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ARTICLE

Does everyone have equal voting power?

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Abstract
Banzhaf (Rutgers Law Rev 19:317–343, 1965) shows by citing a few examples that distortion in the bargaining power of the legislators may exist in the legislative body under weighted voting rules. Such a distortion leads to the incidence of unequal voting power of the citizens. In this paper, we investigate whether the distortion in the bargaining power is confined to a few weighted voting rules or is a general phenomenon. We show if the number of dominant political views (parties) present in a country is at most four, then there exists no weighted voting rule such that the distortion in the bargaining power can be avoided.

Keywords Weighted voting rule · Banzhaf voting power · Proportional voting rule · Equal voting power · Distortion

JEL Classification D72 · D71

1 Introduction

The federal structure of a country devolves legislative power to its constituent units—states. Each state is divided into a number of constituencies, and the population of the constituencies elects representatives by exercising their voting rights. The elected representatives consequently use their legislative power to formulate laws. In many countries, population size determines the number of constituencies to ensure equal representation of the citizens in the legislative body. Citizens' voting power gets materialized through the legislators (representatives) whom they elect. It is

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needless to say if a legislator does not have bargaining power in the legislative body, the population of the corresponding constituency does not enjoy any power either.

The bargaining power of the legislators are closely associated with the voting rule in question. Under weighted voting rules, legislators are assigned weights, and a quota is specified for passage of an agenda. If the total weight of the legislators supporting the agenda exceeds or equals the quota, the agenda gets passed, otherwise, it fails. Banzhaf (1965) shows that in many voting bodies such as in The New Jersey Senate, Nassau County, the voting power of the legislators, which reflect their bargaining strength, do not go in tandem with their popular supports. He shows if the voting power of the legislator is defined as the number of coalitions[1] in which the legislator is pivotal,[2] and the weight is assigned based on the relative population size of the constituency, then it is possible to have situations where a legislator representing a significant number of people in a constituency does not have any voting power or some of the legislators have the same voting power despite having represented different population sizes in their respective constituencies. These situations clearly indicate the distortion in the bargaining strength of the legislators in the legislative body, in the sense that the voting power of the legislators is not proportional to their underlying popular supports. As a consequence, the citizens of different constituencies do not enjoy equal voting power.

In India, a somewhat similar situation is noticed in the legislative bodies of many states—Assam, Manipur, Nagaland, Telangana, West Bengal. In these states, a single party forms a majority in the legislative body. In West Bengal, for instance, three major parties—All India Trinamool Congress (AITC), Bharatiya Janata Party (BJP), and Left Front (LF) contest the recent legislative assembly election 2021. AITC wins a whopping majority with 213 constituencies out of a total of 292. BJP and LF occupy only 77 and 1 constituencies, respectively. It is now evident that in the legislative body if a motion is passed based on a majority voting, legislators belonging to BJP and LF would hardly have any bargaining power to influence the outcome. The same situation prevails in legislative bodies of the other states mentioned above.

We illustrate this distortion in the bargaining power formally with the help of the following examples.

Let there be three legislators LA, LB, and LC representing constituencies A, B, and C with the population sizes—40, 000, 10, 000, and 10, 000, respectively. Let the weights assigned to the legislators based on the relative population sizes be 4, 1, 1, and the quota (Q) for passing an agenda be 3. The following table shows the coalitions in which a legislator is either pivotal(P) or non-pivotal(NP).

[1] Given a set of legislators, any subset of it is a coalition. A coalition is winning if the total weight of the members in the coalition is at least the quota. Thus, a winning coalition can force an agenda to pass with the support of all the members in the coalition. A coalition is losing if it is not winning.

[2] A legislator is pivotal in a coalition, if the removal of the legislator from the coalition turns a winning coalition into a losing one or the inclusion of the legislator in the coalition turns a losing coalition into a winning one.

Coalition	LA	LB	LC
$\{LA, LB, LC\}$	P	NP	NP
$\{LA, LB\}$	P	NP	NP
$\{LA, LC\}$	P	NP	NP
$\{LB, LC\}$	P	NP	NP
$\{LA\}$	P	NP	NP
$\{LB\}$	P	NP	NP
$\{LC\}$	P	NP	NP
$\emptyset$	P	NP	NP

Consider the first coalition-$\{LA, LB, LC\}$. This is a winning coalition as the total weight of the members in the coalition exceeds the quota. Notice, only LA is pivotal in $\{LA, LB, LC\}$. If he/she is removed from the coalition, i.e., $\{LB, LC\}$, it becomes a losing coalition. The same does not hold with the other legislators in the coalition. Likewise, for the coalition $\{LA, LB\}$, again, LA is pivotal. It is immediate, in all eight coalitions, only LA is pivotal.[3] If the voting power of a legislator is defined as the total number of coalitions in which he/she is pivotal, then the voting powers of LA, LB, and LC become 8, 0, and 0, respectively. An implication of this is that the population of the constituencies B and C do not have any voting power despite having a significant share of the total population.

An analogous example can be constructed to reflect further on the similar distortion in the bargaining power. The following example shows that the legislators representing different population sizes have equal voting power.

Let $Q = 2.5$ and the weights assigned to LA, LB, and LC on the basis of the relative population sizes be 2,2,1, respectively.

Coalition	LA	LB	LC
$\{LA, LB, LC\}$	NP	NP	NP
$\{LA, LB\}$	P	P	NP
$\{LB, LC\}$	NP	P	P
$\{LA, LC\}$	P	NP	P
$\{LA\}$	NP	P	P
$\{LB\}$	P	NP	P
$\{LC\}$	P	P	NP
$\emptyset$	NP	NP	NP
Voting Power	4	4	4

In the examples discussed above, there are two weighted voting rules with quota—3, 2.5, and weights- (4,1,1), (2,2,1), respectively. Both the rules follow the principle of proportional representation (the weights of the legislators are the relative population sizes of their respective constituencies). It turns out, for none of

[3] Since there are only three legislators, the total number of coalitions that can be formed is 2^3.

these proportional voting rules, the relative voting powers of the legislators correspond to the relative population sizes of the constituencies, implying a distortion in the bargaining power of the legislators. It is, therefore, pertinent to ask: is the incidence of distortion confined to a few weighted voting rules of the kind discussed above, or is it a general phenomenon? In other words, does there exist any weighted voting rule (not necessarily a proportional one)[4] such that the relative voting powers of the legislators correspond to the relative population sizes of their respective constituencies ensuring equality of voting power among the citizens?

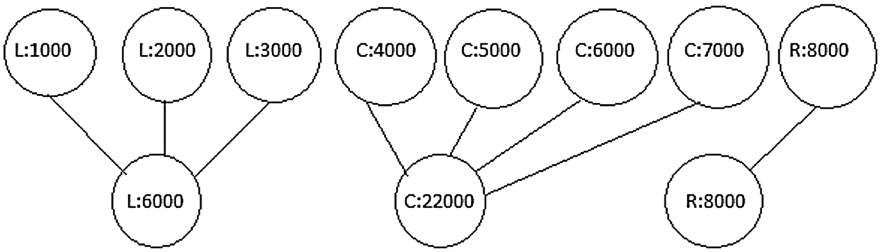
In this paper, we show, if there exist at most four dominant political views (parties) in a country and the population sizes of the constituencies represented by different political parties are distinct, then it is impossible to avoid distortion in the bargaining power of the legislators. At least two parties with unequal popular supports must have the same voting power or at least one of them has zero voting power. We use Banzhaf's definition of voting power to obtain the result.[5]

The requirement of distinct sizes of the population of the constituencies is a reasonable one and holds in almost all political scenarios. We explain the above result with the following example.

Suppose there are eight constituencies in a country, and there prevail three political views- left(L), centre(C), and right(R) (one can think of these political views as political parties representing different political ideologies). The constituencies have population sizes 1000, 2000, 3000,...,8000, respectively. Let three constituencies elect three legislators having 'left' political view, and four constituencies elect four legislators having 'centrist' view and the last elect a legislator with a 'right' political view. It is reasonable to assume that the legislators subscribing to the same political view (party) have the same voting preference. Thus, in the above political scenario, the three constituencies that elect three 'left' legislators can be construed as a single constituency with a representative legislator having a 'left' political view and consequently representing six thousand people. Likewise, the other four constituencies electing 'centrist' legislators can be considered as a single constituency having a representative legislator with a 'centrist' view and representing twenty-two thousand people. The last constituency electing a 'right' legislator has eight thousand people.

[4] Notice, weighted voting rules, by definition, do not require the weights necessarily to be equal to the relative population sizes of the constituencies. The weight could be any non-negative real number.

[5] The results also hold for Shapley–Shubik's definition of voting power. Unlike Banzhaf (1965) and Shapley and Shubik (1954) consider a voting game where voters vote in order. As soon as a majority has voted for an agenda, it is declared passed, and the member who has voted last is considered pivotal. Banzhaf and Shapley–Shubik voting indices have some similar properties. Thus, the result holding for Shapley–Shubik index comes as no surprise.



It follows from the above structure that the three representative legislators from the political parties—L, C, R represent population sizes—6000, 12000, and 8000, respectively. In such a political scenario the result of this paper (Theorem 1) shows, given a quota, there is no weight assignment for the representative legislators such that the distortion in the bargaining power of the legislators can be eliminated (or given a weight assignment for the representative legislators, there is no quota such that distortion in the bargaining power can be avoided).[6] This further implies that the citizens belonging to different constituencies do not enjoy equal voting power (Corollary 1).

Notice, in the above example, if the weights of the representative legislators are determined based on the relative number of seats (constituencies) won by the respective political parties—a case of proportional voting rule—it is immediate from the results (Theorem 1, Corollary 1) that the equality of voting power among the citizens cannot be realized. This implies that the results of this paper are more general and not confined to the proportional voting rules.

Furthermore, following similar arguments, it is possible to extend the implications of the results to the voting scenarios, where different political parties together form a coalition either to support an agenda or to form the government. Voting preferences of the members of the coalition, of course, need to be identical to draw such an inference.

There is substantial literature related to voting power and its measurements. Three broad strands of research have emerged in this area. One of the strands deals with the measures of voting power. It originates from the pioneering contributions by Penrose (1946), Shapley and Shubik (1954) and Banzhaf (1965), and later on by Coleman (1971), Brams and Affuso (1976), Deegan and Packel (1978), Holler (1982), Felsenthal and Machover (1998), Heard and Swartz (1998), Barua et al. (2007) and Turnovec (2007), to name a few.

Another strand looks at the properties of voting indices and provides characterizations of some of these indices. Holler (1985), for example, introduces a randomized voting rule as a solution to the problem of achieving equality between voter's weight

[6] To capture realistic political scenarios, we assume, no single party wins all the constituencies. Hence, the constituencies are shared among at least two political parties.

and his voting power and analyses the properties of such rules. Leech and Leech (2006) look at the sensitivity of voting indices when people form blocs. They consider the Penrose voting index and investigate how voting power varies as bloc size varies. Tchantcho (2008), on the other hand, provides a characterization of complete voting games allowing abstention as an alternative and looks at the sensitivity (influence) of the Shapley–Shubik and the Banzhaf–Coleman voting indices with respect to these voting games. Kirsch and Langner (2010) introduce a new way to represent the Penrose-Banzhaf and the Shapley–Shubik indices in terms of the minimum winning coalition. A similar exercise is carried out by Lange and Koczy (2013) for the Banzhaf and the Shapley–Shubik indices.[7]

The third strand focusses on the paradoxes of voting power that arise in different voting bodies. Banzhaf (1965), as discussed before, shows how variations in the population sizes of the constituencies sometimes result in paradoxical situations in relation to the voting power of the legislators. Brams and Affuso (1976) show that in the European Economic Community, when Ireland, Denmark, and Great Britain were admitted as members, the voting power of Luxembourg increased even though its fraction of the votes decreased. They call this the ‘Paradox of New Members’. Dreyer and Schotter (1980), along similar lines, show that in the reassignment of voting weights, thirty-eight countries got their voting weights reduced, yet they gained in voting power. They claim that such a paradox is largely prevalent in many voting rules discussed in Fischer and Schotter (1978). Laruelle and Valenciano (2002) present an empirical analysis of the voting procedure used by the European Council of Ministers. They calculate the voting power of the member states and the EU citizens and use them to compute the inequality among the EU citizens. Irrespective of the voting procedure used, their results show, the inequality among citizens persists.[8]

This paper contributes to the second and third strands of the literature. There are four sections in this paper. Section 2 contains basic notations and definitions for discussing the results. Section 3 provides the results relating to unequal voting power. Section 4 concludes the paper.

2 Preliminaries

Let a country be divided into n electoral constituencies. There exist $2 \leq k \leq 4$ dominant political parties in the country. $L = \{l_1, l_2, \dots, l_k\}$ denotes the set of representative legislators of k political parties. Typically, l_i denotes the legislator from the party i, $i \in \{1, 2, .., k\}$. Let m_i denote the number of constituencies won by the party i, $\sum_{i=1}^{k} m_i = n$, and $(n_i^1, n_i^2, \dots, n_i^{m_i})$ denote the vector of population sizes of the constituencies won by the party i. Let $n_i = \sum_{j=1}^{m_i} n_i^j$. n_i is the size of the

[7] Also see: Freixas and Lucchetti (2016), Leech (2013), Kurz (2012), Lindner (2008), Lindner and Owen (2008), Lindner and Machover (2004), Tolle (2003) and Fischer and Schotter (1978).

[8] Also see: Deemen and Rusinowska (2003) and Gelmen (2004).

total population of the constituencies won by the party i. We assume, n_i's are distinct.

2^L is the collection of all subsets of L. Any member of 2^L is called a coalition. Let $S_{-i,j}$ denote a coalition such that $l_i, l_j \notin S_{-i,j}$.

We define a weight function, w, as: $w : L \mapsto \mathbb{R}_+$. w assigns a non-negative weight to every legislator. Let $\{w_1, w_2, \dots, w_k\}$ represent a weight assignment of k legislators.

Let $W \subseteq 2^L$ be the collection of all winning coalitions. $S \in 2^L$ is winning if it can pass a quota. Formally, given a quota Q,

$$S \in W \leftrightarrow \sum_{l_i \in S} w_i \geq Q.$$

We assume that the grand coalition is always a winning coalition i.e., $L \in W$.

S is loosing *iff* it is not winning.

A weighted voting rule associated with L, a weight function w, and a quota $Q \in \mathbb{R}_+$ is a pair (L, V), where, $V(S)$ represents the worth of the coalition and is defined as follows:

$$\begin{aligned} &\forall S \in 2^L, \\ &\qquad V(S) \ = 1 \text{ if } S \in W \\ &\qquad\qquad\ \ = 0 \text{ if } S \notin W \end{aligned}$$

We formally define a pivotal voter as given in Banzhaf (1965).

A legislator l_i is pivotal in a coalition $S \in 2^L$ with respect to a weighted voting rule (L, V), if -removal of l_i turns a winning coalition, S, into a losing one, i.e., $V(S) = 1$ and $V(S - \{l_i\}) = 0$, or inclusion of l_i turns a losing coalition, S, into a winning one, i.e., $V(S) = 0$ and $V(S \cup \{l_i\}) = 1$.

$(v_1, v_2, \dots, v_k)$ denotes the vector of number of coalitions in which legislators are pivotal with respect to a weighted voting rule (L, V). According to Banzhaf (1965), $(v_1, v_2, \dots, v_k)$ is the vector of voting power of the legislators. Typically, v_i represents the voting power of l_i.

3 Unequal voting power: results

In this section, we show, the citizens do not enjoy equal voting power when there exist four dominant political parties. We consider Banzhaf's definition of voting power to obtain the result. In Lemma 1, we, first, establish that legislators with higher weights enjoy at least the same power as those with lower weights. We then prove Theorem 1, the result relating to distortion in bargaining power, using Lemma 1.

Lemma 1 *For any weighted voting rule, if $w_i \leq w_z$ for $l_i, l_z \in L$, then $v_i \leq v_z$.*

Proof Let l_i, l_z be two legislators with weights w_i, w_z, respectively. If $w_i = w_z$, it is immediate that $v_i = v_z$. Without loss of generality, suppose $w_i < w_z$. Let l_i be pivotal in a coalition $S \in 2^L$ with respect to a weighted voting rule (L, V). This implies $V(S) = 1$ and $V(S - \{l_i\}) = 0$, or $V(S) = 0$ and $V(S \cup \{l_i\}) = 1$.

Case (i): $V(S) = 1$ and $V(S - \{l_i\}) = 0$. If $l_z \in S$, then it is immediate from the definition of a pivotal voter that l_z is also pivotal in S. If $l_z \notin S$, then there exists another coalition with respect to S, i.e., $(S \cup \{l_z\}) - \{l_i\}$ in which l_z is pivotal.

Case (ii): $V(S) = 0$ and $V(S \cup \{l_i\}) = 1$. If $l_z \notin S$, then it is immediate from the definition of a pivotal voter that l_z is also pivotal in S. If $l_z \in S$, then there exists another coalition with respect to S, i.e., $(S \cup \{l_i\}) - \{l_z\}$ in which l_z is pivotal.

This implies that the voting power of l_z is no less than that of l_i. (1)

Cases (i) and (ii), in view of (1), establish the claim. □

Further, consider the following example. Let (L, V) be a weighted voting rule with quota $Q = 3$. Let $L = \{l_j, l_i, l_z\}$, and the weight assignment be $(w_j, w_i, w_z) = (1, 2, 3)$. In this example, it can be easily verified, $(v_j, v_i, v_z) = (2, 2, 6)$. (2)

Notice, given $w_z > w_i$, it is evident from (2) that $v_z > v_i$.

Lemma 1 shows that the voting power of a legislator does not decrease as his weight increases keeping everything else equal. Theorem 1 shows, for any weighted voting rule, either there exist at least two legislators who have the same voting power or there exists a legislator who has zero voting power when $2 \le k \le 4$. Since the population sizes represented by the legislators are assumed to be distinct, Theorem 1, therefore, implies that the relative voting powers of the legislators do not match the relative population sizes.

It, thus, follows that the citizens do not have equal voting power, as claimed in Corollary 1.

Theorem 1 *Let $2 \le k \le 4$. In every weighted voting rule, the voting powers of at least two legislators are the same or the voting power of at least one of them is zero.*

Proof Let (L, V) be any weighted voting rule with a quota Q. Notice, for any two legislators $l_i, l_j \in L$, if $w_i = w_j$, then $v_i = v_j$ follows immediately. Let all w_i's be distinct. Suppose, there exist at least two legislators, $l_i, l_j \in L$, such that $w_i, w_j \ge Q$. Under this case, both the legislators l_i, l_j have same voting power as they are pivotal in same number of coalitions. This is because, whenever l_i is pivotal in a coalition S, either l_j is pivotal in the same coalition S, or there exists another coalition with respect to S (as discussed in Lemma 1) in which l_j is pivotal, and vice versa.

Thus, $w_i, w_j \ge Q$ implies, l_i, l_j have same voting power. (3)

If $k = 2$, i.e., there are only two legislators, l_1, l_2, then we have either $w_1, w_2 < Q$ or ($w_1 \ge Q$ or $w_2 \ge Q$). The former implies, $v_1 = v_2$, and the latter implies, in view of (3), either $v_1 = v_2$ or ($v_1 = 0$ or $v_2 = 0$).

Given (3), suppose, at least three legislators in L have weights less than Q. (4)

Let, without loss of generality, $w_i < w_{i+1}; i \in \{1, 2, ..., k-1\}$.

Consider, $\{l_1, l_2\}$. If $v_1 = v_2$, then the result follows. Suppose, $v_1 \neq v_2$. Since $w_2 > w_1$, in view of Lemma 1, $v_2 > v_1$. This implies, there exists a coalition $S_{-1,2}$ in which l_2 is pivotal but l_1 is not. In other words, there exists $S_{-1,2}$ such that $[V(S_{-1,2} \cup \{l_2\}) = 1$ and $V(S_{-1,2}) = 0]$ and $V(S_{-1,2} \cup \{l_1\}) = 0$. (5)

Notice, $S_{-1,2} \neq \emptyset$ in view of (4) and $w_i < w_{i+1}; i \in \{1, 2, ..., k-1\}$.

Likewise, suppose $v_3 > v_2$, then there exists $S_{-2,3}$ such that $[V(S_{-2,3} \cup \{l_3\}) = 1$ and $V(S_{-2,3}) = 0]$ and $V(S_{-2,3} \cup \{l_2\}) = 0$ (6)

If $S_{-2,3} \in \{\{l_1\}, \emptyset\}$, then (6) implies, $V(\{l_1, l_3\}) = 1$ or $V(\{l_3\}) = 1$. This, together with $V(S_{-1,2} \cup \{l_1\}) = 0$ (from (5)), leads to a contradiction. Thus, $S_{-2,3} \in \{\{l_1, l_4\}, \{l_4\}\}$.

Suppose, $S_{-2,3} = \{l_1, l_4\}$.

(6) implies, $V(\{l_1, l_2, l_4\}) = 0$ and $V(\{l_1, l_3, l_4\}) = 1$.

Notice $S_{-1,2} \in \{\{l_3\}, \{l_4\}, \{l_3, l_4\}\}$. If $S_{-1,2} \in \{\{l_3\}, \{l_4\}\}$, then in view of $V(S_{-1,2} \cup \{l_2\}) = 1$ (from (5)), it contradicts $V(\{l_1, l_2, l_4\}) = 0$. If $S_{-1,2} = \{l_3, l_4\}$, then in view of $V(S_{-1,2} \cup \{l_1\}) = 0$, it contradicts $V(\{l_1, l_3, l_4\}) = 1$. It, therefore, follows $S_{-2,3} = \{l_4\}$ and $S_{-1,2} = \{l_3, l_4\}$ in view of (5). (7)

Again, between l_3, l_4, there exists $S_{-3,4}$ such that $[V(S_{-3,4} \cup \{l_4\}) = 1$ and $V(S_{-3,4}) = 0]$ and $V(S_{-3,4} \cup \{l_3\}) = 0$. Likewise, $S_{-3,4} \in \{\{l_1, l_2\}, \{l_1\}, \{l_2\}, \emptyset\}$. This, in view of (5) and (7), leads to a contradiction. Therefore, we must have $v_3 = v_4$. This establishes the claim. □

It may be noted, if the number of constituency is more than four then the same voting powers for at least two legislators can not be guaranteed. Consider the following example:

Let $k = 5, \{w_1, w_2, w_3, w_4, w_5\} = \{1, 2, 3, 4, 5\}, Q = 7$.

Voting power of the legislators: $\{v_1, v_2, v_3, v_4, v_5\} = \{2, 6, 10, 14, 18\}$.

The following corollary holds due to Theorem 1.

Corollary 1 *For* $2 \leq k \leq 4$, *there is no weighted voting rule such that the citizens enjoy equal voting power.*

Proof The proof is immediate. Theorem 1 shows, when there are at most four political parties, either at least two legislators have the same voting power or at least one has zero voting power. Given the distinct sizes of the populations represented by the legislators, this implies that the relative voting powers of the legislators do not correspond to relative population sizes. Thus, unequal voting power among the citizens prevails. □

In many countries, the party or the coalition that forms the government has the majority in the legislative body. An interesting observation in this context is, if the members of the party (coalition) in the legislative body have the same voting preference, then the rest of the members in the house who are not part of the ruling party have no voting power under the majority voting rule. The ruling coalition alone plays a decisive role. It, therefore, follows, equal voting power for all the citizens cannot be realized if the house functions based on the majority rule.

4 Conclusion

In this paper, we have tried to explore whether equal voting power (as defined by Banzhaf (1965)) among the citizens is possible under weighted voting rules. The results (Theorem 1, Corollary 1) show, if a country has at most four dominant political views and the population sizes represented by different political parties are distinct, then there always exists distortion in the bargaining strength of the legislators, which, in turn, leads to the incidence of unequal voting power of the citizens. It may be investigated, the results obtained under Banzhaf's definition of voting power also hold under Shapley and Shubik's definition of voting power as they share some similar properties namely, null player and Anonymity properties.[9]

In many countries, the ruling coalition has the majority in the legislative body. When a motion is passed based on the majority rule, irrespective of the number of dominant political parties present in the country, the ruling coalition enjoys absolute voting power, and the rest have no power at all.

In short, this paper establishes that the incidence of unequal voting power of the citizens prevails across all weighted voting rules and is hard to overcome.

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[9] In a weighted voting rule, a player is called null if he/she is not pivotal in any of the coalitions. Null player property requires a null player to remain null under the permutation of the players. Anonymity requires voting power to be insensitive to the permutation of the players.

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