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ARTICLE

N-firm oligopolies with pollution control and random profits

Akio Matsumoto[1] · **Ferenc Szidarovszky**[2]

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Abstract
An n-firm oligopoly is introduced in which, in addition to production levels, the pollution emissions are also included. A regulator cannot monitor individual emission volumes of firms, so uniform incentives are introduced to firms to reduce pollution concentrations. The regulator cannot observe the exact concentrations, so the incentives are also uncertain. Therefore, each firm considers random profit with expectations that it is maximized by minimizing variances or standard deviations. This idea leads to a multi-objective optimization problem for each firm, so two different concepts are applied as a solution. The unique positive Nash equilibrium is proven in all cases examined, and the effects of the environmental tax rate on industry output, prices, and pollution emission levels are analyzed.

Keywords NPS pollution · Environmental policy · Cournot oligopoly · Isoelastic demand · Multiobjective optimization

1 Introduction

Following the pioneering work of Cournot (1838), a great number of scientists were dealing with the different variants of oligopolies. Earlier results up to the mid-70s were summarized in Okuguchi (1976) where mainly linear models were discussed.

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✉ Akio Matsumoto
akiom@tamacc.chuo-u.ac.jp

Ferenc Szidarovszky
szidarka@gmail.com

1 Department of Economics, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo 192-0393, Japan

2 Department of Mathematics, Corvinus University, Fövám tér 8, Budapest 1093, Hungary

Two notable developments have been done since then. One is an introduction of nonlinearity into an oligopoly framework. Nonlinear models with their asymptotic behavior, including several modifications and variants of the classical model, were analyzed in Bischi et al. (2010). The other is the inclusion of environmental issues. More and more attention has been given to environmental policies to control environmental degradation in recent decades. The primary sources of the degradation are greenhouse and the pollution emission by industries using nonrenewable energy sources. The governments and other regulators are interested in creating incentives for the firms to reduce pollution emissions. Two kinds of polluters are usually considered, point source (PS) polluters and non-point source (NPS) polluters. In the case of PS polluters, the regulator knows the individual emission levels, so that it can place punishments or subsidies individually. In the case of NPS polluters, the regulator can monitor only the total emission level produced by the industry, so the traditional approach cannot be used.

The regulator can effectively use a traditional environmental tax to curb emissions in the literature on PS pollution. Haruna and Goel (2018) studied optimal pollution abatement in a mixed duopoly when the firms have a linear price function and perform emission reduction R &D with imperfect appropriation. The optimal strategies for public and private firms are determined and compared. Gama (2020) ranked the widely used environmental command-and-control instruments in output, profit, consumer surplus, and social welfare in an n-firm framework. Sagasta and Usategui (2018) compared the optimal emission taxes on selling and renting firms in a durable goods oligopoly with *n*-firm. Wu and Chen (2013) compared PS and NPS pollutions in water quality and determined that NPS pollution made larger contribution to nutrient loads.

Controlling NPS pollution, Segerson (1988) suggested monitoring the ambient concentration of pollutants and introducing uniform taxes or subsidies to the firms. Therefore, the regulator defines an environmental standard, and implements an ambient charge policy. Accordingly, it charges taxes if the concentration is above this standard or gives subsidies if the concentration is below it. Ganguli and Raju (2012) examined it in the Bertrand duopoly and noticed that larger ambient tax could result in higher concentration. Raju and Ganguli (2013) examined it in Cournot duopoly and numerically showed the effectiveness of ambient charges. Ishikawa et al. (2019) showed that the ambient charge effect in *n*-firm Bertrand oligopolies depends upon the number of firms, their abatement technologies, and the degree of substitutability of the differentiated products. Matsumoto et al. (2018) presented a generalization of the effective ambient charge in the *n*-firm Cournot framework. Matsumoto et al. (2018) constructed a multi-stage game in which duopoly firms determine their optimal output and select the optimal abatement technologies, whereas the regulator determines the optimal tax. Matsumoto and Szidarovszky (2021) introduced hyperbolic demand function in a Cournot oligopoly without product differentiation and demonstrated the condition under which the ambient charge is effective.

The basic model in this paper is an *n*-firm Cournot oligopoly where the firms produce homogeneous goods and emit pollution. The first novelty of the model is adopting a nonlinear price function instead of linear price functions that were assumed to simplify the mathematical analysis in most studies. The second and more critical novelty is an introduction of uncertainty. All models introduced and discussed earlier were based on the simplifying assumption that the regulator can observe the pollution concentration exactly, and analytical considerations are confined to the behavior of the firms to avoid the uncertainty influence or limited to a duopoly case. However, these are not the cases in real life, since concentration might depend on the location where measurement is made, and every measuring equipment faces certain inaccuracies. Since this error is not known exactly by the firms, they consider a random error that either adds to or multiplies the true concentration. Therefore, the profit of each firm is also random, and its expectation is the major objective to be maximized. To ensure best or close to best outcome, the variance or the standard deviation of the profit is also minimized. Therefore, each firm faces two conflicting objectives. Two concepts are applied in this paper: multi-objective optimization and the idea of certainty equivalent. One-stage games are analyzed where the firms cannot modify the exogeneous clearing technology. In the first case, the equilibrium is unique, and in the second case, there is the possibility of multiple equilibria. The effects of the ambient charges are also analyzed on output and emission levels.

This paper is developed as follows. The next section introduces the mathematical model with two different ways to incorporate error terms. They are discussed in Sects. 3 and 4. The additive case is equivalent to the deterministic model, so we concentrated on the second approach of a random multiplier. Section 4 is divided into two parts depending on the solution concept. For each case, equilibrium analysis is presented, and the dependence of the individual outputs of the firms, industry output, product unit price, and pollution emission levels on the tax rate is examined in Sects. 5 and 6. Section 7 discusses the duopoly case, and Sect. 8 offers conclusions and further research directions.

2 Model

An n-firm oligopoly is considered without product differentiation and with hyperbolic demand. There are several ways to introduce hyperbolic (or isoelastic) price functions. Assume that a representative consumer has a logarithmic utility function, $U(Q) = \ln(\alpha Q)$, where Q is the demand, then it maximizes the net benefit $\ln(\alpha Q) - pQ$, where p is the price. Maximizing this function with respect to Q gives the first-order condition

$$\frac{1}{\alpha Q}\alpha - p = 0,$$

or

$$p = \frac{1}{Q}.$$

Another way is based on the assumption that the total buying budget is normalized to unity, and the consumers want to buy as much product as possible subject to the budget constraint. This leads to the equation $pQ = 1$ or $p = 1/Q$. Assuming constant elasticity

$$\frac{dQ}{dp}\frac{p}{Q} = r$$

implying that

$$\int \frac{r}{p} dp = \int \frac{1}{Q} dQ$$

or

$$r \ln p = \ln Q + \ln c$$

resulting in

$$p = (cQ)^{\frac{1}{r}} \text{ and if } r = -1, \text{ then } p = \frac{1}{c}\frac{1}{Q}.$$

Bischi et al. (2010) showed a collection of examples illustrating how the selections of price and cost functions affect the existence and uniqueness of equilibria. The case of hyperbolic price functions was discussed in detail, and its relations to market share and rent-seeking games were also discussed.

Let x_i denote the output of firm i and $Q = \sum_{i=1}^{n} x_i$ the industry output. The price function is assumed to be

$$p = \frac{1}{Q},$$

under the assumption that total demand equals the industry output. The production processes of the firms require pollution emissions. With the appropriate unit, we can assume that one unit of production emits one unit of pollution. To avoid penalties, the firms abate parts or the entire amounts of pollution they produce. Let $1 - e_i$ denote the pollution reduction coefficient of firm i, $0 \le e_i \le 1$, where $e_i = 0$ refers to the case of no pollution emission, and $e_i = 1$ shows fully discharged case, when no abatement is done. Here, the unit of e_i and $1 - e_i$ is emission level/production volume. That is, $e_i x_i$ is the individual emission level of firm i, while $(1 - e_i)x_i$ is abated. Based on the production volume, the regulator is able to asses to the total pollution produced by the firms; however, it cannot determine the total emission level, since parameters e_i are decided by the firms without reporting them to the regulator. Let c_i denote the unit production cost of firm i, and assume that the regulator defines an emission standard, $\bar{E}$, and a penalty or reward rate θ(like environmental tax rate). The unit of c_i is money/production volume, that of $\bar{E}$ is emission level, and θ is measured in money/emission level. If the amount of emission exceeds $\bar{E}$, then the firms pay a uniform penalty which is the θ-multiple of

$$\sum_{i=1}^{n} e_i x_i - \bar{E}.$$

If the total emission is lower than $\bar{E}$, then the firms receive a uniform reward, which is the θ-multiple of

$$\bar{E} - \sum_{i=1}^{n} e_i x_i.$$

Therefore, the payoff function of firm i becomes

$$PR_i = \frac{x_i}{\sum_{i=1}^{n} x_i} - c_i x_i - \theta\left(\sum_{i=1}^{n} e_i x_i - \bar{E}\right). \tag{1}$$

The price function is $1/\sum_{i=1}^{n} x_i$, which is measured in money/production volume, so all terms represent money: revenue, cost, and environmental tax.

In this case, an n-player game is defined, where the firms are the players, and the strategy and payoff of firm i are x_i and PR_i, respectively. This deterministic game ignores the fact that the regulator cannot measure the exact amount of the total emission level of pollution. Therefore, firm i believes that the regulator will charge or reward it with an amount calculated with a randomly measured total emission level. There are two simple ways to model this uncertainty, either assuming that the measured value is

$$\sum_{i=1}^{n} e_i x_i + \varepsilon_i \text{ or } \left(\sum_{i=1}^{n} e_i x_i\right)\varepsilon_i,$$

where ε_i is based on the subjective judgement of firm i. Therefore, the random payoff of firm i has the form

$$\pi_i = \frac{x_i}{\sum_{i=1}^{n} x_i} - c_i x_i - \theta\left(\sum_{i=1}^{n} e_i x_i + \varepsilon_i - \bar{E}\right) \tag{2}$$

or

$$\bar{\pi}_i = \frac{x_i}{\sum_{i=1}^{n} x_i} - c_i x_i - \theta\left(\left(\sum_{i=1}^{n} e_i x_i\right)\varepsilon_i - \bar{E}\right). \tag{3}$$

The unit of ε_i in (2) is the same in which emission is measured. In (3), ε_i is a positive real number. In the case of payoff (2), it is assumed that

$$Exp(\varepsilon_i) = 0 \text{ and } Var(\varepsilon_i) = \delta_i^2,$$

and in the case of (3), it is assumed that

$$Exp(\varepsilon_i) = 1 \text{ and } Var(\varepsilon_i) = \delta_i^2.$$

These cases can be called "the error addition" and "the error multiplication" cases. If measurement error is independent of the order of magnitude of the measured quantity, then "error addition" is the right approach. If error is a certain proportion (e.g., percent) of the measured quantity T, then the measurement is $T + \varepsilon T = T(1 + \varepsilon)$. In the first case, ε_i can be considered as the measurement error; in the second case, $\varepsilon_i - 1$ is the relative measurement error.

3 The case of error addition

Firm i wants to maximize its expected profit, and at the same time, it is willing to secure this payoff level by minimizing its standard deviation (or variance). Therefore, firm i wants to maximize

$$Exp\left(\pi_i\right) = \frac{x_i}{\sum_{i=1}^{n} x_i} - c_i x_i - \theta\left(\sum_{i=1}^{n} e_i x_i - \bar{E}\right) \tag{4}$$

and minimize

$$D\left(\pi_i\right) = \theta\delta_i \text{ or } Var\left(\pi_i\right) = \theta^2\delta_i^2 \tag{5}$$

which are constants, so firm i has no control on these values. Therefore, firm i maximizes payoff (4) which leads to the deterministic case being discussed in the literature by many authors. Therefore, we ignore this case.

4 The case of error multiplication

Consider now the random payoff function (3). Similarly to the previous case, firm i wants to maximize its expectation $Exp(\bar{\pi}_i) = Exp(\pi_i)$, and minimize its standard deviation or variance. Notice that

$$D\left(\bar{\pi}_i\right) = \theta\left(\sum_{i=1}^{n} e_i x_i\right)\delta_i \text{ and } Var\left(\bar{\pi}_i\right) = \theta^2\left(\sum_{i=1}^{n} e_i x_i\right)^2 \delta_i^2. \tag{6}$$

Both the standard deviation and the variance of $\bar{\pi}_i$ are strictly increasing in x_i, so smallest standard deviation and smallest variance occur when $x_i = 0$ meaning no production. In practice, it means minimal output volume that guarantees survival of the firm. That is, firm i faces two conflicting objectives. In the multi-objective optimization literature, there are many concepts and methods to deal with multiple objectives (see for example, (Szidarovszky et al. 1986), (Zarghami and Szidarovszky 2011)) including the lexicographic method, ε-constraint, weighting, distance and direction-based methods. The most popular concept is weighting. Let $\alpha_i > 0$ denote the subjective importance of the variance or standard deviation in comparison to expectation, then the composite objective function becomes

$$\varphi_i = \frac{x_i}{\sum_{i=1}^{n} x_i} - c_i x_i - \theta\left(\sum_{i=1}^{n} e_i x_i - \bar{E}\right) - \alpha_i \theta\left(\sum_{j=1}^{n} e_j x_j\right)\delta_i \tag{7}$$

or

$$\bar{\varphi}_i = \frac{x_i}{\sum_{i=1}^{n} x_i} - c_i x_i - \theta\left(\sum_{i=1}^{n} e_i x_i - \bar{E}\right) - \alpha_i \theta^2\left(\sum_{j=1}^{n} e_j x_j\right)^2 \delta_i^2. \tag{8}$$

In the first case, α_i is a real value, while in the second case, its unit is 1/money to have same units in all terms of the composite objective function. The motivation for the first case is based on the fact that expectation and standard deviation have the same dimension, therefore comparable. The second case is the realization of the concept of certainty equivalent of random payoffs.

Notice that if f_1 and f_2 are the objectives, then the weighting method usually is written in the from of optimizing $\gamma f_1 + (1-\gamma) f_2$, $(0 < \gamma < 1)$ which is equivalent to optimizing

$$f_1 + \frac{1-\gamma}{\gamma} f_2 = f_1 + \alpha f_2.$$

This form shows the relative importance of f_2 in comparison to f_1 .

4.1 Minimizing standard deviations

It is now assumed that in the n-player game the payoff of firm i is given by Eq. (7). To find the equilibrium of the game, the best responses of the firms have to be first determined. The marginal payoff of firm i is the following:

$$\frac{\partial \varphi_i}{\partial x_i} = \frac{y_i}{\left(y_i + x_i\right)^2} - c_i - \theta e_i - \alpha_i \theta e_i \delta_i, \tag{9}$$

where $y_i = \sum_{j \neq i} x_j$ is the output of the rest of the industry from the point of view of firm i. Introduce the notation

$$A_i = c_i + \theta e_i + \alpha_i \theta e_i \delta_i.$$

Since at $x_i = 0$

$$\frac{\partial \varphi_i}{\partial x_i} = \frac{1}{y_i} - A_i,$$

the best response of firm i is as follows:

If $y_i \geq 1/A_i$, then $x_i = 0$; otherwise, $x_i \geq 0$ and solves the first-order condition

$$\frac{y_i}{\left(x_i + y_i\right)^2} - A_i = 0$$

or

$$x_i = \sqrt{\frac{y_i}{A_i}} - y_i. \tag{10}$$

That is

$$R_i(y_i) = \begin{cases} 0 \text{ if } y_i \geq \dfrac{1}{A_i}, \\ \sqrt{\dfrac{y_i}{A_i}} - y_i \text{ otherwise.} \end{cases} \tag{11}$$

We can also rewrite the best response of firm i in terms of the industry output. If $x_i = 0$, then $y_i = Q$, and if $x_i > 0$, then it solves equation

$$x_i = \sqrt{\frac{Q - x_i}{A_i}} - y_i$$

or

$$Q = \sqrt{\frac{Q - x_i}{A_i}},$$

implying that

$$x_i = Q - A_i Q^2. \tag{12}$$

Hence, the best response of firm i can be rewritten as follows:

$$\bar{R}_i(Q) = \begin{cases} 0 \text{ if } Q \geq \dfrac{1}{A_i}, \\ Q - A_i Q^2 \text{ otherwise.} \end{cases} \tag{13}$$

The industry output has to satisfy equation

$$\sum_{i=1}^{n} \bar{R}_i(Q) - Q = 0. \tag{14}$$

The left-hand side is continuous; at $Q = 0$, it is zero with positive derivative if at least two firms have positive outputs. In addition, for $Q > \max\{1/A_i\}$, it is negative, so there is at least one solution for Q. The equilibrium outputs of the firms are $x_i = \bar{R}_i(Q)$. In a boundary equilibrium, one or more firms have zero equilibrium outputs. Ignoring these firms, the game is reduced to the same type of game with less firms. Clearly, the equilibrium outputs of the remaining firms give equilibrium of the reduced game.

Assume first that the equilibrium is positive. Adding Eq. (12) for all values of i

$$Q = nQ - Q^2 A$$

with $A = \sum_{j=1}^{n} A_j$, so

$$Q^* = \frac{n-1}{A}, \tag{15}$$

and the corresponding output values are given by (12). The equilibrium is positive if

$$Q^* - (Q^*)^2 A_i > 0 \text{ for all } i$$

or

$$A_i < \frac{A}{n-1} \text{ for } i = 1, 2, ..., n. \tag{16}$$

If this relation is violated, then there is no positive equilibrium. Notice that

$$\frac{\partial^2 \varphi_i}{\partial x_i^2} = -\frac{2y_i}{(y_i + x_i)^3} < 0,$$

so (12) is the best response of firm i, so we have the following result:

Theorem 1 Under conditions (16), the unique equilibrium is positive. The equilibrium outputs of the firms are given by (12) with $Q = Q^*$.

Notice that (16) can be rewritten as

$$A_i < \frac{n-1}{n-2} \frac{1}{n-1} \sum_{j \neq i} A_j,$$

showing that none of the A_i values can be much larger than the average of the other A_j values. Notice that if firm i increases one of the values of α_i, c_i, e_i or the regulator increases the value of θ, then A_i increases. If its value violates this condition, then there is no interior equilibrium anymore. Also, increasing A_i implies the increase of A resulting in decrease of Q and therefore the increase of price.

If the uncertainty level of at least one firm increases, then for these firms, A_i increases, while for the other firms, A_j remains unchanged. Therefore, A increases, implying that the industry output decreases. The individual output level of firm i is given as

$$x_i = Q - A_i Q^2.$$

Notice first that

$$\frac{\partial A}{\partial \delta_i} = \frac{\partial A_i}{\partial \delta_i} = \alpha_i e_i \theta > 0,$$

and since

$$Q = \frac{n-1}{A},$$

$$\begin{aligned} \frac{\partial x_i}{\partial \delta_i} &= \frac{\partial Q}{\partial \delta_i} - Q^2 \frac{\partial A}{\partial \delta_i} - 2A_i Q \frac{\partial Q}{\partial \delta_i} \\ &= \frac{n-1}{A^2}\left(\frac{2(n-1)A_i}{A} - n\right), \end{aligned}$$

which is negative if

$$A_i < \frac{n}{2(n-1)} A.$$

In the case of duopoly $n = 2$, this condition reduces to $A_i < A$, which always holds.

The possible boundary equilibria can be found in the following way. Let $S \subseteq \{1, 2, ..., n\}$ be a subset of the firms. Define

$$A_S = \sum_{i \in S} A_i,$$

and let

$$Q_S = \frac{m-1}{A},$$

where m is the number of firms in S, and for $i \in S$, let

$$x_i^S = Q_S - A_i Q_S^2.$$

If $x_i^S > 0$ for all $i \in S$, then a positive equilibrium is obtained for the reduced game on S. We need then to check if $x_j^S = 0$ ($j \notin S$) are the corresponding equilibrium strategies for all firms outside S. This is the case if the marginal profits of these firms are nonpositive at zero, that is, if

$$\frac{1}{\sum_{i \in S} x_i^S} - c_j - \theta e_j - \alpha_j \theta e_j \delta_j \leq 0 \ (all\ j \notin S).$$

Then x_ℓ^S ($\ell = 1, 2, ..., n$) give an equilibrium of the n firm game. If $S = \{1, 2, ..., n\}$, then there is no firm outside S, so only the first part has to be checked. This procedure can be repeated for all subsets S of the firms to detect all possible boundary equilibria.

Consider now the modified game with payoff functions

$$\bar{\varphi}_i = \frac{x_i}{\sum_{i=1}^n x_i} - c_i x_i - \theta e_i x_i - \alpha_i \theta e_i \delta_i x_i$$

which has the same marginal profit of all firms than in the original game. Therefore, the best responses of the firms are also same in the two games, implying that they have identical equilibria. This modified game satisfies the existence and uniqueness conditions of Szidarovszky and Okuguchi (1997), so the n-firm oligopoly with payoff function (7) also has a unique equilibrium.

4.2 Minimizing variances

Now, we assume that the payoff of firm i is given by (8). We are looking again for a positive equilibrium. The first-order condition for firm i is given as

$$\frac{\partial \varphi_i}{\partial x_i} = \frac{y_i}{\left(y_i + x_i\right)^2} - c_i - \theta e_i - 2\alpha_i \theta^2 \delta_i^2 \left(\sum_{j=1}^n e_j x_j \right) e_i = 0. \tag{17}$$

In addition to $Q = \sum_{j=1}^n x_j$, introduce the notation $S = \sum_{j=1}^n e_j x_j$, then from (17), we have

$$\frac{Q - x_i}{Q^2} - c_i - \theta e_i - 2\alpha_i \theta^2 \delta_i^2 S e_i = 0$$

or

$$x_i = Q - Q^2 \left(C_i + B_i S \right), \tag{18}$$

where

$$C_i = c_i + \theta e_i \text{ and } B_i = 2\alpha_i \theta^2 \delta_i^2 e_i.$$

Adding Eq. (18) for all i to have

$$Q = nQ - Q^2 \left(\sum_{j=1}^n C_j + S \sum_{j=1}^n B_j \right)$$

or

$$Q \left(\sum_{j=1}^n C_j + S \sum_{j=1}^n B_j \right) = n - 1. \tag{19}$$

Multiply Eq. (18) by e_i and add the resulting equations for all values of i

$$S = Q\sum_{j=1}^{n} e_j - Q^2\left(\sum_{j=1}^{n} e_jC_j + S\sum_{j=1}^{n} e_jB_j\right). \tag{20}$$

Combining (19) and (20), and using relation

$$S = \frac{n-1-Q\sum_{j=1}^{n} C_j}{Q\sum_{j=1}^{n} B_j}, \tag{21}$$

we have

$$\frac{n-1-Q\sum_{j=1}^{n} C_j}{Q\sum_{j=1}^{n} B_j} = Q\sum_{j=1}^{n} e_j - Q^2\left(\sum_{j=1}^{n} e_jC_j + \frac{n-1-Q\sum_{j=1}^{n} C_j}{Q\sum_{j=1}^{n} B_j}\sum_{j=1}^{n} e_jB_j\right). \tag{22}$$

Introduce the simplifying notation

$$B = \sum_{j=1}^{n} B_j,\ C = \sum_{j=1}^{n} C_j,\ E = \sum_{j=1}^{n} e_j$$
$$\bar{B} = \sum_{j=1}^{n} e_jB_j,\ \bar{C} = \sum_{j=1}^{n} e_jC_j,$$

and simplify (22) to the following form:

$$\frac{n-1-QC}{QB} = QE - Q^2\left(\bar{C} + \frac{n-1-QC}{QB}\bar{B}\right),$$

which gives a cubic equation for unknown Q

$$P(Q) = Q^3\left(\bar{C}B - \bar{B}C\right) + Q^2\left[(n-1)\bar{B} - BE\right] - QC + (n-1) = 0. \tag{23}$$

The constant term is positive, the linear coefficient is negative, and the signs of the first two leading coefficients are indeterminate. Therefore, it is very hard to deal with this general case. A cubic polynomial has one or three real roots. The following result can be used to detect the number of roots. Assume that the cubic polynomial has the general form

$$\alpha x^3 + \beta x^2 + \gamma x + \delta,$$

then it has three distinct real roots if and only if

$$-27\alpha^2\delta^2 + 18\alpha\beta\gamma\delta - 4\alpha\gamma^3 - 4\beta^3\delta + \beta^2\gamma^2 > 0.$$

In our case

$$\alpha = \bar{C}B - \bar{B}C,\ \beta = (n-1)\bar{B} - BE,\ \gamma = -C \text{ and } \delta = n-1,$$

so checking this condition seems very complicated without obtaining significant new results. To make the mathematics more tractable have the following assumption:

1. **Assumption 1**. For all i, $e_i \equiv e$.

In this case

$$\bar{C}B - \bar{B}C = e\left(\sum_{j=1}^{n} C_j \sum_{j=1}^{n} B_j\right) - e\left(\sum_{j=1}^{n} B_j \sum_{j=1}^{n} C_j\right) = 0$$

and

$$(n-1)\bar{B} - BE = (n-1)e\sum_{j=1}^{n} B_j - ne\sum_{j=1}^{n} B_j = -eB.$$

Therefore, (23) becomes quadratic:

$$eBQ^2 + CQ - (n-1) = 0, \tag{24}$$

which has a unique positive root

$$Q^* = \frac{-C + \sqrt{C^2 + 4(n-1)eB}}{2eB}. \tag{25}$$

Therefore, $S^* = eQ^*$ and from (18)

$$x_i^* = Q^* - (Q^*)^2\left(C_i + eB_iQ^*\right), \tag{26}$$

which is positive if

$$eB_i(Q^*)^2 + C_iQ^* - 1 < 0. \tag{27}$$

The left-hand side is quadratic and has one positive root

$$\bar{Q}_i = \frac{-C_i + \sqrt{C_i^2 + 4eB_i}}{2eB_i}, \tag{28}$$

and (27) holds if $Q^* < \bar{Q}_i$ for all i.

Theorem 2 If $Q^* < \bar{Q}_i$ for all i, then, under Assumption 1, there is a unique positive equilibrium given by (25) and (26).

Notice that B_i increases in δ_i and C_i does not depend on δ_i. Therefore, B_i increases in δ_i and all other B_j and all C_i do not depend on δ_i.Therefore, B increases based on (28),Q^* decreases if δ_i increases. In other words, the increase of the uncertainty level of any firm has a decreasing effect on the industry output. To examine the dependence of x_i on δ_i, notice first that from

$$Q = \frac{2(n-1)}{C + \sqrt{C^2 + 4(n-1)eB}},$$

we have

$$\sqrt{C^2 + 4(n-1)eB} = \frac{2(n-1)}{Q} - C,$$

and

$$\begin{aligned}\frac{\partial Q}{\partial \delta_i} &= \frac{-2(n-1)}{\left(C + \sqrt{C^2 + 4(n-1)eB}\right)^2} \frac{4(n-1)e}{2\sqrt{C^2 + 4(n-1)eB}} \frac{\partial B}{\partial \delta_i} \\ &= \frac{4\alpha_i \delta_i e^2 \theta^2}{CQ - 2(n-1)} Q^3.\end{aligned}$$

Hence

$$\frac{\partial Q}{\partial \delta_i} = XQ^3 \text{ with } X = \frac{4\alpha_i \delta_i e^2 \theta^2}{CQ - 2(n-1)},$$

which is clearly negative, since

$$Q < \frac{2(n-1)}{C}.$$

Furthermore

$$\begin{aligned}\frac{\partial x_i}{\partial \delta_i} &= \frac{\partial Q}{\partial \delta_i} - 2Q\frac{\partial Q}{\partial \delta_i}\left(C_i + eB_iQ\right) - Q^2\left(4\alpha_i\delta_i e^2\theta^2 Q + eB_i \frac{\partial Q}{\partial \delta_i}\right) \\ &= Q^3X\left(1 - 2Q\left(C_i + eB_iQ\right)\right) - Q^2\left(4\alpha_i\delta_i e^2\theta^2 Q + eB_iXQ^3\right) \\ &= Q^3\left[X\left(1 - 2QC_i - 3eB_iQ^2\right) - X(CQ - 2(n-1))\right] \\ &= -XQ^3\left[3eB_iQ^2 + Q(C + 2C_i) - (2n-1)\right].\end{aligned}$$

This is negative if the square bracketed term is negative or

$$3eB_iQ^2 + (C + 2C_i)Q - (2n-1) < 0.$$

Clearly, Q and B_i depend upon δ_i; therefore, this condition is difficult to be interpreted.

We can use a similar algorithm to the method outlined in the previous case to detect all possible boundary equilibria. We can illustrate the condition $Q^* < \bar{Q}_i$ in the case of symmetric firms. Then, $B = nB_i, C = nC_i$, and the condition means that

$$\frac{-nC_i + \sqrt{n^2C_i^2 + 4(n-1)enB_i}}{2enB_i} < \frac{-C_i + \sqrt{C_i^2 + 4eB_i}}{2eB_i}.$$

Multiply both sides by $2enB_i$ to get

$$-nC_i + \sqrt{n^2C_i^2 + 4(n-1)neB_i} < -nC_i + \sqrt{n^2C_i^2 + 4en^2B_i},$$

which holds, since

$$4(n-1)n < 4n^2.$$

The condition also holds, if B_i and C_i of the different firms are sufficiently close to each other.

We can now compare the two equilibria by comparing the equilibrium industry outputs (15) and (25), which are now denoted by Q_1^* and Q_2^*. Under Assumption 1, $Q_1^* < Q_2^*$ if

$$\frac{n-1}{A} < \frac{-C + \sqrt{C^2 + 4(n-1)eB}}{2eB} \tag{29}$$

or

$$\frac{2eB(n-1)}{A} + C < \sqrt{C^2 + 4(n-1)eB}.$$

Since both sides are positive, this is equivalent to the following:

$$\frac{4e^2B^2(n-1)^2}{A^2} + \frac{4eB(n-1)C}{A} + C^2 < C^2 + 4(n-1)eB.$$

After cancelling C and simplifying by $4eB(n-1)$, we have

$$\frac{eB(n-1)}{A^2} + \frac{C}{A} < 1.$$

Under Assumption 1

$$B = 2e\theta^2 \sum_{i=1}^{n} \alpha_i\delta_i^2,\ C = \sum_{i=1}^{n} c_i + n\theta e, A = C + \left(\sum_{i=1}^{n} \alpha_i\delta_i\right)\theta e,$$

then we have

$$eB(n-1) < A(A-C)$$

or

$$\sum_{i=1}^{n} c_i > \frac{eB(n-1)}{\theta e \sum_{i=1}^{n} \alpha_i\delta_i} - \theta e \sum_{i=1}^{n} \alpha_i\delta_i - n\theta e. \tag{30}$$

Therefore, $Q_1^* < Q_2^*$ if and only if the sum of the marginal costs of the firms is larger than the threshold given by the right hand side of inequality (30), meaning that with small marginal production costs Q_1^* are larger, while with large marginal costs, Q_2^* is the larger, implying that the price at Q_i^* is larger than at Q_j^* if $Q_i^* < Q_j^*$. We note that there is a one-to-one correspondence between industry output Q and total emission eQ , so increase in Q is equivalent with increase of total emission.

In this comparison, we assumed that the same α_i values are selected in models (7) and (8). This comparison is hard to be interpreted, since in (7), standard deviation is compared to expectation, and in (8), variance is compared to expectation, so their meanings are different.

5 Effect of environmental taxes on industry output

Consider first (15). Notice that for all i, A_i strictly increases in θ, so A is increasing, as well. Therefore, Q^* decreases as θ increases. Hence, increasing the environmental tax rate results in the decrease of the industry output which makes the unit price higher.

Consider Eq. (25). Notice first that it can be rewritten as

$$Q^* = \frac{4(n-1)eB}{2eB\left[C + \sqrt{C^2 + 4(n-1)eB}\right]} = \frac{2(n-1)}{C + \sqrt{C^2 + 4(n-1)eB}}.$$

As θ increases, both B_i and C_i increase, so both B and C become larger. Therefore, the denominator of Q^* increases, showing that the increase of the environmental tax rate results in the decrease of the industry output, making the unit price larger.

6 Effect of environmental taxes on emissions

Consider the first equation (15). Notice first that

$$Q^* = \frac{n-1}{A} \text{ and } \frac{dQ^*}{d\theta} = -\frac{n-1}{A^2}\frac{dA}{d\theta} = -\frac{Q^*}{A}\frac{dA}{d\theta}.$$

In addition

$$\frac{\partial A_i}{\partial \theta} = e_i + \alpha_i e_i \delta_i = \frac{A_i - c_i}{\theta}$$

and

$$\frac{\partial A}{\partial \theta} = \frac{\sum_{i=1}^n A_i - \sum_{i=1}^n c_i}{\theta} = \frac{A-c}{\theta},$$

where $c = \sum_{i=1}^n c_i$. Therefore, from (14) with $Q = Q^*$

$$\begin{aligned}\frac{\partial x_i}{\partial\theta} &= \frac{\partial Q^*}{\partial\theta} - 2Q^*\frac{\partial Q^*}{\partial\theta}A_i - (Q^*)^2\frac{\partial A_i}{\partial\theta}\\ &= -\frac{Q^*}{A}\frac{\partial A}{\partial\theta} + 2\frac{(Q^*)^2}{A}\frac{\partial A}{\partial\theta}A_i - (Q^*)^2\frac{\partial A_i}{\partial\theta}\\ &= -\frac{Q^*}{A}\frac{A-c}{\theta} + 2(Q^*)^2\frac{A-c}{A\theta}A_i - (Q^*)^2\frac{A_i-c_i}{\theta},\end{aligned}$$

which has the same sign as

$$\begin{aligned}&-(A-c) + 2Q^*(A-c)A_i - Q^*A(A_i-c_i)\\ &= -(A-c) + 2\frac{(n-1)}{A}(A-c)A_i - (n-1)(A_i-c_i)\\ &= -(A-c)\left(1-\frac{2(n-1)A_i}{A}\right) - (n-1)(A_i-c_i).\end{aligned}$$

Hence

$$sign\left[\frac{\partial x_i}{\partial\theta}\right] = -sign\left[(A-c)\left(1-\frac{2(n-1)A_i}{A}\right) + (n-1)(A_i-c_i)\right], \tag{31}$$

which is clearly negative if

$$A_i < \frac{A}{2(n-1)}. \tag{32}$$

Under this condition, the output of firm i decreases with increasing value of θ.This condition is only sufficient. The sufficient and necessary condition is that (31) is negative. If x_i decreases, then the emission discharged by firm i also decreases. However, this is not necessarily the case for the total discharged emission.

It is easy to show that this sufficient condition cannot hold for all firms. Assume it does, then adding (32) for all values of i, we get

$$A < \frac{nA}{2(n-1)}$$

or

$$2(n-1) < n,$$

which holds only for $n < 2$. Since the total emission is $E = \sum_{i=1}^{n} e_i x_i$,we have

$$\frac{\partial E}{\partial\theta} = \sum_{i=1}^{n} e_i\frac{\partial x_i}{\partial\theta} = \sum_{i=1}^{n} e_i\left\{-\frac{Q^*}{A}\frac{A-c}{\theta} + 2(Q^*)^2\frac{A-c}{A\theta}A_i - (Q^*)^2\frac{A_i-c_i}{\theta}\right\},$$

which has the same sign as

$$\sum_{i=1}^{n} e_i \left\{ -(A-c)\left(1 - \frac{2(n-1)A_i}{A}\right) - (n-1)\left(A_i - c_i\right) \right\}$$
$$= -(A-c)E + \frac{2(n-1)(A-c)u}{A} - (n-1)u + (n-1)v$$

with

$$E = \sum_{i=1}^{n} e_i,\ u = \sum_{i=1}^{n} e_i A_i,\ v = \sum_{i=1}^{n} e_i c_i.$$

For the sake of mathematical simplicity, assume that $e_i = e$ ($i = 1, 2, ..., n$), then this expression simplifies to the following:

$$-(A-c)ne + \frac{2(n-1)(A-c)Ae}{A} - (n-1)Ae + (n-1)ce.$$

This is negative if

$$\begin{gathered} 2(A-c)(n-1) - (n-1)A + (n-1)c \\ = A((n-2) - (n-1)) + c(n-1-(n-2)) \\ = -A + c < 0, \end{gathered}$$

meaning that increasing the environmental tax rate decreases the total emission.

In the case of (25), we know that Q^* decreases in θ, so this is the case for the total discharged emission level eQ^*. This does not imply that the individual discharged emission levels decrease for all firms.

7 The duopoly case

Assume now that $n = 2$, when $A = A_1 + A_2$. From (15), we have

$$Q^* = \frac{1}{A},$$

and both outputs are positive, since $A_i < A_1 + A_2$ for $i = 1, 2$. Therefore, in duopolies, no boundary equilibrium exists if standard deviation represents uncertainty. The interior equilibrium is as follows:

$$x_1^* = \frac{1}{A} - \frac{A_1}{A^2} = \frac{A_2}{A^2} \tag{33}$$

and

$$x_2^* = \frac{1}{A} - \frac{A_2}{A^2} = \frac{A_1}{A^2}. \tag{34}$$

The total emission is

$$E = \frac{e_1 A_1 + e_2 A_2}{A^2}, \tag{35}$$

where

$$A_1 = c_1 + \theta e_1 + \alpha_1 \theta e_1 \delta_1,$$
$$A_2 = c_2 + \theta e_2 + \alpha_2 \theta e_2 \delta_2.$$

The total output increases if A decreases, which is the case if at least one of parameters, $c_i,\ e_i,\ \delta_i,\ \alpha_i\ (i = 1, 2)$ and θ decreases.

Next, we examine how the individual emission levels of the firms depend upon θ. Notice first that

$$\frac{\partial x_1^*}{\partial \theta} = \frac{\left(e_2 + \alpha_2 e_2 \delta_2\right)A^2 - 2AA_2\left(e_1 + \alpha_1 e_1 \delta_1 + e_2 + \alpha_2 e_2 \delta_2\right)}{A^4},$$

which has the same sign as

$$\left(A_2 - c_2\right)\left(A_1 + A_2\right) - 2A_2\left(A_1 + A_2 - c_1 - c_2\right),$$

or using notation $b_i = e_i + \alpha_i e_i \delta_i\ (i = 1, 2)$, we have

$$\theta b_2\left(c_1 + \theta b_1 + c_2 + \theta b_2\right) - 2\left(c_2 + \theta b_2\right)\left(\theta b_1 + \theta b_2\right).$$

Simplifying by θ gives

$$-\theta\left(b_2^2 + b_1 b_2\right) + \left(b_2 c_1 - b_2 c_2 - 2c_2 b_1\right), \tag{36}$$

which is clearly negative if $b_2 c_1 - b_2 c_2 - 2c_2 b_1$ is negative. In this case, the increase of θ decreases the production volume of firm 1. Otherwise, define

$$\theta^* = \frac{b_2 c_1 - b_2 c_2 - 2c_2 b_1}{b_2^2 + b_1 b_2}, \tag{37}$$

and x_1 decreases in θ if $\theta > \theta^*$ and increases if $\theta < \theta^*$. We can note that at least one firm's output always decreases in θ, since it is impossible that for both firms, the second case occurs, that is

$$b_2 c_1 - b_2 c_2 - 2c_2 b_1 > 0,$$
$$b_1 c_2 - b_1 c_1 - 2c_1 b_2 > 0,$$

and by adding these inequalities, we have

$$-b_2 c_1 - b_1 c_2 - b_1 c_1 - b_2 c_2 > 0,$$

which is an obvious contradiction.

Since (36) is a positive constant multiple of $\partial x_1^* / \partial \theta$, we then see that $\partial E / \partial \theta$ has the same sign as

$$\sum_{i=1}^{2} \left\{ -\theta(b_1 + b_2)e_i b_{3-i} + e_i \left(b_{3-i}c_i - b_{3-i}c_{3-i} - 2c_{3-i}b_i \right) \right\}.$$

The coefficient of θ is always negative. The constant term can be written as

$$CO = -\left(e_2 b_1 c_1 + e_1 b_2 c_2\right) + b_2 c_1 \left(e_1 - 2e_2\right) + b_1 c_2 \left(e_2 - 2e_1\right).$$

If $CO \le 0$, then E decreases in θ. Otherwise, let

$$\theta^{**} = \frac{CO}{(b_1 + b_2)\left(e_1 b_2 + e_2 b_1\right)}, \tag{38}$$

then

$$\frac{\partial E}{\partial \theta} < 0 \text{ if } \theta > \theta^{**} \text{ and } \frac{\partial E}{\partial \theta} > 0 \text{ if } \theta < \theta^{**},$$

meaning that the regulator has to select sufficiently large θ values to force the firms to lower their overall emission.

Now, we turn to model (8). In the duopoly case

$$\bar{C} = e_1 C_1 + e_2 C_2,\ \bar{B} = e_1 B_1 + e_2 B_2,$$
$$B = B_1 + B_2 \text{ and } C = C_1 + C_2.$$

Therefore, the leading coefficient of the cubic equation (23) is

$$\begin{aligned} \alpha =& \bar{C}B - \bar{B}C \\ =& \left(e_1 C_1 + e_2 C_2\right)\left(B_1 + B_2\right) - \left(e_1 B_1 + e_2 B_2\right)\left(C_1 + C_2\right) \\ =& \left(e_1 - e_2\right)\left(C_1 B_2 - B_1 C_2\right). \end{aligned}$$

Hence

$$\alpha = 2\theta^2 \left(e_1 - e_2\right)\left[\left(c_1 + \theta e_1\right)\alpha_2 e_2 \delta_2^2 - \left(c_2 + \theta e_2\right)\alpha_1 e_1 \delta_1^2\right]. \tag{39}$$

The quadratic coefficient has the form

$$\begin{aligned} \beta =& \left(e_1 B_1 + e_2 B_2\right) - \left(e_1 + e_2\right)\left(B_1 + B_2\right) \\ =& - e_1 B_2 - e_2 B_1 < 0. \end{aligned} \tag{40}$$

The linear coefficient is

$$\gamma = -C = -\left(C_1 + C_2\right) < 0 \tag{41}$$

with positive coefficient term, $\delta = n - 1$. The derivative of the cubic function $P(Q)$ is

$$P'(Q) = 3\alpha Q^2 + 2\beta Q + \gamma,$$

which is always negative if $\alpha < 0$. In this case, $P(Q)$ strictly decreases, and since $P(0) > 0$, there is a unique positive root. If $\alpha > 0$, then the discriminant of $P'(Q)$ is

$$D = 4\beta^2 - 12\alpha\gamma > 0,$$

and the stationary values are

$$\frac{-2\beta \pm \sqrt{D}}{6\alpha}, \tag{42}$$

and since $D > 4\beta^2$, one root is negative and the other positive. If $\bar{Q}$ is the positive root, then we have the following possibilities:

If $P(\bar{Q}) > 0$, then there is no positive root.
If $P(\bar{Q}) = 0$, then $Q^* = \bar{Q}$ is a root with multiplicity 2.
If $P(\bar{Q}) < 0$, then there are two positive roots, λ_1 and λ_2 with $\lambda_1 < \bar{Q} < \lambda_2$.

This analysis cannot be used in general, since in that case

$$\begin{aligned}\beta &= (n-2)\sum_{i=1}^n e_i B_i - \sum_{i=1}^n \sum_{j\neq i}^n e_i B_j \\ &= \sum_{i=1}^n e_i \left[(n-2)B_i - \sum_{j\neq i}^n B_j\right],\end{aligned}$$

the sign of which is indeterminate. In the special case of $e_i = e$

$$\beta = e\{(n-2)B - (n-1)B\} = -eB < 0$$

as we saw it before.

In the case of duopoly, relation (31) is examined next

$$(A-C)\left(1 - \frac{2A_i}{A}\right) + A_i - c_i = \left(A_1 - c_1 + A_2 - c_2\right)\left(1 - \frac{2A_i}{A_1 + A_2}\right) + A_i - c_i.$$

If $A_i \le A_j$, then this expression is positive, implying that x_i decreases if θ increases. Assume next that $A_i > A_j$, then this expression can be rewritten as

$$\left(A_i - c_i\right)\frac{2A_j}{A_1 + A_2} + \left(A_j - c_j\right)\frac{A_j - A_i}{A_1 + A_2}.$$

Notice that the first term is positive, the second is negative, and this expression is positive if

$$A_i - c_i > \frac{A_i - A_j}{2A_j}\left(A_j - c_j\right),$$

then x_i decreases in θ, otherwise does not depend on θ (in case of equality) or increases in θ (if this relation is violated with strict inequality).

8 Concluding remarks

This paper introduces an extension of classical Cournot oligopolies where the pollution emission volumes of the firms are also considered. The regulator introduces incentives to the firms to reduce pollution concentration. However, it is unable to monitor the individual emission levels; only the combined level is available. Therefore, uniform environmental taxes and rewards are introduced, which are included in the profit functions of the firms. An n-person game is defined where the firms are the players, the production outputs are the strategies, and the profit functions are the payoffs. It is also assumed that the regulator can measure the pollution concentration only with certain error, which is unknown by the firms. Randomizing the error leads to random profits. Their expectations are maximized, and to secure maximal or close to maximal profits, the variances or standard deviations of the profit functions are minimized. Two solution concepts were introduced by two assumptions on the error term. In all cases, the existence of the unique Nash equilibrium is proved under certain conditions. The main findings in this paper are as follows:

(1) An increase of the environmental tax rate has a decreasing effect on the industry output.
(2) If variance is minimized and the firms have identical pollution reduction coefficient, then an increase of the environmental tax rate decreases the total emission.
(3) An increase of the uncertainty level (measured by the variance or standard deviation of the measurement error of the pollution concentration by the regulator) decreases the industry output and conditions were given for a similar effect on the individual output levels of the firms.

In the analysis, the weighting method was selected, but any other method to solve multi-objective optimization problems is also applicable. For example, in the selection of the ε-constraint method, the expected profit is maximized subject to the constraint that the standard deviation or the variance is bounded from above. In this case, there is no difference in choice of the second objective. Comparing solutions of any multi-objective optimization problem is very problematic. Most methods provide non-dominated solutions, and if we consider two solutions with objective values (f_1,f_2) and (f_1^*,f_2^*), such that $f_1 > f_1^*$, then $f_2 < f_2^*$, then without additional subjective preference information, it is impossible to decide which solution is better.

The study reported in this paper can be extended in several directions. Instead of the classical Cournot model, other variants, like models with product differentiation, and labor managed oligopolies can be selected. The simple price and cost functions could be replaced with more realistic function forms. Dynamic extensions are also interesting research topics. Furthermore, in this paper, we assume that the abatement technologies are fixed for analytical simplicity. To move forward one more step, we should construct a game in which the technology level is a decision variable. A two-stage game in which the firm determines the output at the second stage and then selects an optimal abatement technology or a one-stage game in which the output and the technology are simultaneously determined.

Declarations

Conflict of interest On behalf of the authors, the corresponding author states that there are no conflicts of interests.

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