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ECONOMIC ANALYSIS OF LAW, POLITICS, AND REGIONS

Referendums for secession domino and redistribution

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Abstract This paper presents an investigation of voting behavior and whether sequential secession occurs through democratic procedures, or not. Characteristics of our model that are not found in earlier studies are the sequential timing of votes and the introduction of income differences among regions. The main results of this study demonstrate that a *domino effect* induces sequential secession when neighboring countries become independent. Furthermore, we analyze two model expansions: an approval vote for secession referendums by regions and a redistribution policy. Results show that voters often change their votes between secession referendums and the approval vote, that redistribution of income decreases secession, and that it is more effective when income differences are larger.

Keywords Secession domino · Referendum · Voting · Redistribution · Fiscal policy · Local public good · Downs–Hotelling model

JEL Classification D72 · H23 · H77

1 Introduction

The United Kingdom held its "United Kingdom European Union membership referendum" in June 2016. Surprisingly, the votes for secession won the election. Furthermore, some EU member countries such as Italy, The Netherlands, and France discussed secession and claimed to hold referendums. A series of such

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secessions is sometimes called *secession domino*, such as that which took place during the collapse of the Soviet Union. One purpose of this paper is investigation of the domino effects of sequential secession. The main feature of this study is to establish the model of such a sequential voting for secession. The sequential voting model enlightens us about recent events occurring in the real world. How does it affect the secession of regions in the same country to the other regions? Results show that the secession of a neighboring region promotes the secession of others.

Many sub-national regions such as Scotland (UK) and Catalonia (Spain) have held referendums seeking independence from their respective nations or intend to do so in the future. The Scottish referendum that took place in September 2014 was agreed to in advance by both the Scottish and UK governments. The Scottish referendum was, therefore, democratic. The UK government had the right not disallow the Scottish government's decision to hold the referendum. In general, a higher government has the right to forbid or allow the lower government to hold a vote for secession. Which rule is more efficient for residents? To elucidate this phenomenon, this study investigates voter behavior in sequential voting for secession and assesses the consequences of establishing democratic procedures for holding referendums.

Integration and secession have been examined in the fields of regional economics and political economy. Recent political economic studies of secession, as well as our study, are based on the local public goods literature pioneered by Tiebout (1956) and Buchanan (1965). Actually, Oates (1972) and Buchanan and Faith (1987) are among the earlier works in the latter stream, which adopted a political economy approach to modeling of government. Friedman (1977) uses a rent-maximizing model to study the border of the nations. Bolton and Roland (1997) argue that democracy raises too many secession demands and relations based on economic integration.

By contrast, recent reports[1] of secession by Alesina and Spolaore (1997) and Alesina and Spolaore (2003) described analysis of the stability of the equilibrium number of countries and optimality. Wittman (2000) proposes socially efficient solutions for national borders.

This study builds a simple voting model based on the Downs–Hotelling model (Downs (1957)), which has been examined by many in the field of political economics. Jehiel and Scotchmer (2001) build a more general model of jurisdiction formation to compare constitutional secession and immigration rules. These studies assess the stability of countries. However, unlike previous studies, we discuss sequential votes for secession. Therefore, dynamic aspects of secession problems are understood. Bordignon and Brusco (2001) discuss optimal constitutional secession rules. Secession rules increase the probability of secession. Sato (2017) constructs a model of approving referendums for three regions. This study generalizes the model and enables analysis of sequential secession. In addition, income differences and redistribution policy effects are considered. Approving referendums is a key feature of this study. Many countries have no secession rules or prohibit secession. Some countries and unions have constitutionally defined

[1] Spolaore (2015) offers a useful survey of these studies.

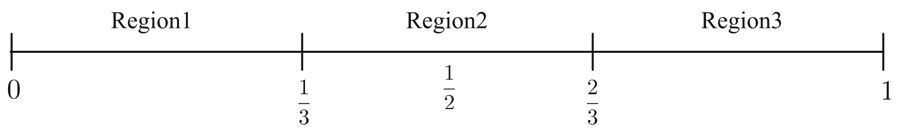

Fig. 1 Country divided into three regions

secession rules. Nevertheless, as in many of these, approval by parliament or some sovereign is necessary. Unilateral secession is prohibited. For example, the EU added an exit clause to Article 50 of the Treaty on European Union by the Lisbon Treaty, which moderately prohibits unilateral withdrawal. To clarify the efficiency of such rules, this study introduces a majority vote of an entire country to approve the regional secession referendum.

Income differences and redistribution are another feature of this paper. Some articles argue relations between income, redistribution, and secession. For instance, Bolton and Roland (1997) presents effects of income-based redistribution to secession. Income differences often raise political conflicts and secession. In recent years, social and political phenomena are apparent by which rich regions separate from poor regions, such as Sandy Springs separating from Fulton Country in the United States. Reportedly[2], residential segregation by income has increased in the US. Accordingly, it is important to inquire about the maldistribution of income and how redistribution affects secession incentives.

The remainder of this paper is organized as described below. Section 2 presents a simple example of the model used for the analyses described in this paper. Section 3 presents the model and derivation of the first-best outcome. Section 4 presents analyses of the equilibrium by voting for secession. Section 5 introduces an approval vote for secession referendums. Section 6 considers relations between redistribution policy and secession. Section 7 concludes this paper.

2 Model of three regions

We start our analysis by consideration of a simple example with three regions. Subsequently, we generalize the model of N regions in later sections.

A country comprises three regions distributed over the interval of [0, 1]. Regions 1–3 are regions of the country as illustrated in Fig. 1. We assume that the regions are of the same size: $s = 1/3$. The country inhabitants are distributed uniformly in [0, 1]. We assume that the inhabitants are not allowed to move to another location, and refer to each by the point, where she lives: $X \in [0, 1]$.

A single local public good—the government—is needed if a region becomes an independent country. We assume that every country's public good cost is the same: K. The cost is constant irrespective of the population and country size. Every individual has the same wage, w, and must pay a per-capita tax t to produce the public good (government). The public good is located at $X^G \in [0, 1]$, as decided by

[2] Fry and Taylor (2012).

majority voting. The utility from public good g decreases as the distance between the public good location X^G and the location of individual X increases. Consequently, the utility function of individual X is

$$U(X, X^G, t) = g\{1 - |X^G - X|\} + w - t.$$

The timing of the game is the following. (1) Only inhabitants in region 1 vote for secession. (2) Only inhabitants in region 2 vote for secession. (3) The public good location is chosen in each independent region.

2.1 Locations of public goods, taxes, and utility

2.1.1 Integrated country

The size (population) of the country is one if all regions are integrated. Consequently, the per-capita tax is $t = K$. The public good location $X^G = 1/2$ is decided by the median voter theorem. The utility of individual X is

$$U^I(X) \equiv U\left(X, \frac{1}{2}, K\right) = g\left\{1 - \left|\frac{1}{2} - X\right|\right\} + w - K, \quad (1)$$

where superscript I denotes "integrated."

2.1.2 Independent region

The new country size is s if the region becomes independent. Consequently, the per-capita tax is $t = K/s$. The public good location is $X^G = s/2$. The utility of individual X in the region is

$$U^S(X) \equiv U\left(X, \frac{s}{2}, \frac{K}{s}\right) = g\left\{1 - \left|\frac{s}{2} - X\right|\right\} + w - \frac{K}{s}, \quad (2)$$

where superscript S denotes "separated."

2.1.3 Integrated region if the other region secedes

The size of the remainder of the region is $1 - s$ if region is independent. Consequently, the per-capita tax is $t = K/(1 - s)$. The public good location is chosen as $X^G = (1 - s)/2$. The utility of individual X in the remainder of the seceded region is

$$U^R(X) \equiv U\left(X, \frac{1+s}{2}, \frac{K}{1-s}\right) = g\left\{1 - \left|\frac{1+s}{2} - X\right|\right\} + w - \frac{K}{1-s}. \quad (3)$$

The key concept of this paper is the "*domino effect*". Region 1 voters decide whether to secede or not by comparing (1) with (2). However, region 2 voters make a decision by comparing (2) with (3). The public good location shifts to the right by $s/2$ if region 1 secedes. This shift promotes secession in the remainder of regions, because the distance from the public good becomes greater. We designate it as *domino effect*.

3 Model of N regions

In this and subsequent sections, we generalize the model and present observations of previous section that are valid for the generalized situation.

A country comprises $N \geq 3$ regions distributed over interval [0, 1]. Each region has the same size[3] of land $s = 1/N$. Let $I = \{i \mid 1, \ldots, N\}$ be a set of regions index. Therefore, the number of elements in I is $|I| = N$. The country inhabitants are distributed uniformly in [0, 1]. We assume that the inhabitants are not allowed to move to another location. We refer to each by the point, $X \in [0, 1]$, where the inhabitant resides. Furthermore, we assume that the region of the smaller index locates more to the left side of interval [0, 1] than regions of a larger index.

These uniform distribution and immobility of inhabitants are the same as those in earlier studies Alesina and Spolaore (1997). The simple distribution clarifies our analysis. We also assume that inhabitants are immobile. That immobility is identical to the exogenous preferences, which is a common assumption of voting model.[4]

3.1 Countries

The country faces a crisis of breaking up. Letting C_j be a set of regions belonging to the same country indexed by j, and letting J be a set of country index j, then initially, $J = \{1\}$ and $C_1 = I$ are all regions integrated to the initial country $1 \in J$. For example, if regions 1 to m are independent from the country, then the new country has index $1, \ldots m \in J$ and for $k = 1, \ldots, m \in J$: $C_k = \{k\}$. The other regions belong to the old country indexed by $m + 1 \in J$ and $C_{m+1} = \{m + 1, \cdots, N\}$. The country structure is a partition of regions I. Therefore, the same region does not belong to two countries: for all $k, l \in J{:}k \neq l \;\Rightarrow\; C_k \cap C_l = \emptyset$. We ignore re-integration processes in this paper; seceding from the initial country represents independence.

3.2 Public goods, costs, and income taxes

A single local public good–the government is necessary if a region becomes an independent country. The public good is located at $X_j^G \in [s \sum_{m=1}^{j-1} |C_m|, s \sum_{m=1}^{j} |C_m|]$, as decided by majority voting. $|C_j|$ is the number of regions belonging to country j. Therefore, $\sum_{m=1}^{j-1} |C_m|$ represents the number of countries located at the left side of country j. We assume that every country's public good cost is K. The cost is constant irrespective of the country size and population.

The public good cost of country $j \in J$ is financed by an income tax. Individuals in the same region $i \in I$ have the same wage, w_i. Individuals in the same country $j \in J$ must pay the same income tax t_j with tax rate τ_j to produce the public good

[3] We assume that the region borders are defined historically such as Bolton and Roland (1997). For analytical simplicity, we assume the same region sizes. See Sato (2017) for a similar model of different sizes of regions.

[4] Rota-Graziosi (2011) argued justification of the immobility assumption.

(government). The tax revenue of country j is $T_j = \sum_{i \in C_j} s\tau_j w_i$. The balanced budget constraint in country j is $T_j = K$. Therefore, we decide the tax rate of country j as

$$\tau_j = \frac{K}{s\sum_{i \in C_j} w_i}.$$

3.3 Preferences

The utility from the public good g decreases as the distance between the public good location X_j^G and the location of individual X increases. Consequently, the utility function of individual X is

$$U_{ij}(X, X_j^G) = g\left\{1 - |X_j^G - X|\right\} + w_i - \tau_j w_i.$$

For analytical convenience, we rewrite the utility function as

$$U_{ij}(X, X_j^G) = u_{ij}(x_j, x_j^G) = g\left\{1 - |x_j^G - x_j|\right\} + w_i - \tau_j w_i, \tag{4}$$

where x_j and x_j^G are the distances from the left side border of country j. Strictly speaking, $x_j = X - s\sum_{m=1}^{j-1}|C_m|$ and $x_j^G = X_j^G - s\sum_{m=1}^{j-1}|C_m|$.

3.4 Social optimum

First, we derive the social optimum in the model. Social welfare W is

$$W = \sum_{j \in J}\left[\int_0^{s|C_j|} g\left\{1 - |x_j^G - x|\right\}\mathrm{d}x + \sum_{i \in C_j} w_i\right] - (n+1)K.$$

Because locations of public goods are the center of each country from a social perspective, the location of the public good in country j is $x_j^G = s|C_j|/2$. Let n be a number of separated regions and country $n+1 \in J$ is the old country. We can arrange social welfare as follows:

$$\begin{aligned}
W &= \sum_{i=1}^{n}\int_0^{s}\left[g\left\{1-\left|\frac{s}{2}-x\right|\right\}\right]\mathrm{d}x \\
&\quad + \int_0^{s(N-n)}\left[g\left\{1-\left|s\frac{N-n}{2}-x\right|\right\}\right]\mathrm{d}x + \sum_{i=1}^{N} w_i - (n+1)K \\
&= n\left[gs\left(1-\frac{s}{4}\right)\right] + (N-n)\left[gs\left(1-\frac{s(N-n)}{4}\right)\right] + \sum_{i=1}^{N} w_i - (n+1)K \\
&= gsN - gs^2\left[\frac{n}{4}+\frac{(N-n)^2}{4}\right] + \sum_{i=1}^{N} w_i - (n+1)K \\
&= g - \frac{g}{N^2}\left[\frac{n}{4}+\frac{(N-n)^2}{4}\right] + \sum_{i=1}^{N} w_i - (n+1)K.
\end{aligned}$$

Therein, it is noteworthy that $s=1/N$. To maximize social welfare, we minimize the following social cost:

$$SC(n) = \frac{g}{N^2}\left[\frac{n}{4} + \frac{(N-n)^2}{4}\right] + (n+1)K.$$

The optimal number of seceding regions[5] is

$$n^{\mathrm{FB}} = N - \frac{1}{2} - \frac{2N^2K}{g}. \tag{5}$$

Social welfare is maximized using a positive integer close to n^{FB} because the number of regions must be an integer. To keep n^{FB} from becoming negative, we presume that

$$\frac{g}{K} \geq \frac{4N^2}{2N-1}. \tag{6}$$

4 Secession by voting

In this section, we consider the equilibrium for secession decided by majority voting. We consider sequential votes. Therefore, the timing of votes is important. We assume the timing as one-sided, which means that secession votes are held from region 1 to region N. To avoid circumstances under which countries have a detached piece of land, we assume only that regions at borders of country can secede. Therefore, region 2 cannot be independent if region 1 does not secede.

If region $N-1$ secedes, then region N need not hold a vote, because region N is already independent. Each region's vote is decided by a median voter, because the median voter theorem is valid in our model. Therefore, we derive the median voter's utility when secession is achieved and when it is not.

The public good is located at $x_j = s/2$ if region $i \in C_j$ secedes from old country j and becomes a new independent country j. Then, the utility of the median voter in region i $(x = s/2)$ is

$$u_{ij}\left(\frac{s}{2}, \frac{s}{2}\right) = g + w_i - \frac{K}{s}. \tag{7}$$

The public good is located at $x_j = s|C_j|/2$ if region $i \in C_j$ does not secede from old country j. Then, the utility of the median voter in region i $(x = s/2)$ is

[5] This equation is derived by the following first-order condition:

$SC'(n) = \frac{g(2N-2n+1)}{4N^2} + K = 0.$ The second-order condition is

$SC''(n) = \frac{g}{2N^2} > 0.$

$$u_{ij}\left(\frac{s}{2},\frac{s|C_j|}{2}\right)=g\left\{1-\left|\frac{s|C_j|}{2}-\frac{s}{2}\right|\right\}+w_i-w_i\frac{K}{s\sum_{k\in C_j}w_k}. \tag{8}$$

Comparing these utilities, one obtains

$$u_{ij}\left(\frac{s}{2},\frac{s}{2}\right)-u_{ij}\left(\frac{s}{2},\frac{s|C_j|}{2}\right)=sg\frac{|C_j|-1}{2}-\frac{K}{s}\left(1-\frac{w_i}{\sum_{k\in C_j}w_k}\right).$$

Secessionists constitute the majority if the equation above is positive. The condition[6] is

$$\frac{g}{K}\geq\frac{2\left(\sum_{k\in C_j}w_k-w_i\right)}{\frac{1}{N^2}\left(|C_j|-1\right)\sum_{k\in C_j}w_k}, \tag{9}$$

where $|C_j|-1>0$, because $|C_j|=1$ means that only region N is in the old country j. Therefore, as the region N has already seceded, the condition (9) is meaningless. This equation suggests that the greater w_i or the less $|C_j|-1$ becomes, the greater the degree of secession becomes. We interpret this observation of $|C_j|-1$ as a *domino effect* and of w_i as a *income effect*. The *domino effect* is that the location of public good shifts to the right if a region of left side secedes. The public good location x_j^G shifts $s|C_j|/2-s\big(|C_j|-1\big)/2=s/2$ toward the right if left side region $k(<i)\in C_j$ seceded. This effect promotes region i secession, because distance from the public good when integration becomes greater.

The higher region income w_i becomes, the greater the degree to which secession is chosen. A richer region tends to choose independence. We designate this as an *income effect*.

These are concluded as the following proposition.

Proposition 1 *The number of secessionists of region i increases with income* w_i *and with the number of previously seceded regions.*

The equilibrium number of seceded countries is an integer close to the (real) number at which inequity (9) becomes zero. For a uniquely defined equilibrium, we assume the following about income.

Assumption 1 For any $i,j\in I$, $w_i>w_j$ if $i>j$.

Under the assumption 1, we derive the equilibrium number of seceding countries. Let n is the (real) number of seceding countries, then we have $|C_j|=N-n$. Therefore, one can rewrite (9) as shown below:

[6] This condition is similar to Rule B of Alesina and Spolaore (1997). The main difference from Alesina and Spolaore (1997) is that they assume approval by the majority in each existing country affected by the border redrawing, whereas, in our model, approval by the majority only in seceding country is needed. Instead of approval by existing countries, this study introduces the approving vote by the whole country in Sect. 5.

$$\frac{g}{K} \geq \frac{2\left(\sum_{k=n+1}^{N} w_k - w_n\right)}{\frac{1}{N^2}(N-n-1)\sum_{k=n+1}^{N} w_k}.$$

The number of seceding regions n^* satisfies

$$n^* = N - 1 - \frac{2N^2K}{g}\left(1 - \frac{w_{n^*}}{\sum_{k=n^*+1}^{N} w_k}\right), \tag{10}$$

where n^* can be larger or smaller than n^{FB}. If the income difference between w_{n^*} and w_{n^*+j} (for any j) is sufficiently large, then $w_{n^*}/\sum_{i=n^*+1}^{N} w_i$ becomes sufficiently small. Therefore, n^* can be smaller than n^{FB}. However, if income differences are sufficiently small and g / K is sufficiently large, then n^* can be larger than n^{FB}. If regional income differences do not exist, then substituting $w_i = w$ for any i, one obtains

$$n^* = N - 1 - \frac{2N^2K}{g}\left(1 - \frac{1}{N-n^*}\right).$$

Solving the equation above[7],

$$n^* = N - \frac{2N^2K}{g}. \tag{11}$$

One can show that $n^* > n^{\mathrm{FB}}$. These observations lead to the following proposition.

Proposition 2 *The number of seceding regions at the equilibrium is smaller than the social optimum if the income differences are sufficiently large. Furthermore, the equilibrium number of seceding regions is larger than the social optimum if the income differences are sufficiently small. When there are no income differences, the number of seceding regions at equilibrium is greater than the social optimum.*

The externality of majority voting and unilateral decision to secede is that the median voter of the seceding region does not consider the utility of the inhabitants of the seceding regions and the rest of the country. This externality distorts the efficiency of the outcome of the voting equilibrium. The externality for the inhabitants of the old country has two effects. The first is increasing taxes due to the decrease in the number of tax payers in the old country j from $|C_j|$ to $|C_j| - 1$. The second is shifting the location of the public good in the old country, which increases the utility of the inhabitants on the right side of the old country, but decreases it on its left side. The average utility increases as the size of the old country shrinks.

More precisely, let us illustrate the externality with respect to the average utility. First, we consider the case with no difference between incomes. Let us increase n slightly, such that it becomes $n + \varepsilon$. Before increasing n, the average distance between the public good and the inhabitants' location is $s(N-n)/4$ in the $N - n$ regions of the rest of the country. A slight change in n causes a positive externality

[7] Here, $n^* = N - 1$ satisfies the equation. However, as $|C_j| - 1 = N - n^* - 1 = 0$, this solution is indefinable.

within the average distance, which is $s(N-n-\varepsilon)/4$. Thus, there is a positive externality with an average amount of $g\{s(N-n)/4 - s(N-n-\varepsilon)/4\}$. On the other hand, the average cost is $K/s(N-n)$ before n is increased. A slight change in n increases the average cost of the inhabitants in the rest of country. The amount of negative externality is $K/s(N-n-\varepsilon) - K/s(N-n)$. Furthermore, the seceding region's inhabitants enjoy an increased average utility from the public good, with a difference of $g\{s(N-n)/4 - s/4\}$. However, the cost of the public good rises to K/s. Then, the average positive externality (difference in utility) of all the inhabitants affected by the secession is

$$\frac{s\varepsilon}{s(N-n)} g\left[\frac{s(N-n)}{4} - \frac{s}{4}\right] + \frac{s(N-n-\varepsilon)}{s(N-n)} g\left[\frac{s(N-n)}{4} - \frac{s(N-n-\varepsilon)}{4}\right]. \tag{12}$$

Furthermore, the average negative externality (difference in cost) is

$$\frac{s\varepsilon}{s(N-n)}\left[\frac{K}{s}\right] + \frac{s(N-n-\varepsilon)}{s(N-n)}\left[\frac{K}{s(N-n-\varepsilon)}\right] - \frac{K}{s(N-n)}. \tag{13}$$

The point at which the positive externality (12) is equal to the negative externality (13) represents the socially optimal level of seceding regions. Then, by solving the equation, one obtains[8]

$$n = N - \frac{1}{2} - \frac{\varepsilon}{2} - \frac{2N^2K}{g}.$$

Now, we will check the average utility of the median voter of a seceding region to demonstrate that this externality is not completely internalized. The average distance from public goods in an integrated country is $s(N-n)/2 - s/2$. This is because the median voter of the seceding region is $x = s/2$ and the average cost of a public good is K/s. Therefore, the increase in utility of the median voter is larger than the increase in the average utility from the secession[9]. Then, we have to compare the median voter's total utility increase from secession with the average. The average variation from secession is (12)–(13) and the median voter's total utility increase is $g\{s(N-n)/2 - s/2\} - \{K/s - K/s(N-n)\}$. Subtracting the former from the latter, we have

$$\frac{\varepsilon[g(2N-2n-\varepsilon-1) - 4N^2K]}{4N(N-n)}.$$

Note that the above is equal to 0 if $n = n^{\mathrm{FB}} - \varepsilon/2$. Substituting $n = n^*$, the above equals $-g^2\varepsilon(1+\varepsilon)/8N^2K$, which is negative for any $\varepsilon > 0$. This implies that the median voter's secession decision causes a negative externality as a whole.

On the other hand, if there are income differences, then the median voter's incentive to secede becomes smaller, because the burden of costs of a public good is

[8] Let $\varepsilon \to 0$, then the n converges to n^{FB}.

[9] This is indicated by $g\{s(N-n)/2 - s/2\} > g\{s(N-n)/4 - s/4\}$ and $g\{s(N-n)/2 - s/2\} > g\{s(N-n)/4 - s(N-n-\varepsilon)/4\}$ for a small ε. They indicate that Eq. (12) is smaller than $g\{s(N-n)/2 - s/2\}$.

small if the average income of the country is high. The cost of public good for the median voter is $w_i K/s\sum_{k\in C_j} w_k$ when region i is integrated. The smaller is $w_i/\sum_{k\in C_j}$, which implies that the income differences are large, the smaller is the cost $w_i K/s\sum_{k\in C_j} w_k$. Therefore, solving $n^* \lesseqgtr n^{FB}$ by (5) and (10), we have the following condition that distinguishes the equilibrium distortion from the social optimum:

$$n^* \lesseqgtr n^{FB} \quad \text{if} \quad \frac{w_{n^*}}{\sum_{k=n^*+1}^{N} w_k} \lesseqgtr \frac{g}{4N^2K},$$

where $w_{n^*}/\sum_{k=n^*+1}^{N} w_k$ denotes the income differences between the seceding region and the rest of the country. The income differences are small, if $w_{n^*}/\sum_{k=n^*+1}^{N} w_k$ is high. If a seceding region's income is sufficiently smaller than the rest of the regions, the positive externality (smaller distances from the public good) is larger than the negative externality (larger average cost of the public good). This effect leads to the size of a country being "too large".

Let us conclude the above discussions. The outcome of the size of a country being "too small" is almost identical to that of previous studies; for example, Alesina and Spolaore (1997), if there are no income differences. However, when income differences are sufficiently large, a country becomes "too large." For this reason, a rich region attracts a poor region under an income tax system through the reduction of burden of the poor region.

5 Approval vote

This section introduces an additional vote to approve secession votes by regions analyzed up to Sect. 4. We consider the case in which the median voter of each region has this approval voting right as a representative. If most representatives $N/2$ vote for prohibiting secession votes, then the status quo, with one country and a public good located at $X^G = 1/2$, is realized. However, because this analysis is perhaps too extremely complicated to solve clearly, we analyze only the case of no income differentiation in this section.

The median voter of region $i \in C_1$ is located at $X = s(i-1) + s/2$. Therefore, her utility for integration is

$$U_{i1}\left(s(i-1)+\frac{s}{2},\frac{1}{2}\right) = g\left\{1-\left|\frac{1}{2}-\left(s(i-1)+\frac{s}{2}\right)\right|\right\} + w - \tau_1 w. \tag{14}$$

If region i secedes, then the utility is the same as that obtained using Eq. (7). Subtracting the above from this result, one obtains the following:

$$u_{ij}\left(\frac{s}{2},\frac{s}{2}\right) - U_{i1}\left(s(i-1)+\frac{s}{2},\frac{1}{2}\right) = \frac{g(N-n)|N-n+1-2i| - 2N^2K(N-n-1)}{2N(N-n)}$$

In that equation, $g(N-n)|N-n+1-2i|/2N(N-n)$ represent effects of a location shift of the public good. $2N^2K(N-n-1)/2N(N-n)$ shows fiscal effects of a population change. Substituting $n=n^*$, one obtains

$$u_{ij}\left(\frac{s}{2},\frac{s}{2}\right)-U_{i1}\left(s(i-1)+\frac{s}{2},\frac{1}{2}\right)=\frac{g+\left|g(1-2i)+2N^2K\right|-2N^2K}{2N}.$$

The region i median voter votes for approval if the difference of utilities above is positive. This condition is

$$i\leq \underline{i}_S\equiv\frac{1}{2}(N+1)-\frac{KN(N-1)}{g}$$

or

$$i\geq \bar{i}_S\equiv\frac{1}{2}(N+1)+\frac{KN(N-1)}{g}.$$

Therefore, the number of regions which vote in approval of secession is divisible into three cases:[10] (1) $\underline{i}_S+n^*-\bar{i}_S$ if $\bar{i}_S\leq n^*$, (2) $\underline{i}_S$ if $\underline{i}_S\leq n^*\leq\bar{i}_S$, (3) n^* if $n^*\leq\underline{i}_S$.

The public good's location of original country j shifts to $X^G=sn+s|C_j|/2=sn+s(N-n)/2$ if region i will not secede. However, the other n regions will secede. Therefore, the utility of the region $i\in C_j$ median voter is

$$\begin{aligned}&U_{ij}\left(s(i-1)+\frac{s}{2},sn+\frac{s(N-n)}{2}\right)\\&\quad=g\left\{1-\left|sn+\frac{s(N-n)}{2}-\left(s(i-1)+\frac{s}{2}\right)\right|\right\}+w-\tau_j w.\end{aligned}$$

Subtracting (14) from the above, we obtain the following.

$$\begin{aligned}&U_{ij}\left(s(i-1)+\frac{s}{2},sn+\frac{s(N-n)}{2}\right)-U_{i1}\left(s(i-1)+\frac{s}{2},\frac{1}{2}\right)\\&\quad=\frac{g(N-n)\{|N+1-2i|+|N+n+1-2i|\}-2nKN}{2N(N-n)}.\end{aligned}$$

Therein, substituting $n=n^*$, one obtains

$$\begin{aligned}&U_{ij}\left(s(i-1)+\frac{s}{2},sn+\frac{s(N-n)}{2}\right)-U_{i1}\left(s(i-1)+\frac{s}{2},\frac{1}{2}\right)\\&\quad=\frac{g\left\{|N+1-2i|-2N\left|1+\frac{1-2i}{2N}-\frac{KN}{g}\right|\right\}+2KN}{2N}.\end{aligned}$$

When the difference above is positive

[10] These conditions of the cases are rewritten as follows: (1) $\bar{i}_S\leq n^*\iff g/K\geq N(3N-1)/(N-1)$, (2) $\underline{i}_S\leq n^*\leq\bar{i}_S\iff 2N(N+1)/(N-1)\leq g/K\leq N(3N-1)/(N-1)$, (3) $n^*\leq\underline{i}_S\iff 2N(N+1)/(N-1)\geq g/K$.

$$i \geq \hat{i}_I \equiv \frac{3}{4}(N+1) - \frac{KN(N+1)}{2g}$$

must hold. Therein, $N - \hat{i}_I$ regions vote for approval.

One can then reasonably infer that there are five types of equilibrium for relations of n^* and the thresholds of i. Actually, $\underline{i}_S < \bar{i}_S$ and $\underline{i}_S \leq \hat{i}_I$ always hold. First, we divide it into two cases: (1) $\underline{i}_S \leq \bar{i}_S \leq \hat{i}_I$ and (2) $\underline{i}_S \leq \hat{i}_I \leq \bar{i}_S$. We can divide the first case into three cases[11]: (1–I) $\underline{i}_S \leq \bar{i}_S \leq \hat{i}_I \leq n^*$, (1–II) $\underline{i}_S \leq \bar{i}_S \leq n^* \leq \hat{i}_I$, (1–III) $\underline{i}_S \leq n^* \leq \bar{i}_S \leq \hat{i}_I$. The second case is divided into two cases. The other cases do not hold: (2–I) $\underline{i}_S \leq n^* \leq \hat{i}_I \leq \bar{i}_S$, (2–II) $n^* \leq \underline{i}_S \leq \hat{i}_I \leq \bar{i}_S$.

Each equilibrium number of approval votes is described in the following lemma.

Lemma 1 *The number of median voters who cast approval secession votes in equilibrium is the following.*

(1–I) : $\underline{i}_S \leq \bar{i}_S \leq \hat{i}_I \leq n^*$

$$\underline{i}_S + N - \bar{i}_S = N - \frac{2KN(N-1)}{g} \quad \text{if} \quad \frac{\mathrm{g}}{\mathrm{K}} \geq \frac{2\mathrm{N}(3\mathrm{N}-1)}{\mathrm{N}-3}$$

(1–II) : $\underline{i}_S \leq \bar{i}_S \leq n^* \leq \hat{i}_I$

$$\underline{i}_S + n^* - \bar{i}_S + N - \hat{i}_I = \frac{g(5N-3) - 2KN(7N-5)}{4g} \quad \text{if} \quad \frac{2\mathrm{N}(3\mathrm{N}-1)}{\mathrm{N}-1} \leq \frac{\mathrm{g}}{\mathrm{K}} \leq \frac{2\mathrm{N}(3\mathrm{N}-1)}{\mathrm{N}-3}$$

(1–III) : $\underline{i}_S \leq n^* \leq \bar{i}_S \leq \hat{i}_I$

$$\underline{i}_S + N - \hat{i}_I = \frac{g(3N-1) - 2KN(N-3)}{4g} \quad \text{if} \quad \frac{2\mathrm{N}(3\mathrm{N}-1)}{\mathrm{N}+1} \leq \frac{\mathrm{g}}{\mathrm{K}} \leq \frac{2\mathrm{N}(3\mathrm{N}-1)}{\mathrm{N}-1}$$

(2–I) : $\underline{i}_S \leq n^* \leq \hat{i}_I \leq \bar{i}_S$

$$\underline{i}_S + N - \hat{i}_I = \frac{g(3N-1) - 2KN(N-3)}{4g} \quad \text{if} \quad \frac{2\mathrm{N}(\mathrm{N}+1)}{\mathrm{N}-1} \leq \frac{\mathrm{g}}{\mathrm{K}} \leq \frac{2\mathrm{N}(3\mathrm{N}-1)}{\mathrm{N}+1}$$

(2–II) : $n^* \leq \underline{i}_S \leq \hat{i}_I \leq \bar{i}_S$

$$n^* + N - \hat{i}_I = \frac{g(5N-3) + 2KN(3N-1)}{4g} \quad \text{if} \quad \frac{\mathrm{g}}{\mathrm{K}} \leq \frac{2\mathrm{N}(\mathrm{N}+1)}{\mathrm{N}-1}$$

[11] Consequently, $n^* < \underline{i}_S \leq \bar{i}_S \leq \hat{i}_I$ does not hold.

We state the following proposition related to approval votes.

Proposition 3 *Some median voters vote against holding secession referendums, but vote for secession in the secession referendum of their own region in some cases. However, some median voters who vote for holding secession referendums vote against secession of their own region in some cases.*

The cause of these contradictory phenomena is the domino effect. The public good location shifts to the right when the left side region secedes, causing the other region of the left side of the public good secession as the distance from the public good becomes larger. We designate this effect as the domino effect. Some regions which decide to secede by the domino effect prefer the integration of the status quo, according to the first half of the proposition. The domino effect causes other changes of votes. The right-hand-side regions are subject to the domino effect as the distance from the public good becomes smaller. Therefore, these regions support the holding of referendums, but the regions themselves do not secede.

Finally, we conclude this section to state the condition of approving secession referendums by most median voters in each region. We can calculate the conditions of the majorities of median voter approving from Lemma 1 and can show that only case (2–II) requires the further condition. The other cases do not. We state the condition as the following lemma.

Lemma 2 *Secession referendums are approved by majority voters, including the median voter, in each region if*

$$\frac{g}{K} \geq \frac{2N(3N-1)}{3(N-1)}. \tag{15}$$

We will check this condition from a social perspective. The social cost in the equilibrium ($n = n^*$) is

$$SC(n^*) = \frac{g^2 + 2gKN + 4gKN^2 - 4K^2N^3}{4gN}.$$

When all regions are integrated, in the status quo, the social cost is

$$SC(0) = \frac{g}{4} + K.$$

From the difference of the above equations, we obtain the following condition by solving $SC(n^*) - SC(0) \leq 0$:

$$\frac{g}{K} \geq \frac{2N^2}{N-1}. \tag{16}$$

Approval secession referendums are efficient if the condition above holds by the social perspective. Comparing (15) with (16), $2N(3N-1)/3(N-1) > 2N^2/(N-$

1) means that the (15) denies secession referendums too much from a social perspective.

Proposition 4 *Approval voting on referendums of secession induces excess suppression of secession.*

6 Redistribution

This section presents investigation of a fiscal policy of redistribution. A redistribution of income is a fundamental means of reducing dissatisfaction.

First, we show that redistribution discourages secession and that countries become large. For tractability, we specify the income distribution as a linear function:

$$w_i = ai + b$$

In that equation, a and b are positive parameters: actually, a represents a difference of income between regions; b is a basic income parameter.

Letting Λ_j be a transfer from country j to its inhabitants, we rewrite the utility in country j (4) as

$$u_{ij}(x_j, x_j^G) = g\{1 - |x_j^G - x_j|\} + w_i + \Lambda_j - \tau_j w_i.$$

The government of country j increases the tax rate to finance the transfer. The tax rate is, therefore,

$$\tau_j = \frac{K + s|C_j|\Lambda_j}{s\sum_{k \in C_j} w_k}.$$

Now, we can show the effect of redistribution by the following derivative[12].

$$\frac{\partial}{\partial \Lambda_j}\left\{u_{ij}\left(\frac{s}{2}, \frac{s}{2}\right) - u_{ij}\left(\frac{s}{2}, \frac{s|C_j|}{2}\right)\right\} = -\frac{a(N - n + 1)}{a(N + n + 1) + 2b} < 0. \tag{17}$$

Therefore, the incentive to secede is decreasing with transfer Λ_j. The further derivative shows that the effect becomes larger as the difference of income a increases:

$$\frac{\partial^2}{\partial \Lambda_j \partial a}\left\{u_{ij}\left(\frac{s}{2}, \frac{s}{2}\right) - u_{ij}\left(\frac{s}{2}, \frac{s|C_j|}{2}\right)\right\} = -\frac{2(N - n + 1)b}{\{a(N + n + 1) + 2b\}^2} < 0. \tag{18}$$

We conclude these observations.

Proposition 5 *Redistribution of income decreases the incentive to secede. The decreasing incentive effect is larger with increased difference in income.*

[12] Derivation of the following formulas in the Appendix.

7 Conclusion

We considered a voting model for secession. The model characteristics are timings of votes, approval votes for secession referendums and income differences among regions. Many earlier studies based on Alesina and Spolaore (1997) analyze a simultaneous game or stability of a nation's formation. We analyzed the sequential votes and showed a *domino effect* that induces a further secession followed by neighboring regions' independence.

From a normative perspective, the equilibrium number of seceded countries can be larger or smaller than the social optimum. This outcome contrasts against those of earlier reports such as that by Alesina and Spolaore (1997), which show that over-secession occurs if a linear distance function of utility is used. Results of the present analyses reveal that the social optimum can be achieved under secession if sequential timing and income differences are considered.

This inefficiency highlights the necessity for another system. We induce approval voting for secession referendums, because almost no countries allow secession in the real world. Therefore, the central government prevents independence of regions if regions hold referendums and secessionists win. Generally speaking, a political agreement between the central government and local government is necessary, so that the outcome of referendum is legally binding. We introduced this point as votes by representatives of the respective regions. According to our results, the approval votes prevent secession when the incentive to secede[13] is low. The one reason is that regions which do not secede vote for approving secession referendums to exclude politically confrontational regions from their country.

Income differences also constitute an important matter in the secession context. Bolton and Roland (1997) investigate income inequities and redistribution policy using a political economic method. That study shows that secession is chosen by regional median voters when political effects are sufficiently large, even if their region is poorer than the integrated country. Our results hold in a more general model. A redistribution policy decreases the incentive to secede; it is more effective when income differences are greater. However, additional research must be conducted to elucidate the political economic aspects of the problem: endogenous decisions of redistribution.

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Appendix

Proof of lemma 2

Proof Letting n_a be the number of approval votes by median voters of each region, then the condition under which approval votes are more than a majority is

[13] Specifically, the incentive to secede is represented as g/K.

$$n_a > \frac{N}{2}.$$

We will check the five cases of Lemma 1.

(1–I) The number of approval votes is

$$n_a = \underline{i}_S + N - \bar{i}_S = N - \frac{2KN(N-1)}{g}.$$

From the condition of this case $g/K \geq 2N(3N-1)/(N-3)$, we have

$$n_a - \frac{N}{2} = \frac{N}{2} - \frac{2KN(N-1)}{g} \geq \frac{N}{2} - \frac{(N-1)(N-3)}{3N-1} = \frac{N^2+7N-6}{2(3N-1)} \geq 0.$$

The last inequity holds as $N \geq 3$.

(1–II) We will check the other cases similarly. From the condition of this case $g/K \geq 2N(3N-1)/(N-1)$, we have

$$n_a - \frac{N}{2} = \frac{g(5N-3) - 2KN(7N-5)}{4g} - \frac{N}{2} \geq \frac{(N+1)(N-1)}{2(3N-1)} \geq 0.$$

The last inequity holds as $N \geq 3$.

(1–III) From the condition of this case $g/K \geq 2N(3N-1)/(N+1)$, we have

$$n_a - \frac{N}{2} = \frac{g(3N-1) - 2KN(N-3)}{4g} - \frac{N}{2} \geq \frac{N^2 - N - 2}{2(3N-1)} \geq 0.$$

The last inequity holds as $N \geq 3$.

(2–I) From the condition of this case of $\frac{g}{K} \geq 2N(N+1)/(N-1)$, we have

$$n_a - \frac{N}{2} = \frac{g(3N-1) - 2KN(N-3)}{4g} - \frac{N}{2} \geq \frac{N-1}{N+1} \geq 0.$$

The last inequity holds as $N \geq 3$.

(2–II) We have

$$n_a - \frac{N}{2} = \frac{3g(N-1) + 2KN(3N-1))}{4g}.$$

In order that the above equation is positive, the following condition must hold.

$$\frac{g}{K} \geq \frac{2N(3N-1)}{3(N-1)}.$$

However, the condition of this case does not include the above, most voters vote against approval if the condition above is not satisfied. Here, we probed the following proposition. □

Derivation of (17) and (18)

Let

$$\Delta = u_{ij}\left(\frac{s}{2}, \frac{s}{2}\right) - u_{ij}\left(\frac{s}{2}, \frac{s|C_j|}{2}\right).$$

The derivative of Δ by λ_j is

$$\frac{\partial \Delta}{\partial \Lambda_j} = \frac{\partial}{\partial \Lambda_j}\left[g\left(1 - \left|\frac{s}{2} - \frac{s}{2}\right|\right) + w_i - \frac{K}{s}w_i - \left\{g\left(1 - \left|\frac{s|C_j|}{2} - \frac{s}{2}\right|\right) + w_i + \Lambda_j - \frac{K + s|C_j|\Lambda_j}{s\sum_{k\in C_j} w_k}w_i\right\}\right].$$

Substituting $|C_j| = N - n$ and $w_i = ai + b$, one obtains

$$\begin{aligned}\frac{\partial \Delta}{\partial \Lambda_j} &= \frac{\partial}{\partial \Lambda_j}\left[g - \frac{K}{s}w_i - \left\{g\left(1 - \left|\frac{s(N-n)}{2} - \frac{s}{2}\right|\right) + w_i + \Lambda_j - \frac{K + s(N-n)\Lambda_j}{s\sum_{k=i+1}^{N}(ak+b)}w_i\right\}\right] \\ &= -\frac{\partial}{\partial \Lambda_j}\left[\Lambda_j - \frac{2\{K + s(N-n)\Lambda_j\}}{s(N-n)\{a(N+n+1)+2b\}}w_i\right] = -\left[1 - \frac{2}{a(N+n+1)+2b}w_i\right] \\ &= -\frac{a(N+n+1)+2b-2w_i}{a(N+n+1)+2b}.\end{aligned}$$

Letting $i = n$, one obtains Eq. (17):

$$\frac{\partial \Delta}{\partial \Lambda_j} = -\frac{a(N+n+1)+2b-2(an+b)}{a(N+n+1)+2b} = -\frac{a(N-n+1)}{a(N+n+1)+2b} < 0.$$

Furthermore, differentiating the above by a, one obtains Eq. (18):

$$\begin{aligned}\frac{\partial^2 \Delta}{\partial \Lambda_j \partial a} &= -\frac{(N-n+1)\{a(N+n+1)+2b\} - (N+n+1)\{a(N-n+1)\}}{\{a(N+n+1)+2b\}^2} \\ &= -\frac{2(N-n+1)b}{\{a(N+n+1)+2b\}^2} < 0.\end{aligned}$$

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