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ORIGINAL RESEARCH

Reliability assessment of multi-computer system consisting *n* clients and the *k*-out-of-*n*: *G* operation scheme with copula repair policy

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Abstract
The reliability features of a computer network system comprising four subsystems under the *k*-out-of-*n*: *G* operational scheme have been studied in this model. All four subsystems are organized in a series configuration and failure rates are assumed to follow exponential distribution. Repairs, on the other hand, follow two different distributions: general distribution and Gumbel–Hougaard family copula, employed to repair partially failed and completely failed states, respectively. Through the transition diagram, using supplementary variable methods, Laplace transforms, the system of first-order partial differential equations is derived and solved. The objective is to obtain the expressions for availability, (MTTF), and cost function. By evaluating the reliability, availability, MTTF, and cost analysis, computations for reliability measures are taken as the specific case. The computation's results are shown in tables and graphs.

Keywords *k*-out-of-*n*: *G* operational scheme · Availability · Reliability · MTTF · Cost analysis · Gumbel–Hougaard family-copula distribution

1 Introduction

Complex systems generally tend to be considered by high diversity and high interconnectedness. Multiple adaptive pathways (known as redundancies) are apportioned with high interconnectedness. This prevents flop from crushing the device. However, because of the high level of interconnectedness, septicemia can spread more easily, ending in system collapse. How do naturally developed complex systems in nature (e.g. rainforests, ecologies, etc.) maintain high connectedness, but limit contagion and thereby prevent system collapse?

In this fast-moving world, the roles of technology in our day-to-day operations, ranging from social, economic, and industrial technology or otherwise cannot be overstressed. The computer network system has become quite a necessity and a critical need of human life. The computer communication network system is made up of many comprehensive and local area computer network systems joined by peripherals. Because new computer network systems are being added at a quick pace, giving an accurate structure of the computer network system is a difficult undertaking. The performance of a computer network system can be predicted in a variety of ways, including transfer time, response time, and the number of users, as well as the transmission media, power supply, hardware, and software that make up the computer network system. Furthermore, the frequency of failure and the time it takes to recover from failure in the event of a disaster are used to assess computer network system reliability. Over the decades, researchers have worked hard to develop an appropriate mechanism. Researchers have concentrated on creating structures that can be repaired due to various configurations, as well as assessing dependability,

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availability, operating costs, and other factors, to satisfy manufacturing businesses' specific demands. Similarly, to come up with a system that has high system availability and low maintenance costs while satisfying the consumers, many researchers have investigated various aspects of the complicated repairable system's reliability for this aim. In 1990, most of the researchers studied system performances of different types of repairable systems, and using one repair policy with general repair distribution. To explore the study of several investigators, Vanderperre (1990) premeditated on the reliability features of a parallel system and used general repair distribution. Gupta and Sharma (1993) studied a model for a two-duplex unit standby system using a general repair policy. Singh et al. (2001) studied complex standby redundant systems involving multi-failure—human failure using general repair policy. Ram and Singh (2008) studied a complex system using (1-out-of-2: *F*) and (1-out-of-*n*: *F*) subsystems and using general repair distributions policy. Ram and Singh (2010) also studied mutually exclusive complex systems and used a general repair policy. In a repairable system, two types of failures are observed i.e., partial failure and complete failure. As a due cause of complete failure, the entire system stops functioning, resulting in a huge loss of the output of the system. Consequently, it needs fast repair to get ready for work. In a realistic situation, a failed system can be repaired in a couple of ways using multi-repair approach. Hence whenever the system experiences a complete shutdown mode, it should be repaired using the copula repair approach.

Copulas are functions that connect the one-dimensional margins of multivariate distribution functions to their one-dimensional margins. Copula research and its application in statistics are a relatively recent, yet rapidly expanding area. To explore the utility of copula in the study of repairability, Singh and Rawal (2014) applied copula distribution, to examine the availability, MTTF, and cost analysis of a complex system with a preemptive resume repair policy. Kumar et al. (2017) conducted studies on the availability and cost analysis of an engineering system containing series subsystems. Ghosh et al. (2017) studied how to reduce the life cycle costs of a modern helicopter by improving the stability and maintainability variables. Gahlot et al. (2018) employed a copula linguistics approach, the premeditated performance of a repairable system under different types of failure, and two types of repair policies. Singh and Ayagi (2018) analyzed complex repairable systems using a preemptive resume repair policy together with a copula approach. Lado and Singh (2019) used Gumbel–Hougaard family copula distribution, to evaluate the profit valuation of a complex repairable system comprising two subsystems in a series configuration. Together with the series and parallel configuration, a specific type of configuration in which out-of-*n* identical *k* units are essential for functioning is defined as a *k*-out-of-*n*: *G*/*F* type of configuration and is found in almost all industrial systems. For the system to be operative, at least *k* of its units out of *n* need to be operational. The remaining $(n-k)$ units are considered redundant units, while the first *k* units are known as fundamental units. Further, a *k*-out-of-*n*: *G* system is equivalent to $(k+1)$-out-of-*n*: *F*, and the (*n*-out-of-*n*: *G*) system has a purely parallel configuration, while (1-out-of-*n*: *F*) is a purely series system. To confine the study of the *k*-out-of-*n*: *G*/*F* types of configuration, Tamegai (1980) studied the *k*-out-of-*n*: *F* repairable systems that have one or two servers and constant failure rates and use general repair policy. Dhillon and Anude's (1994) dynamic structure was examined for the failure of a *k*-out-of-*n*: *G* system. Malik and Bhardwaj (2007) studied the reliability and cost of the 2-out-of-3 redundant system using general repair distribution and waiting time. Singh et al. (2012) studied a system consisting of two subsystems and using *k*-out of *n*: *G* policy and copula. Gulati et al. (2014) studied a reliability system having two units in a parallel configuration and using repairs, general repair and Gumbel–Hougaard family copula distribution. Gahlot et al. (2020) analyzed a system consisting of three identical units under the *k*-out-of-*n*: *G* scheme with copula repair approach. Cost–benefit analysis of a *k*-out-of-*n*: *G* kind of warm standby system under catastrophic failure via copula repair approach was performed by Poonia and Sirohi (2020). A multi-state computer network with five web servers and three database servers system in a series configuration with the implication of copula repair approach was studied by Poonia (2021) via supplementary variable and Laplace transforms. Sirohi et al. (2021) used the Gumbel–Hougaard copula to estimate reliability indices for a complex repairable system in a series configuration with switch and catastrophic failure. A system concerning five clients and two servers as subsystem 1 and subsystem 2 under the *k*-out-of-*n*: *G* scheme was analyzed by Yusuf et al. (2021) with implications of copula repair. Poonia et al. (2021) analyzed the performance of a warm standby *k*-out-of-*n*: *G*, and 2-out-of-4: *G* system in a series configuration using copula repair strategy. The reliability performance prediction of a solar photovoltaic system for rural consumption using Gumbel–Hougaard family copula has been addressed stochastically by Maihulla and Yusuf (2021). Recently, the performance of a complex system consisting of three subsystems in series configuration has been stochastically analyzed by Jibril et al. (2022).

In the present model, system performance was evaluated through reliability measures; the system description consisted of four subsystems in series arrangement. The first subsystem is the client's server which consists *n* clients and the work policy is the *k*-out-of-*n*: *G* to perform aptly. The second subsystem is a load balancer (LB) which balances the load to distributed servers DDS1 and DDS2 which are placed in a parallel configuration and at least one database server is essential to be operative for the system operational mode. Subsystem 3 has two identical distributed database servers (DDS) I and II connected in a parallel configuration. In subsystem 4, CDDS is a centralized distributed database server that manages all of the application data in DDS. The system architecture and state transition diagram in Fig. 1a, b explain the system structure and state conversions from one state to another. The states S_0, S_1, S_2, and S_3 are the states for subsystem 1 operations, S_4 is the state for load balance failure, S_5, S_6 represent states for DDS1 and DDS2 and S_7 indicates CDDS failure (Table 1).

2 Notations, expectations, and assumptions of the system

2.1 Notations

See Table 1.

2.2 Assumption

S_0: Perfect working state, all subsystems are satisfactory.

S_1: Minor degraded state due to failure of one client in subsystem 1. It is a working state as minor partial failure raised and working policy is k-out-of-n: G for subsystem 1.

S_2: Major degraded state after failure of (n–k) clients in subsystem 1 and further failure of a single client would lead to state S_3.

S_3: Complete failed state due to the failure of ($k+1$) clients in subsystem 1.

S_4: Complete failed state due to the failure of load balancer (LB) system not working.

S_5: Major degraded and working state due to the failure of DDS1 in subsystem 3.

S_6: Complete failed state due to the failure of whole subsystem 3.

S_7: Complete failed state due to failure of database distributed centralized server CDDS server.

3 Formulation of the mathematical model

By a probability of considerations and continuity arguments, we can obtain the following set of differential equations governing the present mathematical model.

$$\left[\frac{\partial}{\partial t} + n\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4\right]P_0(t) = \left[\int_0^\infty \varphi_1(x)P_1(x,t)\mathrm{d}x + \int_0^\infty \varphi(y)P_5(y,t)\mathrm{d}y + \int_0^\infty \mu_0(x)\,P_4(x,t)\mathrm{d}x + \int_0^\infty \mu_0(x)\,P_3(x,t)\mathrm{d}x + \int_0^\infty \mu_0(z)\,P_7(z,t)\mathrm{d}z + \int_0^\infty \mu_0(y)\,P_6(y,t)\mathrm{d}y, \right. \tag{1}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + (n-k)\lambda_1 + \lambda_2 + \lambda_4 + 2\lambda_3 + \varphi_1(x)\right]P_1(x,t) = 0, \tag{2}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + (n-k-1)\lambda_1 + \lambda_2 + \lambda_4 + \varphi_1(x)\right]P_2(x,t) = 0, \tag{3}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right]P_3(x,t) = 0, \tag{4}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right]P_4(x,t) = 0, \tag{5}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda_3 + \lambda_2 + \lambda_4 + \varphi(y)\right]P_5(y,t) = 0, \tag{6}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_0(y)\right]P_6(y,t) = 0, \tag{7}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu_0(z)\right]P_7(z,t) = 0. \tag{8}$$

Boundary conditions

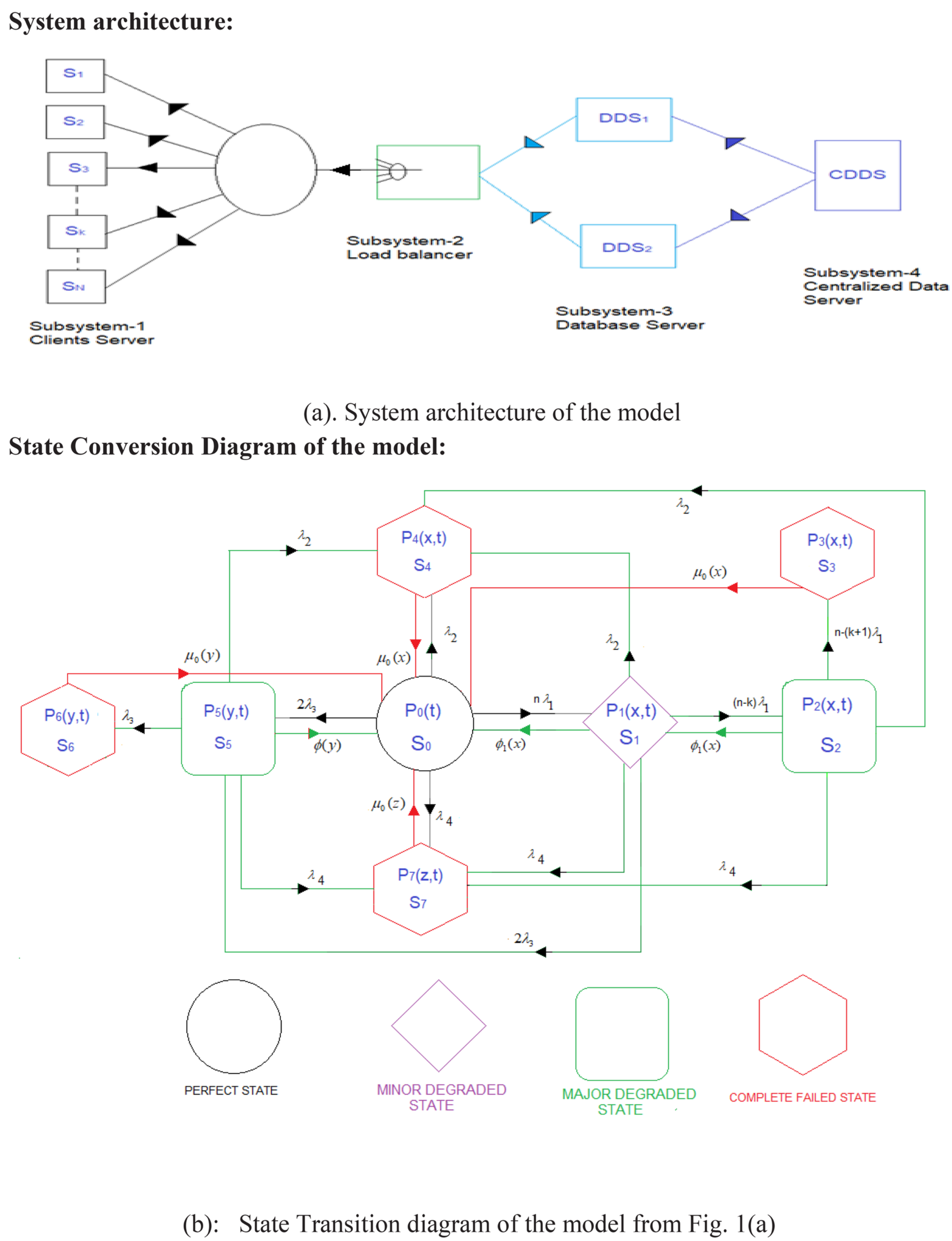

Fig. 1 **a** System architecture of the model. **b** State transition diagram of the model from (**a**)

$$P_1(0,t) = n\lambda_1 P_0(t), \tag{9}$$

$$P_2(0,t) = n(n-k)\lambda_1^2 P_0(t), \tag{10}$$

$$P_3(0,t) = n(n-k)(n-k-1)\lambda_1^3 P_0(t), \tag{11}$$

$$P_4(0,t) = \lambda_2\left[1 + n\lambda_1 + n(n-k)\lambda_1^2 + 2\lambda_3(1+n\lambda_1)\right]P_0(t), \tag{12}$$

Table 1 Nomenclature of symbolic terminology

t	Time variable on the time axis.
s	Laplace transform variable for all proclamations in the mathematical equations.
$\lambda_1/\lambda_2/\lambda_3/\lambda_4$	The failure rate of clients (subsystem 1)/failure rate of load balancer (subsystem 2)/Failure rate of distributed database server (subsystem 3)/failure rate centralized server (subsystem 4)
$\phi_1(x)/\phi(y)$	Repair rate of the unit of subsystem 1/repair rate of a unit of subsystem 2.
$\mu_0(x)/\mu_0(y)/\mu_0(z)$	Repair rates for complete failed states. S_3 and $S_4/S_6/S_7$, respectively
$P_i(t)$	The probability that the system is in S_i state at instants for $i = 0, 1, 2\ldots7$
$\overline{P}_i(s)$,	Laplace transformation of state transition probability $P_i(t)$
$P_i(x, t)$	For the probability that a system is in state S_i for $i = 1\ldots, 7$, the system under repair and elapse repair time is (x, t) with x the repair variable and t the time variable
$P_i(y, t)$	For the probability that a system is in state S_i for $i = 1\ldots, 7$, the system under repair and elapse repair time is (y, t) with y the repair variable and t the time variable
$P_i(z, t)$	For the probability that a system is in state S_i for $i = 1\ldots$, the system under repair and elapse repair time is (z, t) with z the repair variable and t the time variable
E_p (t)	Expected profit during the time interval $[0, t)$
$\mu_0(x)$	The expression of joint probability (failed state S_i to good state S_0) according to the Gumbel–Hougaard family copula definition, $\mu_0(x) = \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}, 1 \le \theta \le \infty$

$$P_5(0, t) = 2\lambda_3(1 + n\lambda_1)P_0(t), \tag{13}$$

$$P_6(0, t) = 2\lambda_3^2(1 + n\lambda_1)P_0(t), \tag{14}$$

$$P_7(0, t) = \lambda_4\left[1 + n\lambda_1 + n(n-k)\lambda_1^2 + 2\lambda_3^2(1 + n\lambda_1)\right]P_0(t). \tag{15}$$

Initials conditions

$$P_0(0) = 1 \text{ and other state probabilities are zero at } t = 0 \tag{16}$$

4 Solution of the model

Taking Laplace transformation of Eqs. (1)–(15) and using Eq. (16), we obtain

$$\begin{aligned}\left[s + n\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4\right]\overline{P}_0(s) &= \left[\int_0^\infty \varphi_1(x)\overline{P}_1(x, s)\mathrm{d}x + \int_0^\infty \varphi(y)\overline{P}_5(y, s)\mathrm{d}y\right.\\ &+ \int_0^\infty \mu_0(x)\,\overline{P}_4(x, s)\mathrm{d}x + \int_0^\infty \mu_0(x)\,\overline{P}_3(x, s)\mathrm{d}x\\ &+ \int_0^\infty \mu_0(z)\,\overline{P}_7(z, s)\mathrm{d}z + \int_0^\infty \mu_0(y)\,\overline{P}_6(y, s)\mathrm{d}y,\end{aligned} \tag{17}$$

$$\left[s + \frac{\partial}{\partial x} + (n-k)\lambda_1 + \lambda_2 + \lambda_4 + 2\lambda_3 + \varphi_1(x)\right]\overline{P}_1(x, s) = 0, \tag{18}$$

$$\left[s + \frac{\partial}{\partial x} + (n-k-1)\lambda_1 + \lambda_2 + \lambda_4 + \varphi_1(x)\right]\overline{P}_2(x, s) = 0, \tag{19}$$

$$\left[s + \frac{\partial}{\partial x} + \mu_0(x)\right]\overline{P}_3(x, s) = 0, \tag{20}$$

$$\left[s + \frac{\partial}{\partial x} + \mu_0(x)\right]\overline{P}_4(x, s) = 0, \tag{21}$$

$$\left[s + \frac{\partial}{\partial y} + \lambda_3 + \lambda_2 + \lambda_4 + \varphi(y)\right]\overline{P}_5(y, s) = 0, \tag{22}$$

$$\left[s + \frac{\partial}{\partial y} + \mu_0(y)\right]\overline{P}_6(y, s) = 0, \tag{23}$$

$$\left[s + \frac{\partial}{\partial z} + \mu_0(z)\right]\overline{P}_7(z, s) = 0, \tag{24}$$

$$\overline{P}_1(0, s) = n\lambda_1\overline{P}_0(s), \tag{25}$$

$$\overline{P}_2(0, s) = n(n-k)\lambda_1^2\overline{P}_0(s), \tag{26}$$

$$\overline{P}_3(0,s) = n(n-k)(n-k-1)\lambda_1^3\overline{P}_0(s), \tag{27}$$

$$\overline{P}_4(0,s) = \lambda_2\left[1+n\lambda_1+n(n-k)\lambda_1^2+2\lambda_3(1+n\lambda_1)\right]\overline{p}_0(s), \tag{28}$$

$$\overline{P}_5(0,s) = 2\lambda_3(1+n\lambda_1)\overline{P}_0(s), \tag{29}$$

$$\overline{P}_6(0,s) = 2\lambda_3^2(1+n\lambda_1)\overline{P}_0(s), \tag{30}$$

$$\overline{P}_7(0,s) = \lambda_4\left[1+n\lambda_1+n(n-k)\lambda_1^2+2\lambda_3^2(1+n\lambda_1)\right]\overline{P}_0(s). \tag{31}$$

Solving (25)–(36) with the help of (37)–(46), one may get

$$\overline{P}_0(s) = \frac{1}{D(s)}, \tag{32}$$

$$\overline{P}_1(s) = \frac{n\lambda_1}{D(s)}\frac{(1-S_{\varphi_1}(s+(n-k)\lambda_1+\lambda_2+\lambda_4+2\lambda_3))}{(s+(n-k)\lambda_1+\lambda_2+\lambda_4+2\lambda_3)}, \tag{33}$$

$$\overline{P}_2(s) = \frac{n(n-k)\lambda_1^2}{D(s)}\frac{(1-S_{\varphi_1}(s+(n-k-1)\lambda_1+\lambda_2+\lambda_4))}{(s+(n-k-1)\lambda_1+\lambda_2+\lambda_4)}, \tag{34}$$

$$\overline{P}_3(s) = \frac{n(n-k)(n-k-1)\lambda_1^3}{D(s)}\frac{(1-S_{\mu_0}(s))}{(s)}, \tag{35}$$

$$\overline{P}_4(s) = \frac{\lambda_2\left[1+n\lambda_1+n(n-k)\lambda_1^2+2\lambda_3(1+n\lambda_1)\right]}{D(s)}\frac{(1-S_{\mu_0}(s))}{(s)}, \tag{36}$$

$$\overline{P}_5(s) = \frac{2\lambda_3(1+n\lambda_1)}{D(s)}\frac{(1-S_{\varphi}(s+\lambda_3+\lambda_2+\lambda_4))}{s+\lambda_3+\lambda_2+\lambda_4}, \tag{37}$$

$$\overline{P}_6(s) = \frac{2\lambda_3^2(1+n\lambda_1)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s}, \tag{38}$$

$$\overline{P}_7(s) = \frac{\lambda_4\left[1+n\lambda_1+n(n-k)\lambda_1^2+2\lambda_3^2(1+n\lambda_1)\right]}{D(s)}\frac{(1-S_{\mu_0}(s))}{s}, \tag{39}$$

$$\begin{aligned} D(s) = {} & s+n\lambda_1+\lambda_2+2\lambda_3+\lambda_4 \\ & -\{n\lambda_1\bar{S}_{\varphi_1}(s+(n-k)\lambda_1+\lambda_2+\lambda_4+2\lambda_3) \\ & +2\lambda_3\left(1+n\lambda_1\right)S_{\varphi}(s+\lambda_3+\lambda_2+\lambda_4) \\ & +\lambda_2\left[1+n\lambda_1+n(n-k)\lambda_1^2+2\lambda_3(1+n\lambda_1)\right]\overline{S}_{\mu_0}(s). \end{aligned} \tag{40}$$

The Laplace transformations of the probabilities that the system is in up (i.e., either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s) = \overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_5(s) \tag{41}$$

$$= \frac{1}{D(s)}\begin{bmatrix} 1+n\lambda_1\dfrac{(1-\overline{S}_{\varphi_1}(s+(n-k)\lambda_1+\lambda_2+\lambda_4+2\lambda_3))}{(s+(n-k)\lambda_1+\lambda_2+\lambda_4+2\lambda_3)} \\ +n(n-k)\lambda_1^2\dfrac{(1-\overline{S}_{\varphi}(s+(n-k-1)\lambda_1+\lambda_4+\lambda_2))}{(s+(n-k-1)\lambda_1+\lambda_4+\lambda_2)} \\ +2\lambda_3(1+n\lambda_1)\dfrac{(1-\overline{S}_{\varphi}(s+\lambda_3+\lambda_2+\lambda_4))}{(s+\lambda_3+\lambda_2+\lambda_4)} \end{bmatrix}, \tag{42}$$

$$\overline{P}_{\text{failed}}(s) = \overline{P}_3(s)+\overline{P}_4(s)+\overline{P}_6(s)+\overline{P}_7(s). \tag{43}$$

5 Analytical study of model

5.1 Availability analysis

When repair follows exponential and general distribution.

Setting $\overline{S}_{\mu_0}(s) = \frac{\mu_0(x)}{s+\mu_0(x)}$, $\mu_0(x) = \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}$, $\overline{S}_{\phi_1}(s) = \frac{\phi_1}{s+\phi_1}$ in Eq. (42) and setting the values of different parameters as $\lambda_1=0.002$, $\lambda_2=0.02$, $\lambda_3=0.01$, $\lambda_4=0.022$, $\varphi=1$, $\theta=1$, $x=1$, $n=50$, $k=30$ and then taking inverse Laplace transform, one can obtain

$$\begin{aligned} (a) = {} & -0.004370691056e^{-1.080000000t} \\ & +0.01834516992e^{-2.769791233t} \\ & -0.1610502486e^{-1.234748565t} \\ & +0.003732471750e^{-1.075979702t} \\ & n=50,\ k=30, \end{aligned}$$

$$\begin{aligned} (b) = {} & -0.002698837430e^{-1.100000000t} \\ & +0.01841727178e^{-2.770103895t} \\ & -0.01828462046e^{-1.250152916t} \\ & +0.001753932297e^{-1.079128739t} \\ & +1.000812254e^{-0.002914450451t} \\ & n=50,\ k=20, \end{aligned}$$

$$\begin{aligned} (c) = {} & 0.01851717582e^{-2.770502055t} \\ & -0.02034819216e^{-1.266111943t} \\ & +0.001759207778e^{-1.081751282t} \\ & +1.003097449e^{-0.003934719794t} \\ & -0.003025640352e^{-1.120000000t} \\ & n=50,\ k=10, \end{aligned}$$

Table 2 Availability analysis for copula repair

Time (t)	Availability (a) ($n=50$, $k=30$)	Availability (b) ($n=50$, $k=20$)	Availability (c) ($n=50$, $k=10$)	Availability (d) ($n=50$, $k=5$)
0	1.0000	1.0000	1.0000	1.0000
10	0.9808	0.9720	0.9644	0.9610
20	0.9635	0.9441	0.9272	0.9196
30	0.9465	0.9170	0.8914	0.8799
40	0.9298	0.8907	0.8570	0.8420
50	0.9133	0.8651	0.8239	0.8058
60	0.8972	0.8403	0.7922	0.7711
70	0.8814	0.8161	0.7616	0.7379
80	0.8659	0.7927	0.7322	0.7061
90	0.8506	0.7699	0.7039	0.6757
100	0.8356	0.7478	0.6768	0.6466

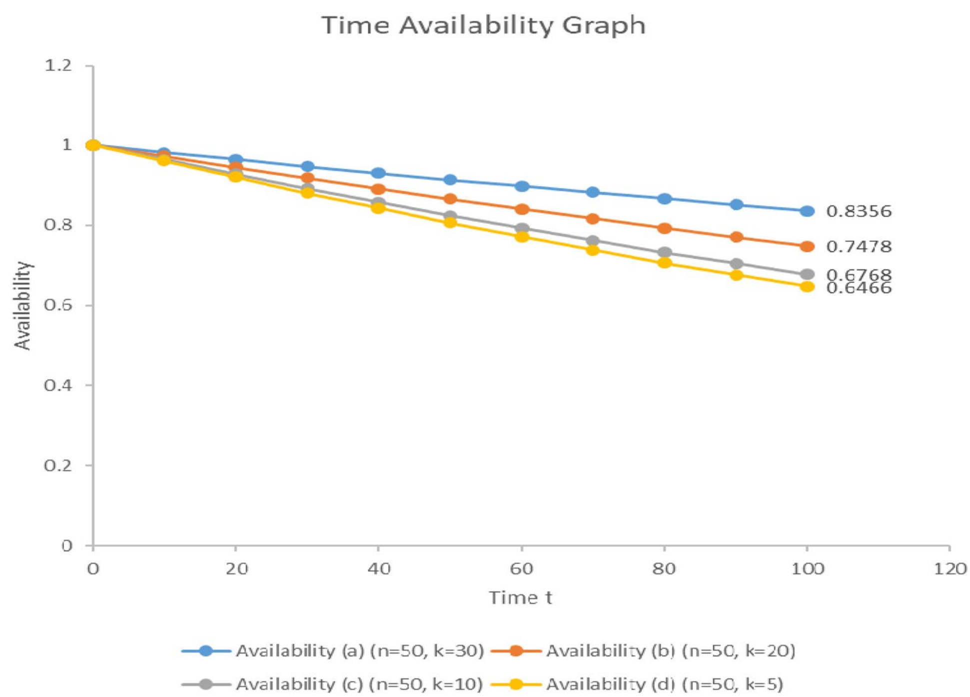

Fig. 2 Variation of availability when repair follows copula distribution

$$
\begin{aligned}
(d) = {} & 0.01857747171e^{-2.770733200t} \\
& - 0.02133520386e^{-1.274265936t} \\
& + 0.001817831986e^{-1.082898227t} \\
& + 1.004193103e^{-0.004402637530t} \\
& - 0.003253202999e^{-1.130000000t} \\
& n = 50,\ k = 5.
\end{aligned}
$$

For, $t=0$, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 units of time, one may get different values of $P_{up}(t)$ as shown in Table 2. If there are n identical units in subsystem 1 in a parallel configuration, then the computations for different values of k, n, are presented in Table 2 and the graphical representation of the availability is given in Fig. 2.

Figure 2 provides the material of availability of the system changes (when repair follows copula distribution) concerning the time when failure rates are fixed at different values. We have analyzed four cases for different values n and k as:

(a) $n=50$, $k=30$, (b) $n=50$, $k=20$, (c) $n=50$, $k=10$ and (d) $n=50$, $k=5$. It can be noticed that the availability decreases; hence, it can be concluded that the availability of the system (when repair follows copula distribution) decreases as the value of the parameters increases, and after a sufficiently long time it converges to zero.

Table 3 Time vs. availability

Time (t)	Availability (a) ($n=50$, $k=30$)	Availability (b) ($n=50$, $k=15$)	Availability (c) ($n=50$, $k=10$)	Availability (d) ($n=30$, $k=5$)
0	1.0000	1.0000	1.0000	1.0000
10	0.9559	0.9474	0.9399	0.9365
20	0.9395	0.9209	0.9046	0.8972
30	0.9233	0.8951	0.8706	0.8596
40	0.9074	0.8701	0.8378	0.8235
50	0.8918	0.8457	0.8063	0.7889
60	0.8765	0.8220	0.7760	0.7558
70	0.8614	0.7990	0.7469	0.7241
80	0.8466	0.7767	0.7188	0.6937
90	0.8320	0.7549	0.6918	0.6646
100	0.8177	0.7338	0.6658	0.6368

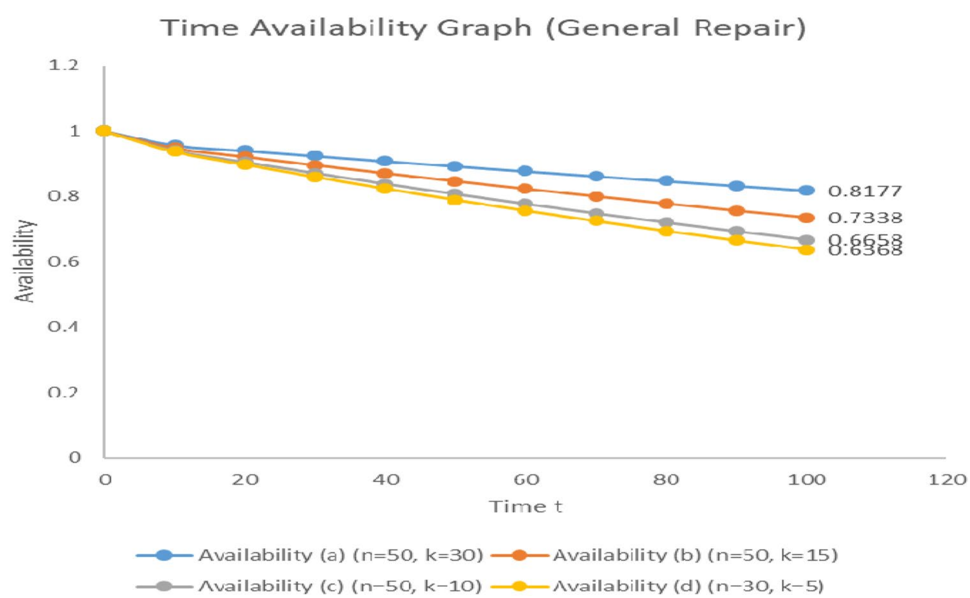

Fig. 3 Variation of availability when repair follows the general distribution

5.2 Availability analysis when the system follows general repair

When the repair follows general distribution, for different values of time variable $t=0$, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 units of time and $\varphi_1(x)=\varphi(y)=1$ and $\mu_0(x)=1$, we get different values of Pup(t) after taking inverse Laplace transform, as shown in Table 3 and corresponding Fig. 3. For similar configuration, (a) $n=50$, $k=30$, (b) $n=50$, $k=20$, (c) $n=50$, $k=10$ and (d) $n=50$, $k=5$, the given expressions are obtained as in (a), (b), (c) and (d).

$$(a) = -0.01640918357e^{-1.080000000t} + 0.009339395761e^{-1.266318299t} + 0.01898355262e^{-1.078833057t} + 0.01546340219e^{-1.017114012t} + 0.9726228329e^{-0.001734632698t} \quad n=50,\ k=30,$$

$$(b) = -0.003601302136e^{-1.100000000t} + 0.004807265361e^{-1.279593433t} + 0.007820210534e^{-1.083536788t} + 0.01627627655e^{-1.018031108t} + 0.9746975497e^{-0.002838670822t} \quad n=50,\ k=20,$$

$$(c) = -0.003700556475e^{-1.120000000t} + 0.0004924485519e^{-1.293593998t} + 0.009690064558e^{-1.087770412t} + 0.01690739010e^{-1.018804271t} + 0.9766106536e^{-0.003831320084t} \quad n = 50,\ k = 10.$$

For different values of time variable $t = 0$, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 units of time, we get different values of $P_{up}(t)$ as shown in Table 2 and Fig. 3

Figure 3 presents the availability variation (when repair follows general distribution) for the time when failure rates are fixed at different values. We have analyzed four cases for different values of n and k,

(a) $n = 50$, $k = 30$, (b) $n = 50$, $k = 20$, (c) $n = 50$, $k = 10$ and (d) $n = 50$, $k = 5$. It can be seen that the availability decreases; hence, it can be concluded that the availability of the system (when repair follows general distribution) decreases as the value of the parameters increases, and after a sufficiently long time converges to zero.

$$(d) = -12.05288136e^{-0.1800000000t} + 12.50000000e^{-0.1720000000t} + 0.18000000000e^{-0.13000000000t} + 0.3728813559e^{-0.06200000000t} \quad n = 50,\ k = 5.$$

Figure 4 as indicated shows the reliability of the system when the repair is not present. By taking all the repairs in an expression of availability as zero, we obtained the expression (a), (b), (c), and (d) of reliability as a function of time and it can be seen that the reliability of the system decreases.

7 Mean time to failure (MTTF) analysis

Setting $\phi_1(x)$, $\phi(y)$ and $\mu_0(x)$ to zero, Eq. (42) and taking the limit of the expression, as s tends to zero one can achieve the MTTF expression as:

$$\text{M.T.T.F.} = \lim_{s\to 0} \overline{P}_{up}(s) = \frac{1 + \frac{n\lambda_1}{(n-k)\lambda_1+\lambda_2+\lambda_4+2\lambda_3} + \frac{n(n-k)\lambda_1^2(1+\lambda_1)}{(n-k-1)\lambda_1+\lambda_4+\lambda_2} + \frac{2\lambda_3(n\lambda_1+1)}{(2\lambda_3+\lambda_2+\lambda_4)}}{\left(n\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4\right)}. \tag{44}$$

6 Reliability analysis

The system performance of a non-repairable system is known as reliability. Therefore, treating all repairs of the system to zero in (43) and the inverse Laplace transform of the resulting expression give us the reliability of the system, and one can obtain the expression (a), (b), (c), and (d) given as (Table 4):

$$(a) = 1.724137931e^{-0.1220000000t} - 1.137019287e^{-0.1800000000t} + 0.04000000000e^{-0.08000000000t} + 0.3728813559e^{-0.06200000000t} \quad n = 50,\ k = 30,$$

$$(b) = 0.07500000000e^{-0.1000000000t} + 2.631578947e^{-0.14200000000t} - 2.079460303e^{-0.18000000000t} + 0.3728813559e^{-0.06200000000t} \quad n = 50,\ k = 20,$$

$$(c) = -5.061770245e^{-0.1800000000t} + 5.555555556e^{-0.1620000000t} + 0.3728813559e^{-0.06200000000t} + 0.1333333333e^{-0.12000000000t} \quad n = 50,\ k = 10,$$

Setting $\lambda_2 = 0.02$, $\lambda_3 = 0.02$, $\lambda_4 = 0.022$ and varying λ_1 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 and 0.10 in (44), one may obtain Table 5 whose column 2 demonstrates variation of MTTF with respect to λ_1.

Setting $\lambda_1 = 0.002$, $\lambda_3 = 0.02$, $\lambda_4 = 0.022$ and varying λ_2 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, and 0.09 in (44), one may obtain Table 5 whose column 3 demonstrates variation of MTTF with respect to λ_2.

Setting $\lambda_1 = 0.002$, $\lambda_2 = 0.02$, $\lambda_4 = 0.022$ and varying λ_3 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, and 0.09 in (44), one may obtain Table 5, whose column 4 shows variation of MTTF with respect to λ_3.

Setting $\lambda_1 = 0.002$, $\lambda_2 = 0.02$, $\lambda_3 = 0.02$ and varying λ_4 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, and 0.09 in (44), one may obtain Table 5, which reveals variation of MTTF with respect to λ_4.

Figure 5 provides the mean time to system failure (MTTF) of the system concerning variations in λ_1, λ_2, λ_3, and λ_4, respectively, when the other parameters have been taken as constants. The variation in MTTF corresponding to failure rate λ_1 decreases slowly, but the variation corresponding to λ_2, λ_3, and λ_4 decreases faster, while the variation of

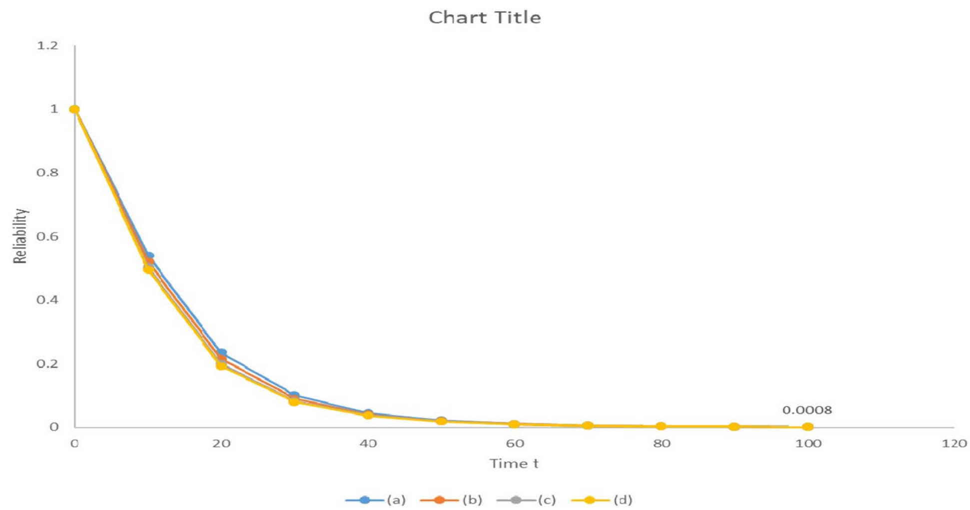

Fig. 4 Variation of reliability for different configurations

Table 4 Time vs. reliability for different configurations

Time t	(a)	(b)	(c)	(d)
0	1	1	1	1
10	0.5396	0.5205	0.5035	0.4956
20	0.2352	0.2150	0.1992	0.1927
30	0.1009	0.0896	0.0819	0.0790
40	0.0451	0.0400	0.0370	0.0360
50	0.0212	0.0192	0.0182	0.0179
60	0.0105	0.0097	0.0094	0.0093
70	0.0053	0.0050	0.0049	0.0049
80	0.0028	0.0027	0.0026	0.0026
90	0.0015	0.0014	0.0014	0.0014
100	0.0008	0.0008	0.0008	0.0008

Table 5 Failure rate vs. MTTF

Failure rate	MTTF (λ_1)	MTTF (λ_2)	MTTF (λ_3)	MTTF (λ_4)
0.01	7.1680	16.2567	15.1449	16.7524
0.02	4.9102	14.1722	14.1723	14.5441
0.03	3.9715	12.5684	13.2222	12.8592
0.04	3.4573	11.2911	12.3444	11.5255
0.05	3.1328	10.2477	11.5500	10.4409
0.06	2.9093	9.3783	10.8365	9.5404
0.07	2.7064	8.6423	10.1966	8.7804
0.08	2.6216	8.0110	9.6220	8.1300
0.09	2.5235	7.4636	9.1046	7.5672
0.10	2.4443	6.9844	8.6371	7.0753

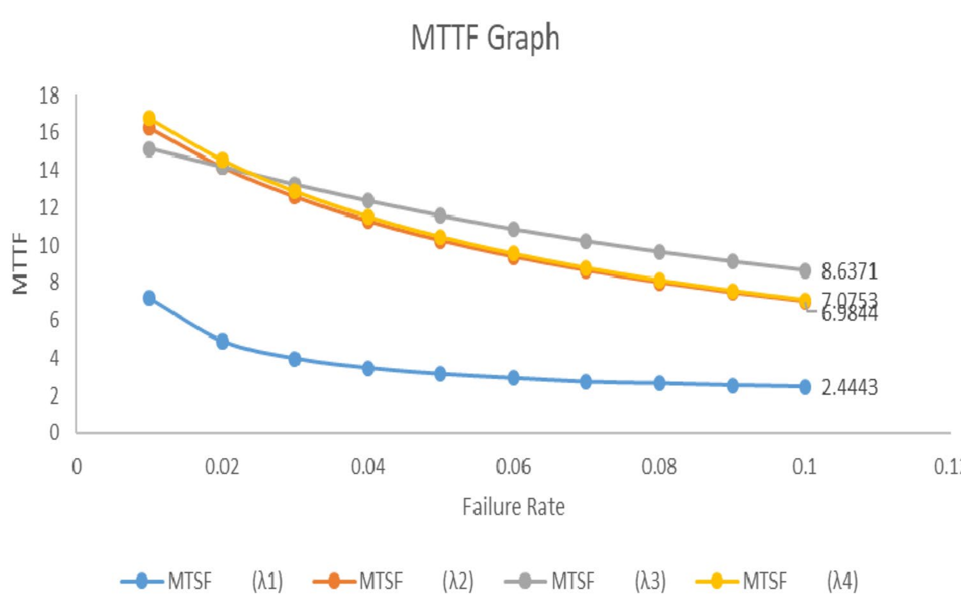

Fig. 5 Failure rate vs. MTTF

Table 6 Time vs. expected profit copula repair approach for $n=50$, $k=30$

Time t	$K_2=0.6$	$K_2=0.5$	$K_2=0.40$	$K_2=0.3$	$K_2=0.20$	$K_2=0.1$
0	0	0	0	0	0	0
10	3.090	4.0898	5.090	6.0898	7.0898	8.0898
20	5.6633	7.6633	9.6633	11.6633	13.6633	15.6633
30	8.0870	11.0870	14.0870	17.0870	20.0870	23.0870
40	10.3794	14.3794	18.3794	22.3794	26.3794	30.3794
50	12.5431	17.5431	22.5431	27.5431	32.5431	37.5431
60	14.5801	20.5801	26.5801	32.5801	38.5801	44.5801
70	16.4926	23.4926	30.4926	37.4926	44.4926	51.4926
80	18.2823	26.2823	34.2823	42.2823	50.2823	58.2823
90	19.9512	28.9512	37.9512	46.9512	55.9512	64.9512
100	21.5011	31.5011	41.5011	51.5011	61.5011	71.5011

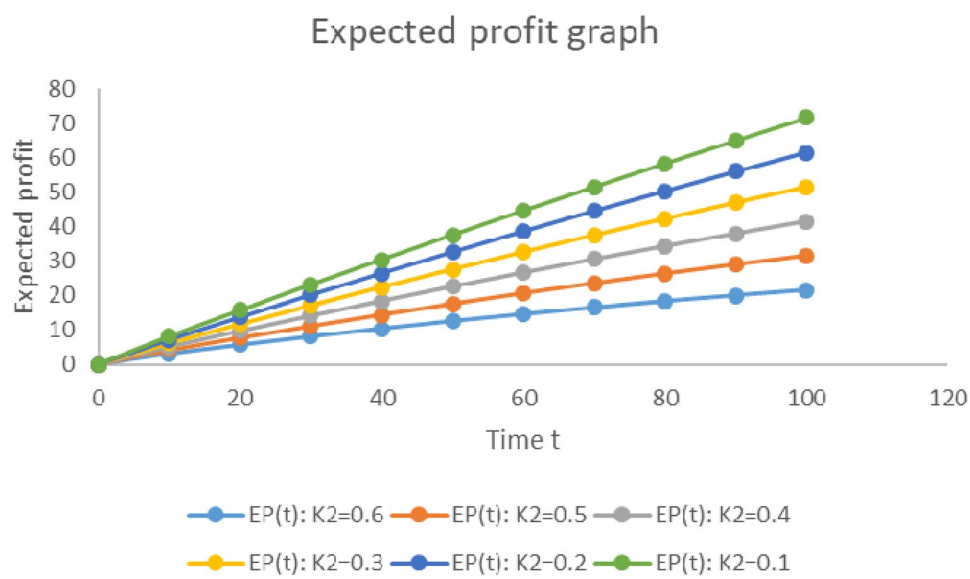

Fig. 6 Time vs. expected profit copula repair approach $n=40$, $k=30$

MTTF concerning λ_1, λ_2, λ_3 and λ_4 seem to be closely related and decreasing slowly.

8 Cost/profit analysis

Let the failure rates of the system be $\lambda_1=0.002$, $\lambda_2=0.02$, $\lambda_3=0.02$, and $\lambda_4=0.022$, mean time to repair be $\varphi(x)=1$ and $x=1$, $\theta=1$, $\varphi(x)=1$ setting in Eq. (42) and taking inverse Laplace transform, one can obtain the availability expression.

Let the service facility be always available, then the expected profit during the interval $[0, t)$ is

$$E_p(t) = K_1 \int_0^t P_{\text{up}}(t)\text{d}t - K_2 t, \tag{45}$$

where K_1 and K_2 are revenue service costs per unit time. Hence,

$$\begin{aligned} E_p(t) = {} & 0.006861404561e^{-1.245189414t} \\ & - 0.4251585799e^{-0.3127453416t} \\ & - 560.3151582e^{-0.001563144460t} \\ & - 0.004272451862e^{-1.076332100t} \\ & + 0.004513638618e^{-1.080000000t} \\ & + 560.7332140\ n = 50,\ k = 30. \end{aligned} \tag{46}$$

Setting $K_1=1$ and $K_2=0.6$, 0.5, 0.4, 0.3, 0.2 and 0.1, respectively, and varying $t=0$, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 units of time, one gets Table 6.

Figure 6 finally shows the expected profit increases concerning the time when the service cost $K_2=0.6$, 0.5, 0.4, 0.3, 0.2, and 0.1, respectively; one can see that as the service cost decreases, the profit increases. Finally, one can observe that when the service cost is low, the expected profit is increased.

9 Result and conclusion

Tables 2 and 3 and Figs. 2 and 3 provide information on how the availability of the complex repairable system changes with regard to the time when failure rates are fixed at different values. Fixing the failure rates as $\lambda_1 = 0.002$, $\lambda_2 = 0.02$, $\lambda_3 = 0.02$, $\lambda_4 = 0.022$, the system availability decreases and the probability of failure increases, with varying time t and ultimately becomes steady to the value zero after a sufficiently long interval of time. As a result, one may confidently forecast the future behavior of a complex system at any time for any given set of parametric values, as evidenced by the model's graphical representation. It has also been found for a fixed value of n, the system availability decreases for lower values of $k = 20$, 10, and 5. Table 3 and the corresponding figure present that general repair was not recommendable when the system was in a complete failed state.

Table 4 presents the reliability of the system for different values of time *t*. It is clear from Table 4 and Fig. 4 that the system efficiency of the non-repairable system is very low, compared to the repairable one. As higher values of time t are approached, the reliability approaches zero. It has also been seen that for reliability computations for fixed values of n and varying *k*, there is not much variation.

Table 5 yields the mean time to failure (MTTF) of the system for variation in λ_1, λ_2, λ_3, and λ_4, respectively, when the other parameters have been taken as constant.

When revenue cost per unit time K_1 is fixed at 1, service costs $K_2 = 0.6$, 0.5, 0.4, 0.3, 0.2, 0.01, profit was calculated and the results are demonstrated by graphs in Fig. 6. One can observe that as service costs decrease, the profit increases. Whenever the service cost increases the net profit starts decreasing for a high value of time.

The present study is focused on performance evaluations for four state repairable systems. The study can also be extended for the *k* state working system under $(k-1)$ minor degraded and *k*th major degraded state. Another interesting model can be also developed with more than two DDS servers and switching devices and another essential component of networking.

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