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REVIEW

Imperfect maintenance and proportional hazard models: a literature survey from 1965 to 2020

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Abstract
Most of the systems are treated as repairable in automobile, nuclear and aviation industries, and often subjected to imperfect maintenance. Reliability assessment of such systems is generally done by observing their failures which could be the consequences of bad environmental conditions, and thus, the mathematical modeling of failure data along with other risk factors becomes essential. The available literature provides mainly two types of models to deal with such situations: (1) traditional imperfect maintenance (TIM) models, and (2) proportional hazard (PH) models (PHM). These are two major categories of models for reliability modelling and analysis of lifetime data and are treated separately. To model and analyze practical industrial problems, at times it is unavoidable to use and combine TIM with PH models. Presently, the literature lacks in providing an organized literature survey on the development of both TIM and PH models together summarized at one place. Considering this as a research gap, we present an extensive literature survey, from 1965 to 2020, on the development of (1) TIM models, and (2) PH models. Through this work, our objective is to create a new paradigm for the academicians to appreciate overall development of above stated categorized models for future work. Most importantly, the yearwise organized framework of the presented paper will contribute as a ready reference for the researchers to appreciate the developments in both the fields.

Keywords Repairable systems · Proportional hazard models · Traditional models · Imperfect maintenance · Literature survey

1 Introduction

Reliability modeling and analysis of repairable systems (Ascher 1968) has always been a challenging task for the industries. Repairable systems are the systems which can be restored to an operating condition by some corrective maintenance (CM)/repair action after a failure occurs (Rigdon and Basu 1989). Moreover, such systems also undergo preventive maintenance (PM) to prevent them from failures. These maintenance actions restore the life of the system to some extent and provide them a new virtual age. This virtual age is dependent on the quality of maintenance actions done on the system. There are mainly three types of maintenance that could be performed on a system, namely: (1) Perfect maintenance; (2) Minimal Maintenance; (3) Imperfect Maintenance.

Perfect maintenance (Briš and Byczanski 2017) brings the systems to as good as new condition whereas minimal maintenance (Crow 1975), (Asher and Feingold 1984) restores the system to as bad as an old condition which means it brings the system at the same stage as it was before the failure. Imperfect maintenance renders the system's age between as good as new and as bad as old conditions. Perfect and minimal maintenance can be considered as two special cases of imperfect maintenance (Pham and Wang 1996), (Rai and Sharma 2017). Reliability modelling of repairable systems considering imperfect maintenance theory is being very much appreciated by researchers (Sharma and Rai 2018a), (Sharma and Rai 2020a) because of its applicability in real-life situations.

Generally, for reliability assessment of repairable systems, its failure profile is observed, but these failures could be the consequences of some bad environmental conditions

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(Rai and Sharma 2018), (Sharma and Rai 2020b). In this field, the development of literature has taken place in two different directions: (1) the development of ***traditional imperfect maintenance (TIM) models***, and (2) development of ***proportional hazard (PH) models (PHM)***. The merits and demerits of these two categories are briefly discussed in the subsequent paragraphs.

Traditional imperfect maintenance models consider imperfect maintenance theory and cater only to failure data of the systems without considering effects of other risk factors. These TIM models have been categorized into two segments (de Toledo et al. 2015) namely: arithmetic reduction of age (ARA) models and, arithmetic reduction of intensity (ARI) models. Arithmetic reduction of age models (Doyen and Gaudoin 2004), assume that the repair action reduces the increment in system age, whereas according to ARI models (Doyen and Gaudoin 2004), repair actions not only reduces the virtual age but also the failure intensity function of the system. Both ARA and ARI models are widely used and cherished by academicians, but the presented work is inclined towards ARA-based TIM models (Sharma and Rai 2020c; Rai and Bolia 2014a, b; Rai and Bolia 2014a, b).

As discussed previously, TIM models consider only time to failure data of a system for reliability analysis. Failure analysis merely based on failure times without considering other risk factors, because of which the failure has occurred, may not provide the accurate prediction of a system lifetime. In such cases, PH models play an imperative role by catering to the effects of risk factors, for example environmental condition, as covariates along with failure data for better prediction of system life (Cox 1972b; Kumar 1995a). Furthermore, the ability of these models to incorporate continuous and categorical variables, which have more than one category without any intrinsic order, along with time-dependent covariates makes it applicable for a more precise reliability analysis.

Both the categories (TIM and PH models) have their own advantages as discussed above. Thus, the combined study of both becomes an essential task to obtain realistic solutions. This approach of integrating models could be very helpful in portraying the overall failure/degradation profile of the system for more precise reliability modelling and analysis and hence can resolve a wider spectrum of real-life industrial problems. Some authors have attempted to combine these fields but still there is a scope for further development. Presently, there is no summarized review available in the literature that will seek the attention of the research fraternity towards these two major fields and this could be considered as a major research gap. Unavailability of a summarized framework and work review in these two fields collectively is the prime motivation of the work presented in this paper.

Hence, to seek the attention of researchers towards clubbing both the major streams, it is vital to provide an organized literature survey on the development of both TIM and PH models together as summarized in Fig. 1. Thus, in this work, we present an extensive but general literature survey on (1) development of various TIM models (focusing mainly on ARA models), and (2) development of PH models.

Our objective is to create a platform for researchers and academician to appreciate the overall development of TIM and PH models and encourage then to work further by combining these two fields. As an outcome of this literature survey, we also highlight the shortcomings of present literature and future scope for the research fraternity working in this field.

The paper is organized as follows; "Development of TIM models" deals with the development of TIM models, "Development of PH models" presents the development of PH models, "Observations and limitations" presents the observations and limitations and "Conclusion and future scope" concludes the paper. The development of TIM models from the year 1965 to 2020 is presented in the next section.

2 Development of TIM models

As discussed earlier, the TIM models consider only failure data without considering other risk factors on the system while performing reliability analysis. The models assume that any type of maintenance action (either PM or CM) restores the life of system and provides it with a new virtual age (Bolia and Rai 2013). This section is dedicated to the TIM models developed from 1965 to till date. The section has been divided into five subsections i.e. development of TIM models: (1) from 1965 to 1990, (2) from 1991 to 2000, (3) from 2001 to 2010, (4) from 2011 to 2020 and the last sub-section provides the details of some review papers in this field.

Fig. 1 Keywords cloud

2.1 From 1965 to 1990

In the history of reliability modelling and analysis, a boom towards the reliability study of complex systems was essentially observed after 1965 with the book written by Barlow and Proschan (Barlow and Proschan 1965). With this, the orientation of researchers shifted more towards modeling and analysis of complex repairable systems. In 1968, Ascher (Ascher 1968) came with the concept of "Bad-As-Old" which was also called time dependent (non-homogeneous) Poisson process. He described that after repair, the failure rate of a repairable system could not be considered as zero, rather it remains same as it was immediately before failure, and such repair is called minimal repair. He also proposed a mathematical model to address as bad as old condition. The same concept of Bad-As-Old dealt by L.H. Crow in 1975 (Crow 1975) assuming that the failure process can be described by power-law process as shown in Eq. (1) and this was called non-homogeneous Poisson process (NHPP).

$$u(t) = a \times \beta \times t^{\beta-1}; a > 0, \beta > 0 \tag{1}$$

where$u(t)$, a and β are the intensity function at time t, scale parameter and shape parameter of the model, respectively.

In this work, Crow proposed the maximum likelihood estimators, goodness to fit test, reliability growth concept and maintenance policies for repairable systems considering NHPP.

With the advancement of technology, academicians started providing more attention towards imperfect maintenance models. The concept of imperfect PM and imperfect repair was introduced which was commonly called as imperfect maintenance.

With minimal repair, the concept of minimal PM was also developed in similar lines by assuming that the restoration of the system depends on the resources available at the time of PM activity (KAY 1976), (Chan and Downs 1978). Nakagawa (1979) developed (p, q) rule in which he assumed that after PM, the system returns to as good as new condition with probability p and returns to as bad as old condition with probability $q(q = 1 - p)$. In 1979, Malik (1979) presented the idea of improvement factor in maintenance scheduling.

Later, Brown and Proschan (1983) developed a new model with similar concept as Nakagawa (1979) for repair activity. They proposed that if the life distribution of a system is F and failure rate is r, then distribution of time between failures (TBF) and correspondence failure rate will be $F_p = 1 - (1 - F)^p$ and $r_p = pr$, respectively. Block et al. (Block et al. 1985) extended the concept of Brown and Proschan (Brown and Proschan 1983) with the assumption that after repair the systems return to as good as new condition with probability $p(t)$ and as bad as old condition with probability $q(t)\left[q(t) = 1 - p(t)\right]$ where t is the age of the system. Shaked and Shanthikumar (1986) introduced the concept of multivariate imperfect repair considering (p, q) rule for a system consist of n components.

Again, in 1986, two revolutionary virtual age models, for imperfect repair, was proposed by Kijima and Sumita. Kijima and Sumita (1986), (Kijima 1989) proposed two virtual age models called Kijima I (KI) and Kijima II (KII) models (as explained below) to define the imperfect repair by introducing a new factor in power-law process called repair effectiveness index (REI) 'q_{CM}' representing repair quality. This process was called generalized renewal process (GRP) in which the repair time was assumed to be negligible.

KI model: This model assumes that the repair restores the accumulated damage during time between *ith* and $(i-1)th$ failure as shown in Eq. (2).

$$V_i = V_{i-1} + q_{CM} y_i (i = 1, 2, \dots .p) \tag{2}$$

KII model: This model assumes that the *ith* repair action at any point of time restores the entire accumulated age since new as shown in Eq. (3).

$$V_i = q_{CM}\left(V_{i-1} + y_i\right)(i = 1, 2, \dots .p) \tag{3}$$

where V_i *is* virtual age after *ith* repair; y_i*is* time between failures; t_i is time of *ith* failure; p is number of failures and q_{CM} is Repair Effectiveness Index (REI) and $V_0 = 0$.

$$q_{CM} = \begin{cases} 0; & \text{for perfect repair} \\ 0 < q_{CM} < 1; & \text{for imperfect repair} \\ 1; & \text{for minimal repair} \end{cases} \tag{4}$$

These Kijima models were very much appreciated by the research fraternity because of its applicability towards repairable systems and adaptability of perfect and minimal repair as its special cases (Yanez et al. 2002).

Thus, in this period, various new concepts of imperfect maintenance were introduced to deal with reliability analysis of repairable systems. After 1990, an exponential growth was observed in this area.

2.2 From 1991 to 2000

In this decade, various extensions of available imperfect maintenance models along with some new models were proposed, such as, in 1991, Kijima and Nakagawa (1991) proposed a shock model for imperfect periodic PM. According to this model, the occurrence of shock causes a system to experience a non-negative damage. Occurrence of each damage adds to the current level damage and the system fails when a cumulative level of damage crosses a threshold level. Authors consider imperfect PM in the sense that PM reduces the damage level by $100(1 - b)\%$, where $0 \le b \le 1$ of total damage. $b = 1$

and 0 for minimal and perfect PM, respectively. In the same year, Festus O. Olorunniwo, Ariwodo Izuchukwu (Olorunniwo and Izuchukwu 1991) developed a mathematical model for preventive and overhaul maintenance activity by incorporating the concept of improvement factors. The improvement factor was used to indicate the quality of maintenance actions.

In 1992, Makis and Jardine (1992) proposed $(p(n,t), q(n,t), s(n,t))$ rule for imperfect maintenance, according to which, after repair, system returns to as good as new condition with probability $p(n,t)$, as bad as old condition with probability $q(n,t)$ or with probability $s(n,t) = 1 - p(n,t) - q(n,t)$ the repair is unsuccessful, where t and n is age of system and number of failures in system.

Later, Finkelstein (1993) developed a general imperfect repair model based on scale transformation. According to this model, if, before failure, the failure distribution of a repairable unit is $F(x)$, then, after repair, it would be $F(ax)$, where a is scale parameter. In 1997, Dagpunar (1997) developed a new model inspiring from KII model. The model assumes that the initial age of the systems is a specific value $V_0 = s$ rather than 0 as assumed in KII model.

Considering three situations i.e. minimal repair, periodic overhaul and complete renewal, Zhang and Jardine (1998) presented a mathematical model to describe the improvement in system due to maintenance as shown in Eq. (5) The authors claim that the improvement due to overhaul does not follow virtual age approach. The authors consider a direct reduction in failure rate and determine the optimal intervals for overhaul by minimizing total cost.

$$v_k(t) = pv_{k-1}(t-s) + (1-p)v_{k-1}(t) \tag{5}$$

where $v_{k-1}(t)$ and $v_k(t)$ are the failure rates of the system before and after overhaul, respectively. p is improvement factor and s is overhaul interval. For $p = 0, v_k(t) = (1-p)v_{k-1}(t)$ and for $p = 1, v_k(t) = pv_{k-1}(t-s)$.

Jack (1998) provided two different age-based models as shown in Eqs. (6)–(9) along with its likelihood function considering CM and PM both as imperfect.

Model I

$$v_{ij} = v_{i-1,j} + \delta_{CM}\left(t_{ij} - t_{i-1,j}\right), \tag{6}$$

$$v_{0j} = v_{0,j-1} + \delta_{PM}\left(v_{n_{j-1},j-1} - v_{0,j-1} + t_{0j} - t_{n_{j-1},j-1}\right), \tag{7}$$

Model II

$$v_{ij} = \delta_{CM}\left(v_{i-1,j} + t_{ij} - t_{i-1,j}\right), \tag{8}$$

$$v_{0j} = \delta_{PM}\left(v_{n_{j-1},j-1} + t_{0j} - t_{n_{j-1},j-1}\right) \tag{9}$$

where t_{ij} = time of ith failure in the jth PM interval ($j = 1, \ldots, k; i = 1 \ldots .n_j$), t_{0j} = time of $(j-1)th$ PM, v_{ij} = virtual age following ith repair in the jth PM interval, v_{0j} = virtual age following $(j-1)th$ PM, with $v_{01} = 0, \delta_{CM} and \delta_{PM}$ are age-reduction factors.

In 1998, Lim et al. (1998) presented an extension of the model developed by Brown and Proschan (1983). The authors proposed a new Bayesian imperfect repair model assuming the probability of perfect repair p as a random variable. Considering a prior distribution for p, they estimated the time between two perfect repairs and its failure rates.

In the year 2000, Marco Scarsini and Moshe Shaked (2000) introduced a model to express the monetary value of an item. The study essentially uses the concept of Kijima models (Kijima 1989) and Kijima and Nakagawa shock model (Kijima and Nakagawa 1991) for the development of the proposed model. The proposed model considers the rate of benefit derived over time wherein the system undergoes repair and PM.

Although, based on the concept of GRP, various models were developed in this period, but the concept of GRP grabbed the attention of researchers in the next decades.

2.3 From 2001 to 2010

The research carried out in this decade was mainly oriented towards exploring the concept of GRP. In 2002, Yan˜ez et al. (2002) discussed the GRP in depth and developed likelihood and maximum likelihood estimators (MLEs) for using KI (GRP) model. Also, they proved that RP and NHPP are the special cases of GRP. To analyze the impact of repair on intensity function, Gasmi et al. in 2003 (Gasmi et al. 2003) proposed a statistical model considering two situations: (1) the system is loaded with the operation, (2) system is in unloaded state wherein system is operating mechanically but failure intensity is lower due to less operating intensity. Three types of repair were considered in modelling: minimal, minor and major where minor and major repairs follow KII model. For the first and second situation, authors assumed KII model and PH models as base models, respectively.

Seo and Bai (2004), proposed a model for the systems which undergo periodic overhauls considering the repair as minimal. The paper uses a fixed multiplier ϴ in Kijima model to capture the effectiveness of the overhaul process and provides cost models when maintenance time is (1) negligible (2) non-negligible. Based on the proposed model, optimal number of overhauls and interval between overhauls are determined.

Cassady et al. (2005), presented a simulation modelling and analysis using the concept of KI and KII models. They proposed a generic availability function and determine cost-based optimal replacement interval for repairable systems.

Again, in 2005, Mettas and Zhao (2005) explored the GRP model to analyze complex multiple repairable systems. They proposed GRP general likelihood function for single and multiple repairable systems and developed fisher information matrix-based confidence bounds for the same.

Pedro Jimenez and Rau'l Villalon (2006), demonstrated that GRP has a property of adaptability and suggested to model GRP as three0-parameter Weibull distribution using the least square method. The authors developed MAPLE language code for the same. Kaminskiy and Krivtsov (2006) proposed Monte Carlo approach for parameter estimation of GRP in 2006. Pascuala and Ortega (2006) proposed a failure rate model as given in Eq. (10), based on an improvement factor P considering three actions i.e. repair, overhaul and replacement, wherein repair and overhaul are considered as minimal and imperfect maintenance, respectively. The authors also determined optimal life-cycle duration and interval for overhauls.

$$\lambda_k(t) = p\lambda_{k-1}\left(t - T_s\right) + (1 - p)\lambda_{k-1}(t) \tag{10}$$

where $\lambda_k(t)$ is failure rate after kth overhaul, p is improvement factor $p \in (0, 1)$, T_s interval time between overhauls (constant).

A new model for general repair was presented by Guo et al. (2007) which considers the repair history of the system into account. The paper provides the closed-form solution for the proposed model. Maxim Finkelstein (2007) discussed some ageing properties of general repair considering the assumptions of KII model. The author proved that the expectation of age at the starting of the next cycle is less than the initial age of the last cycle. Sevˇc´ık (2007) discussed the impact of repair in repairable systems considering that: (1) repair affect the intensity function according to Kijima's models, (2) repair as time-dependent scale transformation.

In 2008, Veber et al. (2008), argued that, in GRP, a finite Weibull distribution with component's weight should be used as a time to first failure distribution instead of normal Weibull distribution and applied EM algorithm for GRP parameter estimation. Later, Yu et al. (2008) developed a KI-based virtual age model for both imperfect CM and PM (Eq. (11)). The paper uses Bayesian method for GRP parameter estimation.

$$V_i = \begin{cases} a_r X_i \; if \; CM \; is \; performed \\ a_p X_i \; if \; PM \; is \; performed \end{cases} \tag{11}$$

where V_i is virtual age after ith maintenance action, X_i is time between two failures, a_r and a_p are the maintainability characteristics of a repairable system.

Moreover, in 2009, Yann Dijoux (2009) developed a reliability model by combining bath tub-shaped ageing with imperfect maintenance (repair). Laurent Doyen (2010) discussed the asymptotic properties of imperfect repair models. Also, with the help of Bayesian approach, Shey-Huei Sheu and Chin-Chih Chang (2010) generalized the multivariate imperfect repair models.

In the next decade also the orientation of research was mainly slanted towards extensions of Kijima models.

2.4 From 2011 to 2020

Development of models in this period was also influenced by GRP models because of its applicability in real world industrial problems. In 2011, Syamsundar et al. (Syamsundar and Naikan 2011) developed ARA and ARI models by combining imperfect repair models and proportional intensity models to obtain more practical results for repairable systems. In 2012, Yuan Fuqing and Uday Kumar (2012) proposed KI and KII models based a new virtual age model (Eq…) which considers REI as a function of time instead of a constant value.

$$v_i = \left(1 - z\left(t_i\right)\right)\left(v_{i-1} + x_i\right) \tag{12}$$

where v_i is virtual age after ith repair, x_i is time between failures $\left(t_i - t_{i-1}\right)$ and $z(t) = exp(-et^c)$ a time-dependent function confined to [0,1]. When $e = 0$, the model is RP model; When c = 0, it becomes KII model; When $e \to \infty, c \to \infty$, the model is NHPP model.

Olexandr Yevkin and Vasiliy Krivtsov (2012) provided an approximate solution for GRP which was simpler than monte carlo simulation technique. Corset et al. (2012) presented Bayesian analysis solution technique for imperfect repair ARA models. Zhi-Ming Wanga and Jian-Guo Yang (2012) developed a nonlinear programing approach for GRP parameter estimation by considering likelihood function as an objective function.

Yu et al. (2013) developed an analytical method for GRP parameters estimation which does not rely on simulation since simulation-based method could be time-consuming method. Based on the developed method, the authors estimated the mean residual life after each repair and calculated the time to the next failure. Moreover, in 2013, Nasr et al. (2013) extended the Kijima virtual age models considering both CM and PM as imperfect and provided the likelihood function and MLEs for both the proposed models and compare the results with Yu et al. work (Yu et al. 2008).

Modified KI model

$$V_i = \sum_{j=1}^{i} a_r^{\delta_j} a_p^{1-\delta_j} x_j \tag{13}$$

Modified KII model

$$\mathrm{V_i} = \sum_{\mathrm{j=1}}^{\mathrm{i}} a_{\mathrm{r}}^{(i-j+1)\delta_{\mathrm{j}}} a_{p}^{(i-j+1)1-\delta_{\mathrm{j}}} \mathrm{x_j} \tag{14}$$

where $\delta_i = \left\{ \begin{array}{l} 1\, ifCM \\ 0\, ifSPM \end{array} \right\}$, x_i is time between maintenance $(i = 1, 2, \ldots, n)$ and n is number of interventions.

Later, for PM interval estimation for deteriorating systems, Harish Garg et al. (2013) provided a cost-based maintenanace model by considering three actions i.e. mechanical service, repair and replacement for a multi-components system. The applicability of the proposed model is nicely explained considering pulping unit of a paper mill as a case.

Peng Wang and Yisha Xiang (2014) presented a new general repair model which incorporates the virtual age concept and repair effectiveness parameters to reflect repair quality. Authors assume the REI as a function of repair quality rather than assuming it as a constant.

In 2015, Rai and Bolia (2015) proposed a failure modes and effect analysis (FMEA) model by replacing occurrence (O) in risk priority number (RPN) with REI and showed that the failure probability of a system gets affected by REI. Later, in 2018, considering Kijima model, Sharma and Rai (2018b) proposed a model to estimate RPN considering REI a function of various subjective factors such as environment, resources, procedure and skill. The authors described the proposed model by taking space station environmental control and life support system (ECLSS) as a case.

Recently, Liu et al. (2020) discussed the asymptotic properties for steady-state virtual age processes. The paper shows that limiting distributions of age, residual life and spread can be generalized to a stable virtual process which is generally described as the ordinary renewal process. The asymptotic distributions are derived mainly for ARA and Brown and Proschan models. Syamsundar et al. (2020b) developed a new alternative scale for reliability analysis considering the throughput or usage of the repairable systems rather than time. Moreover, Syamsundar et al. (2020a) proposed accelerated failure time models to quantify the effects of different factors such as temperature, stress, and pressure on repair process. Sharma and Rai extensively studied the factors which affect REI the most and modeled their inter-dependability using Bayesian networks (Sharma and Rai 2020c), (Sharma and Rai 2020b). Sharma and Rai (2020d), proposed KI based new virtual age model as shown in Eq. (15) and likelihood function which is able to consider each intervention done on the system i.e. CM, scheduled PM and overhaul as imperfect at the same time. The authors divided PM activity into two category: (1) scheduled preventive maintenance (SPM), and (2) Overhaul, and treated them separately while estimating the model parameters.

$$\mathrm{V_i} = \left(\mathrm{V_{i-1}} + \mathrm{q_{CM}x_i}\right)^{\delta_i}\left[\left(\mathrm{V_{i-1}} + \mathrm{q_{SPM}x_i}\right)^{1-h_i}\left(\mathrm{V_{m-1}} + \mathrm{q_O X_m}\right)^{h_i}\right]^{1-\delta_i} \tag{15}$$

where V_i and V_m are virtual age at any ith intervention with $\mathrm{V_0} = 0$ and virtual age at mth overhaul, respectively; $\mathrm{x_i} = \mathrm{t_i} - \mathrm{t_{i-1}}$($\mathrm{t_i}$ is time to ith intervention whether SPM or CM) and $\mathrm{X_m}$ = time between mth and $(m-1)th$ overhauls ($\mathrm{t_m}$ is time to mth overhaul); $\mathrm{q_{CM}}, \mathrm{q_{SPM}}, \mathrm{q_O}$ are restoration factor for CM, SPM, Overhaul, respectively.

Vijay kumar et al. (2021) proposed a novice methodology to select best software reliability growth models by considering the technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) approach. The proposed methodoligy identifies the relative importance criteria and helps the decision makers to select the suitable reliability growth model for a particular application.

In addition, to deal with failure modes-wise censored data in the repairable systems, Sharma and Rai (2021a) proposed a methodology and virtual age models that are based on the concept of Kijima models and demostrated their applicability with the help of data set obtained from the aviation industry. The authors also proposed a progreesive maintenance policy (Sharma and Rai 2021b) considering imperfect maintenance in repairable systems. According to the proposed policy age-based maintenance time (Sharma and Rai 2021c), SPM intervals and overhaul time of a systems should be revised after each service activity.

With the development of various imperfect maintenance models, researchers also paid attention on summarizing the literature in the form of some review papers. The next section summarizes the details of some important review papers available in this field.

2.5 Some review papers in this field

There are some important review papers available in the literature which could be very helpful in understanding the development of model and policies related to imperfect maintenance in repairable systems. For example, a detailed review on early evolution of imperfect maintenance models and maintenance policies were presented by Pham and Wang (1996) and Guo et.al. (2000). Later in 2008, Muralidharan (2008) presented a review on repairable systems and point process models available in the literature. Hussain and Naikan (2010) studied point process maintenance models used for repairable systems. Tanwar et al. (2014) presented an exhaustive literature survey on the ARA and ARI methods available in the literature with a main focus on Kijima type GRP. Yuan and Lu (2015) presented a review on the imperfect repair model for repairable systems.

In the subsequent sections, the development of PH models is presented from the year it was introduced.

3 Development of PH models

Along with the development of TIM models, the researcher realized the need to develop a survival model by considering the effect of various external or internal factors that affect the failure process. In the TIM models, the lifetime of a system is estimated by time to failure data. However, in real-life applications, various other risk factors influence the failure data and are named as covariates. In this section, we present the development of PH models since it was introduced. This section has been divided into four subsections, i.e., development of PH models: (1) from 1972 to 1990, (2) from 1991 to 2000, (3) from 2001 to 2010, and (4) from 2011 to 2020.

3.1 From 1972 to 1990

In this decade, researchers understood the need to consider various risk factors that influence the hazard rate. Cox in 1972 (Cox 1972b) developed a survival model known as proportional hazard model (PHM), which explains that the hazard is a function of the known variables and unknown regression coefficients multiplied by an arbitrary and unknown function of time. Mathematically, the Cox PHM model for a static explanatory variable (time-independent) can be expressed as:

$$\lambda(t;Z) = \lambda_o(t)\,\psi\,(Z\beta) \tag{16}$$

$$\lambda(t;Z) = \lambda_o(t)\,\exp\,(Z\beta) \tag{17}$$

where, $\lambda(t;Z)$ is a hazard rate function of time (t) and covariates (Z) and $\lambda_o(t)$ is a baseline hazard function of time. It provides the hazard rate function of equipment if the effect of covariates is considered to be zero ($Z = 0$). Z is a covariate in a row vector form, and β is a regression parameter in a column vector form. The effect of covariates on the observed failure time is defined by the unknown regression parameter vector β. Covariates of a system have a significant impact on hazard function, which influences the time to failure of a system, as shown in Fig. 2.

However, if two or more explanatory variables do not seem to have a multiplicative effect on the hazard rate, PHM is not applicable. To overcome this, Prentice et al. in 1981 (Prentice et al. 1981), introduced the Stratified PHM, which was based on stratifying the data into levels corresponding to the various inter-failure times in the life cycle for each component. They also proposed the two general classes of models, based on two-time scales; global time t and the time from the immediately preceding failure $t - t_{n(t),}$ respectively:

$$h(t/N(t);Z(t)) = h_{0j}(t)\,\exp\,\left(Z(t)\gamma_j\right) \tag{18}$$

$$h(t/N(t);Z(t)) = h_{0j}\left(t - t_{n(t)}\right)\,\exp\,\left(Z(t)\gamma_j\right) \tag{19}$$

where, $N(t)$ is a random variable for the number of failures in $(0, t]$, and $Z(t)$ denotes the time-dependent

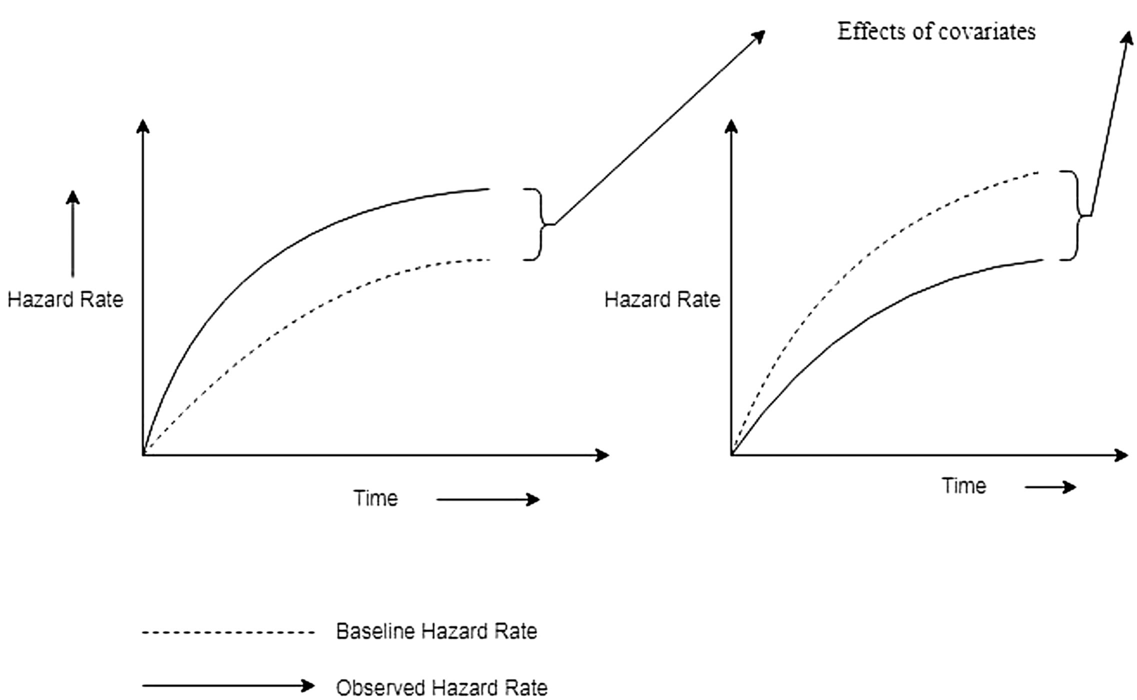

Fig. 2 Effect of covariates on hazard rates

covariates.$h(t/N(t);Z(t))$ and h_{0j} are the intensity functions and the baseline intensity function, respectively, and γ_j is the regression coefficient for the *j*th stratum.

Furthermore, Steve Bennett et al. (1983) developed a proportional odd model that is widely used in the medical field for survival analysis. In PHM, the ratio of hazard rate is constant with respect to time, thus ruling out the situation in which the hazard rate converges to 0 as t tends to infinity. The assumption of PHM becomes unreasonable when the initial effect diminishes with time. The POM property of being convergent hazard function is suitable for survival analysis modeling with time-dependent explanatory variables. Mathematically, it can be expressed as

$$\theta(t) = \frac{F(t)}{1 - F(t)} \tag{20}$$

$$\theta_i(t;Z) = \theta_0(t)\exp\left(\sum_{j=1}^{p} \beta_j Z_{ij}\right) \tag{21}$$

where, $\theta_i(t;Z)$ is the odd function, which is the survival probability of *ith* unit beyond time t with a vector of covariates Z $(Z_{i1}, Z_{i2}, Z_{i3} \dots\dots\dots\dots Z_{ip})^{t}$.

In 1985, Bendell (1985) applied the PHM for reliability assessment. He explained the need for PHM for a repairable or non-repairable system with the help of censored or uncensored data and concluded that PHM has enormous potential and can give invaluable information about the system condition.

Later, Jardine et al. in 1985 (Jardine and Anderson 1985) proposed a special case of PHM known as Weibull PHM. In their model, the failure times are assumed to follow a Weibull distribution. The work reviewed by the authors applies the PHM with the baseline hazard function as a Weibull distribution. This method allows the developed model to estimate the Weibull scale and shape parameters, which further implies that the same Weibull parameters are applicable for the two or more covariate levels. The proposed Weibull PHM is expressed as:

$$\lambda(t;Z) = \lambda_o(t)\exp(Z\beta) = \frac{\gamma}{\theta}\left(\frac{t}{\theta}\right)^{\gamma-1}\exp\left(\sum_{j=1}^{q} \beta_j Z_j\right) \tag{22}$$

where $\gamma \& \theta$ represents the shape and scale parameter of Weibull distribution, respectively.

In 1986, Bendell et al. (1986) studied the reliability analysis of brake discs of high-speed trains using PHM. Brake discs are considered as a non-repairable system that eliminates the complexity associated with repairable systems. They implemented this technique and explained the need for PHM in its simplest form for the system's reliability aspect.

In 1987, Jardine et al. (1987) analyzed the failure data of aircraft and marine engines using Weibull PHM. The authors investigated the effect of concomitant variable as a covariate such as metal particle present in the engine oil. The work showed that the fully parametric model provides information about the equipment age and the concomitant variable's influence on failure time.

In the next decade, researchers mainly concentrated on applying and developing PHM for repairable system reliability analysis.

3.2 From 1991 to 2000

In this decade, various extension models were developed in which PHM was used for reliability analysis for repairable systems and to schedule preventive maintenance for the system.

In 1992, Guo and Love (1992) proposed an imperfect repair proportional intensity model for repairable system reliability assessment. Mathematically, it can be expressed as

$$\lambda\left(t/H_{t-}\right) = \lambda\left(x + V_{N_{t-}}\right)\exp(\gamma Z) \tag{23}$$

where, $\lambda\left(t/H_{t-}\right)$ is the conditional intensity process conditioned on the past history of the process, H_{t-} is the history of the process or available data just prior to time t or the collection of all events observed on [0, t] and $V_{N_{t-}}$ is the virtual age of a system.

In 1995, Kumar (1995a) studied the PHM and its extension model for repairable system reliability assessment. The authors explained the reliability model considering the covariate as explained in the subsequent paragraph. Let $H(t;z1)$ and $H(t;z2)$ be the hazard rates associated with the covariate sets z_1 and z_2, respectively, for any observed time 't'. Then the ratio of the hazard rates is assumed to be constant with respect to time 't' and proportional to each other. Due to this proportionality, the model is known as the proportional hazard model as shown in Eq. (24) below.

$$\frac{H\left(t;Z_1\right)}{H\left(t;Z_2\right)} = \frac{H_o(t)\exp\left(Z_1\beta\right)}{H_o(t)\exp\left(Z_2\beta\right)} = \exp\left[\beta\left(Z_1 - Z_2\right)\right] \tag{24}$$

After discussing Cox PHM for the static explanatory variable, Cox PHM for the dynamic explanatory variable (time-dependent) can be expressed as:

$$\lambda(t;Z) = \lambda_o(t)\exp(Z(t)\beta) \tag{25}$$

where, $Z(t)$ represents the row vector consisting of the covariates, which is time-dependent. If the functional form of $H(t;z)$ is not specified. Then, the likelihood function cannot be derived. Hence, the maximum likelihood equation cannot be formed for regression parameter estimation. Therefore, to estimate the regression parameter (β), cox developed a non-parametric partial likelihood method explained in the subsequent paragraph.

The partial or conditional likelihood is defined as the product of the conditional probabilities of occurrences of a failure event at a time 'x_i', over all such failure events. Let F_{x_i} be the risk set of the failure events and let there be l failure events (censored or uncensored) that have not occurred prior to the failure event at time 'x_i'. Then, mathematically it can be expressed as:

$$\frac{\lambda\left(t_i;Z\right)}{\sum_{l\in F(x_i)}\lambda\left(t_i;Z_l\right)}=\frac{\exp\left(Z_i\beta\right)}{\sum_{l\in F(x_i)}\exp\left(Z_i\beta\right)} \quad (26)$$

$$L(\beta)=\prod_{i=1}^{k}L_i(\beta)=\prod_{i=1}^{k}\frac{\exp(Z_i\beta)}{\sum_{l\in F(x_i)}\exp(Z_i\beta)} \quad (27)$$

where, $L_i(\beta)$ is the conditional probability that a failure occurred at the time t_i and d_i is the number of tied failures at each point. An approximation of the above partial likelihood function is given by

$$L(\beta)=\prod_{i=1}^{k}\frac{\exp(Z_i\beta)}{\left[\sum_{l\in F(x_i)}\exp(Z_i\beta)\right]^{d_i}} \quad (28)$$

The basic and PHM reliability equations are given below as Eq. (29) and Eq. (30):

$$R_0(t)=\exp\left(-\int_0^t\lambda_o(x)\,dx\right)=\exp\left(-H_0(t)\right) \quad (29)$$

$$R(t,x)=\exp\left(-H_0(t)\right)\exp\left(\sum_{j=1}^{q}\beta_j x_j\right) \quad (30)$$

where, $R_0(t)$ is the baseline reliability function dependent only on time,$H_0(t)$ is the cumulative hazard rate, and $R(t,x)$ is the total reliability function dependent on time and covariates.

In 1996, Huamin Liu et al. (Liu and Makis 1996) applied AFTM for reliability analysis of cutting tools under various conditions, representing the cutting tool failure time distribution. They obtained the unknown parameters of the cutting tool either in fixed conditions or variable conditions using the maximum likelihood method. In 1997, Eliashberg et al. (1997) considered purchase time and used mileage as an explanatory variable for PHM for automobile warranty analysis.

In 1997, kobbacy et al. (1997) applied PHM for scheduling preventive maintenance based on the equipment's full condition history. They explained the proposed approach by considering four pumps historical data working in a continuous process industry. They fitted two PHMs, one for life following Preventive Maintenance and another corrective work with suitable explanatory variables.

Percy et al. in 1998 (Percy et al. 1998) introduced the applications of PIM incorporated with various repair history data considered as an explanatory variable. They performed a Bayesian approach to estimate the regression coefficient. The repair data considered as an explanatory variable follows; system age, times since last preventive maintenance (PM) and corrective operation (CO), the total number of PMs and COs, downtime, and severity measure of failure. In their paper, PIM with repair data had a significant role in reliability analysis to schedule PM in a cost-effective manner.

In the next decade, PHM has been applied extensively for the mechanical system under operational, environmental, and maintenance factors as a covariates.

3.3 From 2001 to 2010

During this period, most of the paper focused on selecting appropriate covariate that affects the failure process. Although, few developments were also made in this duration.

In 2001, Rao and Prasad (2001) analyzed the failure data and planned maintenance interval for material handling equipment used in the mining industry, such as loaders, trucks, dozers, and dumpers,. Here, PHM is utilized to study the performance of the repairable system affected by its concomitant variables. The maintenance interval of the equipment is evaluated using the Graphical method.

In 2002, Krivtsov et al. (2002) studied the automobile tire's reliability using a cox survival model. The tire geometry and physical properties are considered an explanatory variable that potentially affects the life of the tire on test. It also explains the application of the linear regression model for failure initiation and propagation modelling. They concluded that the result should be focused on comparing the tire's design based on a reliability point of view rather than predicting a tire's actual field reliability.

In 2002, Prasad and Rao (2002) considered the effect of operating conditions as a covariate for the reliability modeling of a repairable system. They applied the PHM technique to study the failure behavior of electro-mechanical equipment and small DC motor used in underground coal mines at different operating conditions. They performed graphical and analytical methods to determine the optimal preventive maintenance interval with minimal repair and illustrated this model by taking another example of a thermal power unit. Failure due to boiler, electrical, and turbines is selected as covariates.

In 2002, Vlok et al. (2002) considered the vibration monitoring data to determine the optimal replacement policy for a critical item. The authors illustrated the model by choosing circulating pumps used in a coal wash plant. The failure data was collected for two years time period. The study shows that the proportional hazards modeling can be integrated with vibration measurements to obtain a useful decision policy.

Gasmi et al. in 2003 (Gasmi et al. 2003) analyzed the complex repairable system performance operated in loaded and unloaded mode using PHM. They explained the model with the help of hydro power turbine data. The model consists of failure data incorporating both operating conditions and repair effects for maintenance planning. Data are collected for a period of one year. Altogether 466 events are recorded, in which 142 failures, 60 major repairs, 88 minor repairs, and remaining are minimal repair. In their research, the system's failure intensity due to the switching of operating models was analyzed. This model's main aim is to measure the system's failure intensities under various repair actions.

Sun et al. in 2006 (Sun et al. 2006) proposed a new model named as Proportional Covariate Model (PCM) to estimate the hazards of mechanical systems. It assumes that covariates of a system are proportional to the hazard of the system. In this model, covariates are considered as response variables and hazard as an explanatory variable. Mathematically, it can be expressed as;

$$\mathrm{Z}_r(\mathrm{t}) = \mathrm{c}(\mathrm{t})\mathrm{h}(\mathrm{t}) \tag{31}$$

where, $\mathrm{Z}_r(t)$ represents the time-dependent covariate function, $\mathrm{c}(t)$ is the baseline covariate function, which is also time-dependent and $h(t)$ is the hazard function of a system. Covariate function (t) considers both historical failures and historical condition monitoring data. However, this model is not sensitive to operating environmental data.

Elsayed et al. in 2006 (Elsayed et al. 2006) proposed a new model called the extended linear hazard regression model by generalizing the EHL and PH model. The authors considered the effect of proportional-hazards, the time-scale changing as well as the time-varying coefficients to develop this model. In this model, the coefficients are varying linearly with time. Mathematically, it can be expressed as:

$$\lambda(t;z) = \lambda_0(texp\left(\left(\beta_0 + \beta_1 t\right)z\right) exp\left(\left(\alpha_0 + \alpha_1 t\right)z\right) \tag{32}$$

where, α, β are the unknown regression coefficient and $\lambda_0(t)$ is the unspecified baseline hazard function. The covariate z is considered both the time-scale changing effect and hazard multiplicative effect. They analyze the laboratory data of n-type 6H-SiC to explain the time-dependent dielectric breakdown of thermal oxides. The model shows the satisfactory result for long-term operations, when the oxide field is kept below 5 MV/cm at temperatures up to 150 °C.

In 2006, Lin et al. (2006) proposed the application of principal components proportional hazards regression model in condition-based maintenance (CBM) optimization. They considered the principal component (PC) to reduce the number of covariates and eliminate the collinearity among them and demonstrated the model's application by considering the CBM data set for two real data sets obtained from industry: oil analysis and vibration analysis data.

Carr and Wang in 2007 (2008) presented a comparative study of PHM and probabilistic filter approach to evaluate the remaining useful life (RUL) of the system. In their paper, PHM with Markov covariate process is used in which principal component covariates are divided into discrete states. The mid-range value of PC is selected as input to PHM. In the stochastic filter approach, a recursive Bayesian algorithm is used to estimate the RUL based on the condition monitoring information. Finally, the mean square error method is used to compare the accuracy of the two techniques.

In 2007, Huairui R. Guo et al. (2007) proposed a new general repair model which considered the effect of repair such as the expected number of cumulative repair as an explanatory variable. Mathematically, can be given as:

$$\lambda(t;Z) = \lambda_o(t)\ \exp\ (\gamma m(t)) \tag{33}$$

where, $\lambda_o(t)$ is the baseline failure intensity function,exp$(\gamma m(t))$ represents the repair effect with parameter γ and $m(t)$ is the expected number of cumulative repairs.

In 2008, Syamsundar and Naikan (2008) developed a segmented proportional intensity model (PIM) for a maintained system. A segmented model is used when sudden changes in the maintained system are observed. In their work, a failure time was divided into various sub-domain based on the change point. Then, PIM is modelled for each sub-domain. Finally, all subdomain models are combined to represent the segmented model for the maintained system to predict the system failure more precisely.

In 2010, Li and Kott (2010) studied the failure data with heavy-tailed behavior and predicted the RUL using PHM. Two data were selected for analysis; time to failure data and condition monitoring data to analyze the impact of heavy-tailed behavior on RUL prediction. They validate the model by taking a real-world example of printer photoreceptors data.

In 2010, Ming-Yi You et al. (2010) proposed a two-zone PHM by dividing the system into two states. One is the stable zone in which there is a slight variation in the condition monitoring data around its mean value. Another is the degradation zone in which the machine behavior deviates significantly from the normal behavior. The state of the systems is separated by a threshold point T, which is necessary to carry out the analysis in the degradation zone. This model performs a specific analysis when the system enters into the degradation zone. Mathematically it can be expressed as follows:

$$\lambda_j\big(t, Z_j(t)\big) = \begin{cases} \lambda_j(t)\, exp\Big[Z_j(t)'\beta\Big], t < T_j \\ \lambda_j^f(t)\, exp\Big[Z_j^f(t)'\beta^f\Big], t < T_j \end{cases} \tag{34}$$

where λ_j and Z_j are the hazard function and covariates of unit j respectively, and T_j is the threshold point for unit.

In this next decade, PHM is applied extensively to the real-time world problem. Various optimization tools and CBM techniques are combined to increase the PHM computational efficiency and precise reliability analysis.

3.4 From 2011 to 2020

Several other reliability models are developed during this decade, incorporating the covariates considering the virtual age model, Arithmetic reduction age model, arithmetic reduction intensity model, repair effect, etc.

In 2011, E. Lorna Wong et al. (2011) considered the virtual age V_n as a condition monitoring covariate incorporated with Weibull PHM. The result shows a promising effect by incorporating these covariates, which include only regular age and condition monitoring information. The n^{th} repair does not cure the damages that occurred before $(n-1)^{th}$ repair, i.e. damage is cumulative.

Syamsundar and Achutha Naikan in 2011 (Syamsundar and Naikan 2011) proposed PIM for imperfect repair, which includes ARA and ARI model with minimal repair baseline intensity. They illustrated the model by taking an example of crane wheel failure data. The crane wheel failure mainly occurs due to the wear of wheel flanges and this failure factor is considered as covariates. Proportional intensity model with ARA and ARI model are as explained below:

For memory 1, the failure intensity model considers only previous failure data only. Therefore, ARA_1 Proportional intensity model is given by

$$\lambda\big(t/H_{t-}\big) = \lambda\big(t - \rho T_{Nt-}\big)\exp(\gamma Z) \tag{35}$$

The proportional intensity process of ARI_1 is given by

$$\lambda\big(t/H_{t-}\big) = \big(\lambda\big(t\big) - \rho\lambda(T_{Nt-})\big)\exp\big(\gamma Z\big) \tag{36}$$

Where, $\lambda\big(t - T_{Nt-}\big)$ is a baseline hazard function with maximal repair and $\lambda(t)$ is a baseline hazard function with minimal repair.

In 2012, Shyur et al. (2012) applied the Extended Hazard Regression (EHR) model for a case study of aviation events. The airline's safety performance was analyzed by accident data incorporated with safety data. The parameters of this model are estimated by a genetic algorithm. The model shows promising potential in practical application for aviation risk analysis.

En-shun et al. in 2012 (En-shun et al. 2012) studied the proportional hazard model for estimating the RUL of a component based on which maintenance activities are carried out. They developed a Monte-Carlo simulation and particle swarm optimization (PSO) model to estimate maintenance rate and the minimized long-running cost rate by joint optimization of the CBM and spare ordering.

In 2012, Alexandre Mendes and Fard (2012) studied the PHM for time-dependent covariates with repeated events by conducting experiments on various small appliances and tested for a certain period of time. The main aim was to identify the significant variable affecting reliability by analyzing the repeated failure for a single event.

In 2012, Junhong Zhang et al. (2012) explained the uses of PHM to study the effect of different covariates and analyzed the reliability of the aero-engine. They studied the life prediction model based on the continuum damage model (CDM) developed by Chaboche, which is necessary for high cycle fatigue analysis of aero-engine blade. The fatigue limit and material properties are investigated by conducting unsymmetrical fatigue tests of TC4 alloy. The finite element method was carried out for blade stress analysis, which shows that the root has the largest damage during the entire process. Finally, the Cox PHM is used to study the effect of different covariates and analyze the reliability of the aero-engine.

Estelle Deloux et al. in 2012 (Deloux et al. 2012) proposed a generalized PHM for maintenance modelling and optimization to reduce the total maintenance cost of the deteriorating system. In their paper, the Markov chain is used to model the sequence type of maintenance. The system condition is defined based on its previous maintenance activities. Finally, system maintenance efficiency is measured, and its optimal PM policy is formed.

In 2013, Yuan Fuqing and Uday Kumar (2013) studied the effect of repair in the baseline intensity function. They considered both Kijimi I and Kijima II model to accommodate the effectiveness of the repair. Based on these models, the intensity function is described as:

For Kijima I, the PIM model is expressed as:

$$\lambda\big(x_i, Z;q\big) = \frac{\beta}{\theta}\left(\frac{q\sum_{l=0}^{i-1} x_l + x_i}{\theta}\right)^{\gamma-1}\exp\left(\sum_{j=1}^{m}\beta_j Z_j\right) \tag{37}$$

For kijima II, the PIM model is expressed as:

$$\lambda\big(x_i, Z;q\big) = \frac{\beta}{\theta}\left(\frac{V_{i-1} + x_i}{\theta}\right)^{\gamma-1}\exp\left(\sum_{j=1}^{m}\beta_j Z_j\right) \tag{38}$$

They applied the Bayesian approach for parameter estimation. When the number of parameters is larger, one can employ Bayesian inference to estimate the parameters.

In 2013, Rahamat Mohammad et al. (2013) proposed a new model for load-sharing systems using k-out-of-n structure subject to a proportional hazard model. They considered the current load on the component as the multiplicative factor for PHM.

In 2013, Lin Li et al. (2013) developed a multi-zone proportional hazard model. The authors divided the system into different degradation stages. At each degradation stage, a specific baseline hazard rate function was observed based on its condition-monitoring information in the corresponding stage only. For example, in stage 0, the condition-monitoring information is collected for all the samples before stage 1. According to this model, an event can be treated as a failure event when it enters into the degradation process in the corresponding stages, and then traditional PHM modelling can be utilized. Mathematically, it can be expressed as:

$$h^k\left(t;Z^k\left(t^k\right)\right) = h_0^k\left(t^k\right)\ \exp\ \left[\beta^k.Z^k\left(t^k\right)\right] \tag{39}$$

where, t^k is the survival time in stage k. In zone or stage $k(0 \le k \le n-1)$, PHM can be used to estimate the degradation time and the remaining useful time of equipment in that particular zone before entering into the next zone. The overall remaining useful life of the equipment can be estimated by applying PHM for the nth zone.

Qing Zhang et al. in 2014 (Zhang et al. 2014) proposed a mixture weibull PHM to predict the failure of mechanical systems with multiple failures mode. They implemented this technique on high-pressure water descaling pump, which has two failure modes, and compare the proposed model with the traditional Weibull PHM.

In 2014, Tang et al. (2014) analyzed the failure data of power cable (high and medium voltage) using Cox PHM approach. Two covariates are selected for high voltage cable as an installer and joint manufacturer; three covariates are chosen for medium voltage cables such as the method of installation, manufacturer, and cable length, respectively. This model helps to measure the effect of selected covariates on cable and cable joint failures.

In 2016, Lucas Equeter et al. (2016) studied the Cox proportional hazard model for estimation of cutting tool lifespan using cutting speeds as a covariate. They considered the gamma process for tool wear simulation to provide the tool life span at various cutting speeds. The experiment shows that lack of accuracy in some PH fit portions to be related to the irregular spread of fitting data.

In 2017, Wang et al. (2017) analyzed the reliability of roller bearing using Weibull PHM integrated with Principal component analysis (PCA). PCA is used to eliminate the dependency among the variable and reflect the system's performance degradation process. In this paper, PCA is applied to vibration monitoring data for performance analysis. After PCA, three principal components are selected as covariates and integrated with WPHM for reliability assessment. The results show strong stability and practicability for real-time application.

Yuan et al. in 2017 (Yuan et al. 2017) analyzed the accuracy of PHM and compare it with the physical model. They derived the failure model for typical failure mechanisms of engineering systems. The physical model is then converted into a statistical model which is incorporated into the PHM. The results show that the model will become more convicting when the physical model is incorporated into the PHM.

Leila Jafari et al. (2017) proposed a new optimization method to minimize the long-run expected average cost per unit time of the whole system using a PHM integrated with CBM and age information. Authors illustrated the application of the model by considering oil data from the mining industry. In this model, opportunistic maintenance policies are formed using PHM and compared with another model. Thus, the proposed method confirms the superiority among other models.

In 2018, Chrianna I Bharat et al. (2018) validated the Cox PHM model to build trust in its ability to predict failure. They considered graphical methods such as forest plots and nomograms for validation with the help of Kaplan–Meier, and calibration plots. The wastewater pipe data is collected for seven and half years to illustrate data splitting, discrimination and model fit of the Cox PHM model. The result shows that graphical methods improve the ease of interpretation of the model compared to the output obtained in tabular form.

In 2018, Narayanaswamy Balakrishnan et al. (2018) proposed two new models based on Marshall–Olkin distribution are as: modified proportional hazard rates (MPHR) and modified proportional reversed hazard rates (MPRHR) models.

In MPHR, the survival function $F_i\left(x;\lambda_i\right)$ is given by

$$F_i\left(x;\lambda_i\right) = \frac{1-(F^{-}_{(x)})^{\lambda_i}}{1-\alpha^{-}(F^{-}_{(x)})^{\lambda_i}} \text{ for } i = 1 \dots \dots n, \tag{40}$$

where, F^{-} represents the baseline survival function and $x_1 \dots \dots \dots .. x_n$ are the independent lifetime of n components of a system with respective survival functions $F_1{}^{-} \dots \dots \dots .. F_n{}^{-}$ and α is the tilt parameter. Where $\alpha > 0, \alpha^{-} = 1-\alpha$ and $\lambda_i > 0$, i = 1 … … n. For a special case, when α = 0, the model simply reduces to the PHR model. In MPRHR, the distribution function $F_i\left(x;\beta_i\right)$ is given by

$$F_i\left(x;\beta_i\right) = \frac{\alpha(F(x))^{\beta_i}}{1-\alpha^{-}(F(x))^{\beta_i}} \text{ for } i = 1 \dots \dots \dots n, \tag{41}$$

where, $F(x)$ represents the baseline distribution function and $x_1 \dots \dots \dots .. x_n$ are the independent lifetime of n components of a system with respective distribution functions $F_1 \dots\dots\dots .. F_n$ and α is the tilt parameter. Where, $\alpha > 0, \alpha^- = 1 - \alpha$ and $\beta_i > 0, i = 1 \dots\dots n$. For special case, when $\alpha = 0$, model simply reduces to PRHR model.

In 2018, Wim J.C. Verhagen and De Boer (2018) improved the aircraft's statistical reliability assessment using PHM incorporated with operational factors. The authors identified the operational factors that affect the maintenance event based on historical data. They performed extreme value analysis and maximum difference analysis techniques to determine the operational factors that are likely to be the root cause of the failures. Finally, both time-independent and dependent PHM were used for reliability analysis.

In 2019, Aiping Jiang et al. (2019) proposed a decision model for a redundant system. They considered k out of n system and PHM with environmental factor as a covariate for reliability assessment of a redundant system. Based on the reliability analysis, the authors identify the optimal maintenance interval that minimizes the average maintenance cost per unit time for a redundant system.

Recently, in 2020, Rui Zheng and Makis (2020) developed condition-based maintenance (CBM) policy for a system subjected to periodic inspection with dynamic thresholds and multiple maintenance actions. The authors considered both aging and continuous state covariate processes to explains the practical deterioration process. They selected an appropriate maintenance action such as no maintenance, imperfect maintenance, and preventive replacement based on two dynamic thresholds for the covariate at every inspection period. Two failure types are considered as catastrophic failure and minor failure. Finally, the optimization problem is formulated based on the semi-Markov decision process (SMDP) to determine the optimal thresholds that minimize the long-run average cost rate.

Lea Breniere et al. in 2020 (Brenière et al. 2020) proposed an approach that includes time-dependent covariate in the generic virtual age model. They formed a simulation model based on two assumptions: in the first assumption, the covariate is considered as a stepwise constant for analysis, where as in the second assumption, system failure probability is assumed to face failure at each node to form a fine time grid.

In 2020 Chong Chen et al. (2020) proposed a new approach known as Cox proportional hazard deep learning to overcome the data sparsity and data censoring problem in the operational maintenance data analysis. The authors adopted an autoencoder approach for robust representation of nominal data for cox PHM and LSTM (Long Short Term Memory) to predict the time between failure (TBF) based on historical maintenance data. They utilized cox PHM for labeling of censored data. Finally, the TBF prediction model is formed based on LSTM.

Yan-Feng Li et al. in 2020 (Li et al. 2020) proposed a new method for reliability assessment of systems experiencing common cause failure (CCF) in a dynamic environment. They used a Bayesian network for the characterization of CCF in the system. The authors utilized PHM to capture the system's degradation process working under a dynamic environment, and the lifetime distribution and reliability functions are obtained.

Researchers must pay attention to the review papers to comprehend the development and application of PHM for reliability assessment. The review papers available in the literature in this field are mentioned in the next section.

3.5 Some review papers in this field

A few essential review papers are available in the literature that could help the researcher to understand PHM development. In 1994, Dhananjay Kumar and Bengt Klefsjo (1994) presented a review on PHM and its extension model and application for reliability assessment. Later in 2009, Samrout et al. (2009) presented a review on PHM for optimization of maintenance policy. In 2010, Gorjian et al. (2010) presented a collaborative review on covariates for non-parametric and semi-perimetric models applied in the medical and reliability fields. Recently, Alaswad and Xiang in 2017 (Alaswad and Xiang 2017) presented a CBM model review for deterioration processes and PHM for single and multiple systems.

4 Observations and limitations

From the literature survey, a considerable growth is observed in both the fields from 1965 to 2010 as shown in Fig. 3, but both the fields were treated separately. From the year 1965 to 2010, numerous models were developed to deal with complex systems but more orientation was given towards TIM. Moreover, during the period from 2011 to 2020, it can be observed that, though various imperfect maintenance models have been developed, but PH models got much appreciation in terms of industrial applications. However, at the same time, we found that the assumption of imperfect maintenance was neglected by most of the researchers while applying PH models in real-life industrial problems.

Some authors attempted to combine both types of models. For example, the model proposed by (Guo and Love 1992), Syamsundar et al. (Syamsundar and Naikan 2011), Yuan Fuqing and Uday kumar (Fuqing and Kumar 2013) and Lea Breniere et al. in 2020 (Brenière et al. 2020) attempted to combine PH models with virtual age models by considering only CM as imperfect whereas in a practical scenario, for mechanical systems, PM and CM both should be treated as imperfect. On the other hand, Sharma and Rai (2020d), Yu et al. (2008) and Nasr et al. (2013) proposed a virtual age model by treating different interventions as imperfect but did

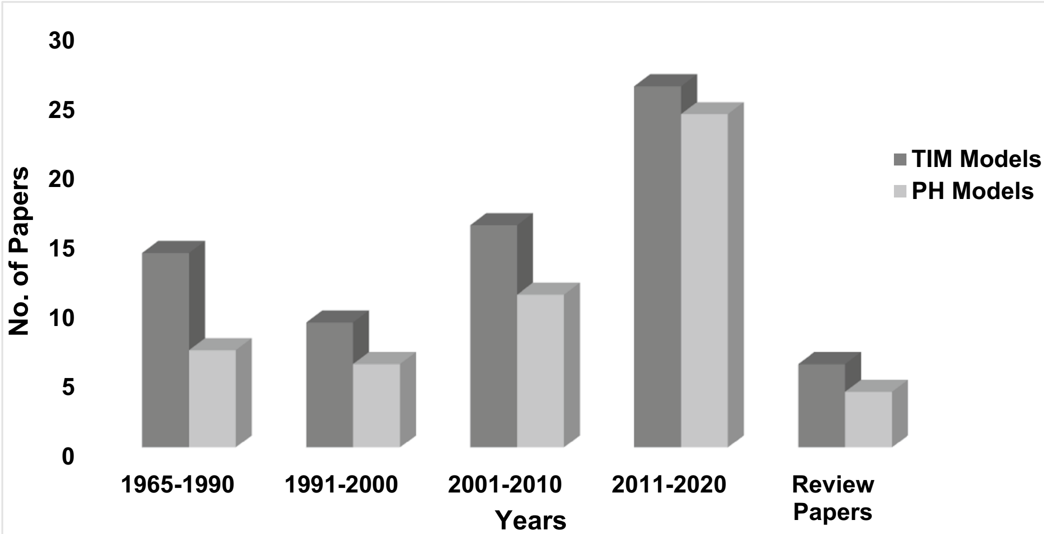

Fig. 3 Development of TIM and PH models in the literature

not include PHM in the developed model. Some consider PH models with CBM techniques but lacks in combining them with imperfect maintenance models. Moreover, artificial intelligence (AI) algorithm is also widely used for data preprocessing in this decade, it can be incorporated with PIM to improve the data sparsity and data censoring problem in the operational maintenance data analysis for accurate reliability assessment for the repairable systems.

Thus, observing the overall development in these two fields, it could be concluded that the available literature still lacks in combining TIM with PH models effectively which could be considered a big gap in this area as a future scope.

5 Conclusion and future scope

Traditional imperfect maintenance and proportional hazard models are two major and important streams in the reliability analysis of complex systems. Having observed that these two fields are treated separately in the literature, this paper presents an extensive literature survey on the development of TIM and PH models. The aim of this literature survey is to attract the researchers towards combining these two fields. We have mainly considered ARA-based TIM models and PH models developed by various researchers, from the year 1965 to 2020 for reliability assessment.

As a major finding from this survey, a boom in the models development is observed in the decade of 2011–2020 in both the fields. The authors considered alternative scales for reliability estimation, introduced imperfect maintenance activities such as overhaul, assumed various factors that affect the performability of systems in TIM models. At the same time, authors used combined CBM with PH models and used various AI techniques for data processing. Although, these models (TIM and PH models) are more or less treated separately. Thus, as a future scope, more attention might be given towards combining these two major streams so that more reliable and practical results could be obtained for the industries dealing with large, complex and critical mechanical systems.

Hence, this literature survey could be very helpful as it provides a ready reference for academicians who are working or willing to work in this field of imperfect proportional hazard maintenance (IPHM).

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