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ORIGINAL ARTICLE

Experimental investigation of performances of different optimal controllers in active vibration control of a cantilever beam

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Abstract
Active vibration control using smart materials has been a subject of research for about 2 decades. In active vibration control, the controller plays an important role in attenuating vibration of the flexible structure. In the present study, a cantilever beam structure is used to study the effect of various optimal controllers using a single pair of actuator and sensor which is placed in a collocated manner at the root of the beam. Simulation and experimental studies are carried out using three optimal controllers, viz. Linear quadratic regulator (LQR), linear quadratic Gaussian (LQG) and H-infinity. The simulation study is carried out using ANSYS© and MATLAB© for all the three controllers. The experimental results are obtained using LabVIEW© programs developed by the authors. The LQR optimal control gain is calculated using state and output feedback control laws. Considering the process and measurement noises the Kalman gain is calculated and the LQG regulator is obtained further by combining the LQR and Kalman gains. The H-infinity controller is designed by considering weighting function which maintains system response and error signals within the prescribed tolerances. In the present study, H-infinity control is found to be giving robust and better performance.

Keywords Active vibration control · LQR controller · LQG controller · H-infinity controller · Cantilever beam · Experiments

Introduction

In diverse applications like space and aircraft structures, satellites, cars, bridges, etc., undesired vibration is a major cause of problems. The effects of such vibrations are varied. Minor effects may include annoyance due to noise in automobiles, machines, etc. Major effects are felt in applications like space structures where precise behavior of the structure is desired and any deviation from the required behavior may result in major expense. Under such conditions, controlling vibration becomes very important. Vibration control is a challenging branch of mechanical engineering. In particular, active vibration control using smart materials is attracting much interest around the world. Active vibration control is the active application of force in an equal and opposite manner to the forces inflicted by external vibration (Preumont 2011). To sense the external vibrations and apply the active force in real time, smart materials are used. Smart materials are the materials that respond with significant change in a property upon application of an external driving signal (Tzou et al. 2004). Such materials can act as sensors which sense the disturbances in the structures and as actuators which are capable of applying the controlling forces.

Khot et al. (2012) carried out simulation study for the active vibration control of a cantilever beam using proportional-integral-derivative (PID)-based output feedback controller. Zhang et al. (2008) studied the active vibration control of a cantilever beam using linear quadratic regulator (LQR) optimal control theory. Khot et al. (2012) also used LQR optimal controller in the simulation study for active vibration control of a cantilever beam. Zhang et al. (2009) studied the active vibration control of a beam using the linear quadratic Gaussian (LQG) and H-infinity optimal controllers and compared the results of both the controllers. Liu et al. (1999) carried out the simulation study for the

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active vibration control of laminated composite plates using integrated piezoelectric transducers. Haider and Al-Hussain (2015) studied the active vibration control of a smart cantilever plate subjected to harmonic excitation. Liu and Liaw (2004) carried out the experimental study of the active vibration control of a smart cantilever beam using proportional controller. Khot et al. (2013) carried out experiments for the active vibration control of a cantilever beam using PID controller. A high-frequency switch mode power converter is designed to generate high voltage required for the actuator to produce the control force. The experimental results are then verified by the simulation results obtained in MATLAB©. Wu et al. (2014) also carried out experimental study and numerical simulation of active vibration control of a highly flexible beam using piezoelectric transducers.

Khot and Khan (2015) carried out the simulation study of active vibration control of a cantilever beam using LQR, LQG, and H-infinity optimal controllers with state and output feedback control laws. The simulation results of all the three controllers are compared and it is concluded that the H-infinity controller has a better close loop dynamic performance than the LQR and LQG controllers. However, they have not carried out any experiment in this regard. Therefore, in the present study, the performances of different types of optimal controllers like LQR, LQG, and H-infinity are investigated and compared for active vibration control of a smart cantilever beam through both simulation and experiments. In simulation, first modal analysis is carried out in ANSYS© and eigen values and eigen vectors are extracted from the results. Then the mathematical modelling is done in MATLAB© using state-space approach. The optimal controllers LQR, LQG, and H-infinity are further designed using the state and output feedback control laws. In experimental study, LabVIEW© platform is used for designing the controllers. The controller output is amplified using bipolar amplifier and is given to the actuator for suppressing the vibration of the beam. Further, the simulation and experimental results are compared. The sections are also formed in the same order in the manuscript.

Modal analysis

Modal analysis is the process of determining the inherent dynamic characteristics of a system in the forms of natural frequencies, damping factors, and mode shapes and subsequently using them to formulate a mathematical model for studying its dynamic behavior. The model of a cantilever beam with a single pair of sensor and actuator is built in ANSYS©. The beam has the size of 508 mm × 25.4 mm × 0.8 mm. The pair of sensor and actuator with dimensions 76.2 mm × 25.4 mm × 0.305 mm is selected for the analysis. The element type chosen for beam is SOLID45 and for piezoelectric patch SOLID5. Mesh size of 70 × 4 × 1, is selected through the mesh convergence analysis. The modal analysis of integrated structure with single pair of actuator and sensor placed in a collocated form at the root of the beam, is carried out in ANSYS© and eigen values of first 10 ranks of frequencies are obtained. Since the node of interest is the tip node, the row of the modal matrix corresponding to the tip node is retained. The eigen vectors corresponding to the tip are then identified. These eigen values and eigen vectors are used further to construct state-space models in MATLAB©.

Mathematical modelling

The mathematical model of the cantilever beam is constructed in MATLAB©. The Eigen values and Eigen vectors obtained through the modal analysis are used to construct state-space matrices. The input–output state-space equations are:

$$x = Ax + Bu, \quad \text{and} \tag{1}$$

$$y = Cx + D, \tag{2}$$

where x is the column vector representing the state of the system, y is the output matrix, u is the input matrix, A is the system matrix, B is the force matrix, C is the output matrix, and D is the direct transmission matrix. Here,

$$A = \begin{bmatrix} 0 & 1 & \ldots & \ldots & \ldots \\ -\omega_1^2 & -2\zeta_1\omega_1^2 & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & 0 & 1 \\ \ldots & \ldots & \ldots & -\omega_n^2 & -2\zeta_n\omega_n^2 \end{bmatrix}, \tag{3}$$

$$B = \begin{bmatrix} 0 \\ F_{p1} \\ 0 \\ F_{p2} \\ \ldots \\ F_{pn} \end{bmatrix}, \tag{4}$$

$$C = [Xn_1 \ldots Xn_n 0], \quad \text{and} \tag{5}$$

$$D = [0], \tag{6}$$

where n is the number of modes, ζ_i are the damping constants, and F_{Pi} are the forces in principle coordinates.

The system matrices A, B, C, and D are used to form state-space model of the beam, using 'ss' function in MATLAB©. The transient and frequency responses are plotted in MATLAB© using 'lsim' and 'bode' functions, respectively. The reduced model is also constructed by considering only those modes of frequencies which are significantly contributing to the overall response. This

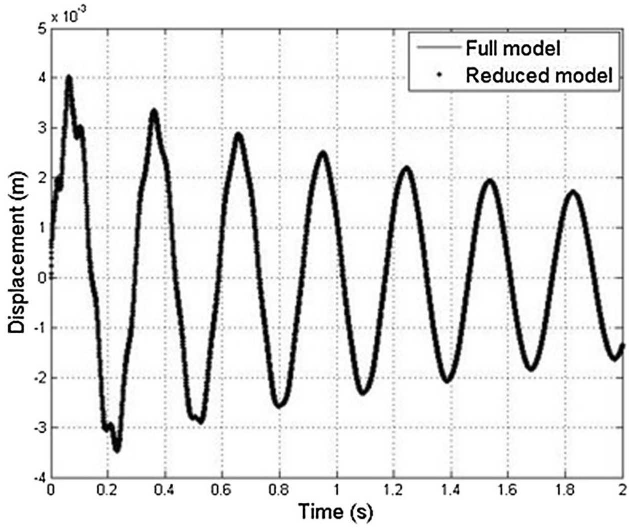


Fig. 1 Transient responses

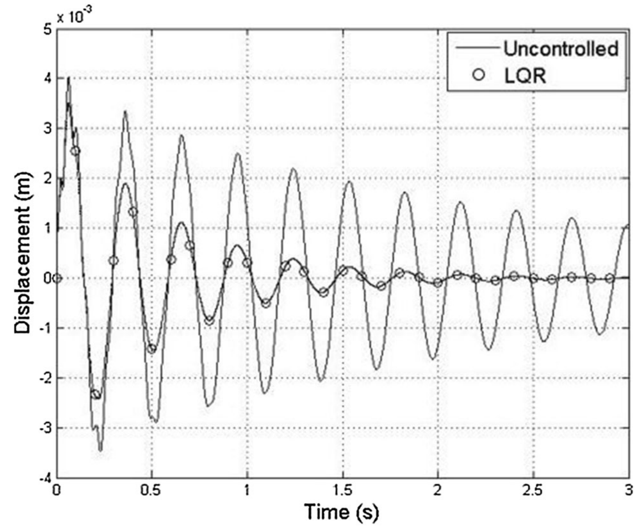


Fig. 2 Transient response obtained in the case of LQR controller governed by state feedback law

ranking is done on the basis of dc gain values. The Transient responses of full and reduced models of the open loop system are given in Fig. 1. From Fig. 1, it is clear that the response of reduced model of the system closely matches with that of full model. Thus, instead of full model, a reduced model consisting of only five modes with highest dc gain values are used to represent the system.

Simulation study

In the closed-loop system with a controller, the actuator produces controlling force which acts on the nodes at the ends of the actuator in the X direction (UX), while the exciting force is applied and output displacement is measured at the tip of the beam along the Z direction (UZ). Therefore, the eigen vectors pertaining to the UX and UZ displacements are required to construct the X_n matrix. Thus, the first few rows of the X_n matrix correspond to the UX displacement of the nodes on one end of the actuator, next few rows correspond to the UX displacement of the nodes on the other end of the actuator, and last row corresponds to the UZ displacement of the tip node. Using the eigen values and X_n matrix (eigen vector) the state-space modelling of close-loop system is formed (Khot and Khan 2015). The optimal controller design using state and output feedback control laws is explained in the following subsections:

State feedback law

Linear quadratic regulator (LQR) optimal controller

The linear quadratic regulator (LQR) controller is designed to minimize the following quadratic cost function,

$$J = \int_0^\infty \left[x^T Q x + u^T R u\right] \mathrm{d}t, \tag{7}$$

where Q and R are suitably chosen positive semi-definite weighting matrices and u is the control force to be applied. The state feedback control law is used here to design the controller. The control force is calculated as,

$$u = -Kx, \tag{8}$$

where K is the optimal controller gain, which is calculated as,

$$K = R^{-1} B^T P, \tag{9}$$

where P is symmetric positive semi-definite solution to the following algebraic Riccati equation.

$$PA + A^T P + Q - PBR^{-1} B^T P = 0. \tag{10}$$

The LQR controller gain for the state feedback law is calculated using the 'lqr' function in MATLAB©. Q and R are taken as 1 and 1e−7, respectively (Tewari 2002) in the simulation.

The closed-loop model is then obtained, which is given below.

$$\dot{x} = (A - BK)x + Bu. \tag{11}$$

The transient response obtained in the case of LQR controller with the state feedback law is plotted in MATLAB© using 'lsim' function and it is given in Fig. 2. The settling time of the tip of the cantilever beam with the LQR controller is 2.4 s.

Linear quadratic Gaussian (LQG) optimal controller

Linear quadratic Gaussian (LQG) controllers are based upon a linear plant, a quadratic objective function, and an assumption of white noise that has normal or Gaussian probability distribution. A state-space realization of the optimal controller for regulating a noisy plant with state-space representation is given as (Tewari 2002),

$$\dot{x} = Ax + Bu + Fv \quad \text{and} \tag{12}$$

$$y = Cx + Du + z, \tag{13}$$

where v and z are the process noise vector and measurement noise, respectively. The design of LQG optimal controller with the state feedback law is explained here. The regulator gain K and Kalman filter which filters the noises are required to design the LQG optimal controller. The regulator gain K is calculated from Eq. 9 by solving the algebraic Riccati equation Eq. 10. Kalman filter is designed for the plant assuming a known control input u, a measured output y, white noises v and z, and with known power spectral densities. The Kalman filter is designed to provide an optimal estimate of the state-vector x (Tewari 2002). The Kalman filter gain L is calculated using following equation.

$$L = R_e C^T z^{-1}, \tag{14}$$

where R_e is the optimal covariance matrix and solution of the following Riccati equation.

$$AR_e + R_e A^T - R_e C^T z^{-1} C R_e + BwB^T = 0. \tag{15}$$

Combination of separately designed LQR controller and Kalman filter gives optimal controller as,

$$\dot{x} = (A - BK - LC + LDK)x + Ly. \tag{16}$$

where K and L are the LQR optimal regulator and Kalman filter gains, respectively. The Kalman gain L is calculated in MATLAB© using 'kalman' function. Here v and z are taken as ρBB^T and $C^T C$ (Tewari 2002). The LQG optimal regulator which is combination of K and L is calculated in MATLAB© using 'reg' function. The transient response obtained in the case of the LQG controller using the state feedback law is given in Fig. 3. The settling time in LQG controller is 2.1 s which is lesser than that obtained in the case of LQR controller because it filters out the noise in the system.

H-infinity controller

The H-infinity optimal controller provides high disturbance noise rejection and maintains the response of the system and error signals within a particular limit. The state feedback control signal for a system with state-space form, is $u = -kx$. Therefore, it is desired to find the state-variable feedback gain K such that the closed-loop system $Acs = A - Bk$ is asymptotically stable. The state feedback gain K is calculated as,

$$K = R^{-1}\left(B^T P + L\right), \tag{17}$$

where $P > 0$ and it is a solution of the following algebraic Riccati equation,

$$PAcs + Acs^T P + \frac{1}{\gamma 2} PDD^T P + Q + K^T RK = 0. \tag{18}$$

The controller gain K is calculated in MATLAB© using 'hinfopt' function. The transient response of the closed-loop plant is given in Fig. 4. The settling time of the

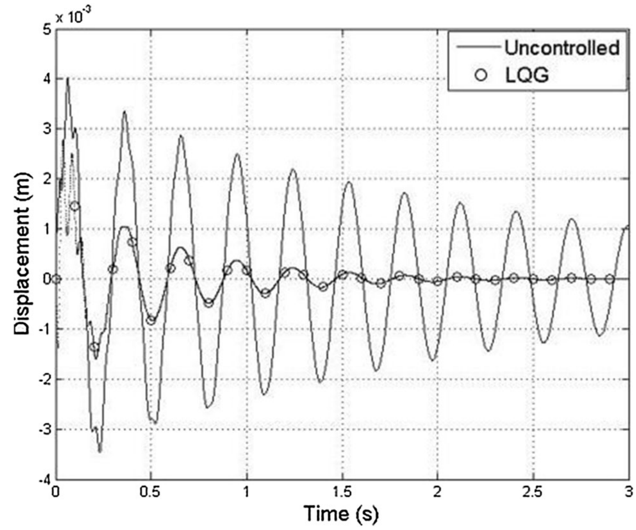


Fig. 3 Transient response obtained in the case of LQG controller governed by state feedback law

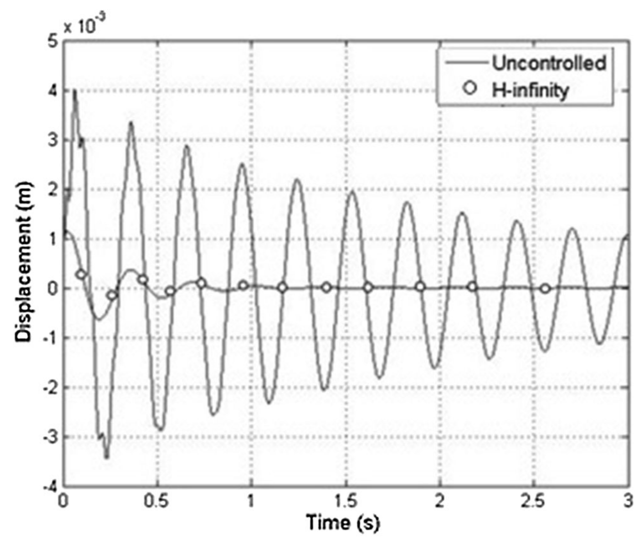


Fig. 4 Transient response obtained in the case of H-infinity controller governed by state feedback law

closed-loop model with H-infinity controller is 1.2 s. From the transient responses, it is clear that the H-infinity controller has a better closed-loop dynamic performance than LQR and LQG optimal controllers.

Output feedback law

The use of output feedback allows flexibility and simplicity of implementation. Moreover, in practical applications, full state measurements are not usually possible. The restricted-measurement of output feedback is of extreme importance in practical controller design applications.

Linear quadratic regulator (LQR) optimal controller

In output feedback control law, the output $y(t)$ is used rather than the state-vector $x(t)$, which is included in the objective function for minimization. The reason for this may be either a lack of physical understanding of some state variables, which makes it difficult to assign weightage to them, or that the desired performance objectives are better specified in terms of the measured output (Tewari 2002). The algebraic Riccati equation for LQR output feedback law is,

$$PA + A^T P - PBR^{-1}B^T P + \left[Q - SR^{-1}S\right] = 0, \quad (19)$$

where $[Q{-}SR^{-1}S]$ is a positive semi-definite matrix. The MATLAB© function 'lqry' is used to solve the output-weighted linear, quadratic optimal control problem. The transient response obtained in the case of LQR controller with output feedback law is shown in Fig. 5. It can be seen from the transient responses with state and output feedback control laws as shown in Figs. 2 and 7 that settling times in both the cases are approximately same which are 2.4 s and 2.6 s, respectively. The only advantage of using output feedback law is that it applies control gain directly to sensor output. Therefore, no estimation of state variables is involved in this approach, and thus it simplifies the internal complexity of the controller (Khot and Khan 2015).

Linear quadratic Gaussian (LQG) optimal controller

In the case of LQG controller with output feedback law, the regulator gain is calculated using LQR output control law as explained earlier. The output matrix C becomes $C \times A$, direct transmission matrix D becomes $C \times B$, and these are used to construct state-space model of the plant. The remaining design procedure of LQG controller with output feedback law is same as for LQG controller with state feedback law (Khot and Khan 2015). The transient response obtained in the case of LQG controller with output feedback law is given in Fig. 6. It can be seen from the transient responses with state and output feedback laws as shown in Figs. 4 and 8 that the settling times in both the cases are almost same which are 2.3 s and 2.21 s, respectively.

H-infinity controller

To find a constant output feedback gain K, one may define the value function given by,

$$\int_0^\infty \left[x^T\left(Q + C^T K^T RKC\right)x - \gamma^2 d^T d\right]\mathrm{d}t, \quad (20)$$

with $KC = R^{-1}B^T P + L$, where, $P > 0$ and $P^T = P$ is a solution of following algebraic Riccati equation,

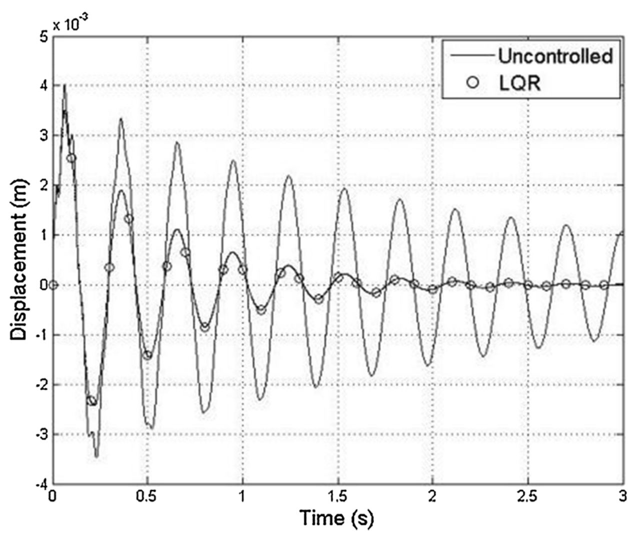


Fig. 5 Transient response obtained in the case of LQR controller governed by output feedback law

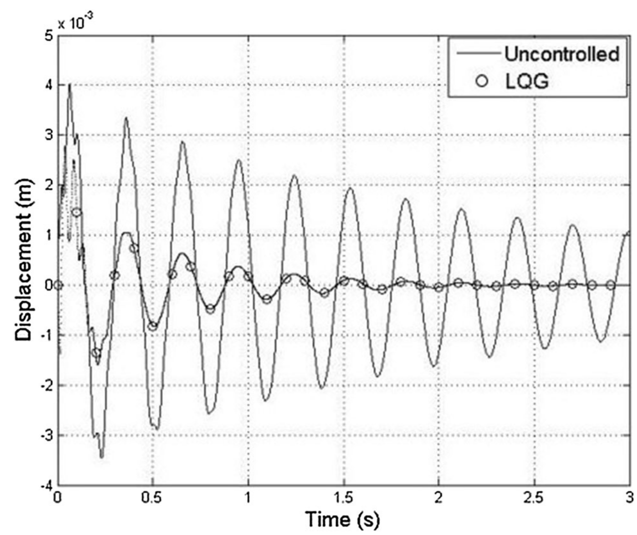


Fig. 6 Transient response obtained in the case of LQG controller governed by output feedback law

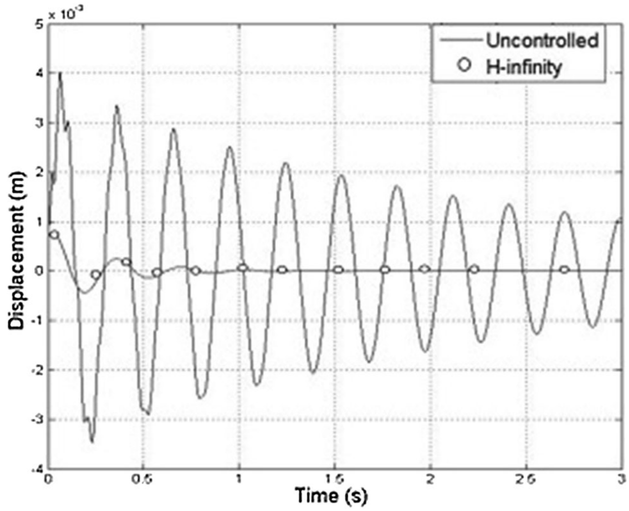


Fig. 7 Transient response obtained in the case of H-infinity controller governed by output feedback law

$$PA + A^T P + \frac{1}{\gamma 2} + PDD^T P - PBR^{-1} B^T P + L^T R^{-1} L = 0. \tag{21}$$

The regulator gain K is calculated in MATLAB© using 'hinfsyn' function (Khot and Khan 2015). The transient response obtained in the case of H-infinity controller with output feedback law is given in Fig. 7. The settling time of the closed-loop model with H-infinity controller is 1 s. Thus, the H-infinity controller has a better closed-loop dynamic performance over LQR and LQG controllers. The following session deals with the experimentation part of the study.

Experimentation

Experimental setup

An experimental setup is developed to investigate the active vibration control of a cantilever beam using different optimal controllers. This is basically achieved by building a closed-loop control system. The experimental setup is shown in Fig. 8. It consists of following components.

Structure

The structure selected for the present study is a cantilever beam made of aluminium (Al). It is of size 510 mm × 26 mm × 1.6 mm with piezoelectric sensor and actuator mounted onto it in a collocated way. The maximum strain developed is at the root of the beam. Thus, for better vibration control sensor and actuator are mounted at the root of the beam.

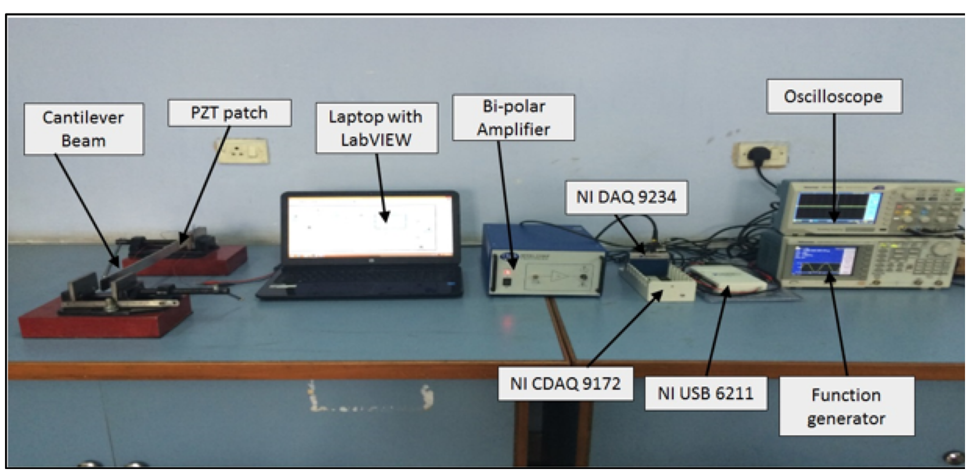
Sensor/actuator

Sensor and actuator used for the experimental work are made of Piezoelectric material of PZT-SP 5H type and have dimensions 75 mm × 25.4 mm × 1 mm. When the beam vibrates, the Piezoelectric sensor senses the vibration and produces corresponding analog voltage. The sensor output is measured using LabVIEW© hardware/software. The controller output after amplification, is given to the actuator which suppresses the vibration of the structure.

Data acquisition devices

The input to the controller should be in the digital form, however, the output from the piezoelectric sensor is in the analog form (voltage). Similarly, the controller output from the computer is in the digital form which is required to be

Fig. 8 Experimental setup



converted into the analog form before giving to the actuator through an amplifier. Therefore, input and output data acquisition devices (DAQ) modules are required. NI DAQ 9234 is used as an input module with NI 9172 chassis which can hold up to eight C series input/output modules. NI DAQ 9234 is an analog input module with an input range of $\pm$ 5 V. NI USB 6211/NI 9263 is used as an output module. The output voltage range of NI USB 6211/NI 9263 is $\pm$ 10 V.

Bipolar amplifier

The magnitude of the output voltage is too small to actuate the piezoelectric actuator. Therefore, a Bipolar amplifier (Model 2100 Hf, 300 Vpp capacity) is used to amplify the actuator input voltage. The Bipolar amplifier has an amplification factor of 50 and thus, the input signal to it is restricted to $\pm$ 3 V.

Piezoelectric accelerometer

A Piezoelectric accelerometer (Model 4504A) is mounted at the free end of the beam to measure the open- and closed-loop transient responses of the beam with the help of FFT Analyzer.

Function generator

A Tektronix AFG3051C single-channel arbitrary Function Generator is used to generate a sine wave to excite the beam during open loop testing.

Oscilloscope

A tektronix TBS1102B-EDU digital storage oscilloscope is used to view the sensor/controller output.

Controller

In the experimental study, LQR, LQG, and H-infinity Controllers are used to control the vibration of the cantilever beam. These controllers are designed using LabVIEW© platform. While carrying out experiments, several issues came up and those are resolved appropriately. To make the reader aware of these issues and their remedies, authors have explained them in the following section:

Experimental issues

Mechanism for providing uniform excitation displacement

In experimental study, displacement is provided at the tip of the beam to excite it. This uncontrolled transient response is compared with those obtained using three controllers. Thus, in each case the uncontrolled transient response must be kept uniform and therefore, a mechanism for providing uniform displacement is designed. For this purpose, a machine vice mounted with a steel ruler is fabricated. To maintain uniform displacement every time tip of the beam is touched to the movable plate of the vice set at a particular distance and released. The same procedure is followed for all the controllers.

Selection of sampling rate

Active vibration control systems are isolation systems that dynamically react to incoming vibration. The active isolation component consists of vibration sensors, control electronics, and actuators. The piezoelectric sensor converts kinetic vibration energy into electrical signal which is transmitted to the control electronics. The electronics reconcile and process the signal from the sensor using an algorithm. The electronics then send a cancellation signal to the actuator. The actuator generates vibration which is equal and out of phase in relation to the incoming vibration. Thus, while carrying out the experiments, to get controlled signal out of phase with respect to the sensor signal the sampling rate of both input module (sensor signal) and output module (controller signal) should be same.

BNC TO BNC 10× attenuation probe design

The oscilloscope enables waveforms to be viewed in a graphical format. The basic type of oscilloscope probe is 1× probe. The 1× probes are suitable for many low-frequency and amplitude applications. It is so called because this type of probe does not attenuate the incoming voltage. However, higher levels of impedance are required to achieve better accuracy. To achieve this, attenuators are built into the end of the probe that connects with the circuit under test. The most common type of probe with a built in attenuator gives an attenuation of ten, and it is known as a 10× oscilloscope probe. The attenuation enables the impedance presented to the circuit under test to be increased by a factor of ten, and this enables more accurate measurements to be made. In particular, the level of capacitance seen by the circuit is reduced which further reduces the high-frequency loading of the circuit by the probe. The 10× probe is designed in such a way that it consists of a connector to interface with the oscilloscope (a BNC connector) and connector to interface with the function generator or amplifier (a BNC connector) so that large-amplitude signals should not be clipped. The probe designed can be used both as 10× and 1× probes.

Current amplifier design

During experimentation, the controller output is given to the amplifier for amplification and then sent to the actuator for controlling the vibration effectively. However, the controller signal from DAQ output module NI USB 6211/NI 9263 is not sufficient in terms of current to get amplified in the Bipolar Amplifier. Therefore, a current amplifier (OPA 548) is implemented to boost the current output of the controller signal to the required level and then given to the amplifier. The OPA548 device is a low-cost, high-voltage, and high-current operational amplifier which is ideal for driving a wide variety of loads. The OPA548 device operates from either single or dual power supply for design flexibility. It is internally protected against over-temperature conditions and current overloads. In addition, it can be designed to provide an accurate and user-selected current limit.

Experimental investigation

A closed-loop system is constructed to investigate the performances of different optimal controllers in active vibration control of a cantilever beam. The closed-loop system is shown in Fig. 9. Initially 10 mm displacement is given at the end of the beam. The sensor output is measured using NI DAQ 9234. The sensor output is given to the controller in the LabVIEW© software. The controller output signal after amplification is provided to the actuator. In experiments only output feedback control law is used since getting information about the states is very difficult. The experiments are conducted for LQR, LQG, and H-infinity controllers.

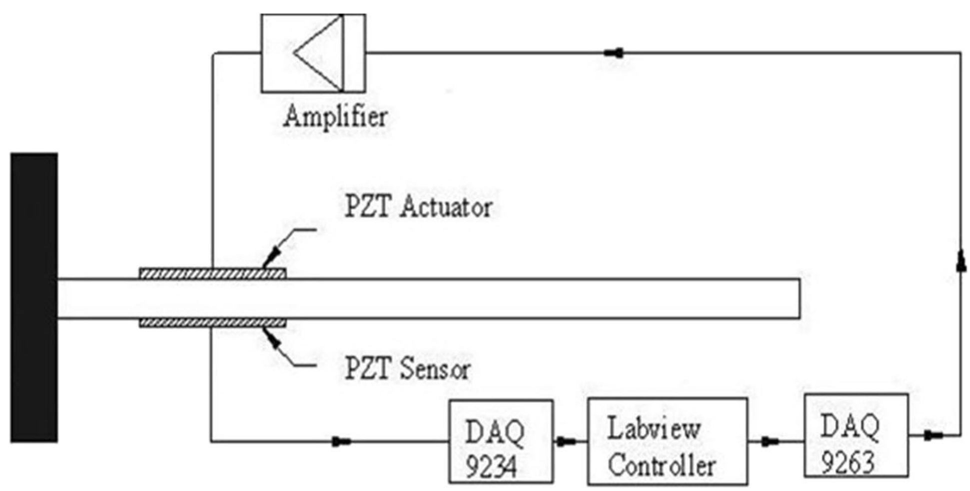


Fig. 9 Closed-loop control system for the vibration control of cantilever

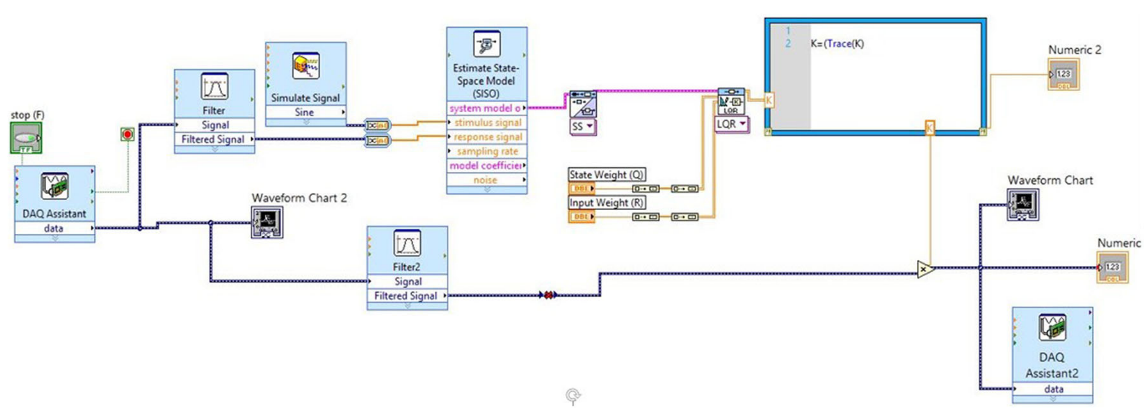


Fig. 10 LQR controller block diagram in LabVIEW©

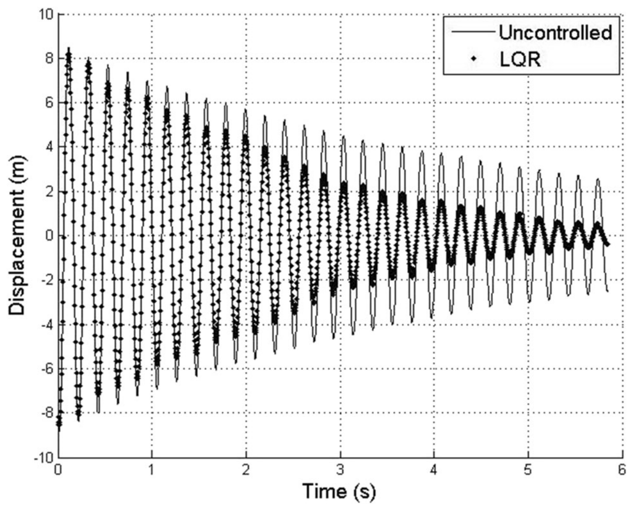


Fig. 11 Closed-loop response obtained using LQR controller

LQR controller

The closed-loop control system is constructed in LabVIEW© to actively control the vibration of a cantilever beam using LQR controller as shown in Fig. 10. The sensor signal is acquired using NI DAQ 9234. Butterworth filter is used to cancel out high-frequency noise. The system identification is done using the estimated state-space model. The parameters Q and R are selected such that the controller gain is below $\pm$ 3. The state-space model and parameters are given to the LQR controller for estimating the gain. The controlled signal is given to NI USB6211/NI 9263. This controlled signal after amplification is given to the actuator for suppressing the vibration. The controlled response obtained using the LQR controller is compared with the uncontrolled one and it is shown in Fig. 11. The settling time in this case, is 6.5 s.

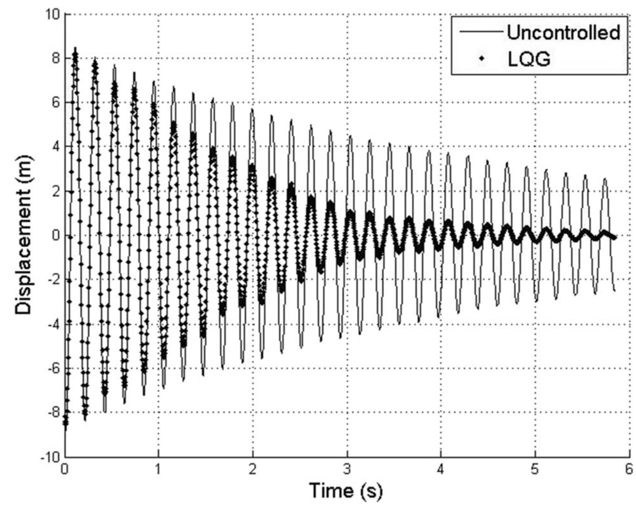


Fig. 13 Closed-loop response obtained using LQG controller

LQG controller

The closed-loop control system constructed to actively control the vibration of a cantilever beam using LQG controller is shown in Fig. 12. The LQG controller is designed by combining the optimal regulator gain K of LQR controller with the optimal observer gain L (Kalman filter). In the LQG controller block diagram, Kalman filter is designed considering white noise with known power spectral densities which filters out noise in the system. Combination of separately designed optimal regulator and Kalman filter gives optimal compensator. The controlled

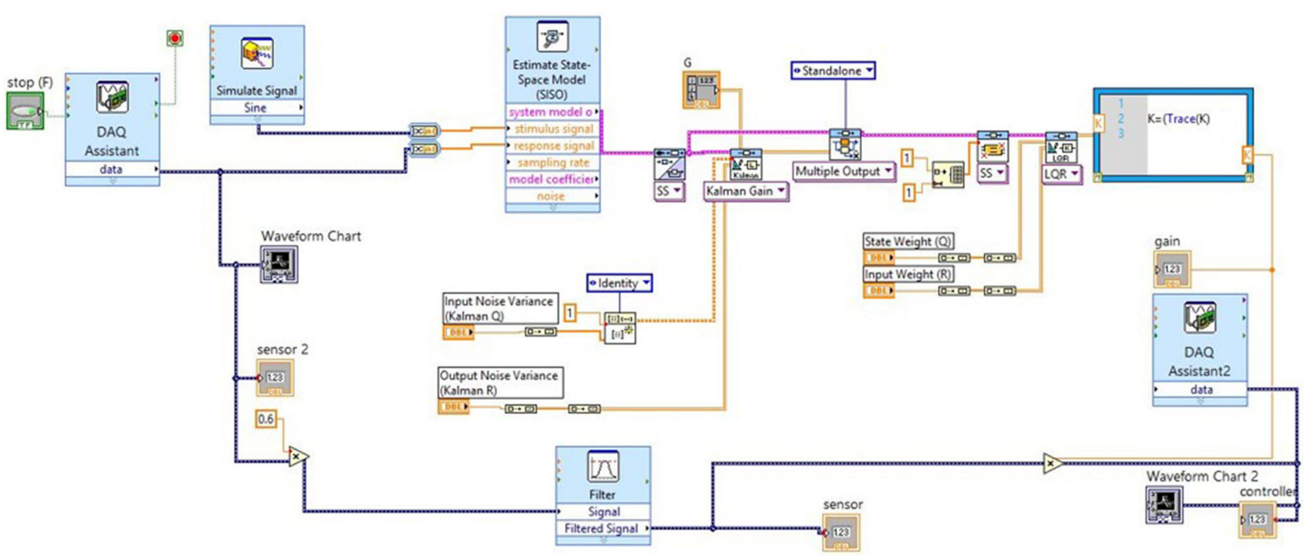


Fig. 12 LQG controller block diagram in LabVIEW©

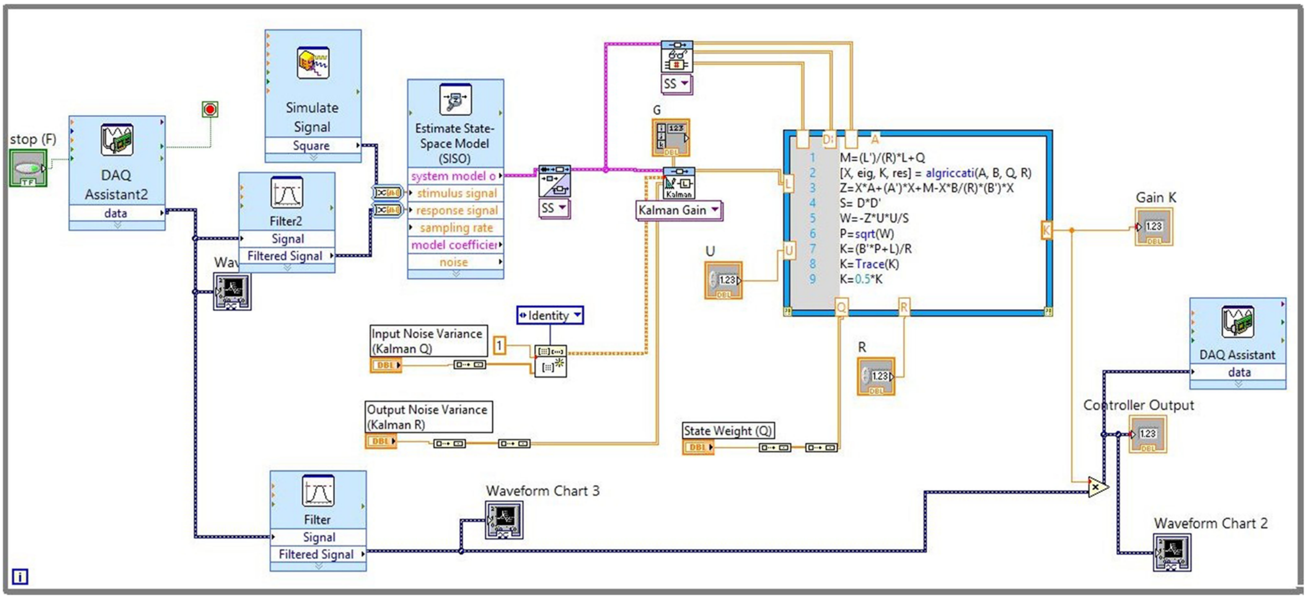


Fig. 14 H-Infinity controller block diagram in LabVIEW©

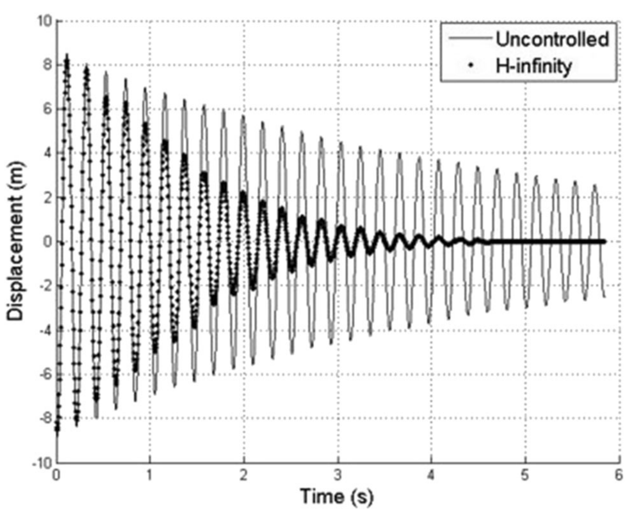


Fig. 15 Closed-loop response obtained using H-infinity controller

response obtained using the LQG controller is compared with the uncontrolled one and is shown in Fig. 13. The settling time in this case, is 5.8 s, which is lesser as compared to the LQR controller. This is because it filters out the white noise in the system.

H-infinity controller

The LQG controller developed using LQR optimal regulator and Kalman filter exhibits good performance; however, robustness to process and measure noise cannot be guaranteed. The H-infinity controller provides a closed-loop response of the system according to the design specifications, such as model uncertainty, disturbance attenuation at higher frequencies, required bandwidth of the closed-loop plant, etc. Practically H-infinity controllers are of higher order which may lead to large control effort requirement. The closed-loop control system constructed to actively control the vibration of a cantilever beam using the H-infinity controller is shown in Fig. 14. The controlled response obtained using the H-infinity controller is compared with the uncontrolled one and is shown in Fig. 15. The settling time in this case is 4.7 s. From the transient responses shown in Figs. 11, 13, and 15 it is clear that the H-infinity controller has a better closed-loop dynamic performance than the LQR and LQG controllers.

Conclusion

In the present study, active vibration control of a cantilever beam is investigated using LQR, LQG, and H-infinity optimal controllers with state and output feedback control laws. Both simulation and experimental studies are carried out. In the simulation study, finite-element model of the cantilever beam integrated with a single collocated pair of piezoelectric actuator and sensor, is prepared. The modal analysis is carried out in ANSYS©, and the eigen values and eigen vectors are obtained which are further imported in MATLAB© and state-space model is built. Both full and reduced state-space models are constructed and they were found to be in good agreement as far as their frequency and transient responses are concerned. Thus, the reduced model is used to design controllers with state and output feedback control laws. The transient responses obtained in the cases of various controllers are compared and it is found that the

H-infinity controller has a better closed-loop dynamic performance over LQR and LQG controllers. This is because, H-infinity controller shows better performance in terms of sensitivity and provides high disturbance rejection, providing stability for any operating condition. In the experimental study, experimental issues like mechanism for providing uniform excitation displacement to the beam, selection of sampling rate of input and output modules, BNC to BNC 10× attenuation probe design, and current amplifier design are resolved. In experimentation LabVIEW© platform is used for designing the controllers. The controller output is amplified using a bipolar amplifier. The amplified voltage is given to the actuator for effectively controlling the vibration. The experimental results of all the three controllers are compared and it is concluded that the H-$\propto$ controller has a better closed-loop dynamic performance over LQR and LQG controller and this experimental result is in accordance with the simulation result.

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Compliance with ethical standards

Conflict of interest The authors declare no conflict of interest in preparing this article.

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