Life Cycle Reliability and Safety Engineering (2024) 13:189–206
https://doi.org/10.1007/s41872-024-00255-9

ORIGINAL RESEARCH

A probabilistic model for evaluating the operational safety and operational safety analysis by the maximum likelihood method: application on the cameroonian company of petroleum depot of garoua

Cedric Tatsinkou Fogang[1] · **Wolfgang Nzié**[2] · **Aslain Brisco Ngnassi Djami**[2]

Received: 14 September 2023 / Accepted: 30 April 2024 / Published online: 13 May 2024

Abstract

This article presents an operational safety assessment model and analysis in cases where failure rate and repair rate vary over time, using the maximum likelihood analytical method as a method for estimating reliability parameters and uses Gumbel's graphic paper for the estimation of maintainability parameters. The proposed approach takes into account both reliability and maintainability analyses, unlike current approaches in the literature which perform either reliability or maintainability analyses. A Weibull law for a reliability study has the advantage of being generalized and can also model, under certain conditions, one of the most frequent cases of distribution of technical repair times, namely the logarithmic normal distribution, which lends itself well to comparison with the laws of extreme values. The hypothesis acceptance test used is the Kolmogorov–Smirnov test. The application on the equipment of the Cameroonian Company of Petroleum Depots (CCPD) of Garoua is based on a FMEA analysis and provides parameters and curves after operation of the time between failures and technical times to repair (TTR).

Keywords FMECA · Operational safety · Technical Time to Repair · Time between failures · Weibull

Abbreviations

FMECA	Failure Modes Effets and Criticality Analysis
PRA	Preliminary Risk Analysis
FTA	Fault Tree Analysis
TBF	Time Between Failures
TTR	Technical Times to Repair
CCPD	Cameroonian Company of Petroleum Depots

1 Introduction

The activity of evaluation, whether of a given concept, theory, value, parameter or set of parameters, does not escape the daily lives of men. Sometimes, even unconsciously, we evaluate. What changes is the domain or context. In a family or industrial setting, for example, we can estimate the amount of resources needed or gains in terms of finances, materials, consumables, etc. for a period of time. We will be able to estimate what we need to consume to maintain a good physical fitness for a day or even longer; a head of household will be able to assess what he needs for his children's schooling for one or more years; A civil engineer will be able to estimate the amount of money needed to construct a building, or what he will be able to earn after the construction of a building, and/or what he has earned after the construction of said building. As for the mechanical engineering designer, he will be able to evaluate, as in the latter case, the costs related to the design of a piece of equipment or a production system, but also several other parameters such as time, quality and operational safety, which are constraints that should not be neglected nowadays with the

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✉ Cedric Tatsinkou Fogang
tatsinkoucedric@gmail.com

Wolfgang Nzié
wnzie@yahoo.fr

Aslain Brisco Ngnassi Djami
ngnassbris@yahoo.fr

[1] Department of Mechanical Engineering, National Advanced School of Agro - Industrial Sciences, University of Ngaoundere, Ngaoundere, Cameroon

[2] Department of Fundamental Sciences and Techniques of Enginner, School of Chemical Engeneer and Mineral Industry, University of Ngaoundere, Ngaoundere, Cameroon

competitiveness of the global market and the complexity of the systems designed due to the integration of various technologies (Guillerm 2011). It is in this context that the notion of robustness takes place. This is because a robust production system must be able to cope with disruptions in order to maintain its performance at a high level (Ntricker and Lanza 2014). However, its disruptions take place throughout the life cycle of the system, and therefore both when it is in operation and when it is down (the quality of maintenance has an influence on the lifespan of a system). For this reason, the operational safety can be seen as an indicator or parameter to validate the robustness of a system.

Basically, there are three main concepts that help to understand the concept of operational safety (Avizienis et al. 2000). These are the attributes of operational safety, the impediments to operational safety, and the means by which operational safety is achieved. Operational safety attributes refer to the quantifiable, evaluable properties that characterize the performance of the system. These are reliability, maintainability, availability, and security. Reliability refers to continuity of service, i.e., “the ability of an entity to perform a required function, under given conditions, during a given period of time” (NF X 60–500). Maintainability refers to the ability to be repaired and evolved. It is “the ability of an entity to be maintained or restored, within a given period of time, to a state in which it can perform a required function, when maintenance is performed under given conditions, with prescribed procedures and means” (NF X 60–500). Availability is “the ability of a good to perform a required function under given conditions, at a given time during a given time interval, assuming that the supply of external needs is assured” (Nzié 2006). Safety is defined for a machine as its ability to perform a function without harming the health and integrity of the operator (Nzié 2006). Obstacles to operational safety refer to undesirable and unexpected causes or results of operation (Ciame 2009). It can be a failure, a mistake or a mistake (Avizienis et al. 2000). Failure is the “cessation of an entity’s ability to perform a required function” (DIN EN 13306 2010); Fault is the supposed cause of an error (Villemeur 1991); The error is the part of a system’s state that can cause a failure (Laprie 1995). As far as the means of operational safety are concerned, they refer to all the techniques used in order to avoid the presence of faults and thus avoid potential failures. It can be measures to prevent, tolerate, eliminate or predict faults (Avizienis et al. 2000). Fault prevention refers to actions aimed at preventing the occurrence or introduction of faults; The purpose of the fault tolerance action is to give the system the ability to deliver an acceptable service, even in the presence of faults; the elimination of faults refers to the reduction of the presence of faults; Defect prediction refers to the estimation of the presence, creation, and consequences of defects. Although operational safety is considered to be the science of failures (Villemeur 1991; Zwingelstein 1995), It can also be understood on the one hand as the ability of a system to have its functional performance (reliability, maintainability, availability) and not to generate risk (security), and on the other hand as the set of activities for assessing the suitability of a system. As a result, the assessment of operational safety involves the estimation of its attributes of reliability, maintainability, availability and safety.

A number of recent studies relating to operational safety studies of identifiable systems can be mentioned. Among them, we have the work related to computer networking systems such as, modeling the work reliability of a computer system with priority to hardware repair over hardware replacement, and upgrading software subject to technical control and MRT from the semi-Markov regenerative point technique (Kumar and Malik 2014); Availability study and analysis of power systems in the sugar industry based on variables under different types of failure and general repair (Kadyan and Kumar 2015); Analysis of the reliability and performance of an industrial system on the basis of a free warranty policy under a period of Niwas and Garg (2018). For the studies of the systems based on the copula approach (joint probability distribution), we can mention, the analysis of the availability, the MTTF (Mean time to Failure) and the costs of a system with two units in a series configuration with the failure of the controller and the man with different types of failure and two types of repair of the Gumbel-Hougaard family (Singh et al. 2013); the study of the cost evaluation based on the performance of a system composed of two subsystems in series Variable and priority additional repair of the first defective unit (Lado et al. 2018); the study of some reliability characteristics of a 2 out of 4 consecutive linear system connected to a support device for operation using the Kolmogorov method (Yusuf et al. 2018); Analysis of the network environment using a reliability approach using the Markov process that converts the Markov process to a non-Markov process (Kuldeep et al. 2017); the study of a complex repairable system composed of two subsystems in serial configurations under k-out-of-n: G/F diagram with different types of failure and two types of repair using Gumbel-Hougaard (Gahlot et al. 2020). reliability analysis of distributed hardware-software systems (Vijayalakshmi 2015); Fault tolerance study (Sari and Akkaya 2015); the study of reliability measurements, sensitivity analysis of a coal handling unit for a thermal power plant consisting of two subsystems in series configuration using additional variable techniques (Kumar and Ram 2013); Analysis of a system comprising two subsystems in serial configuration with different types of failures and copula repair approach (Abdul and Singh 2019); Prediction of the reliability of a distributed system with homogeneity in the software and the server using a joint probability distribution via the copula approach (Raghav et al. 2021).

From the above, we can again notice a strong mark of the attributes of dependability. However, the evaluation of its parameters often involves the use of probability distributions. Discrete probability distributions (binomial distribution and Poisson distribution) are used to quantify the failure under stress (Lyonnet 2006), that occurs when an entity refuses to change state when asked to do so (Villemeur 1991). Continuous probability distributions (Weibull's distribution, exponential distribution, normal distribution, lognormal distribution, Gamma's distribution, and Gumbel's distribution) are associated with continuous random variables, and are used to quantify the duration of a feature's operation (Villemeur 1991). However, the assumption made for the statistical distribution of technical repair times and operating times assumes that there is an estimator for each of the parameters. The hypothesis of invariability of the repair rate imposes a law of exponential maintainability, which is not, however, the most frequent in practice (Dhillon 2006). In addition, the lognormal distribution, which is the most common (Bellaouar 2014; Morice 1968), can be modeled by a two-parameter Weibull distribution. The two-parameter Weibull reliability distribution is also modelled, using the maximum likelihood method (Perreault and Bobee 1992; Lannoy 1994), which consists in establishing an analytic function called likelihood, based on the probability density function, whose variables are the parameters that we want to estimate, so that the maximum of said function is obtained at the estimated estimators. The maximum likelihood method has the advantage of providing an unbiased, efficient, consistent, and robust estimator (Lloyd and Lipow 1962). Two distributions allow a study of maintainability according to the size *N* of the sample. These are the lognormal distribution for large samples and the distribution of extreme values (LVE), including Gumbel, for small samples. This paper proposes a model for assessing operational safety and analysis in the case where operation and repair times are variable over time, using the maximum likelihood analytical method and the Gumbel graphical method as parameter estimation methods.

2 Methods

Safety analyses are important to ensure proper design and monitoring of production systems. The probabilistic approach is an effective way to account for uncertainties, and has so far been addressed for reliability, maintainability and safety studies. We therefore propose in the following paragraphs, a model to consider them together.

2.1 Reliability and maintenability: operational safety fundamental parameters

The reliability function over a period of time t is the difference between the cumulative distribution function where $t \to \infty$ and the cumulative distribution function in the period of time t or, alternately, it is the subtraction of the cumulative distribution function of failure over a period of time t from unity.

$$R(t) = P\left(T_f > t\right) = 1 - F(t) \quad (1)$$

The hazard rate function is a representation of the failure rate pattern of the ratio between a particular probability density function (p. d. f.), and its cumulative distribution function (c. d. f.) or its reliability function. For continuous random variables, the cumulative distribution function is defined by

$$F(t) = \int_0^t f(t)dt \quad (2)$$

where: f (t) = probability density function of the distribution of value t over the interval 0 to t. The hazard rate function is then defined as

$$\lambda(t) = \frac{f(t)}{R(t)} \quad (3)$$

Thus the reliability function is defined by:

$$R(t) = e^{-\int_0^t \lambda(t)dt} \quad (4)$$

The maintainability function $M(t)$, for any distribution, is expressed by the following relationship

$$M(t) = P\left(T_r < t\right) = \int_0^t g(t)dt \quad (5)$$

g(t) is the probability density function of the maintenance (repair) time. Obviously maintainability is as a cumulative distribution function as the complementary of reliability F(t). Thus the maintainability function is defined also by the following expression, thanks to (1) and (4):

$$M(t) = 1 - e^{-\int_0^t \mu(t)dt} \quad (6)$$

The repair rate function is analogically defined by:

$$\mu(t) = \frac{g(t)}{1 - M(t)} \quad (7)$$

Given that many workplace accidents occur during maintenance operations or as a result of a reliability problem (Nzié 2006), guaranteeing the security of equipment in operation implies taking the necessary measures to ensure reliability and maintainability (Nzié et al. 2017).

Table 1 FMEA centrifugal pump

Material: Centrifugal pump		Failure characteristic								Result of the study
Organ	Function	Failure mode	Cause of failure	Effect of default	Detection	Criticality				Action to take
						F	G	N	C	
Braid trim	Ensure the tightness of the pump	Leak	Exceeded lifetime	Poor seal drop in pump flow	Visual	2	3	2	12	Replace packings
Safety valve	Limits the pressure in a hydraulic circuit	Stuck	Incorrect adjustment of the calibration screw	Noisy operation, Bottom safety valve	Visual	2	1	2	4	Correctly set the calibration screw
Purge valve	Allows the pump to be purged	Blocked	Bad assembly	The pump does not purge	Visual	2	1	2	4	Unclog and correctly install the bleed valve
Pressure probes	Indicates the value of the pressure in the circuit	Provides no indication	Lifetime exceeded on	No indication of suction and discharge pressures	Visual	1	2	2	4	Replace the pressure probe
		Incorrect indications	Accidental		Visual	1	2	2	4	
Valves of the pump system	Ensure the opening and closing of the circuit	Does not open or close	Seizure/Corrosion	No flow at the outlet of the pump	Visual	2	3	3	18	Lubricate the pump with an appropriate grease
Check valve	Ensure fluid flow in one direction only	Stay open	Stuck	Pump depriming	Visual	3	3	2	18	Loosen the valve
Cable gland	Seal the pump	High leakage	Defective seal	Lack of tightness	Visual	3	2	3	18	Replace seals
			Incorrect tightening of the stuffing box packing follower		Visual	2	3	3	18	Correctly tighten the stuffing box gland
Packing follower	Seal the pump	Loose	Bad assembly	Very large leak in the stuffing box	Visual	2	3	3	18	Correctly fit the stuffing box packing follower

Material: Centrifugal pump electric motor		Failure characteristic								Result of the study
Organ	Function	Failure mode	Cause of failure	Effect of default	Detection	Criticality				Action to take
						F	G	N	C	
Bearings	Rotating guide	Large clearance or breakage	Natural wear	Performance losses	Noise and heating	1	3	2	6	Replace
			Bad lubrication			1	3	2	6	Replace
Rotor	Rotating part	Breaking the windings	Wear	Performance losses	Visual	1	3	3	9	Correct the fault
Stator	Static part	Breaking of the windings or Poorly insulated windings	Wear	Performance losses	Visual	1	3	3	9	Correct the fault
Motor contactor	Power switch	Does not make contact between terminals	Wear	The motor is not powered	Visual	1	3	3	9	Replace contactor
Fuses and circuit breakers	Protects against short circuit and overload	Instantly melts or skips on startup	Short circuit in the motor	The motor is not powered	Visual	3	2	2	12	Eliminate the short circuit
			Inappropriate caliber		Visual					Properly calibrate the protective device

Table 1 (continued)

Material: Centrifugal pump electric motor		Failure characteristic								Result of the study
Organ	Function	Failure mode	Cause of failure	Effect of default	Detection	Criticality				Action to take
						F	G	N	C	
Junction box	Provides food	The engine starts hard	Bad coupling	No training	Visual	1	2	2	4	Correct connection
Fan	Provide cooling	Insufficient ventilation	Clogged ventilation corridors	No cooling	Visual	1	3	1	3	Clear the ventilation corridors
Material: Lubrication circuit		**Failure characteristic**								**Result of the study**
Organ	**Function**	**Failure mode**	**Cause of failure**	**Effect of default**	**Detection**	**Criticality**				**Action to take**
						F	**G**	**N**	**C**	
Oil pan	Store the oil	Leak/pierced	Material aging	Oil loss, engine overheating and wear of parts	Visual	1	4	2	8	Repair the leak by welding or waterproof gluing
Oil pump	Increase oil pressure	Don't aspire	No oil in the crankcase	Heating and wear of parts	Visual	2	4	4	32	Fill the crankcase with oil
			Is not primed							Prime the pump
Pressure control valve	Check oil pressure	Abnormal increase in pressure	Defective control valve	Pipe burst	Visual on display board	2	3	3	12	Replace pressure control valve
Oil filter	Retain oil impurities	Allow impurities to pass	Torn filter cartridge	Component heating	Visual on bulletin board	3	3	2	18	Replace filter cartridge
		Does not filter	Clogged filter	Pressure increase						Clean the filter cartridge
Crankshaft bearings	Guide the crankshaft in rotation	Abnormal noise	Worn bearing	Crankshaft damage	Audible	1	4	4	16	Replace bearings
Circulation piping	Channel the oil circulation	Leak/pierced	Material aging	Oil loss and poor lubrication	Visual	2	4	2	16	Repair by waterproof welding of perforated surfaces
Bearings Camshaft	Guide the camshaft in rotation	Abnormal noise	Worn shaft bearing	Camshaft misalignment and cam wear	Audible	2	4	3	24	Bearing replacement and lubrication
Connecting rod bearings	Guide the connecting rod in rotation	Abnormal noise	Worn connecting rod bearing	Connecting rod wear and engine vibration	Audible	2	4	3	24	Bearing replacement and lubrication
Oil tappet to rocker arm	Pushes the oil to the rocker arm	Does not squirt oil	Clogged rocker arm rod	Overheating of the rocker arms	Sensitive	1	3	3	9	Unclog the push rods
Rocker arm bearings	Guide the rocker arm in rotation	Abnormal noise	Worn rocker arm bearing	Rocker arm wear and vibration of nearby components	Audible	2	4	3	24	Bearing replacement and lubrication

In this case, security can be expressed as follows (Ciame 2009):

$$\begin{aligned} S\left(t_f, t_r\right) &= P\left[\left(T_f > t_f\right) \cup \left(T_r < t_r\right)\right] = R\left(t_f\right) + M\left(t_r\right) - R\left(t_f\right)M\left(t_r\right) \\ &= e^{-\int_0^{t_f} \lambda(t)dt} + 1 - e^{-\int_0^{t_r} \mu(t)dt} - e^{-\int_0^{t_f} \lambda(t)dt}\left(1 - e^{-\int_0^{t_r} \mu(t)dt}\right) \end{aligned} \tag{8}$$

$$S\left(t_f, t_r\right) = 1 + e^{-\int_0^{t_r} \mu(t)dt}\left(e^{-\int_0^{t_f} \lambda(t)dt} - 1\right) \tag{9}$$

t_f and t_r are respectively the variables operational and repair time of the production system

2.1.1 Probabilistic operational safety assessment model

Let us name $OpS(t_f, t_r)$ the operational safety function. From the equations of its fundamental parameters, reliability and maintainability given in (4) and (6), and considering the expression of safety given to Eq. (8) we express it as follows:

$$\begin{aligned} OpS\left(t_f, t_r\right) =& P\left[\left(T_f > t_f\right) \cup \left(T_r < t_r\right) \cup \left(T_f > t_f\right) \cup \left(T_r < t_r\right)\right] \\ &= 1 - \left[\left(R\left(t_f\right) - 1\right)\left(1 - M\left(t_r\right)\right)\right]^2 \end{aligned} \tag{10}$$

Thus is deduced after some transformations the mathematical expression of operational safety.

$$OpS\left(t_f, t_r\right) = 1 - \left[(e^{-\int_0^{t_f} \lambda(t)dt} - 1)e^{-\int_0^{t_r} \mu(t)dt}\right]^2 \tag{11}$$

By analogy with Eqs. (3) and (7), the risk rate function can be deduced, after calculating the probability density of the functional safety function, which also makes it possible to calculate the function of the confidence rate.

Let us name now $l\left(t_f, t_r\right)$, $\varphi\left(t_f, t_r\right)$ and $\psi(t_f, t_r)$, the functions of probability density, risk rate and confidence rate, respectively. We can express them as follows

$$l\left(t_f, t_r\right) = \frac{\partial^2 OpS\left(t_f, t_r\right)}{\partial t_f \partial t_r} = 4f\left(t_f\right)\left[R\left(t_f\right) - 1\right]g\left(t_r\right)\left[1 - M\left(t_r\right)\right] \tag{12}$$

$$\varphi\left(t_f, t_r\right) = \frac{l\left(t_f, t_r\right)}{OpS\left(t_f, t_r\right)} \tag{13}$$

$$\psi\left(t_f, t_r\right) = 1 - \varphi\left(t_f, t_r\right) \tag{14}$$

3 Operational safety analysis by the maximum likelihood method

3.1 Maximum likelihood method

Either to estimate a parameter θ of a statistical distribution law on the basis of observations $t_1, t_2, ..., t_n$ of the random variable time. The likelihood $L(t_1, t_2, ..., t_n;\theta)$ is defined as the probability of observing the durations $t_1, t_2, ..., t_n$ for a given value of the parameter θ. The maximum likelihood method consists in taking as estimator of the parameter θ, the value of θ which maximizes the likelihood.

For a reliability analysis, the likelihood function is given by the following expression (Kumar and Malik 2014):

$$L\left(t_1, t_2, ..., t_n;\theta\right) = \prod_i^n f(t_i, \theta) \tag{15}$$

With $t_1, t_2, ..., t_n$: observed failure times; $\theta = (\theta_1, \theta_2, .., \theta_r)$: the vector of parameters sought; $f(t_i, \theta)$: the failure probability density function.

If $L(t, \theta)$ is differentiable and if the maximum of the likelihood function exists at $\widehat{\theta}$, then it satisfies the following equation:

$$\left.\frac{\partial L\left(t_1, ..., t_n;\theta_1, \ldots, \theta_r\right)}{\partial \theta_i}\right|_{\theta_i = \widehat{\theta}_i} = 0 \tag{16}$$

Or in logarithmic form by the following equation:

$$\left.\frac{\partial ln(L\left(t_1, ..., t_n;\theta_1, \ldots, \theta_r\right))}{\partial \theta_i}\right|_{\theta_i = \widehat{\theta}_i} = 0 \tag{17}$$

With $i = 1, ..., r$

3.2 Probabilistic evaluation model

For a variable failure rate, the following relation may describe Weibull's law for the reliability function (Dhillon 2006):

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\theta}} \tag{18}$$

With θ, the shape parameter and α, the scale parameter.

The maintainability function for the law of extreme values is expressed by:

$$M(t) = e^{-e^{-a(t-u)}} \tag{19}$$

With a, the inverse of the slope and u, the location parameter read from the Gumbel graph paper.

Thus, the operational safety expression given to Eq. (10) takes the following form:

$$OpS\left(t_f, t_r\right) = 1 - \left[\left(e^{-\left(\frac{t}{\alpha}\right)^{\theta}} - 1\right)\left(1 - e^{-e^{-a\left(t_r - u\right)}}\right)\right]^2 \tag{20}$$

Table 2 FMEA of the lubrication circuit

Material: Oil lubrication circuit		Failure characteristic								Result of the study
Organ	Function	Failure mode	Cause of failure	Effect of default	Detection	Criticality				Action to take
						F	G	N	C	
Diesel tank	Storing diesel	Leaking/drilled	Material aging	Circuit not supplied, diesel pump cavitation	Visual	1	4	2	8	Repair the leak by welding or gluing
Feed pump	Pumping diesel under pressure	Don't aspire	No fuel	No fuel	Visual	3	4	2	24	Refuel
			Is not primed							Prime the pump
Diesel filter	Retain diesel impurities	Does not filter	Blocked	Does not filter	Visual	3	4	2	24	Clean the filter cartridge
			Torn filter cartridge	Leave the impurities						Replace filter cartridge
Injection pump	Inject diesel fuel into the rocker arms	Don't aspire	No fuel	No power to rocker arms	Visual	3	4	2	24	Refuel
			Is not primed		Visual					Prime the pump
Injection valves	Allow diesel injection into the chamber	Blocked	presence of impurities in the circuit	Bad combustion	Visual	2	4	3	24	Clean to unclog the injection valve
Excess diesel valves	Allow diesel return to the tank	Blocked	Presence of impurities in the return circuit	Saturation of the diesel return pipe	Visual	1	3	2	6	Clean to unclog the valve
Diesel circulation piping	Allow the circulation of diesel fuel	Leak	Material aging	Fuel loss	Visual	2	4	2	16	Repair the leak by welding or gluing
		Blocked	Excessive presence of impurities in the circuit	Bursting of pipes	Visual					Unclog the pipe
Glow plugs	Preheat diesel	Does not preheat diesel	Defective spark plugs	No or poor combustion	Visual	3	3	3	27	Replace glow plugs
Oil pressure sensor	Convert pressure to electrical signal	Absence or bad conversion	Exceeded lifetime	Display of incorrect values	Visual	2	3	3	18	Replace the oil pressure sensor
Injection pump	Inject the oil into the circuit	Don't aspire	Absence of oil in the circuit	Circuit heating	Visual	2	4	3	24	Check the presence of oil in the circuit
Acceleration Gear	Allow speed increase	Seizing up and overheating	Lack of lubrication	Tooth wear and breakage	Visual	1	4	3	12	Ensure proper oil lubrication of the gears
Piston shaft sleeve	Protects piston and cylinder from wear	Wear	Exceeded lifetime	Piston damage	Visual	1	3	3	9	Replace with a new shirt

Table 3 FMEA of the air cooling circuit

Material: Air cooling circuit		Failure characteristic								Result of the study
Organ	Function	Failure mode	Cause of failure	Effect of default	Detection	Criticality				Action to take
						F	G	N	C	
Engine oil pipe-exhaust thermostat	Channel the oil circulation	Leak/pierced	Material aging	Absence or poor cooling, oil contamination	Visual	1	4	3	12	Repair by waterproof welding of perforated surfaces
Exhaust thermostat air piping	Channel airflow	Leak/pierced	Material aging	Absence or poor cooling	Visual	1	4	3	12	Repair by waterproof welding of perforated surfaces
Exhaust thermostat	Controls and regulates outlet temperature	Does not regulate temperature	Faulty thermostat	Temperature rise	Visual	2	3	2	12	Check and replace thermostat
Fan	Blows air to cool the motor	Don't spin	Broken drive belt	Abnormal motor overheating	Visual	2	4	1	8	Replace the broken belt with a new one of the same characteristics

3.3 Application of the maximum likelihood method to weibull's law of operational safety

Relation (15) assumes knowledge of the probability density function, which is defined as the time derivative of the dependability function. Starting from relation (12) we can deduce the Weibull operational safety law given in relation (21) below:

$$l\left(t_f, t_r\right) = 4\left[-\frac{\theta}{\alpha}\left(\frac{t_f}{\alpha}\right)^{\theta-1} e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\left(e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}} - 1\right)\right] * \left[ae^{-a\left(t_r-u\right)}e^{-e^{-a\left(t_r-u\right)}}\left(1 - e^{-e^{-a\left(t_r-u\right)}}\right)\right] \tag{21}$$

The application of relation (15) for the likelihood on the probability density function given in relation (21) leads to the following relation:

$$L\left(t_1, t_2, \dots, t_n;\theta\right) = \prod\nolimits_{i=1}^{n} l\left(t_i, \theta, \alpha\right) = \prod\nolimits_{i=1}^{n} l\left(t_{fi}, t_{ri}, \theta, \alpha\right)$$

$$= (4)^n \prod\nolimits_{i=1}^{n}\left[-\frac{\theta}{\alpha}\left(\frac{t_f}{\alpha}\right)^{\theta-1} e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\left(1 - e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\right)\right] *$$

$$\prod\nolimits_{i=1}^{n}\left[ae^{-a\left(t_r-u\right)}e^{-e^{-a\left(t_r-u\right)}}\left(1 - e^{-e^{-a\left(t_r-u\right)}}\right)\right] \tag{22}$$

By introducing the natural logarithm in each member of the relation (22), we obtain the following relation:

$$\begin{aligned} lnL\left(t_1, t_2, \dots, t_n;\theta\right) &= nln4 + nln\theta - nln\alpha \\ &+ (\theta - 1)\sum_{i=1}^{n} ln\left(\frac{t_{fi}}{\alpha}\right) + \sum_{i=1}^{n} ln\left(1 - e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\right) \\ &+ nlna + \sum_{i=1}^{n}\left[-a\left(t_{ri} - u\right)\right] - \sum_{i=1}^{n}\left[-e^{-a\left(t_{ri}-u\right)}\right] \\ &+ \sum_{i=1}^{n} ln\left(1 - e^{-e^{-a\left(t_r-u\right)}}\right) \end{aligned} \tag{23}$$

By performing the member-to-member partial derivatives of expression (23) with respect to the scale parameter α and the shape parameter θ, we successively obtain the following relations:

$$\frac{\partial lnL}{\partial \alpha} = -\frac{n\theta}{\alpha} + \sum_{i=1}^{n}\frac{\theta}{(\alpha)^{\theta+1}}\left(t_{fi}\right)^{\theta} - \sum_{i=1}^{n}\frac{\left(\frac{\theta}{\alpha}\right)\left(\frac{t_{fi}}{\alpha}\right)e^{-\left(\frac{t_{fi}}{\alpha}\right)^{\theta}}}{\left(1 - e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\right)} \tag{24}$$

Table 4 FMECA of diesel engine auxiliary elements

Material: Auxiliary elements of the diesel engine		Failure characteristic								Result of the study
Organ	Function	Failure mode	Cause of failure	Effect of default	Detection	Criticality				Action to take
						F	G	N	C	
Battery	Provide electrical power for engine starting	The engine will not start	Uncharged battery	Engine unavailability	Visual	3	4	2	24	Charge the battery
Air filter	Filters combustion air	Presence of impurities in the combustion air	Torn filter cartridge	Bad combustion	Visual (combustion smoke)	3	4	2	24	Replace filter cartridge
		Does not filter	Clogged filter	Incomplete combustion						Blow out the filter cartridge with pressurized air
Control panel	Displays engine operating parameters	The indicators do not evolve	Short circuit, break in the electrical circuit	Poor appreciation of operating parameters	Visual	3	4	2	24	Investigate the cause of the short circuit and repair it
			Sensor failure							Replace sensors
Alternators	Recharges the discharged battery	Does not charge the battery	Broken alternator belt	The engine will not start	Visual	2	4	2	16	Fit a new belt
			Broken output cables		Visual					Connect the output cables
Starter	Allows the engine to start	Do not start the engine	Faulty starter	The motor does not turn	Visual	2	4	3	24	Repair or replace starter
Belt	Transmits power between moving organs	Cut off	Exceeded lifetime	Power not transmitted	Visual	3	3	2	18	Replace belt
Temperature sensing	Translates a physical quantity into an electrical signal	Defective	Operates in an unsuitable environment (high physical magnitude range)	Incorrect transmitted signal	Visual	2	3	3	18	Check then replace the sensor

Fig. 1 View of fire pump unit No. 1 and 3

Fig. 2 View of fire pump unit No. 2

$$\frac{\partial lnL}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n} ln\left(\frac{t_{fi}}{\alpha}\right) - \sum_{i=1}^{n} \left(\frac{t_{fi}}{\alpha}\right)^{\theta} ln\left(\frac{t_{fi}}{\alpha}\right) - \sum_{i=1}^{n} \frac{\left(\frac{t_{fi}}{\alpha}\right) ln\left(\frac{t_{fi}}{\alpha}\right) e^{-\left(\frac{t_{fi}}{\alpha}\right)^{\theta}}}{\left(1 - e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\right)} \tag{25}$$

The estimators of the scale parameters α and form θ are those, which maximize the likelihood function in its form in relation (23). This means that they are the solutions of the system of equations below:

$$\begin{cases} \frac{\partial lnL}{\partial \alpha} = 0 \\ \frac{\partial lnL}{\partial \theta} = 0 \end{cases} \tag{26}$$

By carrying the expressions (24) and (25) in the system of Eqs. (26), the latter takes the following form:

$$\begin{cases} -\frac{n\theta}{\alpha} + \sum_{i=1}^{n} \frac{\theta}{(\alpha)^{\theta+1}} (t_{fi})^{\theta} - \sum_{i=1}^{n} \frac{\left(\frac{\theta}{\alpha}\right)\left(\frac{t_{fi}}{\alpha}\right) e^{-\left(\frac{t_{fi}}{\alpha}\right)^{\theta}}}{\left(1-e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\right)} = 0 \\ \frac{n}{\theta} + \sum_{i=1}^{n} ln\left(\frac{t_{fi}}{\alpha}\right) - \sum_{i=1}^{n} \left(\frac{t_{fi}}{\alpha}\right)^{\theta} ln\left(\frac{t_{fi}}{\alpha}\right) - \sum_{i=1}^{n} \frac{\left(\frac{t_{fi}}{\alpha}\right) ln\left(\frac{t_{fi}}{\alpha}\right) e^{-\left(\frac{t_{fi}}{\alpha}\right)^{\theta}}}{\left(1-e^{-\left(\frac{t_f}{\alpha}\right)^{\theta}}\right)} = 0 \end{cases} \tag{27}$$

The system of Eqs. (27) can be solved by successive approximations following indications with starting values (Lannoy 1994; Lloyd and Lipow 1962).

Table 5 TBF and TTR readings for each module

TBF/FPU No. 1 (hours)	TTR/FPU No. 1 (hours)	TBF/FPU No. 2 (hours)	TTR/FPU No. 2 (hours)	TBF/FPU No.3 (hours)	TTR/ FPU No. 3 (hours)
43.5	2.1	27.3	1.4	19.5	1.20
27.2	1.15	39.7	1.15	31.2	0.5
98.3	2.5	18.2	2	78.3	3.1
21.1	0.8	57.7	0.5	51.1	1.25
14.1	5.5	96.3	0.6	94.1	1
19.4	1.75	88.4	1	19.4	1.1
28.8	0.5	123.9	0.5	26.8	2
25.9	0.5	75.3	1.5	103.1	1.25
21.4	1.5	83.8	7	8.4	1
12.7	1.2	64.7	0.9	48.1	0.7
29.3	0.6	71.7	1.5	72.9	1.2
56.7	0.75	25.7	0.5	36.2	3.6
33.5	1	42.8	0.7	17.9	1.4
64.4	2	56.4	1.4	45.6	2.4

Table 6 Different parameters for each FPU

Fire pump unit (FPU)	Parameters			
	α	θ	a	u
FPU No. 1	35.344429	1.312945	0.6833333	1
FPU No. 2	62.10244	1.64276	0.725	0.825
FPU No. 3	46.695112	1.233374	0.79333333	1.07

4 Results and discussion: application to the equipment of the CCPD in garoua

The FMEA studies of the equipment of the CCPD of Garoua given in Tables 1, 2, 3 and 4 show that the feed pump, the fuel and oil filter, the injection pump, all the bearings of the diesel and gasoline engines have criticalities greater than 12. They are therefore considered critical according to the company.

Also, the criticality values observed in the FMEA tables allow us to note that diesel and gasoline engine assemblies are critical equipment for the fire protection room; pumps and its various auxiliaries in general are critical equipment for the pump; and the diesel engine of the emergency generator is the critical equipment of this room. Figures 1 and 2 give a view of the different fire pump unit.

Table 5 provides information on the time between failures and technical times to repair for its equipment over a two-year period.

Table 7 Kolmogorov–Smirnov test for FPG No. 1

i	Ordered TBFs	$F(t) = \frac{i-0.3}{N+0.4}$	$R(t) = e^{-(\frac{t}{\alpha})^{\theta}}$	$F_r(t) = 1 - R(t)$	$\lvert F_r(t) - F_{th}(t) \rvert$
1	12.7	0.048611	0.941387	0.058613	0.010002
2	14.1	0.118056	0.921209	0.078791	0.03926
3	19.4	0.1875	0.811289	0.188711	0.001211
4	21.1	0.256944	0.765274	0.234726	0.02222
5	21.4	0.326389	0.756672	0.243328	0.08306
6	25.9	0.395833	0.613932	0.386068	0.00976
7	27.3	0.465278	0.565933	0.434067	0.03121
8	28.8	0.534722	0.5138	0.4862	0.04852
9	29.3	0.604167	0.496396	0.503604	0.10056
10	33.5	0.673611	0.354434	0.645566	0.02805
11	43.5	0.743056	0.10746	0.89254	0.149484
12	56.7	0.8125	0.007822	0.992178	0.179678
13	64.4	0.881944	0.000871	0.999129	0.117185
14	98.3	0.951389	2.68E-11	1	0.048611

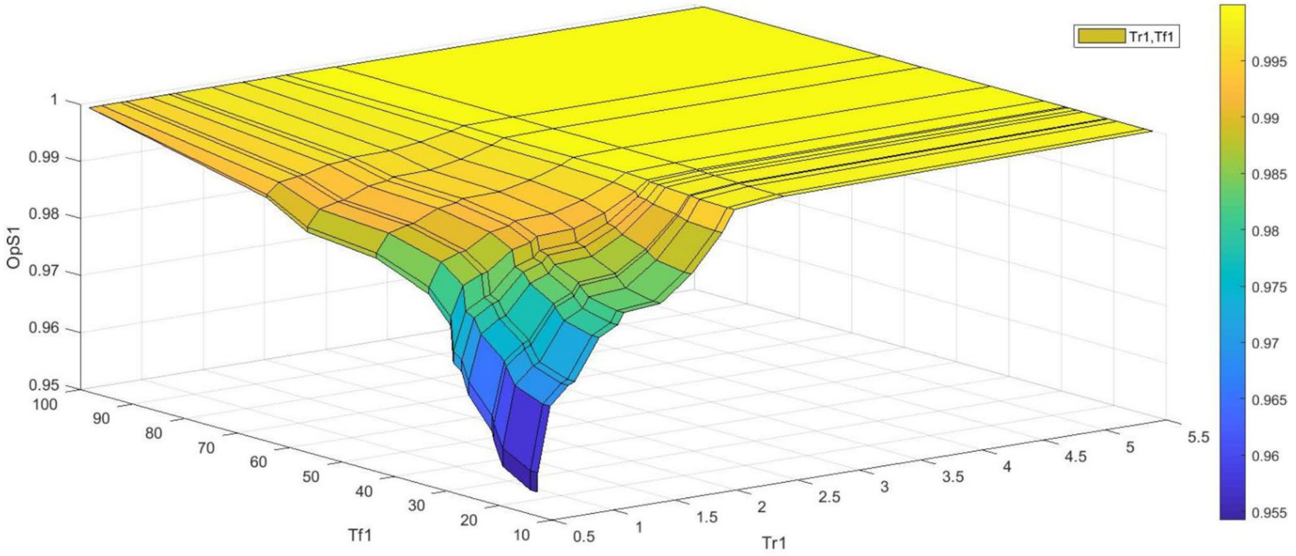

Fig. 3 Operational safety curve for FPU No. 1

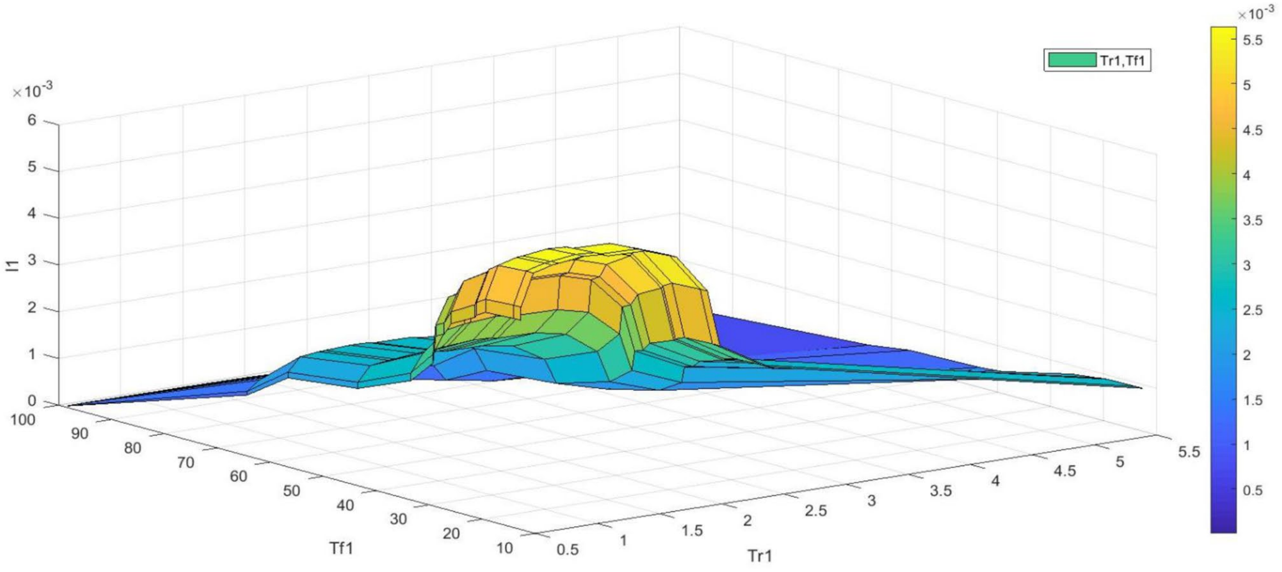

Fig. 4 Probability density curve for FPU No. 1

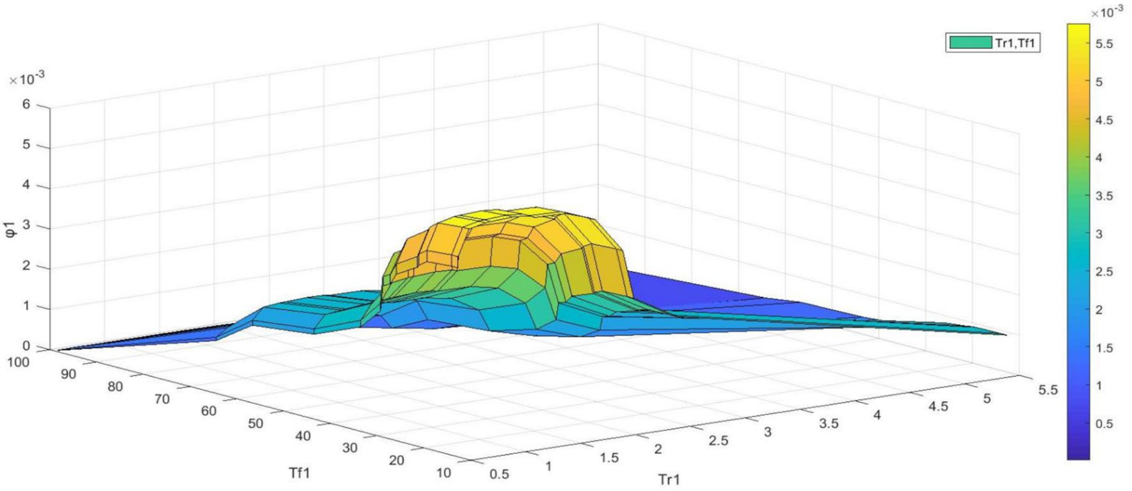

Fig. 5 Risk rate curve for FPU No. 1

The following expression is used to estimate the initial values of θ (Niwas and Garg 2018). The mean of each sample considered as the initial value for α.

$$\frac{\sum t_i}{n} = \left(\frac{\sum {t_i}^{\theta}}{n}\right)^{\frac{1}{\theta}} * \Gamma\left(1 + \frac{1}{\theta}\right) \tag{28}$$

For the fire pump unit No.1, we have:

$$\frac{\sum t_i}{n} = 35.45 \tag{29}$$

For $\theta = 1.2$ we have:

$$\left(\frac{\sum {t_i}^{\theta}}{n}\right)^{\frac{1}{\theta}} = 36.70261 \tag{30}$$

$$\Gamma\left(1 + \frac{1}{\theta}\right) = 0.9664 \tag{31}$$

And,

$$\left(\frac{\sum {t_i}^{\theta}}{n}\right)^{\frac{1}{\theta}} * \Gamma\left(1 + \frac{1}{\theta}\right) = 35.4694 \tag{32}$$

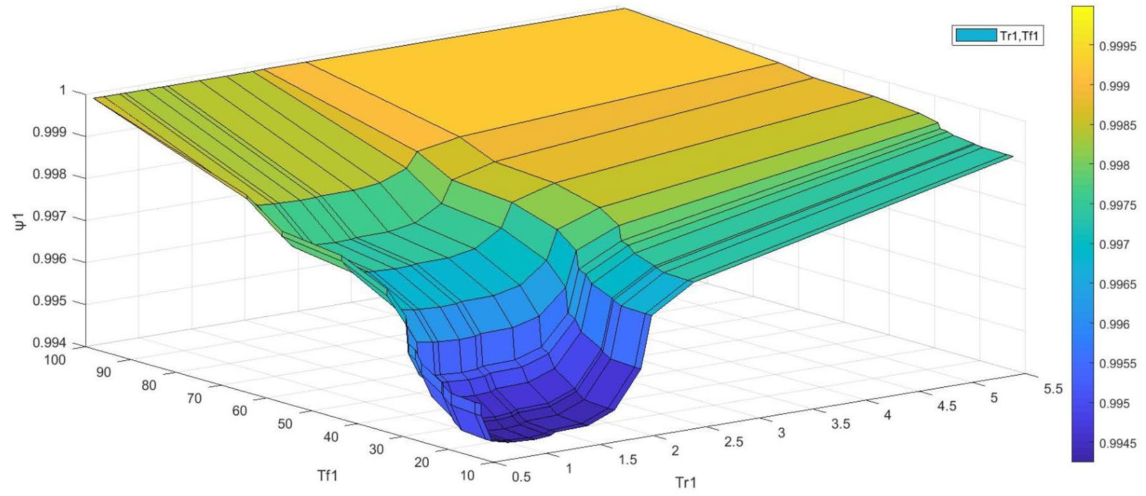

Fig. 6 Confidence rate curve for FPU No. 1

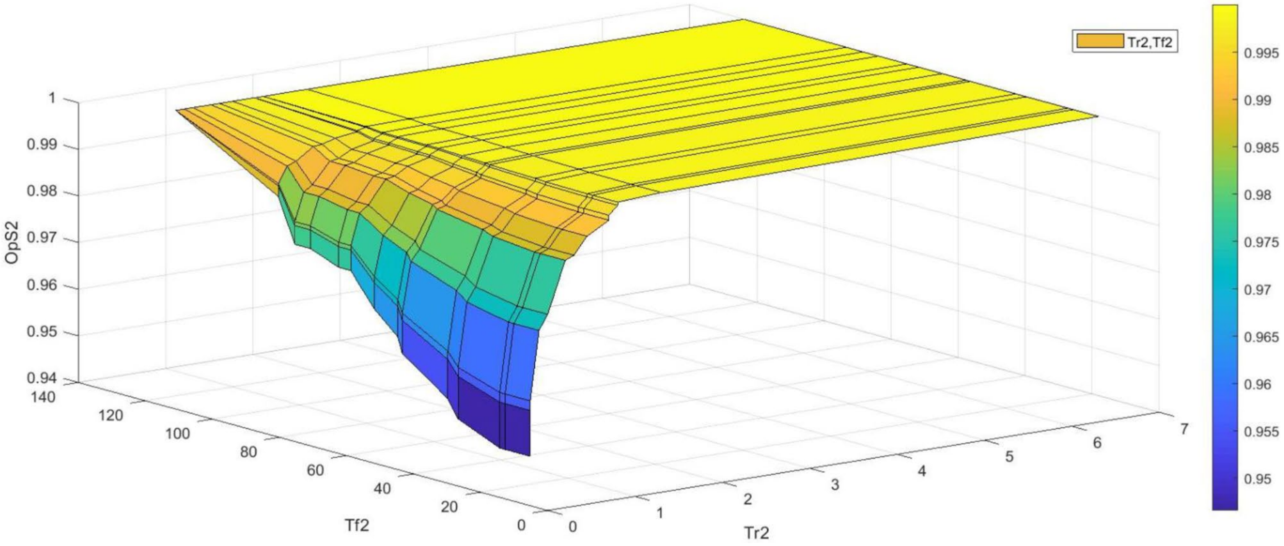

Fig. 7 Operational safety curve for FPU No. 2

The values given by the expressions (29) and (32) being close, we can take 1.2 as the initial value for FPU No. 1. For the FPUs No. 2 and 3, we obtain as initial values 1.5 and 1.2 respectively. With R under calculator by successive approximations, we obtain the values of Table 6 for θ and α.

The different θ values obtained for each FPU allow us to realize that its equipment is already in the aging phase of its life cycles ($\theta > 1$), with a slightly advanced state compared to the others for FPU n°2. However, it is more difficult to predict a failure at this stage of a system's life cycle because failures are random, hence the need to pay more attention to them to ensure better maintenance.

According to Table 7, we have: $D_{n,\alpha} = D_{14,005} = 0.3489$ et $D_{max} = 0.179$. Since $D_{max} < D_{14,005}$, the hypothesis of a Weibull distribution with a 5% risk of being wrong is therefore accepted. $D_{max} = 0.154$ For FPU No. 2 and $D_{max} = 0.065$ for the FPU No. 3, which also makes it possible to validate for its two other FPUs the hypothesis of a Weibull distribution with a risk of error of 5%. Therefore, we can write the following expressions for each FPU. We can therefore have the expressions of $OpS\left(t_f, t_r\right)$,$\varphi\left(t_f, t_r\right)$ and

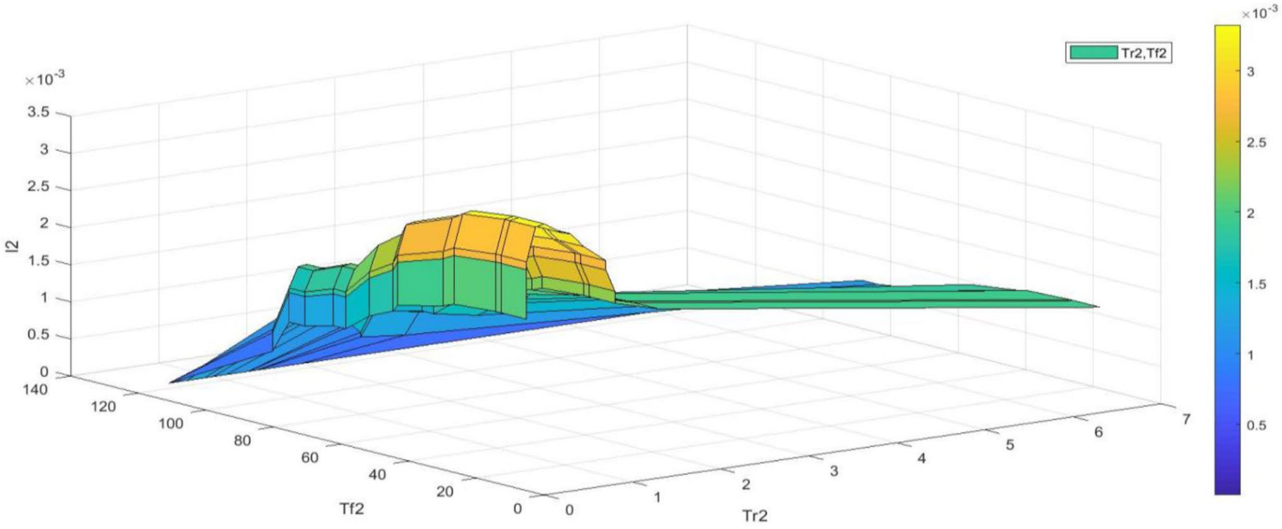

Fig. 8 Probability density curve for FPU No. 2

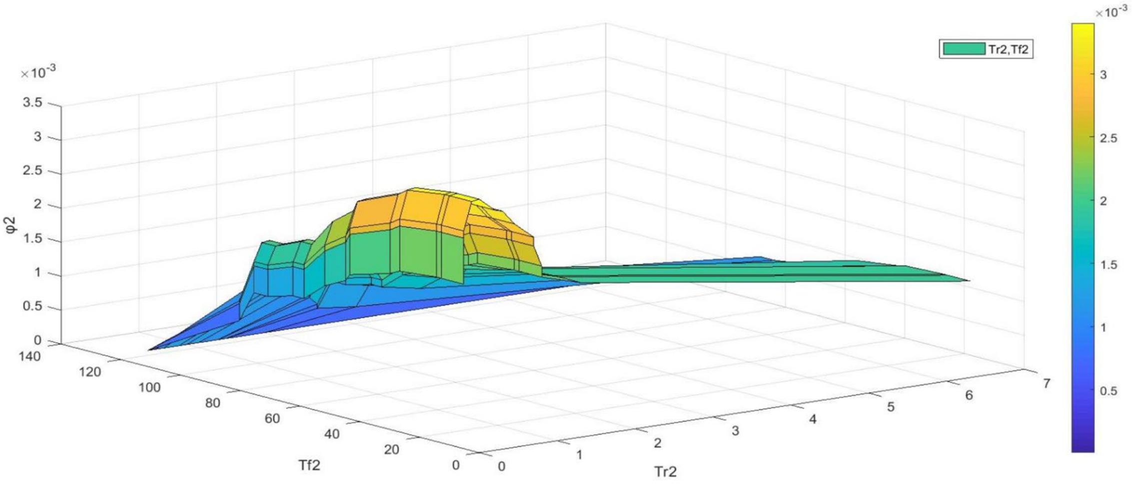

Fig. 9 Risk rate curve for FPU No. 2

$\psi\left(t_f, t_r\right)$ for each FPU. Their different curves are given in Figs. 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

$$OpS_1\left(t_f, t_r\right) = 1 - \left[\left(e^{-\left(\frac{t_f}{35.344}\right)^{1.312}} - 1\right)\left(1 - e^{-e^{-0.683(t_r-1)}}\right)\right]^2 \tag{33}$$

$$l_1\left(t_f, t_r\right) = -4\left[0.037\left(\frac{t_f}{35.344}\right)^{0.312} e^{-\left(\frac{t_f}{35.344}\right)^{1.312}}\left(e^{-\left(\frac{t_f}{35.344}\right)^{1.312}} - 1\right)\right] \left[0.683e^{-0.683(t_r-1)} e^{-e^{-0.683(t_r-1)}}\left(1 - e^{-e^{-0.683(t_r-1)}}\right)\right] \tag{34}$$

$$\varphi_1\left(t_f, t_r\right) = -4\left[\frac{0.037\left(\frac{t_f}{35.344}\right)^{0.312} e^{-\left(\frac{t_f}{35.344}\right)^{1.312}}\left(e^{-\left(\frac{t_f}{35.344}\right)^{1.312}} - 1\right)}{1 - \left[\left(e^{-\left(\frac{t_f}{35.344}\right)^{1.312}} - 1\right)\left(1 - e^{-e^{-0.683(t_r-1)}}\right)\right]^2}\right] * \left[0.683e^{-0.683(t_r-1)} e^{-e^{-0.683(t_r-1)}}\left(1 - e^{-e^{-0.683(t_r-1)}}\right)\right] \tag{35}$$

$$\psi_1\left(t_f, t_r\right) = 1 + 4\left[\frac{0.037\left(\frac{t_f}{35.344}\right)^{0.312} e^{-\left(\frac{t_f}{35.344}\right)^{1.312}}\left(e^{-\left(\frac{t_f}{35.344}\right)^{1.312}} - 1\right)}{1 - \left[\left(e^{-\left(\frac{t_f}{35.344}\right)^{1.312}} - 1\right)\left(1 - e^{-e^{-0.683(t_r-1)}}\right)\right]^2}\right] *$$

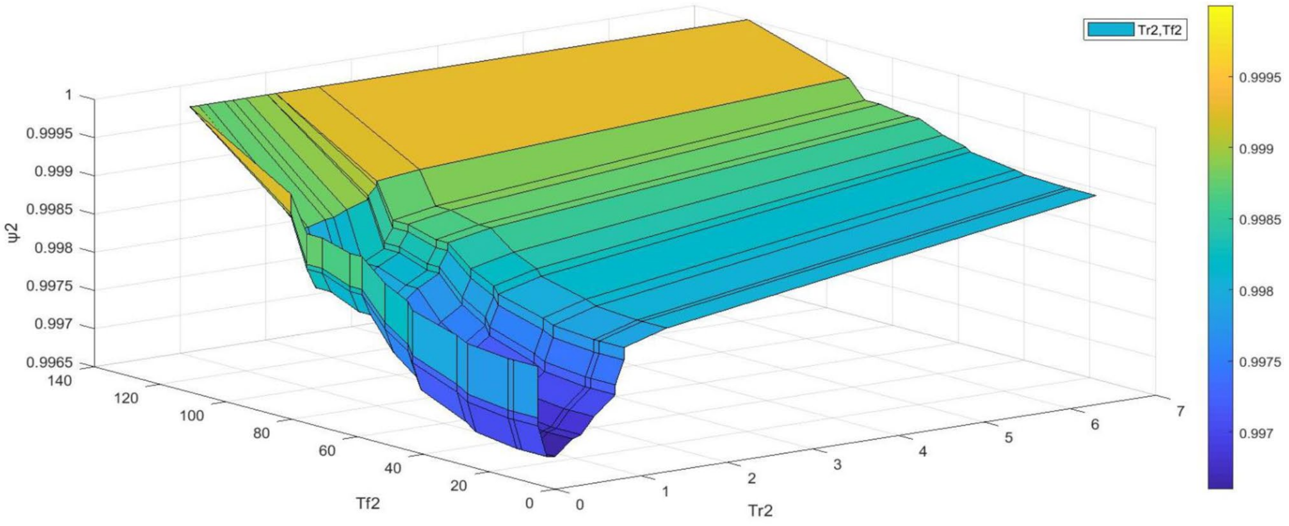

Fig. 10 Confidence rate curve for FPU No. 2

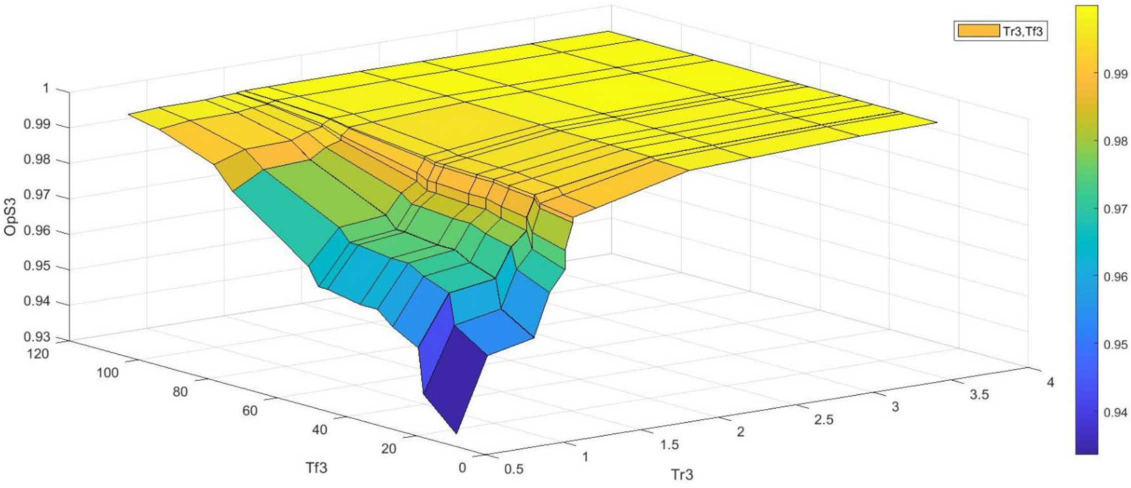

Fig. 11 Operating safety curve for FPU No. 3

$$\left[0.683e^{-0.683(t_r-1)}e^{-e^{-0.683(t_r-1)}}\left(1-e^{-e^{-0.683(t_r-1)}}\right)\right] \tag{36}$$

$$OpS_2\left(t_f,t_r\right)=1-\left[\left(e^{-\left(\frac{t}{62.102}\right)^{1.643}}-1\right)\left(1-e^{-e^{-0.725(t_r-0.825)}}\right)\right]^2 \tag{37}$$

$$\varphi_2\left(t_f,t_r\right)=-4\left[\frac{0.026\left(\frac{t_f}{62.102}\right)^{0.643}e^{-\left(\frac{t_f}{62.102}\right)^{1.643}}\left(e^{-\left(\frac{t_f}{62.102}\right)^{1.643}}-1\right)}{1-\left[\left(e^{-\left(\frac{t}{62.102}\right)^{1.643}}-1\right)\left(1-e^{-e^{-0.725(t_r-0.825)}}\right)\right]^2}\right]*$$

$$l_2\left(t_f,t_r\right)=-4\left[0.026\left(\frac{t_f}{62.102}\right)^{0.643}e^{-\left(\frac{t_f}{62.102}\right)^{1.643}}\left(e^{-\left(\frac{t_f}{62.102}\right)^{1.643}}-1\right)\right]\left[0.725e^{-0.725(t_r-0.825)}e^{-e^{-0.725(t_r-0.825)}}\left(1-e^{-e^{-0.725(t_r-0.825)}}\right)\right] \tag{38}$$

$$\left[0.725e^{-0.725(t_r-0.825)}e^{-e^{-0.725(t_r-0.825)}}\left(1-e^{-e^{-0.725(t_r-0.825)}}\right)\right] \tag{39}$$

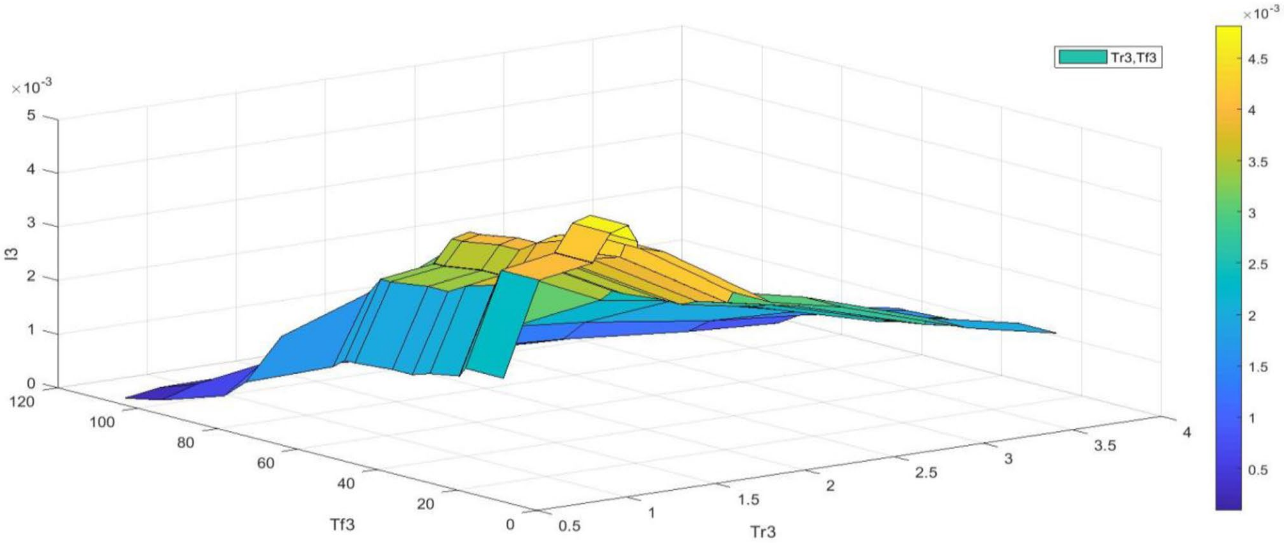

Fig. 12 Probability density curve for FPU No. 3

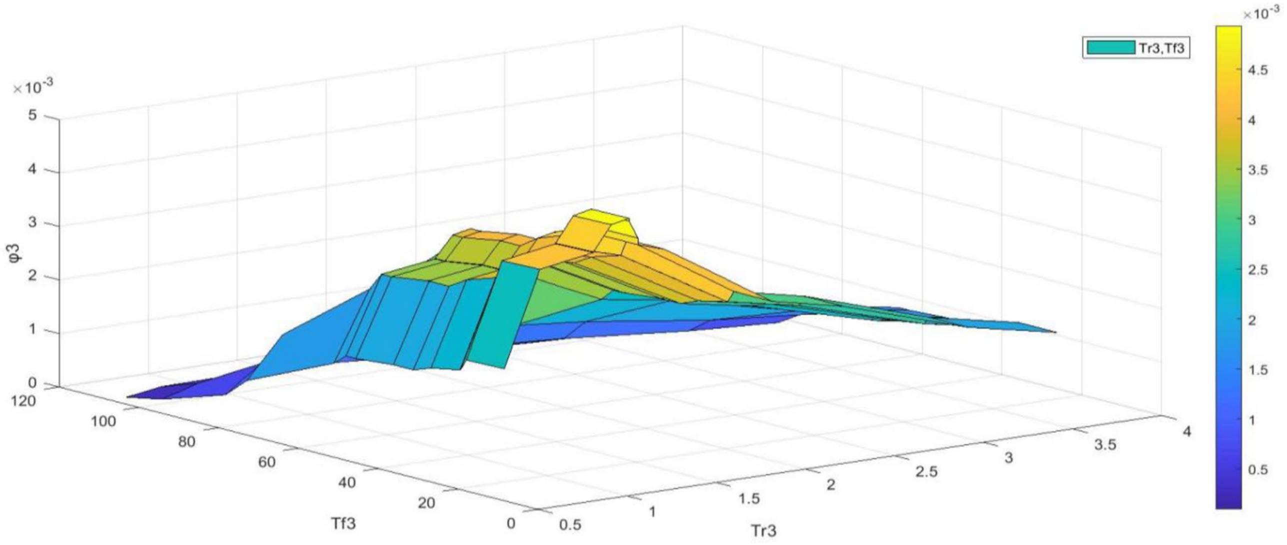

Fig. 13 Risk rate curve for FPU No. 3

$$\psi_2\left(t_f,t_r\right)=1+4\left[\frac{0.026\left(\frac{t_f}{62.102}\right)^{0.643}e^{-\left(\frac{t_f}{62.102}\right)^{1.643}}\left(e^{-\left(\frac{t_f}{62.102}\right)^{1.643}}-1\right)}{1-\left[\left(e^{-\left(\frac{t}{62.102}\right)^{1.643}}-1\right)\left(1-e^{-e^{-0.725(t_r-0.825)}}\right)\right]^2}\right]*$$

$$\left[0.725e^{-0.725(t_r-0.825)}e^{-e^{-0.725(t_r-0.825)}}\left(1-e^{-e^{-0.725(t_r-0.825)}}\right)\right] \quad (40)$$

$$OpS_3\left(t_f,t_r\right)=1-\left[\left(e^{-\left(\frac{t_f}{46.695}\right)^{1.223}}-1\right)\left(1-e^{-e^{-0.793(t_r-1.07)}}\right)\right]^2 \quad (41)$$

$$l_3\left(t_f,t_r\right)=-4\left[0.026\left(\frac{t_f}{46.695}\right)^{0.223}e^{-\left(\frac{t_f}{46.695}\right)^{1.223}}\left(e^{-\left(\frac{t_f}{46.695}\right)^{1.223}}-1\right)\right]$$
$$\left[0.793e^{-0.793(t_r-1.07)}e^{-e^{-0.793(t_r-1.07)}}\left(1-e^{-e^{-0.793(t_r-1.07)}}\right)\right] \quad (42)$$

$$\varphi_3\left(t_f,t_r\right)=-4\left[\frac{0.026\left(\frac{t_f}{46.695}\right)^{0.223}e^{-\left(\frac{t_f}{46.695}\right)^{1.223}}\left(e^{-\left(\frac{t_f}{46.695}\right)^{1.223}}-1\right)}{1-\left[\left(e^{-\left(\frac{t_f}{46.695}\right)^{1.223}}-1\right)\left(1-e^{-e^{-0.793(t_r-1.07)}}\right)\right]^2}\right]*$$

$$\left[0.793e^{-0.793(t_r-1.07)}e^{-e^{-0.793(t_r-1.07)}}\left(1-e^{-e^{-0.793(t_r-1.07)}}\right)\right] \quad (43)$$

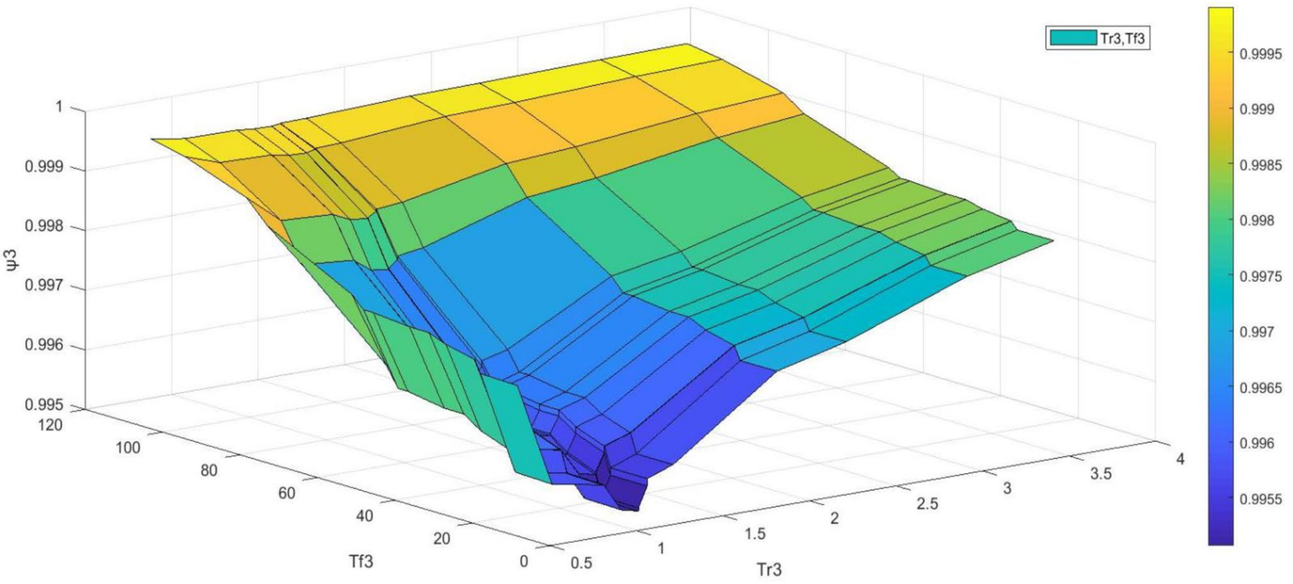

Fig. 14 Confidence rate curve for FPU No. 3

$$\psi_3\left(t_f, t_r\right) = 1 + 4\left[\frac{0.026\left(\frac{t_f}{46.695}\right)^{0.223} e^{-\left(\frac{t_f}{46.695}\right)^{1.223}}\left(e^{-\left(\frac{t_f}{46.695}\right)^{1.223}} - 1\right)}{1 - \left[\left(e^{-\left(\frac{t_f}{46.695}\right)^{1.223}} - 1\right)\left(1 - e^{-e^{-0.793(t_r-1.07)}}\right)\right]^2}\right] *$$

$$\left[0.793e^{-0.793\left(t_r-1.07\right)} e^{-e^{-0.793\left(t_r-1.07\right)}}\left(1 - e^{-e^{-0.793\left(t_r-1.07\right)}}\right)\right] \quad (44)$$

By observing the different curves obtained, we can see that the different operational safety curves all have the same look. The same applies to probability densities, risk rates and confidence rates. For operational safety curves, we have an area where the probability that the system is safe presents uncertainties, relatively between 0.93 and 0.98 with repair times between 0.5 and 2 h, and operating times greater than or equal to 45. The other part of the different operational safety curves gives probabilities relatively close to 1. It can also be seen that for the different safety curves, in the area where the probability of the system being safe is low, the probability density and the rate of risk of non-security are significantly greater, and the confidence rate is greater. The reverse also occurs when we are in the zone where this probability is close to 1. It is practically zero for FPUs No. 2 and 3, with values close to 10^{-3}. The fact that the different FPUs have an overall probability of being 1 and therefore 100% safe is also indicative of a good design of its equipment, guaranteeing good reliability and maintainability. This fact can be perceived by observing the confidence rate curves, which have values that oscillate around 99.65% and 100% (see Figs. 13 and 14).

5 Conclusion

Setting up a functioning production system is the challenge of any designer. In this document, our concern was to propose a probabilistic model for assessing operational safety and to show how it can be exploited, supported by the practical case of the equipment of the CCPD of Garoua. The exploitation of probability distributions allowed us to implement a model. The equations and curves obtained for each pumping unit considered in the practical case allow us to consider this model in a very positive way, insofar as they sufficiently describe what is real. Although it is not easily exploitable and requires knowledge of the times between failure and technical times to repair of the system, we propose in perspective, the implementation of techniques allowing the exploitation of the model at the design stage.

Acknowledgements This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. The authors are grateful to the Cameroonian Company of Petroleum Depots of Garoua for having made available to them the data relating to their equipment.

Author contributions Cedric Tatsinkou Fogang: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Formal analysis, Conceptualisation. Wolfgang Nzié: Writing – review & editing, Writing – original draft, Validation, Supervision. Aslain Brisco Ngnassi djami: Writing – review & editing, Formal analysis, Validation.

Data availability The authors do not have permission to share data.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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