interest** On behalf of all the authors, the corresponding author states that there is no conflict of interest.

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