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ORIGINAL RESEARCH

Profit analysis of a two-unit cold standby system model operating under different weather conditions

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Abstract

Generally, each machine or products are used by the human being up to its maximum capacity. If a system is performing beyond its capacity/defined conditions by the manufacturer called, the system is working under abnormal weather conditions. While, a system is performing within its capacities/stated conditions set by manufacturer called as the system working in abnormal weather conditions, for example; a car is functioning exceeding its accommodating capacity can be termed as working under abnormal weather conditions; a hydraulic machine exceeding its weight uplifting capacity of 500 tons by lifting 600 tons is termed as working under abnormal weather conditions. To overcome in such a situation, only effective maintenance strategies and suitable structure design of redundant system are the crucial factors which keep the standby system operational without failures for longer period of time. In fact, proper functioning of service mechanism and the reliability of system are strongly associated with each other. In the present paper, a water supply system simulate that is functioning of two-unit cold standby system with facilities of preventive maintenance, inspection and repair operating under different weather conditions with priority to preventive maintenance over inspection. The units are identical in nature. The single server only works under normal weather conditions capable of performing three operations inspection, repair and preventive maintenance responds to system instantly. The replacement of units is suggested if repair is impossible to perform during inspection. The operative unit undergoes for preventive maintenance after a specific time of operation. Repair of the unit is done by the server at its complete failure. All random variables are statistically independent. It is assumed that the failure rate and rate by which system undergoes for preventive maintenance are constant whereas the inspection rate, repair rate and maintenance rate follows negative exponential distribution. The expressions/graphs for several reliability measures are derived/depicted in steady state using regenerative point technique and semi-Markov process to determine the nature of the system.

Keywords Effect of priority · Standby system · Preventive maintenance · Repair · Inspection and weather conditions

Abbreviations

E	The set of regenerative states $\{S_0, S_1, S_2, S_3, S_4, S_5 \text{ and } S_6\}$
O/Cs	The unit is operative/cold stand by
α_0	The rate by which unit undergoes for preventive maintenance after a specific operative time 't' {called maximum operation time}
λ	Constant failure rate of the unit
a/b	Probability of repair/replacement after inspection
′(dash)	Used to represent alternative result
–	Used to stop all mechanical activity due to abnormal weather
β/β_1	Constant abnormal weather rate/normal weather rate
$f(t)/F(t)$	Pdf/cdf of preventive maintenance time of unit
$g(t)/G(t)$	Pdf/cdf of repair time of a failed unit
$h(t)/H(t)$	Pdf/cdf of inspection time of a failed unit

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$Pm/\overline{Pm}$	The unit is under preventive maintenance/preventive maintenance of the unit is stopped due to abnormal weather conditions
$Fur/\overline{Fur}$	The failed unit is under repair/repair of unit is stopped due to abnormal weather
$Fui/\overline{Fui}$	The failed unit is under inspection/inspection is stopped due to abnormal weather
$PM/\overline{PM}$	The unit is continuously under preventive maintenance/continuous preventive maintenance is stopped due to abnormal weather conditions
$FUI/\overline{FUI}$	The failed unit is continuously under inspection from previous state/continuous inspection is stopped due to abnormal weather conditions
$FUR/\overline{FUR}$	The failed unit is continuously under repair/repair is stopped due to abnormal weather conditions
Pmm/PMm	The unit is under continuous preventive maintenance resumed from previous state which was stopped in between due to abnormal weather
Furr/FURr	The failed unit is under continuous repair resumed from previous state which was stopped in between due to abnormal weather
Fuii/FUIi	The failed unit is under continuous inspection resumed from previous state which was stopped in between due to abnormal weather
wPm/wFi	The unit is waiting for preventive maintenance/inspection
WPm/WFi	The unit is continuously waiting for preventive maintenance/repair/inspection
$Q_{i,j;k(r,s)}$	Pdf/cdf of direct transition time from regenerative state S_i to regenerative state S_j or to a failed state S_j visiting state S_k once and more times states S_r and S_s
$p_{i,j}$	Probability of transition from state S_i to S_j
$p_{i,j;k(r,s)}$	Probability of transition from state S_i to S_j visiting state S_j, S_k once and more times states S_r and S_s
$m_{i,j}$	The unconditional mean time taken by the system to transit from any regenerative state S_i when it (time) is counted from epoch of entrance into that state S_j. Mathematically it can be written as $m_{ij} = \int_0^\infty td[Q_{ij}(t)] = -q_{ij}^{*'}(0)$
⊗/⊕	Symbol for Laplace–Stieltjes convolution/Laplace convolution
*/~	Symbol for Laplace–Stieltjes transform/Laplace transform
μ_i	The mean Sojourn time in state S_i this is given by $\mu_i = E(t) = \int_0^\infty P(T > t)dt = \sum_j m_{ij}$, where T denotes the time to system failure
$W_i(t)/R_i(t)$	Probability that the server is busy in state S_i up to time t without making any transition to any other regenerative state or before returning to the same state via one or more non-regenerative stage

1 Introduction

Proper and punctual working of water supply system is very important to all citizens of a city. The supply system works without failure behind this, the idea of inspection, preventive maintenance, priority and repair of identical or non-identical units under different weather conditions have been discussed by the researchers including, (Osaki and Asakura 1970) obtained a two-unit standby redundant system with repair and preventive maintenance. (Srinivasan and Gopalan 1973) discussed probability analysis of a two-unit system with warm standby and single repair facility. (Dhillon and Natesan 1983) analyzed stochastically outdoor power system in fluctuating environment. (Gupta and Goel 1991) obtained profit analysis of two-unit cold standby system with abnormal weather condition. (Chander 2005) analyzed reliability models with priority for operation and repair with arrival time of server. (Malik and Barak 2009) discussed reliability and economic analysis of a system operating under different weather conditions. (Kumar et al. 2012) discussed cost analysis of a two-unit cold standby system subjected to degradation, inspection and priority. (Kishan and Jain 2012) presented a two non-identical unit standby system model with repair, inspection and post-repair under classical and Bayesian viewpoints. (Kadyan and Ramniwas 2013) discussed cost–benefit analysis of a single-unit system with warranty for repair. (Deswal and Malik 2015) explained reliability measures of a system of two non-identical units with priority subject to weather conditions. Recently, (Barak and Barak 2016) discussed impact of abnormal weather conditions on various reliability measures of a repairable system with inspection. (Barak et al. 2017a, b) analyzed stochastically a cold standby system with conditional failure of server. (Barak et al. 2017a, b) discussed stochastic analysis of two-unit redundant system with priority to inspection over repair. Barak et al. 2018) analyzed stochastically a two-unit system with standby and server failure subject to inspection.

Keeping these studies in mind, a water supply system consists of two identical units (as in Fig. 1) in which the unit may fail directly from normal mode. Initially, one unit is operative and other is in spare as cold standby. There is a single server who attends the system immediately whenever required. Server is capable of performing three operations i.e. preventive maintenance, inspection and repair. The preventive maintenance is carried out after a maximum operation time. Repair of unit is done at complete failure. Inspection facility is available before repair/replacement of the failed unit. Priority is given to preventive maintenance over inspection. Unit works as new after repair/preventive maintenance. Server starts, restarts or resumes its duty in normal weather only. Operations preventive maintenance and repair are stopped in abnormal weather to protect the system from unnecessary damage. It is assumed that the rate of change of weather, failure rate and the rate by which system undergoes for preventive maintenance or inspection are constant. The distributions for preventive maintenance, repair time, inspection time are taken as arbitrary with different distributions. Graphical and numerical inferences are explained in detail. All random variables are statistically independent.

2 Transition probabilities and mean Sojourn times

Simple probabilistic consideration yields the following expressions for non-zero elements. In particular case; let $f(t) = \theta e^{-\theta t},\ g(t) = \phi e^{-\phi t}$. The transition probabilities obtained are as follows:

$$p_{i,j} = Q_{ij}(\infty) = \int_0^\infty q_{i,j}(t)\mathrm{d}t \tag{1}$$

$$p_{0,1} = \frac{\alpha_0}{\alpha_0+\lambda},\ p_{0,2} = \frac{\lambda}{\alpha_0+\lambda},\ p_{1,0} = \frac{\theta}{\lambda+\beta+\theta+\alpha_0},\ p_{1,3} = \frac{\beta}{\lambda+\beta+\theta+\alpha_0},\ p_{1,9} = \frac{\lambda}{\lambda+\beta+\theta+\alpha_0}$$

$$p_{1,10} = \frac{\alpha_0}{\lambda+\beta+\theta+\alpha_0},\ p_{2,0} = \frac{b\eta}{\lambda+\beta+\eta+\alpha_0},\ p_{2,4} = \frac{a\eta}{\lambda+\beta+\eta+\alpha_0},\ p_{2,6} = \frac{\beta}{\lambda+\beta+\eta+\alpha_0}$$

$$p_{2,23} = \frac{\alpha_0}{\lambda+\beta+\eta+\alpha_0},\ p_{2,31} = \frac{\lambda}{\lambda+\beta+\eta+\alpha_0},\ p_{3,1} = p_{5,4} = p_{6,2} = \frac{\beta_1}{\lambda+\beta_1+\alpha_0},\ p_{3,13} = p_{5.37} = p_{6,27} = \frac{\alpha_0}{\lambda+\beta_1+\alpha_0}$$

$$p_{3,13} = p_{5.37} = p_{6,27} = \frac{\alpha_0}{\lambda+\beta_1+\alpha_0},\ p_{3,15} = p_{5,36} = p_{6,29} = \frac{\lambda}{\lambda+\beta_1+\alpha_0}$$

$$p_{4,0} = \frac{\phi}{\phi+\alpha_0+\lambda+\beta},\ p_{4,5} = \frac{\beta}{\phi+\alpha_0+\lambda+\beta},\ p_{4,17} = \frac{\lambda}{\phi+\alpha_0+\lambda+\beta},\ p_{4,20} = \frac{\alpha_0}{\phi+\alpha_0+\lambda+\beta}$$

$$p_{7,2} = p_{9,2} = p_{10,1} = p_{12,1} = p_{14,1} = p_{16,2} = \frac{\theta}{\theta+\beta},$$

$$p_{7,8} = p_{9,8} = p_{10,11} = p_{12,11} = p_{14,13} = p_{16,15} = \frac{\beta}{\theta+\beta}$$

$$p_{17,2} = p_{19,2} = p_{20,1} = p_{22,1} = p_{26,1} = p_{34,2} = p_{35,2} = p_{38,1} = \frac{\phi}{\phi+\beta}$$

$$p_{17,18} = p_{19,18} = p_{20,21} = p_{22,21} = p_{26,37} = p_{34,36} = p_{35,36} = p_{38,37} = \frac{\beta}{\phi+\beta}$$

$$p_{23,1} = p_{25,1} = p_{28,1} = p_{30,2} = p_{31,2} = p_{33,2} = \frac{b\eta}{\eta+\beta}$$

$$p_{23,24} = p_{25,24} = p_{28,27} = p_{30,29} = p_{31,32} = p_{33,32} = \frac{\beta}{\eta+\beta}$$

$$p_{23,26} = p_{25,26} = p_{28,26} = p_{30,34} = p_{31,34} = p_{33,34} = \frac{a\eta}{\eta+\beta}$$

$$p_{8,7} = p_{11,12} = p_{12,13} = p_{13,14} = p_{15,16} = p_{18,19} = p_{21,22} = p_{24,25} = p_{27,28} = p_{29,30} = p_{32,33} = p_{36,35} = p_{37,38} = 1$$

$$\begin{aligned}
p_{1,1;10} &= \frac{\theta\alpha_0}{(\beta+\theta)(\lambda+\theta+\beta+\alpha_0)},\ p_{1,1;10(11,12)} = \frac{\beta\alpha_0}{(\beta+\theta)(\lambda+\theta+\beta+\alpha_0)},\\
p_{1,2;9} &= \frac{\theta\lambda}{(\beta+\theta)(\lambda+\theta+\beta+\alpha_0)},\ p_{1,2;9(8,7)} = \frac{\beta\lambda}{(\beta+\theta)(\lambda+\theta+\beta+\alpha_0)},\\
p_{2,2;23} &= \frac{\alpha_0\theta}{(\beta+\theta)(\lambda+\eta+\beta+\alpha_0)},\ p_{2,2;23(24,25)} = \frac{\beta\alpha_0}{(\beta+\theta)(\lambda+\eta+\beta+\alpha_0)},\\
p_{2,2;28} &= \frac{b\lambda\eta}{(\beta+\eta)(\lambda+\eta+\beta+\alpha_0)},\ p_{2,2;28(29,30)} = \frac{b\lambda\beta}{(\beta+\eta)(\lambda+\eta+\beta+\alpha_0)}\\
p_{2,2;28,31} &= \frac{a\lambda\beta\phi}{(\beta+\eta)(\beta+\phi)(\lambda+\eta+\beta+\alpha_0)},\ p_{2,2;28(29,30)31} = \frac{a\lambda\beta\phi}{(\beta+\phi)(\beta+\eta)(\lambda+\eta+\beta+\alpha_0)}\\
p_{2,2;28,31(32,33)} &= \frac{a\lambda\eta\beta}{(\beta+\phi)(\eta+\lambda+\beta+\alpha_0)(\eta+\beta)}\\
p_{2,2;28(29,30)31(32,33)} &= \frac{a\lambda\beta^2}{(\beta+\phi)(\lambda+\eta+\beta+\alpha_0)(\beta+\eta)},\ p_{3,1;(13,14)} = p_{5,1;(34,35)} = \frac{\alpha_0}{\lambda+\beta_1+\alpha_0}\\
p_{3,2;(15,16)} &= p_{5,2;(32,33)} = \frac{\lambda}{\lambda+\beta_1+\alpha_0}\\
p_{4,1;20} &= \frac{\alpha_0\phi}{(\beta+\phi)(\lambda+\phi+\beta+\alpha_0)},\ p_{4,2;17} = \frac{\lambda\phi}{(\beta+\phi)(\lambda+\phi+\beta+\alpha_0)}\\
p_{4,1;20(21,22)} &= \frac{\alpha_0\beta}{(\beta+\phi)(\lambda+\phi+\beta+\alpha_0)},\ p_{4,2;17(18,19)} = \frac{\lambda\beta}{(\beta+\phi)(\lambda+\phi+\beta+\alpha_0)},\\
p_{6,2;(24,25)} &= \frac{\alpha_0}{(\lambda+\beta_1+\alpha_0)},\ p_{6,2;(26,27)} = \frac{a\alpha_0\phi}{(\beta+\phi)(\alpha_0+\lambda+\beta_1)}\\
p_{6,2;(26,27)31} &= \frac{a\lambda\phi}{(\beta+\phi)(\alpha_0+\lambda+\beta_1)},\ p_{6,2;(26,27)31(32,33)} = \frac{a\lambda\beta}{(\beta+\phi)(\alpha_0+\lambda+\beta_1)}
\end{aligned} \tag{2}$$

The mean Sojourn times (μ_i and μ'_i) is the state S_i are

$$\mu_0 = \frac{1}{\alpha_0+\lambda},\ \mu_1 = \frac{1}{\alpha_0+\lambda+\beta+\theta},\ \mu_2 = \frac{1}{\alpha_0+\lambda+\beta+\eta},\ \mu_3 = \mu_5 = \mu_6 = \frac{1}{\alpha_0+\lambda+\beta_1}$$

$$\mu_4 = \frac{1}{\alpha_0+\lambda+\beta+\phi} \tag{3}$$

and $\mu'_1 = \frac{\theta\beta_1+(\lambda+\alpha_0)(\beta+\beta_1)}{\theta\beta_1(\lambda+\beta+\theta+\alpha_0)}$, $\mu'_2 = \frac{\eta\theta\phi\beta_1+(\alpha_0\eta\phi+\lambda\theta\phi+a\lambda\theta\eta)(\beta+\beta_1)}{\theta\phi\eta\beta_1(\lambda+\beta+\eta+\alpha_0)}$

$$\mu'_3 = \frac{\theta\beta_1+(\lambda+\alpha_0)(\beta+\beta_1+\theta)}{\theta\beta_1(\lambda+\beta_1+\alpha_0)},\ \mu'_4 = \frac{\beta_1\phi+(\alpha_0+\lambda)(\beta+\beta_1)}{\phi\beta_1(\lambda+\beta+\alpha_0+\phi)}$$

$$\mu'_5 = \frac{\phi\beta_1+(\lambda+\alpha_0)(\beta+\beta_1+\phi)}{\phi\beta_1(\lambda+\beta_1+\alpha_0)},\ \mu'_6 = \frac{\beta_1\eta\theta\phi+\alpha_0(\theta+\beta)\eta\phi+\lambda(\eta+\beta)\theta\phi+a\lambda(\beta+\beta_1)\eta\theta}{\theta\phi\eta\beta_1(\lambda+\beta_1+\alpha_0)} \tag{4}$$

3 Reliability and mean time to system failure (MTSF)

Let $\phi_i(t)$ be the c.d.f. of first passage time from the regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for: $\phi_i(t)$.

$$\phi_0(t) = q_{0,1}(t) \otimes \phi_1(t) + q_{0,2}(t) \otimes \phi_2(t)$$

$$\phi_1(t) = q_{1,0}(t) \otimes \phi_0(t) + q_{1,3}(t) \otimes \phi_3(t) + q_{1,9}(t) + q_{1,10}(t)$$

$$\phi_2(t) = q_{2,0}(t) \otimes \phi_0(t) + q_{2,4}(t) \otimes \phi_4(t) + q_{2,6}(t) \otimes \phi_6(t) + q_{2,23}(t) + q_{2,28}(t)$$

$$\phi_3(t) = q_{3,1}(t) \otimes \phi_1(t) + q_{3,13}(t) + q_{3,15}(t)$$

$$\phi_4(t) = q_{4,0}(t) \otimes \phi_0(t) + q_{4,5}(t) \otimes \phi_5(t) + q_{4,17}(t) + q_{4,20}(t)$$

$$\phi_5(t) = q_{5,4}(t) \otimes \phi_4(t) + q_{5,32}(t) + q_{5,34}(t)$$

$$\phi_6(t) = q_{6,2}(t) \otimes \phi_2(t) + q_{6,24}(t) + q_{6,26}(t). \quad (5)$$

Taking L.T. of above relation (6.6.1) and solving for $\tilde{\phi}_0(t)$. We have

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \quad (6)$$

The reliability of the system model can be obtained by taking Laplace inverse transformation of (6.6.2). The mean time to system failure (MTSF) is given by:

$$A_3(t) = M_3(t) + [q_{3,1}(t) + q_{3,1;(13,14)}(t)] \oplus A_1(t) + q_{3,2;(15,16)}(t) \oplus A_2(t)$$

$$A_4(t) = M_4(t) + q_{4,0}(t) \oplus A_0(t) + [q_{4,1;20}(t) + q_{4,1;20(21,22)}(t)] \oplus A_1(t) + [q_{4,2;17}(t) + q_{4,2;17(18,19)}(t)] \oplus A_2(t) + q_{4,5}(t) \oplus A_5(t)$$

$$A_5(t) = M_5(t) + [q_{5,1;(34,35)}(t)] \oplus A_1(t) + [q_{5,2;(32,33)}(t)] \oplus A_2(t) + q_{5,4}(t) \oplus A_4(t)$$

$$A_6(t) = M_6(t) + [\, q_{6,2}(t) + q_{6,2;(24,25)}(t) + q_{6,2;(26,27)}(t) + q_{6,2;(26,27)31}(t) + q_{6,2;(26,27)31(32,33)}] \oplus A_2(t), \quad (8)$$

where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\alpha_0+\lambda)t},\ M_1(t) = e^{-(\alpha_0+\lambda+\beta)t}\overline{F(t)},\ M_2(t) = e^{-(\alpha_0+\beta+\lambda)t}\overline{H(t)}$$
$$M_4(t) = e^{-(\alpha_0+\beta+\lambda)t}\overline{G(t)} \text{ and } M_3(t) = M_5(t) = M_6(t) = e^{-(\alpha_0+\beta+\lambda)t} \quad (9)$$

Taking Laplace transformation of above relations (8) and (9) and solving for $A_0^*(s)$, the steady state availability is given by

$$\text{MTSF} = \lim_{s\to 0} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{\left[\begin{array}{l} [\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1\}\{(\eta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1\} \\ \{(\phi+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1\}] + [\alpha_0\{(\eta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1\}(\lambda+\alpha_0+\beta_1+\beta) \\ \{(\phi+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1\}] + [\lambda\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1\}(\lambda+\alpha_0+\beta_1+\beta)\} \\ \{(\phi+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1\}] + [a\lambda\eta(\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\beta+\beta_1) \\ \{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1)\}] \end{array}\right]}{\left[\begin{array}{l} [(\lambda+\alpha_0)\{(\eta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1)\}\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1)\} \\ -\theta\alpha_0(\lambda+\alpha_0+\beta_1)\{(\eta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1)\} \\ -b\eta\lambda(\lambda+\alpha_0+\beta_1)\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1)\}] \\ [(\phi+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1] - [a\eta\lambda\phi(\lambda+\alpha_0+\beta_1)^2\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1) - \beta\beta_1)] \end{array}\right]} \quad (7)$$

4 Steady state availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at $t=0$. The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(t) + q_{0,1}(t) \oplus A_1(t) + q_{0,2}(t) \oplus A_2(t)$$

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sN_0^*}{D_0^*} = \frac{N_0}{D_0'} \quad \left(\frac{0}{0}\text{ form}\right) \quad (10)$$

$$\text{where } N_0 = \lim_{s\to 0} N_0^* \quad \text{and} \quad D_0' = \lim_{s\to 0} D_0'^* \quad (11)$$

$$A_1(t) = M_1(t) + q_{1,0}(t) \oplus A_0(t) + [q_{1,1;10}(t) + q_{1,1;10(11,12)}(t)] \oplus A_1(t) + [q_{1,2;9}(t) + q_{1,2;9(8,7)}(t)] \oplus A_2(t) + q_{1,3}(t) \oplus A_3(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \oplus A_0(t) + [q_{2,2;23}(t) + q_{2,2;23(24,25)}(t) + q_{2,2;28}(t) + q_{2,2;28,31}(t) + q_{2,2;28(29,30)}(t) + q_{2,2;28(29,30)31}(t) + q_{2,2;28,31(32,33)}(t) + q_{2,2;28(29,30)31(32,33)}] \oplus A_2(t) + q_{2,4}(t) \oplus A_4(t) + q_{2,6}(t) \oplus A_6(t)$$

$$N_0 = \frac{\begin{bmatrix} \{(\phi+\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ \{(\eta+\lambda)(\lambda+\alpha_0+\beta_1)+\lambda\beta\} \end{bmatrix}}{[(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^3(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)(\lambda+\alpha_0+\eta+\beta)]} \tag{12}$$

$$D_0' = X + T(P_1 + P_2 + P_3) \tag{13}$$

where

$$X = \frac{\left[\eta\{(\theta+\alpha_0+\lambda+\beta)(\lambda+\alpha_0+\beta_1)-(\alpha_0(\alpha_0+\lambda+\beta+\beta_1)+\beta\beta_1)\right]}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\eta+\beta)}$$

$$T = \frac{\left[(\phi+\alpha_0+\lambda+\beta)(\lambda+\alpha_0+\beta_1)-\beta_1\beta\}\right]}{[(\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\phi+\beta)]}$$

$$P_1 = \frac{\begin{bmatrix} [\{(\theta+\alpha_0+\lambda+\beta)(\lambda+\alpha_0+\beta)-\beta\beta_1\}-\alpha_0(\alpha_0+\lambda+\beta+\beta_1)\} \\ \{(\eta+\alpha_0+\lambda+\beta)(\lambda+\alpha_0+\beta)-\beta\beta_1\}-\lambda(\alpha_0+\lambda+\beta+\beta_1)\}]-\alpha_0\lambda(\alpha_0+\lambda+\beta+\beta_1)^2 \end{bmatrix}}{(\lambda+\alpha_0+\beta_1)^2(\theta+\alpha_0+\lambda+\beta)(\eta+\alpha_0+\lambda+\beta)}$$

$$P_3 = \frac{\begin{bmatrix} [\lambda\{(\theta+\alpha_0+\lambda+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}][\beta+\beta_1][(\alpha_0+\lambda+\beta+\beta_1)\{\alpha_0\eta\phi+\lambda\theta\phi+a\eta\theta\eta) \\ +(\alpha_0+\lambda+\beta_1)\eta\theta\phi] \end{bmatrix}}{[\beta_1\eta\phi(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^2(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\eta+\beta)]} \tag{14}$$

$$P_2 = \frac{\left[\alpha_0\eta(\beta+\beta_1)\{(\alpha_0+\lambda+\theta+\beta+\beta_1)(\alpha_0+\lambda)+\beta_1\theta\}\right]}{[\theta\beta_1(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\eta+\beta)]}$$

$$B_4^P(t) = q_{4,0}(t) \oplus B_0^P(t)(t) + [q_{4,1;20}(t) + q_{4,1;20(21,22)}(t)] \oplus B_1^P(t) + [q_{4,2;17}(t) + q_{4,2;17(18,19)}(t)] \oplus B_2^P(t) + q_{4,5}(t) \oplus B_5^P(t)$$

$$B_5^P(t) = [q_{5,1;(34,35)}(t)] \oplus B_1^P(t) + [q_{5,2;(32,33)}(t)] \oplus B_2^P(t) + q_{5,4}(t) \oplus B_4^P(t)$$

$$B_6^P(t) = W_6^P(t) + [q_{6,2}(t) + q_{6,2;(24,25)}(t) + q_{6,2;(26,27)}(t) + q_{6,2;(26,27)31}(t) + q_{6,2;(26,27)31(32,33)}] \oplus B_2^P(t), \tag{15}$$

5 Busy period analysis for server due to preventive maintenance

(a) Let $B_i^p(t)$ be the probability that the server is busy in preventive maintenance of the unit at an instant 't' given that system entered state i at $t=0$. The recursive relations for $B_i^p(t)$ are as follows

$$B_0^P(t) = q_{0,1}(t) \oplus B_1^P(t) + q_{0,2}(t) \oplus B_2^P(t)$$

$$B_1^P(t) = W_1^P(t) + q_{1,0}(t) \oplus B_0^P(t) + [q_{1,1;10}(t) + q_{1,1;10(11,12)}(t)] \oplus B_1^P(t) + [q_{1,2;9}(t) + q_{1,2;9(8,7)}(t)] \oplus B_2^P(t) + q_{1,3}(t) \oplus B_3^P(t)$$

$$B_2^P(t) = W_2^P(t) + q_{20}(t) \oplus B_0^P(t) + [q_{2,2;23}(t) + q_{2,2;23(24,25)}(t) + q_{2,2;28}(t) + q_{2,2;28,31}(t) + q_{2,2;28(29,30)}(t) + q_{2,2;28(29,30)31}(t) + q_{2,2;28,31(32,33)}(t) + q_{2,2;28(29,30)31(32,33)}] \oplus B_2^P(t) + q_{2,4}(t) \oplus B_4^P(t) + q_{2,6}(t) \oplus B_6^P(t)$$

$$B_3^P(t) = W_3^P(t) + [q_{3,1}(t) + q_{3,1;(13,14)}(t)] \oplus B_1^P(t) + q_{3,2;(15,16)}(t) \oplus B_2^P(t)$$

where $W_i(t)$ be the probability that the server is busy in state S_i for preventive maintenance up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states and $\lim_{s\to 0} W_1^{P*}(s) = \frac{(\alpha_0+\lambda+\theta)}{\theta(\alpha_0+\lambda+\theta+\beta)}$, $\lim_{s\to 0} W_2^{P*}(s) = \frac{(\alpha_0)}{\theta(\alpha_0+\lambda+\eta+\beta)}$

$$\lim_{s\to 0} W_3^{P*}(s) = \frac{(\alpha_0+\lambda)}{\theta(\alpha_0+\lambda+\beta_1)},\ \lim_{s\to 0} W_6^{P*}(s) = \frac{\alpha_0}{\theta(\alpha_0+\lambda+\beta_1)} \tag{16}$$

Solving for $B_0^{p*}(s)$, the time for which server is busy due to preventive maintenance is given by

$$B_0^p(\infty) = \lim_{s\to 0} sB_0^{*p}(s) = \lim_{s\to 0} \frac{M_1^{p*}}{D_0'^*} = \frac{M_1^P}{D_0'} \tag{17}$$

where

$$M_1^P = \lim_{s\to 0} M_1^{P*}(s) \text{ and } D_0' = \lim_{s\to 0} D_0'^*(s) \tag{18}$$

$$M_1^P(t)=\frac{[\alpha_0\{(\theta+\lambda+\beta+\alpha_0)(\lambda+\beta_1+\alpha_0)-\beta\beta_1\}\{(\phi+\lambda+\beta+\alpha_0)(\lambda+\beta_1+\alpha_0)-\beta\beta_1\}\{(\eta+\lambda)(\lambda+\beta_1+\alpha_0)+\beta\lambda\}]}{[\theta(\lambda+\alpha_0)(\theta+\lambda+\beta+\alpha_0)(\phi+\lambda+\beta+\alpha_0)(\eta+\lambda+\beta+\alpha_0)(\lambda+\beta_1+\alpha_0)^3]} \tag{19}$$

and D'_0 has already been defined in (13)

6 Busy period analyses for server due to inspection and repair

Let $B_i^R(t)$ be the probability that the server is busy in inspection or repair of the unit at an instant 't' given that system entered state S_i at $t=0$. The recursive relations for $B_i^R(t)$ is as follows

$$B_0^R(t)=q_{0,1}(t)\oplus B_1^R(t)+q_{0,2}(t)\oplus B_2^R(t)$$

$$B_1^R(t)=q_{1,0}(t)\oplus B_0^R(t)+[q_{1,1;10}(t)+q_{1,1;10(11,12)}(t)]\oplus B_1^R(t)+[q_{1,2;9}(t)+q_{1,2;9(8,7)}(t)]\oplus B_2^R(t)+q_{1,3}(t)\oplus B_3^R(t)$$

$$B_2^R(t)=W_2^R(t)+q_{20}(t)\oplus B_0^R(t)+[q_{2,2;23}(t)+q_{2,2;23(24,25)}(t)+q_{2,2;28}(t)+q_{2,2;28,31}(t)+q_{2,2;28(29,30)}(t)+q_{2,2;28(29,30)31}(t)+q_{2,2;28,31(32,33)}(t)+q_{2,2;28(29,30)31(32,33)}]\oplus B_2^R(t)+q_{2,4}(t)\oplus B_4^R(t)+q_{2,6}(t)\oplus B_6^R(t)$$

$$B_3^R(t)=[q_{3,1}(t)+q_{3,1;(13,14)}(t)]\oplus B_1^R(t)+q_{3,2;(15,16)}(t)\oplus B_2^R(t)$$

$$B_4^R(t)=W_4^R(t)(t)+q_{4,0}(t)\oplus B_0^P(t)+[q_{4,1;20}(t)+q_{4,1;20(21,22)}(t)]\oplus B_1^R(t)+[q_{4,2;17}(t)+q_{4,2;17(18,19)}(t)]\oplus B_2^R(t)+q_{4,5}(t)\oplus B_5^R(t)$$

$$B_5^R(t)=W_5^R(t)+[q_{5,1;(34,35)}(t)]\oplus B_1^P(t)+[q_{5,2;(32,33)}(t)]\oplus B_2^P(t)+q_{5,4}(t)\oplus B_4^P(t)$$

$$B_6^R(t)=W_6^R(t)+[\,q_{6,2}(t)+q_{6,2;(24,25)}(t)+\,q_{6,2;(26,27)}(t)+q_{6,2;(26,27)31}(t)+\,q_{6,2;(26,27)31(32,33)}]\oplus B_2^R(t) \tag{20}$$

where $W_i^R(t)$ be the probability that the server is busy in state S_i due to repair up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states and $\lim_{s\to 0} W_2^{R*}(s)=\frac{\phi\eta(\beta+\eta)(\beta+\phi)+\lambda\{a\eta^2(\beta+\phi)+\beta\phi(\beta+\phi)+a\eta\beta^2\}}{\phi\eta(\beta+\eta)(\beta+\phi)(\alpha_0+\lambda+\eta+\beta)}$

$$\lim_{s\to 0} W_4^{R*}(s)=\frac{(\alpha_0+\lambda+\phi)}{\phi(\alpha_0+\lambda+\beta+\phi)},\ \lim_{s\to 0} W_5^{R*}(s)=\frac{(\alpha_0+\lambda)}{\phi(\alpha_0+\lambda+\beta_1)}$$
$$\lim_{s\to 0} W_6^{R*}(s)=\frac{\lambda(\phi+a\eta)}{\phi(\alpha_0+\lambda+\beta_1)} \tag{21}$$

Solving for $B_0^{R*}(s)$, the time for which server is busy due to preventive maintenance is given by

$$B_0^R(\infty)=\lim_{s\to 0}\ sB_0^{R*}(s)=\lim_{s\to 0}\frac{M_2^{R*}}{D_0'^{*}}=\frac{M_2^R}{D_0'} \tag{22}$$

$$M_2^R=\lim_{s\to 0} M_2^{R*}(s) \text{ and } D_0'=\lim_{s\to 0} D_0'^{*}(s) \tag{23}$$

Using relations (21) and (22) into (23)

$M_1^R=GH$

where $G=\frac{[\lambda[\{(\theta+\lambda+\beta+\alpha_0)(\lambda+\beta_1+\alpha_0)-\beta\beta_1\}]]}{[(\theta+\lambda+\beta+\alpha_0)(\lambda+\beta_1+\alpha_0)(\lambda+\alpha_0)]}$

$$H=\frac{\left[\begin{array}{r}(\phi+\alpha_0+\lambda+\beta)(\lambda+\alpha_0+\beta_1)-\beta_1\beta\}[\{(\lambda+\beta_1+\alpha_0)\eta+\lambda\beta\}(\beta+\eta)(\beta+\phi)(\phi+a\eta)+\\ \lambda(\lambda+\beta_1+\alpha_0)\{(a\eta^2+\beta\phi)(\beta+\phi)+a\eta\beta^2\}]\end{array}\right]}{\phi\eta(\beta+\eta)(\beta+\phi)(\lambda+\beta+\eta+\alpha_0)(\lambda+\beta+\phi+\alpha_0)(\lambda+\beta_1+\alpha_0)^2} \tag{24}$$

and D'_0 has already been defined in (13).

7 Expected number of visits by the server due to preventive maintenance and due to inspection, repair

Let $N_i^P(t)$ be the expected number of preventive maintenance and repair of unit by the server in (0, t] given that the system entered the regenerative state i at $t=0$. The recursive relations for $N_i^P(t)$ are given by

$$N_0^P(t) = Q_{0,1}(t) \oplus (N_1^P(t) + \delta_{PK}) + Q_{0,2}(t) \oplus (N_2^P(t) + \delta_{RK})$$

$$\begin{aligned} N_1^P(t) =& Q_{1,0}(t) \oplus N_0^P(t) + [Q_{1,1;10}(t) \\ &+ q_{1,1;10(11,12)}(t)] \oplus N_1^P(t) \\ &+ [Q_{1,2;9}(t) + Q_{1,2;9(8,7)}(t)] \oplus N_2^P(t) \\ &+ Q_{1,3}(t) \oplus N_3^P(t) \\ N_2^P(t) =& q_{20}(t) \oplus N_0^P(t) + [q_{2,2;23}(t) \\ &+ q_{2,2;23(24,25)}(t) + q_{2,2;28}(t) + q_{2,2;28,31}(t) + q_{2,2;28(29,30)}(t) \\ &+ q_{2,2;28(29,30)31}(t) + q_{2,2;28,31(32,33)}(t) \\ &+ q_{2,2;28(29,30)31(32,33)}] \oplus N_2^P(t) + q_{2,4}(t) \oplus N_4^P(t) \\ &+ q_{2,6}(t) \oplus N_6^P(t) \\ N_3^P(t) =& [q_{3,1}(t) + q_{3,1;(13,14)}(t)] \oplus N_1^P(t) + q_{3,2;(15,16)}(t) \oplus N_2^P(t) \\ N_4^P(t) =& q_{4,0}(t) \oplus N_0^P(t) + [q_{4,1;20}(t) + q_{4,1;20(21,22)}(t)] \oplus N_1^P(t) \\ &+ [q_{4,2;17}(t) + q_{4,2;17(18,19)}(t)] \oplus N_2^P(t) + q_{4,5}(t) \oplus N_5^P(t) \\ N_5^P(t) =& [q_{5,1;(34,35)}(t)] \oplus N_1^P(t) + [q_{5,2;(32,33)}(t)] \\ &\oplus N_2^P(t) + q_{5,4}(t) \oplus N_4^P(t) \\ N_6^P(t) =& [q_{6,2}(t) + q_{6,2;(24,25)}(t) + q_{6,2;(26,27)}(t) + q_{6,2;(26,27)31}(t) \\ &+ q_{6,2;(26,27)31(32,33)}] \oplus N_2^P(t) \end{aligned} \tag{25}$$

($K=P$, for preventive maintenance; $K=R$, for inspection and repair of the units).

Solving for $\tilde{N}_0^P(s)$. The expected no of preventive maintenance per unit time are, respectively, of given by

$$N_0^P(\infty) = \lim_{s\to 0} s\tilde{N}_0^P(s) = \frac{\tilde{M}_3^P}{\tilde{D}'_0} = \frac{M_3}{D'_0} \tag{26}$$

$$M_3 = \lim_{s\to 0} \tilde{M}_3^P(s) = \frac{\left[\begin{array}{l} [\eta\alpha_0\{(\phi+\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}[\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ -\{\alpha_0(\alpha_0+\lambda+\beta+\beta_1)\}]-[a\eta\lambda\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ (\alpha_0+\lambda+\beta+\beta_1)] \end{array}\right]}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^2(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)(\eta+\alpha_0+\phi+\beta)} \tag{27}$$

and D'_0 has already been defined in (13).

Solving for $\tilde{N}_0^R(s)$. The expected no of inspection and repair per unit time are, respectively, given by

$$N_0^R(\infty) = \lim_{s\to 0} s\tilde{N}_0^R(s) = \frac{\tilde{M}_4^R}{\tilde{D}'_0} = \frac{M_4}{D'_0} \tag{28}$$

where

$$M_4 = \lim_{s\to 0} \tilde{M}_4^R(s) = \frac{\left[\begin{array}{l} [\eta\lambda\{(\phi+\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}[\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ -\{\alpha_0(\alpha_0+\lambda+\beta+\beta_1)\}]-[a\eta\lambda\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ (\alpha_0+\lambda+\beta+\beta_1)] \end{array}\right]}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^2(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)(\eta+\alpha_0+\phi+\beta)}$$

and D'_0 has already been defined in (13).

8 Profit analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^R - K_2B_0^P - K_3N_0^R - K_4N_0^P \tag{29}$$

Let $K_0=(5000)$: revenue per unit up-time of the system,

$K_1=(400)$: cost per unit time for which server is busy due preventive maintenance,

$K_2=(500)$: cost per unit time for which server is busy due to repair and inspection,

$K_3=(350)$: cost per visit per unit time repair and inspection, and $K_4=(300)$: cost per visit per unit time preventive maintenance.

9 Discussion

To verify whether the cold standby system with priority to preventive maintenance over inspection is profitable or not, the numerical and graphical behavior of mean time to system failure, availability and profit function has been studied in Figs. 2, 3, 4, respectively. The application of this model in the industry as well as in the water supply system by taking particular values to the parameters like (α_0, β, β_1, λ, ϕ, η and θ).

Figure 2 is constructed to depict the graphical behavior of the MTSF (mean time to system failure).Thus, mean time to system failure increases swiftly with the increase

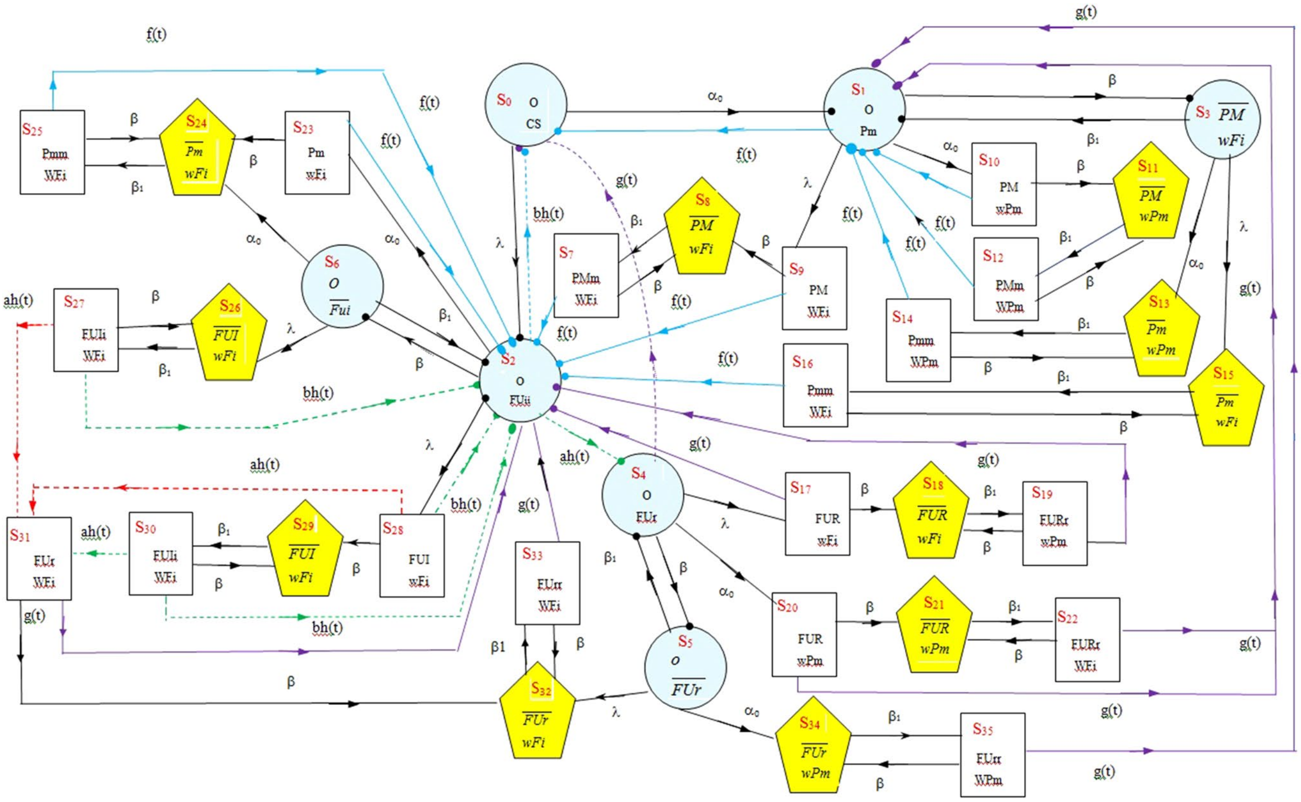


Fig. 1 State transition diagram

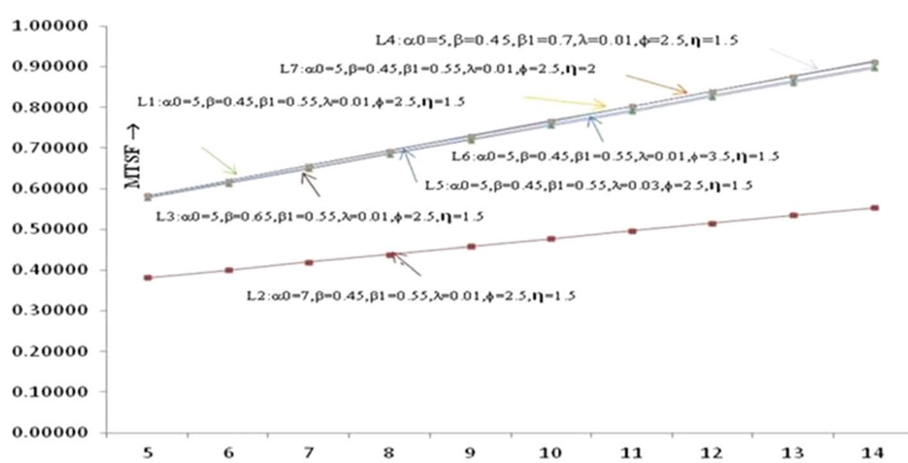


Fig. 2 Preventive maintenance rate (θ)

of preventive maintenance rate θ. The curve L_2 indicated when the rate by which the unit goes for preventive maintenance after completions of pre-specific maximum operation time α_0 changes from 5 to 7 the MTSF declined sharply, but in increasing manner as preventive maintenance rate θ increasing.

Figure 3 highlights graphical behavior of availability of the system Vs preventive maintenance rate θ. There is relatively steep rise in values of availability against parameter β_1 in comparison to other parameters. Second line L_2 of this table shows the when the rate α_0 change from 5 to 7 then the value of availability of the system rapidly declined from the range (0.48–0.72) to (0.33–0.63) The curve name $L_1(\alpha_0 = 5,\ \beta = .45,\ \beta_1 = .55,\ \lambda = .01,\ \phi = 2.5,\ \eta = 1.5)$ $L_5(\alpha_0 = .5,\ \beta = .45,\ \beta_1 = .55,\ \lambda = .03,\ \phi = 2.5,\ \eta = 1.5)$

Fig. 3 Preventive maintenance rate (θ)

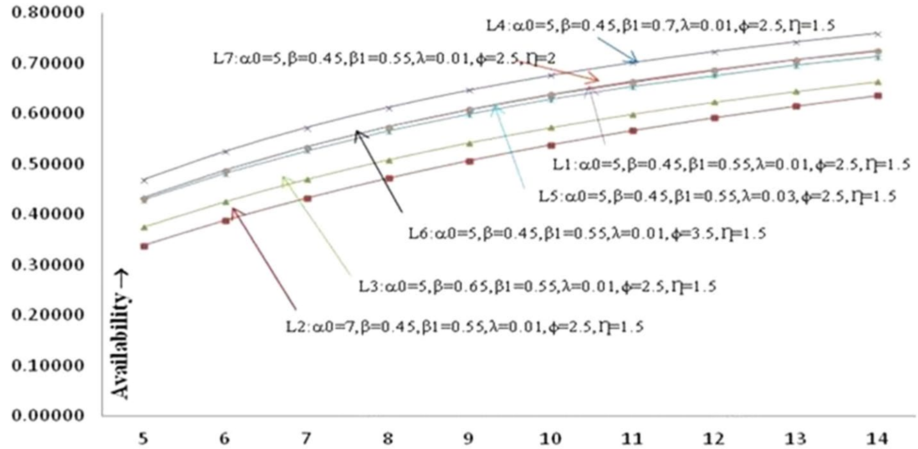
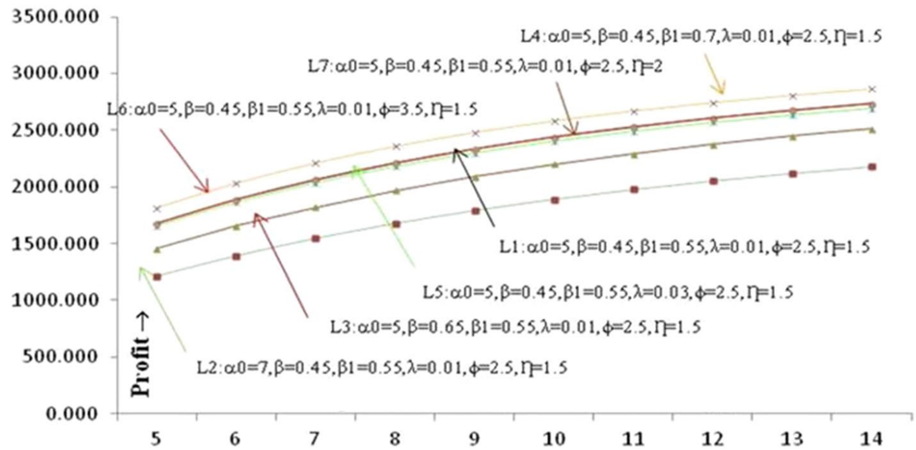


Fig. 4 Preventive maintenance rate (θ)



$L_6(\alpha_0 = .5, \beta = .45, \beta_1 = .55, \lambda = .03$ and $\phi = 3.5, \eta = 1.5)$ and the curve $L_7(\alpha_0 = .5, \beta = .45, \beta_1 = .55, \lambda = .01$ and $\phi = 3.5, \eta = 2)$ overlapping showing the similar impact of failure rate λ and repair rate ϕ and inspection rate η, on availability of system.

Figure 4 depicts the graphical behavior of the profit Vs preventive maintenance rate θ. The effect of different parameters can be observed easily from the graph. The system is more profitable if it works in controlled weather condition. The curve namely curve L_2 ($\alpha_0 = 7, \beta = .45, \beta_1 = .55, \lambda = .01, \phi = 2.5, \eta = 1.5$) indicates that the rate of the specific operation time α_0 increase 5–7 then there is steep fall in values of profit in comparison to other parameter. The curves $L_5(\alpha_0 = .5, \beta = .45, \beta_1 = .55, \lambda = .03, \phi = 2.5, \eta = 1.5)$ $L_6(\alpha_0 = .5, \beta = .45, \beta_1 = .55, \lambda = .03, \phi = 3.5, \eta = 1.5)$ and the curve $L_7(\alpha_0 = .5, \beta = .45, \beta_1 = .55, \lambda = .03, \phi = 2.5, \eta = 2)$ are coinciding curves showing the similar impact of failure rate λ and repair rate ϕ inspection rate η on profit of system.

10 Conclusion

It is concluded that the present model can be made the water supply system more available/beneficial by enhancing the inspection rate of system. Furthermore, by increasing preventive maintenance rate, a considerable profit can be obtained from system. In normal and abnormal weather conditions it is inferred that the system becomes productive when the preventive maintenance rate increases. Consequently, modifying maintenance mechanism adapted by the server followed by prioritizing preventive maintenance over inspection does wonders to the system.

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