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ORIGINAL PAPER

Some Generalizations of $*$-Lie Derivable Mappings and Their Characterization on Standard Operator Algebras

Behrooz Fadaee[1] · **Hoger Ghahramani**[1] · **Heydar Moradi**[1]

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Abstract

We introduce generalizations of $*$-Lie derivable mappings (which are not necessarily linear) on $*$-algebras and then provide characterizations of these generalizations on standard operator algebras. Indeed, if $\mathcal{H}$ is an infinite dimensional complex Hilbert space and $\mathcal{A}$ be a unital standard operator algebra on $\mathcal{H}$ which is closed under the adjoint operation, then we characterize these mappings on $\mathcal{A}$, especially we show that these mappings are linear. Our results are various generalizations of the main result of [W. Jing, *Nonlinear $*$-Lie derivations of standard operator algebras*, Quaestiones Math. 39 (2016), 1037–1046].

Keywords $\star$-Lie derivation · $*$-Lie derivable map · Generalized $*$-Lie 2-derivable map · Left (right) generalized $*$-Lie derivable map · Standard operator algebra

Mathematics Subject Classification 47B47 · 47L10 · 47L30

1 Introduction

Throughout this paper, all of algebras are associative. One of the favourite problems in mathematical is study of relationship between the additive and multiplicative structure of algebras (or rings). In this regard, Martindel in [13] proved that every multiplicative

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✉ Hoger Ghahramani
hoger.ghahramani@yahoo.com; h.ghahramani@uok.ac.ir

Behrooz Fadaee
behroozfadaee@yahoo.com; b.fadaee@sci.uok.ac.ir

Heydar Moradi
heydarmoradi61@gmail.com

[1] Department of Mathematics, Faculty of Science, University of Kurdistan, P.O. Box 416, Sanandaj, Kurdistan, Iran

bijective mapping from $\mathcal{R}$ to an arbitrary ring is additive, by considering suitable sufficient condition on the ring $\mathcal{R}$. Then, the question of which mappings on an algebra (ring) $\mathcal{A}$ are automatically additive or linear was considered and different results were obtained, in this direction we refer the reader to [5, 15–17] and references therein for more details. Among these results, there are especially results about Lie derivations and its generalizations, some of which we mention. Let $\mathcal{A}$ be an algebra and $\mathcal{M}$ be an $\mathcal{A}$-bimodule. A mapping $\delta : \mathcal{A} \to \mathcal{M}$ (not necessarily linear) is called a *Lie derivable map* if

$$\delta([a, b]) = [\delta(a), b] + [a, \delta(b)] \quad (a, b \in \mathcal{A}),$$

where $[a, b] = ab - ba$ is Lie product (or commutator) of a and b. The mapping δ is called a *derivable map* if

$$\delta(ab) = a\delta(b) + \delta(a)b \quad (a, b \in \mathcal{A}),$$

A linear (additive) Lie derivable mapping δ is called *linear (additive) Lie derivation*. If δ is a linear (additive) derivable mapping, then it is called a *linear (additive) derivation*. Lu and Liu in [12] studied Lie derivable maps on $\mathcal{B}(\mathcal{X})$, where $\mathcal{X}$ is a Banach space and $\mathcal{B}(\mathcal{X})$ is the algebra of all bounded linear operators on $\mathcal{X}$. They proved that every Lie derivable map on $\mathcal{B}(\mathcal{X})$ can be expressed as the sum of an additive derivation of $\mathcal{B}(\mathcal{X})$ into itself and a central mapping on $\mathcal{B}(\mathcal{X})$ which vanishes at commutators. This result was generalized to the case of Lie derivable maps on prime rings containing a non-trivial idempotent in [8]. In addition, derivable Lie maps on triangular algebras and generalized matrix algebras have been characterized (see [14, 18]). In the following, some generalizations of derivable Lie maps were also considered and characterized. Some of these generalizations are as follows. Let $\mathcal{A}$ be an algebra, $\mathcal{M}$ be an $\mathcal{A}$-bimodule and $\delta, \tau : \mathcal{A} \to \mathcal{M}$ be mappings. τ is called *left generalized Lie derivable map* with respect to δ whenever

$$\tau([a, b]) = \tau(a)b - \tau(b)a + a\delta(b) - b\delta(a), \quad (a, b \in \mathcal{A}).$$

It should be noted that the *right generalized Lie derivable map* with respect to δ can be defined as follows

$$\tau([a, b]) = a\tau(b) - b\tau(a) + \delta(a)b - \delta(b)a, \quad (a, b \in \mathcal{A}).$$

Since the characterization results of these two types of mapping are similar, usually only the first definition is considered. Fei and Zhang in [6] studied generalized Lie derivable mappings on triangular algebras. In addition, see [3, 7, 10]. Another generalization of Lie derivable maps are generalized Lie 2-derivable maps, which are defined as follows. Let δ be a Lie derivable map. τ is called *generalized Lie 2-derivable map* with respect to δ whenever

$$\tau([a, b]) = [\tau(a), b] + [a, \delta(b)], \quad (a, b \in \mathcal{A}).$$

Under certain sufficient conditions on triangular algebras and generalized matrix algebras, generalized Lie 2-derivable maps have been characterized in [11, 14], respectively (note that, the results are obtained in a more general state for generalized Lie n-derivable maps). Also, for more results in this regard, refer to [1, 2] and references therein.

In the continuation of this study path, mappings on $*$-algebras were considered. Among these maps, it is possible to mention the $*$-Lie derivable map, which is defined as follows. Let $\mathcal{B}$ be a $*$-algebra and $\mathcal{A}$ is a $*$-subalgebra of $\mathcal{B}$ and $\delta : \mathcal{A} \to \mathcal{B}$ be a map. δ is a $*$-*Lie derivable map* if

$$\delta([a, b]_*) = [\delta(a), b]_* + [a, \delta(b)]_*, \quad (a, b \in \mathcal{A}),$$

where $[a, b]_* = ab - ba^*$. In [19] the authors studied $*$-Lie derivable maps on factor von Neumann algebras. They proved that every $*$-Lie derivable map from a factor von Neumann algebra in an infinite dimensional Hilbert space into itself is an additive $*$-derivation. Note that a mapping T between $*$-algebras is called a $*$-*map* if $T(a^*) = T(a)^*$ for all a in the domain. In [9] Jing showed that any $*$-Lie derivable map on standard operator algebra is automatically linear. To be more precise, Jing proved the following theorem.

Theorem 1.1 [9, Theorem 2.14] *Let $\mathcal{H}$ be an infinite dimensional complex Hilbert space, and $\mathcal{A}$ be a standard operator algebra on $\mathcal{H}$ containing the identity operator I. If $\mathcal{A}$ is closed under the adjoint operation and $\delta : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ is a $*$-Lie derivable map, then δ is linear $*$-derivation. Moreover, there exists an operator $T \in \mathcal{B}(\mathcal{H})$ satisfying $T + T^* = 0$ such that $\delta(A) = AT - TA$ for all $A \in \mathcal{A}$.*

Continuing the natural process of these studies and with the idea of the mentioned generalizations of the Lie derivable mapping, we present the following definitions as generalizations of the $*$-Lie derivable mapping.

Definition 1.2 Let $\mathcal{B}$ be a $*$-algebra and $\mathcal{A}$ be a $*$-subalgebra of $\mathcal{B}$, and let $\delta, \tau : \mathcal{A} \to \mathcal{B}$ be mappings.

(i) Let δ be a $*$-Lie derivable map. Then τ is called *generalized $*$-Lie 2-derivable map* with respect to δ if

$$\tau([a, b]_*) = [\tau(a), b]_* + [a, \delta(b)]_*, \quad (a, b \in \mathcal{A}).$$

(ii) τ is called *left generalized $*$-Lie derivable map* with respect to δ whenever

$$\tau([a, b]_*) = \tau(a)b - \tau(b)a^* + a\delta(b) - b\delta(a)^*, \quad (a, b \in \mathcal{A}).$$

(iii) τ is called *right generalized $*$-Lie derivable map* with respect to δ if

$$\tau([a, b]_*) = a\tau(b) - b\tau(a)^* + \delta(a)b - \delta(b)a^*, \quad (a, b \in \mathcal{A}).$$

It is clear each of the defined mapping is a $*$-Lie derivable map if $\delta = \tau$. In addition, note that Definitions 1.2-(ii) and 1.2-(iii) are not symmetrical.

In this paper, we consider the mappings defined in 1.2 on the standard operator algebras in infinite dimensional complex Hilbert spaces, and we determine their structure, especially the result is that these maps are linear. More precisely, we prove the following theorems.

Theorem 1.3 *Let $\mathcal{H}$ be an infinite dimensional complex Hilbert space, $\mathcal{A}$ be a standard operator algebra on $\mathcal{H}$ containing the identity operator I, and $\mathcal{A}$ be closed under the adjoint operation. If $\tau : \mathcal{A} \to B(\mathcal{H})$ is generalized $*$-Lie 2-derivable map with respect to the $*$-Lie derivable map $\delta : \mathcal{A} \to B(\mathcal{H})$, then there exist operators $T, S \in \mathcal{B}(\mathcal{H})$ satisfying $T + T^* = 0$, $T - S \in \mathbb{R}I$ such that $\delta(A) = AT - TA$ and $\tau(A) = AT - SA$ for all $A \in \mathcal{A}$. Especially, τ, δ are linear maps.*

Theorem 1.4 *Let $\mathcal{H}$ be an infinite dimensional complex Hilbert space, $\mathcal{A}$ be a standard operator algebra on $\mathcal{H}$ containing the identity operator I, and $\mathcal{A}$ be closed under the adjoint operation. If $\tau : \mathcal{A} \to B(\mathcal{H})$ is left generalized $*$-Lie derivable map with respect to the $*$-Lie derivable map $\delta : \mathcal{A} \to B(\mathcal{H})$, then there exist operators $T, S \in \mathcal{B}(\mathcal{H})$ satisfying $T + T^* = 0$ such that $\delta(A) = AT - TA$ and $\tau(A) = AT - SA$ for all $A \in \mathcal{A}$. Especially, τ, δ are linear maps.*

Theorem 1.5 *Let $\mathcal{H}$ be an infinite dimensional complex Hilbert space, $\mathcal{A}$ be a standard operator algebra on $\mathcal{H}$ containing the identity operator I, and $\mathcal{A}$ be closed under the adjoint operation. If $\tau : \mathcal{A} \to B(\mathcal{H})$ is right generalized $*$-Lie derivable map with respect to the $*$-Lie derivable map $\delta : \mathcal{A} \to B(\mathcal{H})$, then there exist operators $T, S \in \mathcal{B}(\mathcal{H})$ satisfying $T + T^* = 0$, $(S - T) = (S - T)^*$ such that $\delta(A) = AT - TA$ and $\tau(A) = AS - TA$ for all $A \in \mathcal{A}$. Especially, τ, δ are linear maps.*

Each of above theorems are generalizations of Theorem 1.1. To prove these theorems, it is necessary to consider the maps on standard operator algebras that apply to certain multiplicative properties and determine their structure, especially, we prove that these maps are automatically linear (see, Lemmas 2.1 and 3.1). These results can be interesting in themselves.

Let $\mathcal{H}$ be a Hilbert space. We denote by $\mathcal{B}(\mathcal{H})$ the algebra of all bounded linear operators on Hilbert space $\mathcal{H}$, and $\mathcal{F}(\mathcal{H})$ denotes the algebra of all finite rank operators in $\mathcal{B}(\mathcal{H})$. Recall that a *standard operator algebra* is any subalgebra $\mathcal{A}$ of $\mathcal{B}(\mathcal{H})$ which contains $\mathcal{F}(\mathcal{H})$. We shall denote the identity operator of $\mathcal{B}(\mathcal{H})$ by I. It should be remarked that a standard operator algebra is not necessarily closed in the sense of weak operator topology. Every standard operator algebra is prime and its center is $\mathbb{C}I$. For more information on standard operator algebras, we refer to [4].

2 Proof of Theorem 1.3

The following lemma is a key element in proving Theorem 1.3.

Lemma 2.1 *Let $\mathcal{H}$ be a complex Hilbert space, and $\mathcal{A}$ be a standard operator algebra on $\mathcal{H}$ containing the identity operator I. If $\mathcal{A}$ is closed under the adjoint operation and $\phi : \mathcal{A} \to B(\mathcal{H})$ is a map satisfying $\phi([A, B]_*) = [\phi(A), B]_*$ for any $A, B \in \mathcal{A}$, then $\phi(A) = \phi(I)A$ for all $A \in \mathcal{A}$, where $\phi(I) \in \mathbb{R}I$. Especially, ϕ is linear.*

Proof The proof of Lemma will be organized in a several steps.

Step 1. $\phi(0) = 0$.

Proof

$$\phi(0) = \phi([0, 0]_*) = [\phi(0), 0]_* = 0.$$

□

Step 2. $\phi(\lambda I) \in \mathbb{R}I$, for any $\lambda \in \mathbb{R}$.

Proof Since

$$0 = \phi(0) = \phi([\lambda I, A]_*) = [\phi(\lambda I), A]_* = \phi(\lambda I)A - A\phi(\lambda I)^*,$$

for all $\lambda \in \mathbb{R}$ and $A \in \mathcal{A}$, we have $\phi(\lambda I)A = A\phi(\lambda I)^*$. Letting $A = I$, we see that $\phi(\lambda I) = \phi(\lambda I)^*$. Thus condition $\phi(\lambda I)A = A\phi(\lambda I)^*$ becomes $\phi(\lambda I)A = A\phi(\lambda I)$. It follows that $\phi(\lambda I) \in \mathbb{C}I$, the center of $\mathcal{A}$. Since $\phi(\lambda I)$ is self-adjoint, $\phi(\lambda I) \in \mathbb{R}I$, for any $\lambda \in \mathbb{R}$. □

Step 3. $\phi(-\frac{1}{2}I) = i\phi(\frac{1}{2}iI) = -i\phi(\frac{1}{2}iI)^*$.

Proof Since $[\frac{1}{2}iI, \frac{1}{2}iI]_* = [-\frac{1}{2}iI, -\frac{1}{2}iI]_* = -\dfrac{1}{2}I$ and $[-\frac{1}{2}iI, -\frac{1}{2}I]_* = \frac{1}{2}iI$, we have

$$\begin{aligned}
\phi\left(-\frac{1}{2}I\right) &= \phi\left(\left[\frac{1}{2}iI, \frac{1}{2}iI\right]_*\right) = \left[\phi\left(\frac{1}{2}iI\right), \frac{1}{2}iI\right]_* \\
&= \phi\left(\frac{1}{2}iI\right)\left(\frac{1}{2}iI\right) - \left(\frac{1}{2}iI\right)\phi\left(\frac{1}{2}iI\right)^* \\
&= \frac{1}{2}i\left(\left(\phi\left(\frac{1}{2}iI\right) - \phi\left(\frac{1}{2}iI\right)^*\right)\right), \qquad (2.1)\\
\phi\left(-\frac{1}{2}I\right) &= \phi\left(\left[-\frac{1}{2}iI, -\frac{1}{2}iI\right]_*\right) = \left[\phi\left(-\frac{1}{1}iI\right), -\frac{1}{2}iI\right]_* \\
&= \phi\left(-\frac{1}{2}iI\right)\left(-\frac{1}{2}iI\right) - \left(-\frac{1}{2}iI\right)\phi\left(-\frac{1}{2}iI\right)^* \\
&= \frac{1}{2}i\left(\left(\phi\left(-\frac{1}{2}iI\right)^* - \phi\left(-\frac{1}{2}iI\right)\right)\right), \qquad (2.2)
\end{aligned}$$

and

$$\begin{aligned}
\phi\left(\frac{1}{2}iI\right) &= \phi\left(\left[-\frac{1}{2}iI, -\frac{1}{2}I\right]_*\right) = \left[\phi\left(-\frac{1}{2}iI\right), -\frac{1}{2}I\right]_* \\
&= \phi\left(-\frac{1}{2}iI\right)\left(-\frac{1}{2}I\right) - \left(-\frac{1}{2}I\right)\phi\left(-\frac{1}{2}iI\right)^* \\
&= \frac{1}{2}\left(\phi\left(-\frac{1}{2}iI\right)^* - \phi\left(-\frac{1}{2}iI\right)\right). \qquad (2.3)
\end{aligned}$$

By Eqs. (2.2) and (2.3), we get

$$\phi\left(-\frac{1}{2}I\right) = i\phi\left(\frac{1}{2}iI\right). \tag{2.4}$$

Now by Eqs. (2.1) and (2.4) , we see that

$$i\phi\left(\frac{1}{2}iI\right) = \frac{1}{2}i\left(\phi\left(\frac{1}{2}iI\right) - \phi\left(\frac{1}{2}iI\right)^*\right).$$

So

$$\frac{1}{2}i\phi\left(\frac{1}{2}iI\right) = -\frac{1}{2}i\phi\left(\frac{1}{2}iI\right)^*.$$

Then

$$\phi\left(\frac{1}{2}iI\right) = -\phi\left(\frac{1}{2}iI\right)^*. \tag{2.5}$$

By comparing (2.4) and (2.5), we obtain

$$\phi\left(-\frac{1}{2}I\right) = -i\phi\left(\frac{1}{2}iI\right)^*.$$

□

Step 4. $\phi(iA) = i\phi(I)A$, for any $A \in \mathcal{A}$.

Proof Let $A \in \mathcal{A}$. Since $[\frac{1}{2}iI, A]_* = iA$, by using Step 2 and Step 3, we see that

$$\begin{aligned}
\phi(iA) &= \phi\left(\left[\frac{1}{2}iI, A\right]_*\right) = \left[\phi\left(\frac{1}{2}iI\right), A\right]_* \\
&= \phi\left(\frac{1}{2}iI\right)A - A\phi\left(\frac{1}{2}iI\right)^* \\
&= \phi\left(\frac{1}{2}iI\right)A + A\phi\left(\frac{1}{2}iI\right) \\
&= -i\phi\left(-\frac{1}{2}I\right)A + A\left(-i\phi\left(-\frac{1}{2}I\right)\right) \\
&= -i\phi\left(-\frac{1}{2}I\right)A - i\phi\left(-\frac{1}{2}I\right)A \\
&= -2i\phi\left(-\frac{1}{2}I\right)A.
\end{aligned}$$

Thus

$$\phi(iA) = -2i\phi\left(-\frac{1}{2}I\right)A. \tag{2.6}$$

So by Eq. (2.6) we have,

$$\phi(I) = \phi(i(-iI)) = -2i\phi\left(-\frac{1}{2}I\right)(-iI)$$
$$= -2\phi\left(-\frac{1}{2}I\right). \tag{2.7}$$

By composition (2.6) and (2.7) we see that

$$\phi(iA) = i\phi(I)A.$$

□

Now, by Step 4, we get

$$\phi(A) = \phi(i(-iA)) = \phi(I)A.$$

for all $A \in \mathcal{A}$. From Step 2 it follows that $\phi(I) \in \mathbb{R}I$. The proof is complete. □

Now, we are ready to prove Theorem 1.3.

Proof of Theorem 1.3 According to Theorem 1.1, there exists an operator $T \in \mathcal{B}(\mathcal{H})$ satisfying $T + T^* = 0$ such that $\delta(A) = AT - TA$ for all $A \in \mathcal{A}$. Define the mapping $\phi : \mathcal{A} \to B(\mathcal{H})$ by $\phi = \tau - \delta$. From the assumption it follows that ϕ is a map satisfying

$$\phi([A, B]_*) = [\phi(A), B]_* \quad (A, B \in \mathcal{A}).$$

So by Lemma 2.1, $\phi(A) = \phi(I)A$ for all $A \in \mathcal{A}$, where $\phi(I) \in \mathbb{R}I$. By the definition of ϕ we have $\tau(A) = \delta(A) + \phi(A) = AT - TA + \phi(I)A$ for any $A \in \mathcal{A}$. Set $S = T - \phi(I) \in \mathcal{B}(\mathcal{H})$. So $T - S \in \mathbb{R}I$, and $\tau(A) = AT - SA$ for all $A \in \mathcal{A}$. It is clear that τ, δ are linear maps. The proof is completed. □

3 Proofs of Theorems 1.4 and 1.5

First, we prove the following lemma.

Lemma 3.1 *Let $\mathcal{H}$ be a complex Hilbert space, and $\mathcal{A}$ be a standard operator algebra on $\mathcal{H}$ containing the identity operator I. Suppose that $\mathcal{A}$ is closed under the adjoint operation, and $\phi : \mathcal{A} \to B(\mathcal{H})$ is a map.*

(i) *If ϕ satisfies $\phi([A, B]_*) = \phi(A)B - \phi(B)A^*$ for all $A, B \in \mathcal{A}$, then $\phi(A) = \phi(I)A$ for all $A \in \mathcal{A}$.*
(ii) *If ϕ satisfies $\phi([A, B]_*) = A\phi(B) - B\phi(A)^*$ for all $A, B \in \mathcal{A}$, then $\phi(A) = A\phi(I)$ for all $A \in \mathcal{A}$, where $\phi(I) = \phi(I)^*$.*

Especially, ϕ is linear in both cases.

Proof We have $\phi(0) = 0$, by letting $A = B = 0$ in both cases.

(i) By letting $A = I$, we get

$$0 = \phi(0) = \phi(I)B - \phi(B),$$

for all $B \in \mathcal{A}$. Hence $\phi(A) = \phi(I)A$ for all $A \in \mathcal{A}$.

(ii) Let $A = I$, we see that

$$0 = \phi(0) = \phi(B) - B\phi(I)^*$$

for all $B \in \mathcal{A}$. Set $B = I$, therefore $\phi(I) = \phi(I)^*$. Thus $\phi(A) = A\phi(I)$ for all $A \in \mathcal{A}$, where $\phi(I) = \phi(I)^*$.

□

In the following, we present the proof of Theorems 1.4 and 1.5.

Proof of Theorem 1.4 From Theorem 1.1 it follows that there exists an operator $T \in \mathcal{B}(\mathcal{H})$ satisfying $T + T^* = 0$ such that $\delta(A) = AT - TA$ for all $A \in \mathcal{A}$. Define the mapping $\phi : \mathcal{A} \to B(\mathcal{H})$ by $\phi = \tau - \delta$. From the assumption it follows that ϕ is a map satisfying

$$\phi([A, B]_*) = \phi(A)B - \phi(B)A^* \quad (A, B \in \mathcal{A}).$$

So by Lemma 3.1-(i), $\phi(A) = \phi(I)A$ for all $A \in \mathcal{A}$. From the definition of ϕ we get $\tau(A) = \delta(A)+\phi(A) = AT-TA+\phi(I)A$ for any $A \in \mathcal{A}$. Set $S = T-\phi(I) \in \mathcal{B}(\mathcal{H})$. So $\tau(A) = AT - SA$ for all $A \in \mathcal{A}$. It is clear that τ, δ are linear maps. The proof is completed. □

Proof of Theorem 1.5 From Theorem 1.1 it follows that there exists an operator $T \in \mathcal{B}(\mathcal{H})$ satisfying $T + T^* = 0$ such that $\delta(A) = AT - TA$ for all $A \in \mathcal{A}$. Define the mapping $\phi : \mathcal{A} \to B(\mathcal{H})$ by $\phi = \tau - \delta$. From the assumption it follows that ϕ is a map satisfying

$$\phi([A, B]_*) = A\phi(B) - B\phi(A)^* \quad (A, B \in \mathcal{A}).$$

So by Lemma 3.1-(ii), $\phi(A) = A\phi(I)$ for all $A \in \mathcal{A}$, where $\phi(I) = \phi(I)^*$. From the definition of ϕ it follows that $\tau(A) = \delta(A) + \phi(A) = AT - TA + A\phi(I)$ for any $A \in \mathcal{A}$. Set $S = T + \phi(I) \in \mathcal{B}(\mathcal{H})$. So $S - T$ is a self adjoint operator, and $\tau(A) = AS - TA$ for all $A \in \mathcal{A}$. It is clear that τ, δ are linear maps. The proof is completed. □

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Declarations

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References

1. Ashraf, M., Ansari, M.A.: Multiplicative generalized Lie n-derivations of unital rings with idempotents. Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat. **116**, 20 (2022). https://doi.org/10.1007/s13398-022-01233-5
2. Behfar, R., Ghahramani, H.: Lie maps on triangular algebras without assuming unity. Mediterr. J. Math. **18**, 20 (2021). https://doi.org/10.1007/s00009-021-01836-z
3. Benkovič, D.: Generalized Lie derivations on triangular algebras. Linear Algebra Appl. **434**, 532–1544 (2011)
4. Dales, H.G.: Banach Algebras and Automatic Continuity. Clarendon Press, Oxford (2000)
5. Du, Y., Wang, Y.: Additivity of Jordan maps on triangular algebras. Linear Multilinear Algebra **60**, 933–940 (2012)
6. Fei, X., Zhang, J.: Nonlinear generalized Lie derivations on triangular algebras. Linear Multilinear Algebra **65**, 1158–1170 (2017)
7. Hvala, B.: Generalized Lie derivations in prime rings. Taiwan. J. Math. **11**, 1425–1430 (2007)
8. Jing, W., Lu, F.: Lie derivable mappings on prime rings. Linear Multilinear Algebra **60**, 167–180 (2012)
9. Jing, W.: Nonlinear $*$-Lie derivations of standard operator algebras. Quaest. Math. **39**, 1037–1046 (2016)
10. Liao, P., Liu, C.: On generalized Lie derivations of Lie ideals of prime algebras. Linear Algebra Appl. **430**, 1236–1242 (2009)
11. Lin, W.: Nonlinear generalized Lie n-derivations on triangular algebras. Commun. Algebra **46**, 2368–2383 (2018)
12. Lu, F., Liu, B.: Lie derivable maps on B(X). J. Math. Anal. Appl. **372**, 369–376 (2010)
13. Martindale, W.S., III.: When are multiplicative mappings additive? Proc. Am. Math. Soc. **21**, 695–698 (1969)
14. Wang, Y., Wang, Y.: Multiplicative Lie n-derivations of generalized matrix algebras. Linear Algebra Appl. **438**, 2599–2616 (2013)
15. Wang, L.: Nonlinear mappings on full matrices derivable at zero point. Linear Multilinear Algebra **65**, 2324–2332 (2017)
16. Wong, D., Ma, X., Chen, L.: Nonlinear mappings on upper triangular matrices derivable at zero point. Linear Algebra Appl. **483**, 236–248 (2015)
17. Yang, Y.: Nonlinear Jordan derivations of incidence algebras. Commun. Algebra **49**, 3200–3213 (2021)
18. Yu, W., Zhang, J.: Nonlinear Lie derivations of triangular algebras. Linear Algebra Appl. **432**, 2953–2960 (2010)
19. Yu, W., Zhang, J.: Nonlinear $*$-Lie derivations on factor von Neumann algebras. Linear Algebra Appl. **437**, 1979–1991 (2012)

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