



Stochastic analysis of six wheeler automobile system model with a spare wheel

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Abstract

The present paper deals with a stochastic model of a transport bus/truck running with six identical tyres and one spare wheel considered as standby. Out of six tyres, two tyres are in working position arranged one in each side of front end. Whereas remaining four are arranged in rear end with two of each side in parallel. The bus/truck will fail due to the tyre failure if in addition of spare unit any side of front-end tyre fails or both the tyres working in parallel in either of the rear-end side fail completely. The repair is carried out only when the transport bus/truck fails completely due to the tyre failure. The replacement of a failed tyre with standby spare wheel is carried out with constant rate and during this replacement process the system remains down. For the successfully running of the bus/truck each side of front end tyres and at least one of each side of rear-end tyre must be in good condition. The continuous parametric markov process and supplementary variable technique is used to obtain various economic related measure of system effectiveness. Maximum likelihood approach is used to derive the estimates of unknown parameters for MTSF.

Keywords Standby · Reliability · Availability · MTSF · Profit analysis · Lindley distribution · MLE

1 Introduction

In the realm of automotive engineering, ensuring vehicle reliability is crucial for safety and efficiency, especially when incorporating a spare wheel as a backup feature. The replacement of units, such as tires, engines, or other critical components, plays a pivotal role in enhancing the reliability and availability of the automotive system. Engineers frequently employ redundancy strategies to guarantee system availability and reliability to enhance these characteristics. As a result, during the past several decades, a wide range of standby systems have been built and examined. These research's primary goals have been to provide evaluation methodologies and tools to show the system's reliability, availability, maintainability, and safety. Under reliability

analysis, numerous researchers contributed to the examination of various models by employing supplementary techniques. For instance, Gupta and Singh (2021) examined a Markov chain system model of a power generating system, focusing on redundancy to enhance reliability. In power generating system model three generating units and one transformer unit is sufficient to run the system satisfactorily, the fourth generator and second transformer is added as a redundant unit to enhance the reliability of the system. Goyal et al. (2021) discussed measures of cost-effectiveness and reliability in a windmill water pumping system incorporating warranty and two types of repair facilities to ensure the system's ongoing and satisfactory operation. Additionally, studies by Pundir and Patawa (2019) addressed two repairable dissimilar units' cold standby system waiting for repair facility after failure of system units. Barak et al. (2018) analyzed two unit cold standby system with the facility that the server investigate the failed unit before repair or replacement. Also the refreshment facility available for the server whenever required for his better performance. The cold standby unit may fail from prolonged periods of inactivity or for any other reason, whereas the operational unit may fail straight from normal mode. Kumar et al. (2018) evaluated performance measures of a redundant system

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with preventive maintenance, giving priority to the original unit for repair. Other studies, such as Gupta and Chaudhary (2019), Ram et al. (2013) and Ram and Manglik (2016), explored various aspects of repairable manufacturing systems, including common cause failures, waiting times, and multiple-state complexities. Kumar et al. (2015) employed a probabilistic approach to analyze the reliability of casting processes in foundries, considering factors like availability and mean time to failure. Liu et al. (2015) investigate a repairable cold standby system with functioning vacations and vacation interruption as needed. Each study contributes valuable insights to the broader understanding of reliability and mean time to system failure, highlighting the importance of standby unit and repair of failed unit. Incorporating insights from previous studies, the current study addresses stochastic analysis of a six-wheeler vehicle, which includes a standby spare wheel strategically designed to mitigate tire-related issues.

The purpose of the present study is to explore the reliability, availability, and cost-benefit analysis of a six-wheeler transport bus or truck equipped with a single spare wheel. The article has been structured as follows: Sect. 2, titled “Material and Methods,” outlines the necessary notations, system states, and description of the system model. Section 3 presents the mathematical model of fundamental equation and their Laplace Transformations. In Sect. 4, the focus is on exploring the reliability, mean time to system failure (MTSF), availability and expected system’s profit. Additionally, numerical studies are conducted to provide further clarity on the results obtained. In Sect. 5, Classical estimation has been considered to derive the estimates of unknown parameters for MTSF.

2 Materials and methods

2.1 Symbols and system state

(a) Symbols:

$P_i(t)$	the probability that the system is in state S_i at epoch t ($i = 0$ to 9)
$P_j(x, t)$	the probability that the system is in state S_j at epoch t that has been in this state for x unit duration of time, i.e. x unit time for repair that has been lapsed in state S_j at time t ($j = 7$ to 9).
α	constant failure rate of each tyre and so that the failure time of each tyre follows exponential distribution.

β	constant replacement rate of the tyre.
$\mu_j(x), h_j(x)$	general repair rate and corresponding p.d.f. of repair time of the system

$$h_j(x) = \mu_j(x) \exp \left[- \int_0^x \mu_j(u) du \right]; j = 7, 8, 9$$

(b) To establish the different states of the system, the following symbols have been defined initially:

6 – OG	Six tyres are in good condition and operative.
5 – OG	Five tyres are in good condition.
1 – SG	Spare wheel in good condition
S rep / S F	Spare wheel is under replacement/ in failed mode respectively.
$\bar{A}, \bar{B}, \bar{C}, \bar{D}, \bar{E}, \bar{F}$	A, B, C, D, E, F as represented in block diagram are failed respectively.

2.2 Description of system model

The block diagram of the system model representing the network of the tyres is shown in Fig. 1. Here the tyres A and B of both the side of front end are connected in series system. Two tyres in each side of (C,D) and (E,F) rear end are arranged in parallel configuration.

The transition diagram with possible states of the system model along with transition rates between them is shown in Fig. 2. In this diagram, $S_0, S_2, S_3, S_4, S_5, S_6$ are up states, S_1 is down state and S_7, S_8, S_9 are failed states.

Initially in (state S_0) all the six tyres are in good condition and operative and spare wheel is kept as standby. In state S_1 system in down state where one tyre is failed and replacement of failed unit by spare standby wheel is under process. In state S_2 the bus is in running condition with six tyres in good and operative and spare wheel is failed. In state $S_3/ S_4/ S_5/ S_6$ one rear end tyre C/D/E/F from parallel system has failed but system is still operative. System transit in failed state S_7 if any of the tyre A or B fails from state S_2 . It moves to the failed state S_8 from S_3 or S_4 if tyre D in S_3 or tyre C in S_4 fails. Similarly, system reaches to the failed state S_9 from operative states S_5 or S_6 if tyre F in S_5 or tyre E in S_6 fails. The repairman is busy in two types of activity (i) for replacement of failed tyre when the system in down state S_1 (ii) for repair of the failed tyre when the system in failed state S_7, S_8, S_9 .

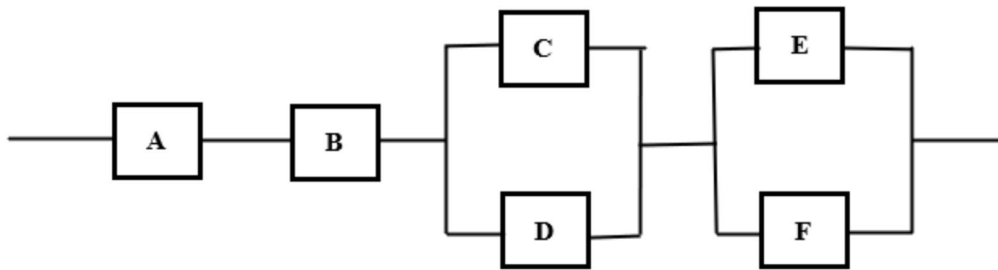


Fig. 1 Block diagram of the system model

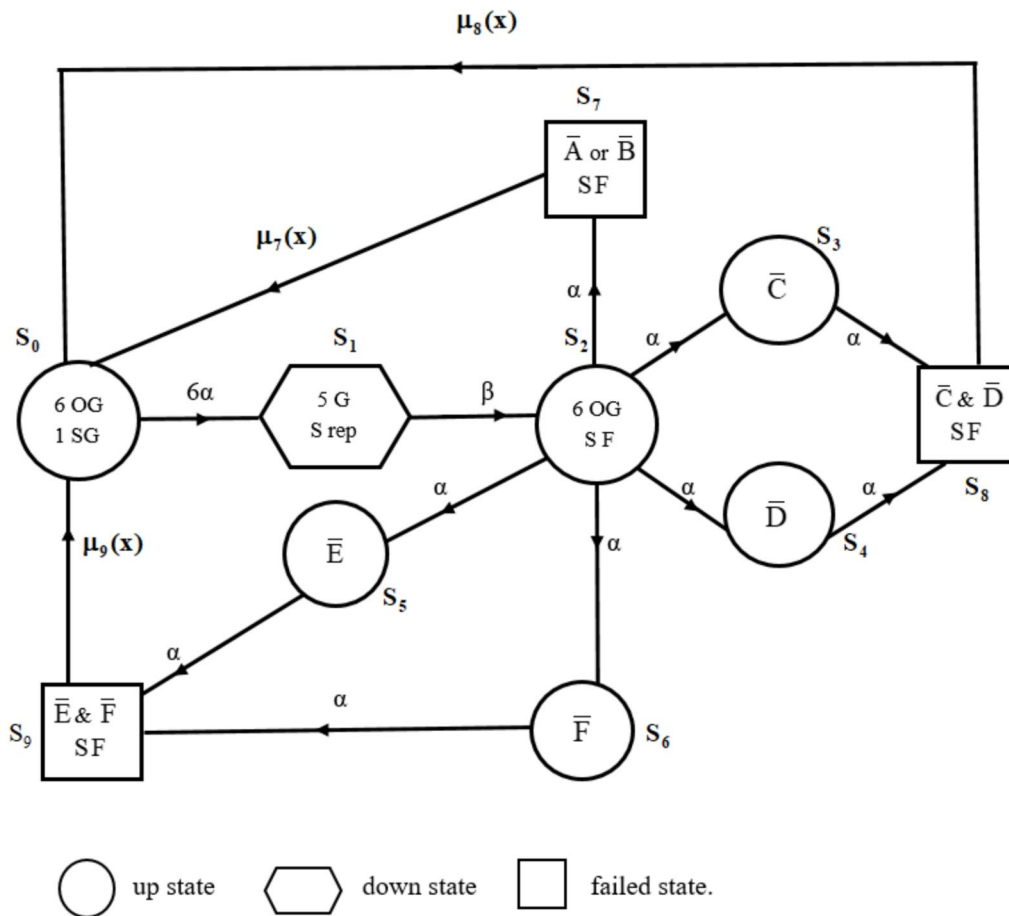


Fig. 2 Diagram depicting the transitions between system states

3 Mathematical model

3.1 Fundamental equations and their laplace transformations

Simple probabilistic examination, utilizing the Poisson process, results in a series of integro-differential equations as outlined below:

$$\left[\frac{\partial}{\partial t} + 6\alpha \right] P_0(t) = \sum_{j=7}^9 \int_0^{\infty} P_j(x, t) \mu_j(x) dx \tag{1}$$

$$\left[\frac{\partial}{\partial t} + \beta \right] P_1(t) = 6\alpha P_0(t) \tag{2}$$

$$\left[\frac{\partial}{\partial t} + 5\alpha \right] P_2(t) = \beta P_1(t) \tag{3}$$

$$\left[\frac{\partial}{\partial t} + \alpha \right] P_i(t) = \alpha P_2(t); i = 3, 4, 5, 6 \tag{4}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_j(x) \right] P_j(x, t) = 0; j = 7, 8, 9 \tag{5}$$

Initial condition

$$P_0(0) = 1, \quad P_i(0) = 0 \quad i = 1 \text{ to } 6 \tag{6}$$

$$P_j(x, 0) = 0 \quad j = 7, 8, 9 \tag{7}$$

Three boundary conditions

$$P_7(0, t) = \alpha P_2(t) \tag{8}$$

$$P_8(0, t) = \alpha P_3(t) + \alpha P_4(t) \tag{9}$$

$$P_9(0, t) = \alpha P_5(t) + \alpha P_6(t) \tag{10}$$

3.2 Solution of the model

In Sect. 3.1, the equations derived are solved utilizing Laplace Transform techniques to ascertain the probability associated with each system state.

$$P_0^*(s) = \left[(s + 6\alpha) - \frac{6\alpha^2\beta}{(s + 5\alpha)(s + \beta)} (s)h_7^*(s) - \frac{12\alpha^3\beta}{(s + 5\alpha)(s + \beta)(s + \alpha)} (h_8^*(s) + h_9^*(s)) \right]^{-1} \tag{11}$$

$$P_1^*(s) = \frac{6\alpha}{s + \beta} P_0^*(s) \tag{12}$$

$$P_2^*(s) = \frac{6\alpha\beta}{(s + \beta)(s + 5\alpha)} P_0^*(s) \tag{13}$$

$$P_k^*(s) = \frac{6\alpha^2\beta}{(s + \alpha)(s + \beta)(s + 5\alpha)} P_0^*(s); k = 3, 4, 5, 6 \tag{14}$$

$$P_7^*(s) = \frac{6\alpha^2\beta}{(s + \beta)(s + 5\alpha)} \left[\frac{1 - h_7^*(s)}{s} \right] P_0^*(s) \tag{15}$$

$$P_8^*(s) = \frac{12\alpha^3\beta}{(s + \alpha)(s + \beta)(s + 5\alpha)} \left[\frac{1 - h_8^*(s)}{s} \right] P_0^*(s) \tag{16}$$

$$P_9^*(s) = \frac{12\alpha^3\beta}{(s + \alpha)(s + \beta)(s + 5\alpha)} \left[\frac{1 - h_9^*(s)}{s} \right] P_0^*(s) \tag{17}$$

4 Numerical example

4.1 Reliability analysis

The reliability of the system is the probability that the system does not reach to any of the failed state during (0,t) which means that the system will be either in any of the up state or it is in down state at time t when the repair of the system is not permissible. So to obtain reliability of the system we regard the failed state S_7, S_8 and S_9 of the system as absorbing. In view of this the expression of reliability $R(t)$ in terms of its Laplace transform is given:

$$R^*(s) = P_0^*(s) + P_1^*(s) + P_2^*(s) + P_3^*(s) + P_4^*(s) + P_5^*(s) + P_6^*(s) \Big|_{\mu_7(x)=\mu_8(x)=\mu_9(x)=0}$$

$$= \frac{1}{(s + 6\alpha)} \left[1 + \frac{6\alpha}{(s + \beta)} + \frac{6\alpha\beta}{(s + \beta)(s + 5\alpha)} + \frac{24\alpha^2\beta}{(s + \alpha)(s + \beta)(s + 5\alpha)} \right] \tag{18}$$

The value of $R(t)$ can be obtained by taking Inverse Laplace Transform (ILT) of the Eq. (18) by using MATLAB for varying values of failure rate $\alpha = 0.03, 0.05, 0.07$ and fixed replacement rate $\beta = 0.9$. The expression of the reliability for the above three values of failure rate α are respectively given as follows:

$$R(t) = 0.0086206e^{(-0.9)t} - 0.25e^{(-0.18)t} + 1.241379e^{(-0.03)t} \tag{19}$$

$$R(t) = 1.2705882e^{(-0.05)t} - 0.3e^{(-0.3)t} + 0.02941176e^{(-0.9)t} \tag{20}$$

$$R(t) = 1.3012048e^{(-0.07)t} - 0.37e^{(-0.42)t} + 0.0737951e^{(-0.9)t} \tag{21}$$

By varying mission time 0 to 140 at step 10 in Eqs. (19–21), we calculated the values of reliability as given in Table 1 for the different values of α . These values of reliability are plotted graphically in Fig. 3 and important conclusion are drawn.

From the curve plotted in Fig. 3, it is revealed that the reliability of the system decreases uniformly as mission time t increases. It approaches to zero for the large value of t as it should be. From critical examination of the reliability graphs, it is observed that at mission time 90, the reliability of the system is 0.0834 which means at time t = 90 the

Fig. 3 Reliability of the system as a function of time t

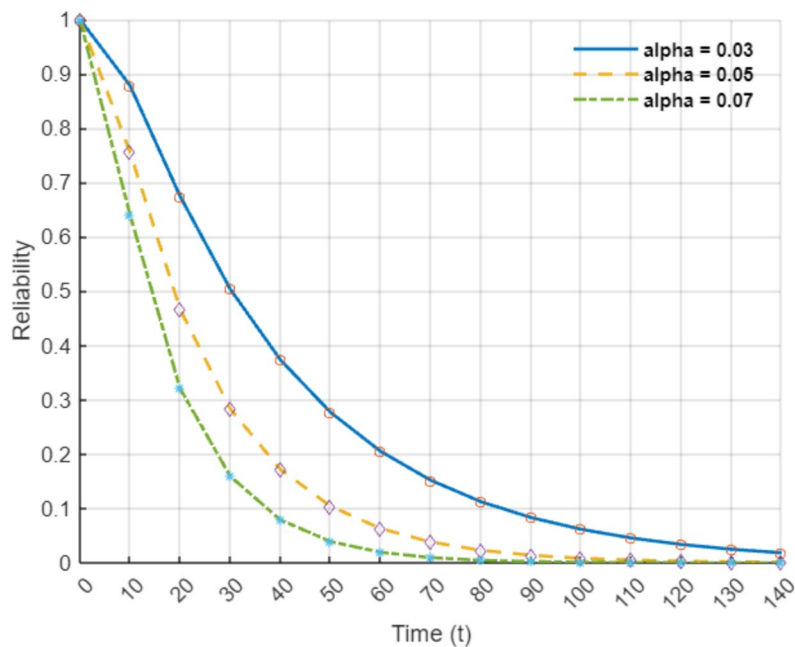


Table 1 Reliability and availability of the system as a function of time t

Mission time (t)	Reliability R(t)			Availability A(t)		
	$\alpha = 0.03$	$\alpha = 0.05$	$\alpha = 0.07$	$\alpha = 0.03$	$\alpha = 0.05$	$\alpha = 0.07$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0.8783	0.7557	0.6405	0.9517	0.9033	0.8565
20	0.6745	0.4667	0.3208	0.8829	0.7960	0.7207
30	0.5036	0.2835	0.1593	0.8175	0.7021	0.6071
40	0.3737	0.1720	0.0791	0.7567	0.6192	0.5114
50	0.2770	0.1043	0.0393	0.7005	0.5462	0.4308
60	0.2058	0.0633	0.0192	0.6484	0.4817	0.3629
70	0.1520	0.0384	0.0097	0.6002	0.4249	0.3057
80	0.1126	0.0233	0.0048	0.5556	0.3747	0.2575
90	0.0834	0.0141	0.0024	0.5143	0.3305	0.2169
100	0.0618	0.0086	0.0012	0.4761	0.2915	0.1828
110	0.0458	0.0052	0.0006	0.4407	0.2571	0.1539
120	0.0339	0.0031	0.0003	0.4080	0.2268	0.1297
130	0.0251	0.0019	0.0001	0.3776	0.2000	0.1092
140	0.0186	0.0012	0.0001	0.3496	0.1764	0.0920

system is reliable only 08%. Similarly, at mission time t = 140 the system is reliable only 01%.

$$\begin{aligned}
 MTSF &= \lim_{s \rightarrow 0} R^*(s) \\
 &= \left[\frac{1}{6\alpha} + \frac{1}{\beta} + \frac{1}{\alpha} \right] \tag{22}
 \end{aligned}$$

4.2 Mean time to system failure (MTSF)

Mean time to system failure (MTSF) is the predicted elapsed time between inherent failures of a system during operation. MTSF may also be defined as the average time to reach into any of the failed state of the system. Taking s tends to zero in Eq. (18) we can get the expression of MTSF as follows:

varying the value of replacement rate (β) as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, for three different values of failure rate α as 0.03, 0.05 and 0.07 in Eq. (22) the values of MTSF are shown in Table 2. These values of MTSF of corresponding to each value of failure rate are plotted graphically in Fig. 4 to draw the important conclusions.

Fig. 4 The trend of mean time to system failure with respect to α and β

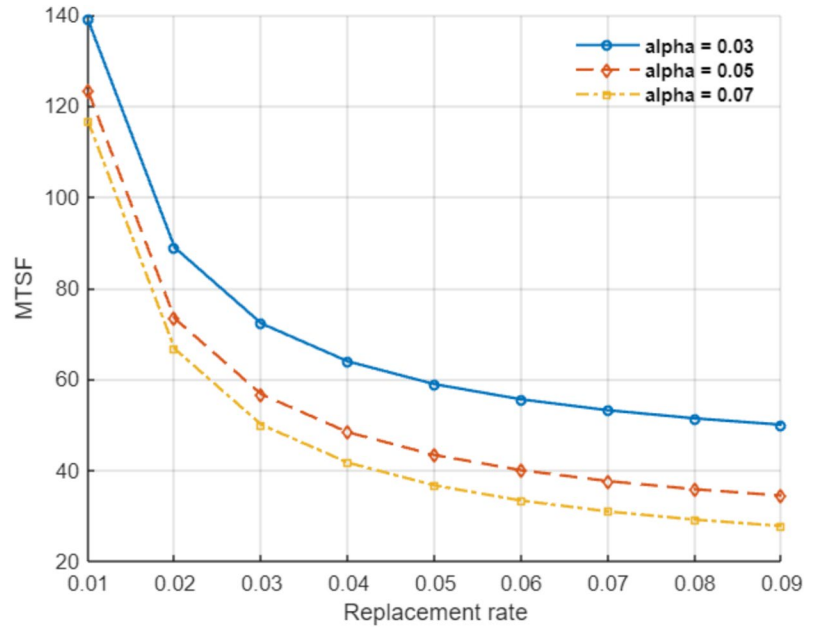


Table 2 Trend of MTSF with respect to α and β

Replacement rate	$\alpha = 0.03$	$\alpha = 0.05$	$\alpha = 0.07$
0.01	138.8889	123.33333	116.6667
0.02	88.88889	73.33333	66.66667
0.03	72.22222	56.66667	50.0000
0.04	63.88889	43.33333	41.66667
0.05	58.88889	43.33333	36.66667
0.06	55.55556	40.0000	33.33333
0.07	53.1746	37.61905	30.95238
0.08	51.38889	35.83333	29.16667
0.09	50.0000	34.44445	27.7788

From the graph plotted in Fig. 4, we observe that the MTSF of the system decrease uniformly as the replacement rate (β) increases and from this we conclude that, as much as the replacement rate of component is high the chances of the system failure is low. Moreover, as the failure rate increases the MTSF decreases irrespective of the value of replacement rate.

4.3 Point-wise availability analysis

It is denoted by $A(t)$ and is defined as the probability that the system as in any of the working (operative) state (S_0 ,

S_2, S_3, S_4, S_5 and S_6) at time t and its value in terms of its Laplace Transform (LT) is given by

$$A^*(s) = P_0^*(s) + P_2^*(s) + P_3^*(s) + P_4^*(s) + P_5^*(s) + P_6^*(s) = \left[1 + \frac{6\alpha\beta}{(s + 5\alpha)(s + \beta)} + \frac{24\alpha^2\beta}{(s + \alpha)(s + \beta)(s + 5\alpha)} \right] P_0^*(s) \tag{23}$$

To calculate the availability of the integrated system, we perform the inverse Laplace transform of Eq. (23). Repair time distribution $h_j(t); j = 7,8,9$ follows Lindley with parameters θ_j ($j = 7,8,9$) then the value of $h_j^*(s) = \frac{(s+\theta_j+1)\theta_j^2}{(1+\theta_j)(s+\theta_j)}$ for three different failure rate $\alpha = 0.03, 0.05, 0.07$ and fixed replacement rate $\beta = 0.9$, and repair parameter $\theta_j = 0.7$ for all $j = 7,8,9$. The expressions of point-wise availabilities for different failure rates 0.03, 0.05 and 0.07 respectively are as follows by using MATLAB:

$$A(t) = 0.01635855e^{(-0.90832185)t} - 0.01964903e^{(-0.21393854)t} - 0.02737489e^{(-0.68001621)t} + 1.03066537e^{(-0.00772338)t} \tag{24}$$

$$A(t) = 0.05443684e^{(-0.3956625)t} - 0.12180939e^{(-0.61718821)t} + 1.02325315e^{(-0.01255624)t} + 0.044119388e^{(-0.92459299)t} \tag{25}$$

$$\begin{aligned}
 A(t) = & 1.01567467e^{(-0.01715178)t} \\
 & + 0.07916223e^{(-0.94933176)t} \\
 & - 0.09483691e^{(-0.56175823)t} \\
 & \cos(0.1602647 t) \\
 & + 0.245197e^{(-0.5617582)t} \\
 & \sin(0.1602647 t)
 \end{aligned}
 \tag{26}$$

Now, varying mission time from 0 to 140 at step 10 in Eqs. (24–26) we obtain the values of point-wise availability of the system in Table 1 for three different values of α . The curves of the system’s availability with respect to the time t for $\alpha = 0.03, 0.05, 0.07$ are sketched in Fig. 5.

From analysis of the availability graph plotted in Fig. 5, reveals a uniform decrease in the system’s availability as the time and failure rate of a tyre increases. From the above analysis it has been concluded that at time $t = 140$ there are 34% chances for the system being operative when $\alpha = 0.03$. Similarly, chances of the system being operative tends to 17% and 09% respectively for $\alpha = 0.05$ and $\alpha = 0.07$, at $t = 140$.

4.4 Busy period analysis of repairman

(i) Due to replacement

Let $B_{rep}(t)$ be the probability that the repairman is busy in the replacement of failed tyre in state S_1 . Then its value in terms of its Laplace Transform is given by

$$\begin{aligned}
 B_{rep}^*(s) &= P_1^*(s) \\
 &= \left(\frac{6\alpha}{s + \beta} \right) P_0^*(s)
 \end{aligned}
 \tag{27}$$

Then expressions of $B_{rep}(t)$ by taking Inverse Laplace Transform of Eq. (27) for varying values of failure rate $\alpha = 0.03, 0.05$ and 0.07 for fixed replacement rate as $\beta = 0.9$ and repair parameter $\theta_j = 0.7$ for all $j = 7, 8, 9$ are respectively given by:

$$\begin{aligned}
 B_{rep}(t) = & 0.241133e^{(-0.213938)t} \\
 & - 0.032684e^{(-0.6800162)t} - 0.230682e^{(-0.908321)t} \\
 & + 0.022232e^{(-0.007723)t}
 \end{aligned}
 \tag{28}$$

$$\begin{aligned}
 B_{rep}(t) = & 0.703051e^{(-0.395662)t} - 0.342227e^{(-0.617881)t} \\
 & + 0.036552e^{(-0.012556)t} \\
 & - 0.397376e^{(-0.924592)t}
 \end{aligned}
 \tag{29}$$

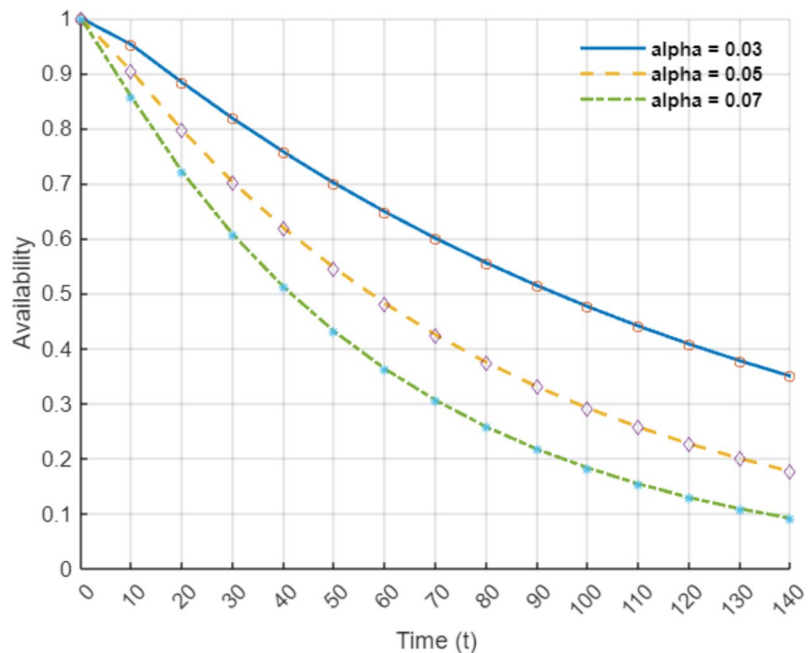
$$\begin{aligned}
 B_{rep}(t) = & 0.050450e^{(-0.0171517)t} \\
 & - 0.561589e^{(-0.949331)t} \\
 & + 0.511138e^{(-0.561758)t} \\
 & \cos(0.1602647 t)
 \end{aligned}
 \tag{30}$$

$$\begin{aligned}
 & + 1.0911126e^{(-0.5617582)t} \\
 & 7 \sin(0.1602647 t)
 \end{aligned}$$

(ii) Due to repair from the failed state

Let $B_r(t)$ be the probability that the repairman is busy in the repair in any of the failed state S_7, S_8 and S_9 . So, by

Fig. 5 Availability of the system as a function of time t



additive law of probability, the probability that the repairman will be busy at epoch t when system initially starts from S_0 as given by in Laplace transform is as follows:

$$B_r^*(s) = \sum_{j=7}^9 P_j^*(s) = \left[\frac{6\alpha^2\beta}{(s+\beta)(s+5\alpha)} \left\{ \frac{1-h_7^*(s)}{s} \right\} + \frac{12\alpha^2\beta}{(s+\alpha)(s+\beta)(s+5\alpha)} \left\{ \frac{(1-h_8^*(s))}{s} + \frac{(1-h_9^*(s))}{s} \right\} \right] P_0^*(s) \tag{31}$$

Then expressions of $B_r(t)$ by taking Inverse Laplace Transform of Eq. (31) for varying values of failure rate $\alpha = 0.03, 0.05$ and 0.07 when $\beta = 0.9$ and $\theta_j = 0.7$ for all $j = 7, 8, 9$ are as follows:

$$B_r(t) = 0.0273748e^{(-0.68001621)t} + 0.019649e^{(-0.213938)t} - 0.0163585e^{(-0.908321)t} - 1.03066e^{(-0.007723)t} + 1.0 \tag{32}$$

$$B_r(t) = 0.121809e^{(-0.6171882)t} - 0.0544368e^{(-0.395662)t} - 1.023253e^{(-0.01255624)t} - 0.044119e^{(-0.924592)t} + 1.0 \tag{33}$$

$$B_r(t) = 0.094836e^{(-0.5617582)t} \cos(0.1602645t) - 0.07916223e^{(-0.9493317)t} - 1.0156746e^{(-0.01715178)t} - 0.24519705e^{(-0.5617582)t} \sin(0.1602647t) + 1.0 \tag{34}$$

4.5 Cost-benefit analysis

Maintaining control over costs is vital to uphold product reliability. If we assume that the service facility remains consistently accessible, we can express the anticipated profit within the timeframe $(0, t]$ as follows:

$$E_p(t) = \text{Total revenue in time interval } (0,t) - ((\text{Expected cost of replacement during } (0,t) + (\text{Expected cost of repair during } (0,t)))$$

$$E_p(t) = I_0 \int_0^t A(u)du - \left(I_1 \int_0^t B_{rep}(u)du + I_2 \int_0^t B_r(u)du \right) \tag{35}$$

where I_0 = the income generated per unit of system up time.

I_1 = the replacement cost per unit of time.

I_2 = the repair cost per unit of time.

Setting the value of $\beta = 0.9, \theta_j = 0.7, I_0 = 100, I_1 = 30, I_2 = 60$ and varying the values of failure rate α by $0.03, 0.05, 0.07, 0.1$ in Eq. (35) respectively we compute the expression of $E_p(t)$ as follows:

For $\alpha = 0.03$ expected profit equation $E_p(t)$ is:

$$E_p(t) = I_0(0.040256e^{(-0.6800162t)} + 0.0918442e^{(-0.213938t)} - 0.0180096e^{(-0.9083218t)} - 133.4474242e^{(-0.00772338t)} + 133.3333) - [I_1(0.0480642e^{(-0.68001621t)} - 1.1271175e^{(-0.21393854t)} + 0.253965e^{(-0.9083218t)} - 2.8786154e^{(-0.0077233t)} + 3.70370) + I_2(t - 0.040256e^{(-0.68001621t)} - 0.091844e^{(-0.213938t)} + 0.0180096e^{(-0.908321t)} + 133.44742e^{(-0.00772338t)} - 133.33333)] \tag{36}$$

For $\alpha = 0.05$ expected profit equation $E_p(t)$ is:

$$E_p(t) = I_0(0.1973618e^{(-0.6171882t)} - 0.1375840e^{(-0.395662t)} - 81.4935416e^{(-0.0125562t)} - 0.4477176e^{(-0.9245929t)} + 81.481481) - [I_1(0.5544974e^{(-0.6171882t)} - 1.7768965e^{(-0.3956625t)} + 2.9110867e^{(-0.0125562t)} + 0.429785e^{(-0.9245929t)} + 3.70370) + I_2(t - 0.197361e^{(-0.61788212t)} - 0.137584e^{(-0.395662t)} + 81.493541e^{(-0.0125562t)} + 0.4477176e^{(-0.9245929t)} - 81.4814814)] \tag{37}$$

Similarly we can obtain the expression of $E_p(t)$ for $\alpha = 0.07$ and $\alpha = 0.1$. The net expected profit $E_p(t)$ for $\alpha = 0.03, 0.05, 0.07$ and 0.1 is shown in Table 3 for different values of t ranging 0 to 140 .

From the analysis of Table 3 and the corresponding Fig. 6, it is concluded that initially the profit increases up to an extent as the time increase and then it starts decreasing. At time $t = 0$, profit is zero. It is also revealed from Table 3 that for $\alpha = 0.03$ profit is maximum at $t = 130$ and after this it starts decreasing. Moreover, for $\alpha = 0.05$, profit is maximum at $t = 80$ where as for $\alpha = 0.07$ profit is maximum at $t = 60$ and for $\alpha = 0.1$ profit is maximum at $t = 40$. It is obvious from the curves that the system goes in loss after $t = 200, 150$ and 110 for $\alpha = 0.05, 0.07$ and 0.1 respectively.

Table 3 Expected profit with respect to failure rate as function of time t

Mission time (t)	$\alpha = 0.03$			$\alpha = 0.05$			$\alpha = 0.07$			$\alpha = 0.1$		
	Revenue	Total cost	Profit	Revenue	Total cost	Profit	Revenue	Total cost	Profit	Revenue	Total cost	Profit
0	0	0	0	0	0	0	0	0	0	0	0	0
10	982	38.1	943.9	960	57	903	937	74.4	862.6	904	98.1	805.9
20	1899	97.2	1801.8	1809	157.8	1651.2	1724	214.2	1509.8	1610	289.2	1320.8
30	2749	193.2	2555.8	2557	317.1	2239.9	2386	426.9	1959.1	2168	566.4	1601.6
40	3535	326.7	3208.3	3216	528.6	2687.4	2944	700.2	2243.8	2608	911.4	1696.6
50	4264	494.1	3769.9	3798	785.7	3012.3	3414	1025.4	2388.6	2955	1310.7	1644.3
60	4938	693.9	4244.1	4312	1082.7	3229.3	3810	1393.5	2416.5	3230	1751.4	1478.6
70	5562	923.7	4638.3	4764	1416.3	3347.7	4143	1798.8	2344.2	3446	2226.3	1219.7
80	6139	1181.1	4957.9	5164	1780.8	3383.2	4424	2234.4	2189.6	3617	2727.3	889.7
90	6674	1463.7	5210.3	5516	2173.2	3342.8	4661	2695.8	1965.2	3752	3249.3	502.7
100	7169	1769.7	5399.3	5826	2590.5	3235.5	4860	3179.1	1680.9	3859	3787.2	71.8
110	7627	2097.9	5529.1	6100	3029.1	3070.9	5028	3681	1347	3943	4338.6	- 395.6
120	8051	2446.2	5604.8	6342	3486.6	2855.4	5170	4197.9	972.1	4010	4899.9	- 889.9
130	8444	2813.1	5630.9	6555	3960.9	2594.1	5289	4728.3	560.7	4062	5469.9	- 1407.9
140	8807	3197.7	5609.3	6743	4450.2	2292.8	5389	5269.8	119.2	4104	6045.3	- 1941.3

5 Classical estimation

In this section, we consider the classical estimation of the model parameters. Suppose that the failure and replacement time distribution are independently distributed as Exponential.

Let $\tilde{W}_1 = (w_{11}, w_{12}, \dots, w_{1n_1})$ are random sample of size n_1 for failure time and $\tilde{W}_2 = (w_{21}, w_{22}, \dots, w_{2n_2})$ are random sample of size n_2 for replacement time and $Y_i = \sum_{j=1}^2 w_{ij}; i = 1, 2$.

Then the joint likelihood function is

$$L = L(W_1, W_2 | \alpha, \beta) = \alpha^{n_1} e^{-\alpha Y_1} \beta^{n_2} e^{-\beta Y_2}$$

The log-likelihood function is

$$\log L = n_1 \log \alpha - \alpha Y_1 + n_2 \log \beta - \beta Y_2$$

By using maximum likelihood approach, the maximum likelihood estimates of (α, β) are

$$\hat{\alpha} = \frac{n_1}{Y_1}, \quad \hat{\beta} = \frac{n_2}{Y_2}$$

Using large sample theory of M.L.E, the asymptotic sampling distribution of (α, β) is $N_2(0, F^{-1})$ where F is observed Fisher Information matrix with diagonal elements as

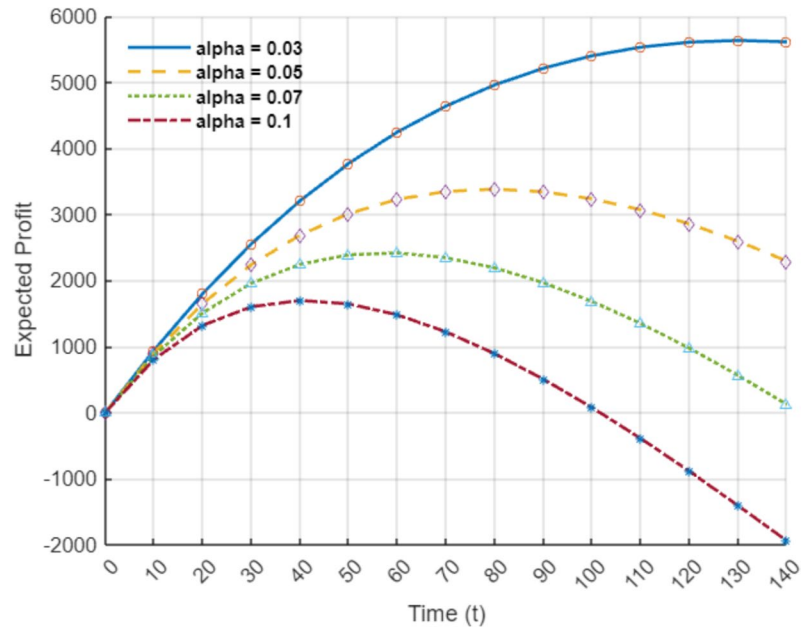
$$F_{11} = E\left(-\frac{\partial^2 \log L}{\partial \alpha^2}\right) = \frac{n_1}{\alpha^2}, \quad F_{22} = E\left(-\frac{\partial^2 \log L}{\partial \beta^2}\right) = \frac{n_2}{\beta^2}$$

and non-diagonal elements as zero. The asymptotic $(1 - \gamma) \times 100\%$ confidence interval for $\Psi = (\alpha, \beta)$ is $\hat{\Psi} + Z_{\gamma/2} \sqrt{V(\hat{\Psi})}$. Here $V(\hat{\Psi})$ is variance of $\hat{\Psi}$ obtained from F and $Z_{\gamma/2}$ is upper $100 \times (\gamma/2)^{th}$ percentile of standard

Table 4 Estimates of MTSF for various values of α and β

α	0.03				0.05				0.07			
	0.02	0.04	0.06	0.08	0.02	0.04	0.06	0.08	0.02	0.04	0.06	0.08
True value	88.889	63.889	55.556	51.389	73.333	48.333	40.000	35.833	66.667	41.667	33.333	29.167
MLE	88.742	63.742	55.401	51.229	73.262	48.263	39.922	35.750	66.626	41.627	33.286	29.114
SE	0.200	0.146	0.133	0.129	0.174	0.108	0.090	0.083	0.166	0.095	0.074	0.066
CI lower	88.350	63.457	55.140	50.977	72.921	48.052	39.744	35.586	66.300	41.441	33.140	28.985
CI upper	89.133	64.028	55.662	51.481	73.603	48.475	40.099	35.913	66.952	41.813	33.431	29.243
Width CI	0.783	0.571	0.522	0.504	0.682	0.423	0.354	0.327	0.652	0.372	0.292	0.258

Fig. 6 The expected profit of the system over time t



normal distribution. The respective asymptotic distribution of $MTSF(M)$ is $N_2(0, M' F^{-1} M)$ where $M' = \left(\frac{\partial M}{\partial \alpha}, \frac{\partial M}{\partial \beta} \right)$.

5.1 Simulation study and result

Now, we shall use the simulation result to discuss the MLE performance of MTSF for the proposed system. We have fixed the sample size $n_1 = n_2 = 100,000$. And generate the samples from exponential distribution for different parameter values and calculate their estimated values. After putting the ML estimates of the parameters in the obtained expression of MTSF (22), we get MLEs of MTSF. All the analysis have been performed using R Software.

Table 4 provide the MLE of MTSF and also their SE and confidence interval for varying values of α and β . From the Table 4 it is observed that as failure rate and replacement rate increases MTSF decreases. Moreover, ML estimates of MTSF are closer to true values.

6 Conclusion

From critical examination of the model, the authors conclude that the system model is highly functional due to the spare wheel as the MTSF decreases when the replacement rate increases. Moreover, as the failure rate increases the MTSF decreases irrespective of the values of replacement rate. It was also determined that profit of the user initially increases up to an extent with the time and start to decrease after a particular time. From the classical estimation ML estimate of MTSF are closer to true value of MTSF. Therefore, the

analyzed results are beneficial for automotive industries to make this system more reliable and more profitable for users.

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Declarations

Conflict of interest The authors have no conflict of interest to declare that are relevant to the content of this article.

Data availability Data sharing not applicable to this article as no dataset were generated or analysed during the current study.

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